Social Security and Retirement across the OECD*

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October/2010
Working Paper 10-07
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October 18, 2010

Abstract

Employment to population ratios differ markedly across OECD countries relative to rates in the U.S., especially for persons aged 55-69. Social security features also differ across the OECD, particularly with respect to replacement rates, entitlement ages and earnings tests. I conjecture that differences in social security features explain many differences in employment to population ratios at older ages. I assess my conjecture quantitatively with a life cycle general equilibrium model of retirement. At ages 60-64 the correlation between my model’s simulations and observed data is about two thirds. The replacement rate and the earnings test explain 90% of observed variability, implying that differences in entitlement ages do not explain differences in employment to population rates at older ages.

Keywords: Social security, retirement, idiosyncratic labor income risk

JEL Codes: E24, H53, J14, J26

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*I am deeply indebted to Richard Rogerson, Berthold Herendorf and Roozbeh Hosseini for their intellectual encouragement. I would also like to thanks Kevin Reffet, Manjira Datta, Josep Pijoan-Mas and Manuel Santos for helpful insights and the participants of the W.P. Carey Department of Economics' Macro Seminar and Brown Bag Seminar, the ITAM seminar, the Universidad Complutense seminar, the Universidad Carlos III seminar and the CEMFI brownbag seminar. Special thanks to Galina Verschagina who patiently discussed my manuscript.
1 Introduction

Employment to population ratios differ markedly across OECD countries. In 2006 the employment ratios ranged from 42% in Turkey to 66% in Norway. Differences are most significant for older people, as illustrated by ratios at ages 60-64, which ranged from 13% in Hungary to 60% in New Zealand for the same year. At the same time, there are large differences on many features of social security systems across the OECD. For example, the net replacement rate\(^1\) ranged from 38% in Mexico to 124% in Turkey, while the entitlement age varied from 55 in Australia to 67 in Norway. Some countries (such as Denmark) do not allow social security benefits to be collected by those who work, whereas others (such as Canada) permit workers to collect benefits. A third group of countries make social security benefits to be means tested. In the US, this means test is called the "earnings test." I use this terminology to refer to any restriction on working and collecting social security benefits at the same time.

My paper seeks to answer two questions: Can differences in social security features account for large differences in employment to population at older ages? And what features of social security are the most important contributors to differences in employment to population? Understanding these two questions has policy ramifications, as demographic projections show that these over 50 will account for more than half of the population in 2050.

To answer to the questions I develop a life cycle general equilibrium model of retirement, with discrete labor choice, idiosyncratic labor income risk and incomplete markets. The model is calibrated to match statistics of the US economy and social security system. A key feature of my model is that I am able to capture heterogeneity in employment by age found in the data, which is a desirable property if we want to study heterogeneity in retirement across nations. For example, in the US, more than 60% of the population is working at age 62, and 40% is still working at age 65. My model is able to capture very well the employment profile from ages 50 to 80.

To evaluate the effects of cross national differences in social security, I solve for the stationary equilibrium of a model calibrated to the US, but with social security systems of each OECD country. I ask what would happen if suddenly the US had a completely different social security system. My main findings are as follows. First, it turns out that differences in social security account for a large portion of the differences in retirement behavior. One way of illustrating this finding is to compare coefficients of variation of employment to population across OECD countries observed in the data with those generated by the model. At ages 60-64 this statistic is .45 in the data and .42 in the model. At ages 65-69 it is .80 in the data and .70 in the model. As a matter of fact, the correlation between data and model predictions is .60 for ages 60-64 and .65 for ages 65-69.

\(^1\)The OECD supplies a variety of measures of generosity and progressivity of social security systems. In particular, the net replacement rate measures promised entitlements relative to average individual earnings at the age of retirement when taxes on these entitlements are taken into account.
This means that my model captures much of the variability found in the data allowing just for cross national variation in social security features. For example, when I run linear regressions between the employment to population at ages 60-64 in the simulations and in the OECD data the $R^2$ equals 92%. This is consistent with the large fraction of the coefficient of variation that my model explains. On top of it, the variability on social security features across the OECD match surprisingly well the employment to population for ages older than 50.

Second, when I ask what the most salient characteristics of social security with a bearing in retirement on retirement are, it turns out that the replacement rate and the ”earnings tests” are of utter importance, while differences in the entitlement age are not. To assess the magnitude of each feature, I shut them down to the US levels one at a time. I find that the coefficient of variation of employment to population at ages 60-64 in the model is .20 when there are differences only in the replacement rate and .22 when countries have differences only in ”earnings tests.” In contrast, the coefficient is .05 when there are solely differences in the entitlement age. It follows that the replacement rate and ”earnings tests” each account for roughly 50% of the variability in the model. I do not find that there are significant interactions between the replacement rate and ”earnings tests” on one side and the entitlement age on the other.

My paper is most related to two streams of literature. The first follows Prescott (2004), who sought to explain large differences in hours of work through differences in the average tax rate for G-7 economies, using a stand-in household growth model. Prescott et. al (2007) and Rogerson & Wallenius (2009) developed a life cycle model with an intensive and extensive margin in labor choice to analyze the effect of a simple tax and transfer system on hours of work. It turns out that their results are similar to those found in Prescott (2004.) Wallenius (2009) extends this framework to include human capital accumulation and studies differences in hours per capita in Belgium, France and Germany that are generated through differences in social security. She finds that social security has large effects on hours of work, mostly through the extensive margin. Similar to the spirit of my work, Guvenen et al. (2009) examine the role of progresivity of the tax code in accounting for evolution of wage inequality in Continental Europe relative to the US. They find that different features of taxes on income, in particular progresivity, are able to account for most differences in wage variance.

Relative to Wallenius (2009) my paper has two important characteristics. First, my model incorporates heterogeneity, and it is able to match the distribution of retirement that is found in the data. In her model, everyone retires at the same age. Second, I compute outcomes for a much larger set of countries. While I also find large effects brought about by social security, heterogeneity reduces the impact on employment to population. These smaller effects can be due to a smaller response.

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2 Many papers have studied the impact of differences in taxes on hours of work. For example, see Ohanian et al. (2007), Rogerson (2007), McDaniel (2009) and Ragan (2005)
of individuals to social security when there is labor income risk, or alternatively to composition
effects; as when there is mortality risk, the weight of older individuals on the total population
is smaller. To investigate the role of heterogeneity I cut the variance of the income risk by half
and recalibrate the model to match the US economy. I find that a country where social security
has twice the replacement rate of that in the US will have an employment to population ratio 3
percentage points below in a world with half the labor income risk. The employment to population
ratio will be 6 percentage points below in a world with the range of idiosyncratic labor income
risks. Furthermore, the employment to population ratio at ages 60-64 will be 9 percentage points
below that in the US in the former case, whereas it will be 25 below in world with the amount of
idiosyncratic labor income risk found in the data. This rules out idiosyncratic labor income risk as
the source of the discrepancy. The next candidates, within my model, are differences in mortality
risk and differences in the structure of the population that they imply\(^3\). To illustrate this point
I shut down mortality risk, assume that each age group has the same weight and recalibrate the
model. I find that a country where social security system has twice the replacement rate of that
in the US would have an employment to population ratio 10 percentage points below the US. An
additional advantage of my model is that it can be used to study how social security affects the
ability of individuals to insure against risk. It can also be used for welfare comparisons. These
applications are left for future extensions.

A second stream of literature studies different positive and normative aspects of social security. I
will not attempt to survey this stream here as it is very extensive\(^4\). The most closely related refer-
ence from this literature is French (2005 & 2007). He develops a model with labor income, health
risk and incomplete markets to study the role of social security into accounting for retirement
behavior in the US. He finds that market incompleteness plus social security are key to under-
standing the retirement behavior. This provides some support to the notion that the assumptions
in my model are important. I depart from his work in that I include general equilibrium. This is
an important extension if we want to study cross national differences in retirement, as different
social security systems imply very different prices.

### 2 Employment and social security in the OECD

This section presents empirical evidence for OECD countries in 2006. I use labor force statistics
by age and sex from the OECD on-line database\(^5\), social security data from “Pensions at Glance

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\(^3\)More experimentation is needed to check the importance of heterogeneity in the OECD.

\(^4\)For example, Gustman & Steinmeier (1986), Stock & Wise (1990), Gruber & Wise (2004, 2007), Coile & Gruber
to mention a few important contributions to the study of social security

\(^5\)http://stats.oecd.org/Index.aspx
2009” and productivity data from the “Total Economy Database.” To study the role of social security in accounting for cross national differences in retirement I collect the formal employment to population ratio and employment to population ratios at ages 55-59, 60-64, 65-69 and 70-74. Even though the general employment to population ratio is not the main focus of my study it is useful as a benchmark to understand the magnitude of cross national differences in employment at older ages.

There are large differences in national employment to population ratios. Turkey has the lowest employment rate at 42%, whereas Norway has the highest at 66% (Figure 1 (a)). Differences become larger for older individuals. For example, if we look at the employment rate at ages 60-64, differences range from 13% in Hungary to 60% in New Zealand (see Figure 1 (b)). The US has an employment to population ratio of 65% overall, and 51% for ages 60-64.

Social security systems are complex, and they differ in many dimensions. For example, consider three countries: Belgium, France and the US (I could have picked any other three). In Belgium to qualify for full social security benefits one has to be at least 65 with no fewer than 45 years of employment, although one may be entitled to reduced benefits if he/she is at least 60 and has worked no fewer than 35 years. Social security depends also on marital status and the presence of dependants. Belgian social security is means tested and includes different types of allowances, such as vacation allowances. In France one needs to be 60 and have been employed at least 40 years to qualify for full benefits but if one entered the labor force at ages 14-16 he/she may qualify

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6The Conference Board and Groningen Growth and Development Center

7I define the employment to population ratio as the ratio of employees age 20-75 to individuals 20-75. My model economy will have an initial age of 20. Few individuals work past age 75. Also the OECD has data limitations beyond ages 70-74.
for full benefits at ages 56-59. One may continue any gainful activity and collect social security but a worker has to wait 6 months out of employment when he/she claims entitlement to cash his/her first social security check. One may also defer social security subject to some conditions, and there are means tested “solidarity pensions” that do not depend on earnings. Social security is based on the best 25 years, it is indexed to cost of living and marital status as well. In the US social security is no simpler than in continental European countries. Individuals are entitled to full benefits at age 65, but they may collect reduced benefits at age 62. Benefit reductions can be compensated if benefits are suspended later; compensation is roughly actuarially fair. Any individual is required to be employed at least 10 years to qualify. Dependents are also entitled to benefits; these depend on family structure.

Given the complexities of the various systems I focus on three key features that are measured by the OECD: replacement rates, entitlement ages and ”earnings tests”. The definition of replacement rate that I use is the ratio of social security net benefits at entitlement age to individual average net earnings at entitlement age for a single male whose average equals the average earnings of the economy (AW hereafter) and has entered employment at age 20 with no career breaks. The assumption about the age of entry to employment in the OECD data is convenient to my model, as my individuals will enter the economy at age 20, and have continuous careers until they are 50 years old at least. The entitlement age is defined by each country’s social security law. It is common to have more than one entitlement age. Across the OECD we may find an early entitlement age, a normal entitlement age and even a deferred entitlement age. The entitlement ages sometimes depend on sex and occupation. For my model I will choose only one entitlement age: the early entitlement age for males. I also abstract from differences in entitlement age by occupation as my model lacks of occupational choice. More information can be found in the Appendix. Finally, I assume that the ”earnings test” for each country is a 0-1 variable. I rely on the work of Duval (2003) and the thorough description of social security systems around the world provided by the US Social Security Administration. Duval computes an implicit tax on continuing to work based on the social security rules of a sample of OECD countries. This tax is measured as the social security benefits that one looses from continuing to work an additional 5 years relative to maximum social security benefit relative to social security benefit if one retires. These three variables are sufficient to capture differences in social security programs across OECD members. For example, a country

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8By focusing in three stylized features of social security I am seeking comparability and I do not imply that useful knowledge can be gained from a careful modelling of each country’s details. This is a daunting task that is left for future research.

9It is the promised benefit payment when different taxes on income and social security benefits are taken into account. There is not a perfect measure of cross national generosity of social security as countries differ in their tax codes and demographics. The OECD provides measures of gross generosity, net generosity, gross public generosity, net generosity and gross/net pension wealth. In the Appendix, I define and show values for these measures. I also provide a number of experiments to address the importance of the measure I use as a benchmark.
that requires more years of employment for full benefits to accrue will have a smaller replacement rate, other things being equal. There are large differences in replacement rates ranging from 38% in Mexico to 124% in Turkey. There are also large differences in entitlement ages, which varies from 55 in Australia to 67 in Norway (Figure 3). Figure 4 shows Duval’s implicit tax on continuing to work to illustrated the differences in rules that allow for collecting social security while the beneficiaries are also working. In the rest of the paper I assume that in each country one is either allowed to collect benefits while working or not at all. I set the following threshold: a country will allow benefits to accrue to those who are working if Duval’s implicit tax on continuing to work is less than 50%. Sometimes country’s social security code prohibit explicitly working if one has claimed benefits. As Duval’s measure is over a 5 year period, this extra information overrule the threshold criteria. Clearly this classification is arbitrary, and more detailed modeling of social security rules regarding collecting benefits while working would be desirable.

**Figure 2.** Replacement Rate and Entitlement Age
This section describes assumptions about demographics, preferences and endowments, technology, social security and market structure.

3.1 Demographics

I assume that population is stable. Population $N$ grows at a constant rate $n$. People live a maximum number of periods $A$, and every period they face a probability of dying, $1 - s_a$, which depends on age only. These assumptions induce a population structure where each age is a constant fraction of the population at every period, $\mu_a$\textsuperscript{10}.

3.2 Preferences and endowments

Every individual has identical preferences over sequences of consumption $\{c_a\}$ and leisure $\{h_a\}$. Consumption must be non negative and I assume that hours of work can take two values: zero or $\bar{h}$. Every individual is endowed with one unit of time each period and has preferences given by:

$$E_0 \left[ \sum_{a=1}^{A} \beta^a \left( \prod_{j=1}^{a} s_j \right) u(c_a, 1 - h_a) \right]$$

\textsuperscript{10}This number is obtained with the following recursion: $\mu_{a+1} = \frac{s_{a+1}}{1 + n} \mu_a$, and I normalize the weights to 1, so $\sum_a \mu_a = 1$. 

Figure 3. Implicit Taxes on Continuing to Work
3.3 Individual productivity

Let $z_{i,a}$ be the productivity of individual $i$ at age $a$. The log of this productivity is the sum of two components: $z_{i,a}^d$, which is a deterministic component that depends on age, and $z_{i,a}^w$, which is a random component and captures heterogeneity within each age group. I assume that the $\log(z_{i,a}^w)$ is characterized by an AR(1) process. It can be written as

$$\log(z_{i,a}) = \log(z_{i,a}^d) + \log(z_{i,a}^w)$$

(2)

$$\log(z_{i,a+1}^w) = \rho \log(z_{i,a}^w) + \epsilon_{i,a+1}$$

(3)

where $\epsilon_{i,a}$ is the innovation that is independently and identically distributed as $N(0, \sigma^2_\epsilon)$

3.4 Technology

There is a representative firm that operates a constant returns to scale technology transforming aggregate capital ($K$) and aggregate efficiency units of labor ($L$) into a homogeneous and perfectly divisible product ($Y$). Capital depreciates at rate $\delta$. Output can be used for either consumption or investment.

3.5 Markets

At each date there are markets for capital, labor and product. There are no insurance markets and no markets for borrowing or lending. However, as in Aiyagari (1994), individuals accumulate precautionary savings.

3.6 Social Security

Social security is defined by two elements. The first is a payroll tax ($\tau$) that is levied on every worker. The second is a function $\phi(\bar{e}_a, h_a, a)$ that characterizes benefit amounts and entitlement conditions. It is a function of average earnings, as the benefit amount will depend on individual average earnings. The function depends on labor choice as social security may restrict the possibility of accruing social security while working. Finally, it depends on age as individuals are not entitled to receive social security until they reach certain age ($\hat{a}$). Further details about social security will be given in the calibration section.
3.7 Accidental Bequests

As individuals may die with positive probability they may leave some capital. I assume that the government collects all this capital and distributes it as lump sum among those individuals alive. I will denote accidental bequests as $B$.

3.8 Recursive Steady State Representation of the Individual Decision

I represent the individual decision problem recursively. Individual state variables are: wealth ($k$), the idiosyncratic component of productivity ($z^w$), average earnings ($\bar{e}$) and age ($a$). Each period, individuals decide how much to consume ($c$), how much capital to hold ($k'$) and employment ($h$). In steady state, taking interest rates ($r$), wages ($w$), payroll tax ($\tau$), social security system ($\phi$) and accidental bequests ($B$) as given, each individual solves the following Bellman equation:

$$V_a(k, z^w, \bar{e}) = \max_{c, k', h} \left[ u(c, 1 - h) + \beta s_{a+1} E_z [V_{a+1}(k, z^{w'}, \bar{e}')] \right]$$

s.t. $c + k' = (1 + r)k + (1 - \tau)wz_a h + \phi(\bar{e}, h_a, a) + B$

3.9 Aggregate State Variable

The aggregate state variable of the economy is a list of measures over individual states $\{\Psi_a(k, z^w, \bar{e})\}$. In steady state, this list is a function of individuals’ policy functions and the idiosyncratic component of productivity.

3.10 Steady State Recursive Competitive Equilibrium

To save notation I collect individual state variables other than age in a vector $x = (k, z^w, \bar{e})$. A stationary recursive competitive equilibrium is a list of functions and scalars: $c_a(x)$, $k'_a(x)$, $h_a(x)$, $V_a(x)$, $\phi(\bar{e}, h_a, a)$, $\Psi_a(x)$, $w, r, \tau, K, L, B$, such that:

1. $c_a(x), k'_a(x), h_a(x)$ and $V_a(x)$ solve equation (3) for every $a = 1, ..., A$.

2. $K$ and $L$ solve the representative firm profit maximization problem, so input prices are given by the first order conditions: $r = F_K(K, L) - \delta$ and $w = F_L(K, L)$.

3. Markets clear:

   (a) $\sum_a \mu_a \int_X [c_a(x) + k'_a(x)] d\Psi_a = F(K, L) + (1 - \delta)K$

   (b) $\sum_a \mu_a \int_X k'_a(x) d\Psi_a = (1 + n)K$

   (c) $\sum_a \mu_a \int_X z_a h_a(x) d\Psi_a = L$
4. The aggregate state is consistent with individual behavior.

5. Social security is balanced:

\[ \tau L = \sum_{a \geq a} \mu_a \int_X \phi(\varepsilon, h_a(x), a) d\Psi_a \]

6. Accidental bequest are distributed evenly among individuals alive:

\[ \sum_a \mu_a (1 - s_{a+1}) \int_X (1 + r) k^d_a(x) d\Psi_a = B(1 + n) \]

4 \hspace{1cm} \text{Calibration}

I calibrate the model to key features of the US economy. Some parameters are selected independently, relying on various data sources and previous research. These are demographics, the individual productivity process, fraction of time worked, labor share and social security system. The remaining parameters, depreciation rate and preferences, are chosen together by solving the steady state equilibrium of the model to match some statistics of the US economy.

4.1 \hspace{1cm} \text{Parameters Calibrated Independently}

I need to choose the growth rate of the population \((n)\), age when individuals enter the economy, length of life \((A)\), probability of survival \((s_a)\), the individual productivity process \((z_{i,a})\), labor share \((\alpha)\), fraction of time worked \((\bar{h})\) and social security system.

4.1.1 \hspace{1cm} \text{Demographics}

I set the population growth rate to be equal the US average of 1.2\% over the period of 1960-2006. This number is taken from the US Census Bureau Statistical Abstract of 2009. Individuals enter the economy at age 20, and they die with probability 1 when they are 94, implying an \(A = 75\). The probability of survival is taken from actuarial tables for males provided by the US Social Security Administration in 2004. Figure 5 shows survival rates for selected life spans as well as stationary population weights that are implied.

4.1.2 \hspace{1cm} \text{Individual productivity process}

Individual productivity \((z_{i,a})\) is characterized by two components: \(z^d_a\), a deterministic component of age, and \(z^w_{i,a}\), a stochastic component.

To characterize the deterministic component, I use annual earnings and annual hours worked for a
sample of white non-disabled males with at least high-school education from IPUMS-CPS\textsuperscript{11} over the period 1992-2006. The selection of the sample is driven by the objective of isolating incentive effects on retirement of social security systems. I drop females as there are issues related to fertility choices and the gender gap that probably affect their employment choices and are unrelated to the incentives social security provide\textsuperscript{12}. High school drop outs are also left out of the sample as they have remarkably different employment behavior and earning dynamics. As my model abstracts from permanent differences, it is a reasonable first step to start without them. Finally, I abstract from disabled individuals as they face a unique set of employment incentives due to insurance factor. Understanding the joint role of disability insurance and social security is an important research topic and should be addressed in the future, but at this stage it would make the model too complicated for the present purposes.

I express annual earnings in $US1982. The empirical literature usually decomposes annual earnings into three different elements: age, time and cohort. A well known problem in this literature is that time and the cohort effects cannot be identified separately without making strong assumptions. Hugget et al. (2009) decompose earnings under three different hypotheses: they assume that either the time effect is zero, the cohort effect is zero or the time effect and the cohort effect are orthogonal. They find that none of the assumptions significantly affect estimation of the age component of earnings. In the steady state, the time effect should be proportional to the time variable, so I assume that earnings grow at a 2% rate due to productivity gains\textsuperscript{13}. I construct hourly wages by dividing annual earnings and annual hours. Then I compute the ratio of mean hourly wage by age

\textsuperscript{11}http://cps.ipums.org
\textsuperscript{12}However, it is worth noting that the deterministic component of productivity for males and females looks alike.
\textsuperscript{13}Hugget et al. (2009) document a growth of wage per hour in the PSID of 1.5% for the period 1969-1992.
to mean hourly wage. This produces a hump-shaped profile. I fit a quadratic polynomial over ages 20-65 weighted by the sample importance of each observation to eliminate sample variability and noise related to selection around retirement age. Finally, I truncate the polynomial to zero when it goes below zero, which happens at age 80\textsuperscript{14}. Estimating labor productivity is a difficult task at older ages, as there are very big selection effects. In the context of my model this does not seem to be crucial as my parameters approximate earnings profiles relatively well until age 75, and beyond this age there are few individuals working. Figure 6 (a) shows the result of the calibration of the deterministic component, and Figure 6 (b) compares CPS annual earnings with the earnings profile of the calibration. The simulated earnings profile is consistent with the earning profile from the CPS for most of the life cycle. The stochastic component of individual productivity is characterized by an AR(1)

$$\log(z_{i,a+1}^w) = \rho \log(z_{i,a}^w) + \epsilon_{i,a+1}$$

with $\epsilon_{i,a+1} \overset{iid}{\rightarrow} N(0, \sigma^2_\epsilon)$. Parameters $\rho$ and $\sigma^2_\epsilon$ are taken from French (2005) and equal .977 and .0141 respectively.

Finally, the fraction of time spent working ($\bar{h}$) is set to .45 of available time in a year. To calculate the available time I assume that individuals can spend 12 hours a day working, which delivers 4380 hours a year and 1971 hours spent at work.

\textsuperscript{14}Note that this does not deliver very different results than assuming that the deterministic component of productivity is given by average earnings by age relative to average earnings, as is frequently done in the literature.
4.1.3 Social security

The social security system is calibrated to that of the US. In my model individuals start collecting benefits at age 62, which is the early entitlement age in the US. There are computational reasons and economic reasons why I make this choice. Ideally I should also include the normal retirement age and entitlement choice to benefits, but that would make computations more time consuming than they already are. This modeling choice also makes economic sense, as in the US individuals are not allowed to borrow against social security income. Therefore asset poor individuals would like to get benefits as soon available. On the other hand, the timing of benefits does not matter much for the rich. Thus, setting the entitlement age at 65 would make my model overestimate the employment rate of asset poor individuals.

I assume that the US has no restrictions on collecting social security while working. If we use the implicit tax on continuing to work obtained by Duval (2003) as a proxy for this restriction, it is one of the smallest across the OECD (12%). One strong penalty on collecting social security while working in the US was the “earnings test”, which consisted of a tax on social security benefits for individuals who claimed entitlement before age 67 while still working. The test established two income thresholds: after the first threshold, $1 of social security benefits was taxed away for every $2 of labor earnings above this threshold; and after the second threshold, $1 of social security benefits was taxed away for every $3 of labor earnings above this second threshold. On top of this arrangement, the US system included an actuarial compensation factor that allowed the individuals to compensate for some benefit lost later on. In 2000, the “earnings test” was reformed. Before the reform, the test applied to people who continued to work and were younger than 67 and the actuarial compensation between ages 65 to 67 was not actuarially fair. After the reform, the test has only been applied to individuals younger than 65, and compensation is actuarially fair. Therefore as a first approximation it is reasonable to abstract from the ”earnings test” and assume that the US has zero restrictions on collecting social security while working.

The social security benefit formula is taken from the US Social Security Administration. It is a piece-wise linear function of average individual earnings ($\bar{e}$) as in Hugget & Ventura (1999), French (2005) or Nishiyama & Smetters (2007). Bend-points are multiples of AW so they can be directly taken to the model economy. US social security replaces 90% of the first $761 monthly, 32% from $761 and through $4,586, and 15% above $4,586. This is equivalent to .2,1.24 and 2.47 in multiples of annualized average earnings (AW). Therefore it is written as

$$\phi(\bar{e}_a, h_a, a) = \begin{cases} 
0 & \text{if } a < 62 \\
\varphi(\bar{e}) & \text{otherwise}
\end{cases}$$

Note that as I assume that there are no restrictions on accruing social security while working; $h_a$
does not play any role. I have made the following additional simplifications: social security takes into account the 35 best years of earnings, while I take the simple average over a lifetime, capped for individual earnings higher than 247% of AW. I characterize individual average earnings by the following formula:

\[ \bar{e}' = \begin{cases} \frac{\bar{e} \cdot (a-1) + \min(wz_{i,a}h,2.47 \cdot AW)}{a} & \text{if } a < \hat{a} \\ \bar{e} & \text{otherwise} \end{cases} \]

I abstract from the feature that US social security requires individuals to be employed for at least 10 years. This is not an issue in my model as everyone works more than 10 years. I also assume that there are no earnings limits on the payroll tax, while in the US earnings above $100,000 are exempt (roughly 3\(AW\)). Still this seems a harmless assumption as the group of individuals earning more than 3\(AW\) is relatively small and social security represents a smaller fraction of their retirement income.

4.1.4 Labor Share

I assume that production technology is Cobb-Douglas, \( Y = K^\alpha L^{1-\alpha} \). Labor share \((1 - \alpha)\) is set to .64 of production, as it is in the NIPA. This number implies the same value in simulations by definition.

4.2 Parameters Calibrated Together

I calibrate preferences and the depreciation rate to match some key moments of the US economy. I assume that the utility function is separable in consumption and leisure, and that it takes the
following form:

\[ u(c, 1 - h) = \frac{c^{1-\sigma}}{1 - \sigma} + \lambda \cdot (1 - h) \]  

(5)

This function is characterized by relative risk aversion (\(\sigma\)) and the weight of leisure (\(\lambda\)).

**Objective.** I choose (\(\sigma, \lambda, \beta, \delta\)) to match the following key statistics in the US: a capital-output ratio of 3.0, an investment-output ratio of .20, a labor share of .64 and the employment to population ratio profile from ages 50 to 80. I calculate the ratio from the same sample of the CPS that I used to calculate hourly wages. I have 33 moments and 4 parameters, so I choose the parameters to minimize square deviation of moments from data and analogous moments simulated by my model. I use a Nelder-Meade algorithm to find the minimum. Even though every parameter may affect any moment, the discount factor \(\beta\) is related to the capital output ratio. Once the algorithm finds a value of the discount factor that makes the capital-output equal 3.00, a value of \(\alpha\) of .36 delivers a labor share of .64 and a value of \(\delta\) of .066 delivers an investment output ratio of .20. The deterministic component of productivity (\(\{z^d_a\}\)), the weight of leisure in utility (\(\lambda\)) and relative risk aversion (\(\sigma\)) interact to determine the level and shape of the employment profile. At first sight, it is not obvious why relative risk aversion plays a role in determining the shape of the employment profile. For a high value of \(\sigma\) (which implies low elasticity of substitution), the drop in employment when individuals receive social security benefits will be smaller than if \(\sigma\) is small, thus the employment profile will be steeper for smaller values of the relative risk aversion coefficient.

### 4.3 Calibration results

Table 1 shows the results of the calibration. Relative risk aversion (\(\sigma\)) is within the range of values found in the literature, which vary from 1 to 8. \(\beta\) is in the low range for life cycle models, but I still get a hump-shaped consumption profile as is shown in Figure 7.

<table>
<thead>
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<th>Parameters from the Calibration</th>
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</thead>
<tbody>
<tr>
<td>(A)</td>
</tr>
<tr>
<td>75</td>
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</table>

The model matches the ratios of capital and investment to output and labor share perfectly. It is also successful in matching employment to population ratio by age. Figure 8 shows the match of employment to population for ages 50-80. After 80 almost no one works and in my model nobody is working. This is a key feature of my model, as it is a prerequisite to analyze cross national differences in employment by age. French (2005) also matches the employment profile but he seems to over-estimate employment to population above age 62. That he attempts to match the wealth
distribution at the same time is the most likely reason for his results. To match the accumulation of wealth in the top wealth quintile one needs a high $\beta$, but this would also induce individuals to retire early. French (2007) partially solves this issue introducing heterogeneous preferences. My model performs well along other dimensions that it was not intended to match. In particular, the replacement rate in my model is 40% and the measure I use in data for the US is 45% (the gross replacement on the other hand is 39%). The equilibrium payroll tax in my model is 10.38%, which is similar to the US payroll tax when the contributions made to medicaid are discounted (9.5%). I take these facts as evidence that my model captures the main features of US social security.
5 Policy experiments

In this section I assess the importance of three key features of social security to explain cross national differences in employment to population rates at older ages using benchmark calibration. I also investigate the most important features of social security, if any are salient. Note that the experiments should be understood as exercises in determining circumstances of Americans that awaken one day under a different social security system that they believed it was always there. The study of transitions is left for future research as it still needs to be determined what should be the optimal policy to implement and computing transitions is very costly.

5.1 Description of the Experiments

Section 2 documented large differences in employment to population rates at older ages across the OECD. It also documented large differences in three key features of social security: replacement rates, entitlement ages and "earnings tests". As I get a very good fit to the US, I use this nation as a benchmark and express all the employment statistics relative to it. This circumvention makes sense as I calibrated for a subsample of the US that has a high employment to population. If I used raw employment to population ratios simulated from my model I would deliver an over-estimated employment to population profile when women, racial minorities, disabled and high school drop outs are included. Therefore I apply the same relative employment to population that I find in model simulations to the whole population. I also express replacement rates of each country relative to the US as I am focusing on the importance of having a social security systems that are more (or less) generous than the US on average. I abstract from differences in the redistribution of each social security system and impose the re-distributive structure of the US. This is a limitation as it would be interesting to understand the separate role of average replacement and replacement rate for different average lifetime earnings. It turns out that disentangling numerically both is involved. For the purpose of this paper it is sufficient that replacement rate and progressivity are positively correlated, as they turn out to be in the data.

To account for differences in employment through differences in social security I solve for the stationary equilibrium of the model with different parameterizations of social security to mimic differences in the replacement rate, entitlement age and "earnings tests". Then I compare the results of simulations with OECD employment data for 2006.

I begin with employment to population at 60-64 because it is the most common age of retirement and conveys a strong message about the performance of the model. Still, there are some countries that have entitlement ages below 60 or above 64, so I group these countries by entitlement ages.

\[15\] In the Appendix, I recalibrate the model to data for males and females and the ability of the model to explain the variability in employment to population at older ages does not change substantially.
and compute employment to population around entitlement ages. This means that if I compare a country like Italy, which has entitlement age of 57, to the US, I use employment to population for ages 55-59. In the Appendix I include figures with country specific performance to match employment profiles for people older than 50. The model captures a substantial amount of the employment behavior of older individuals.

Finally, I pin down the features of social security that are key to generating the large differences in employment to population found in the data by setting to US levels some features of social security while leaving others as they are in the OECD. The measure of variability that I will use is the coefficient of variation.

5.2 Results

5.2.1 Retirement relative to the US

First, differences in social security account for large differences in retirement behavior. This is a surprising result as my model allows for differences along three dimensions of social security only. Figure 9 illustrates the ability of the model to match retirement behavior, measured as the employment to population ratio at age 60-64 relative to the US. In Figure 9 bars are data for OECD countries and dots are model simulations for each OECD country. The data is sorted from low to high employment to population at ages 60-64 relative to the US. The model does a very good job of matching the magnitude of employment to population at older ages, and it roughly preserves the rank of countries in the data. The model is also able to make accurate predictions for many OECD countries. To summarize the predictive performance of the model, I use the correlation
between OECD data and model simulations. For the benchmark simulations, correlation is .60 for males and females. In the Appendix, I present correlations for various assumptions on the sample and measure used for the replacement rate. The model always has high predictive ability, with correlations ranging from .46 to .71 at ages 60-64 and correlations ranging from .31 to .75 for ages 65-69. When I run a linear regression between the OECD data and model benchmark simulations, $R^2$ equals 92%, implying that the model captures a substantial portion of variability in the data. The model over-predicts employment on average (the ratio of simulation to data is 1.10) but this is not surprising, as I abstract from other taxes and transfers that may affect employment at older ages, such as unemployment insurance that can be bridged into early retirement and disability insurance. Austria, Poland, Italy and Czech Republic stand are outliers for ages 60-64. My model accounts for two thirds of employment variability in Turkey, Greece and Finland and over-predicts employment in UK, Ireland and Mexico. Korea and Sweden are under-predicted but the model captures almost all variability. Assuming that there are no measurement issues in OECD data, a few potential factors may account for these discrepancies. First, there are some countries that have entitlement ages that do not fall within the 60-64 range, and my model may capture behavior at entitlement age better. All the countries mentioned above except Turkey, Greece and Finland (with entitlement ages 60,60 and 62 respectively) have entitlement ages below 60 (Italy, Czech Republic and Korea) or above 64 (Austria, Poland, UK, Ireland and Mexico).

I address this issue by computing the employment to population ratio for countries grouped by entitlement age, but it may also be observed in country specific employment ratio profiles plot in the Appendix. Figure 10 shows the fit of employment for countries with entitlement ages lower than 60 (Figure 10 (a)) and entitlement ages greater than 64 (Figure 10 (b)). The model fit is better at

![Figure 10](image)

(a) Entitlement age < 60

(b) Entitlement age > 64

**Figure 10.** Employment to population at the entitlement age
the entitlement age, as is illustrated when I group countries in this way. But why does the model miss some variability in post-entitlement employment of countries with the entitlement age lower than 60 and pre-entitlement employment in countries with entitlement ages higher than 64. One potential weakness in my modeling choices could be assumption about "earnings test". Restricting it to be a 0-1 variable is a decision made for the sake of simplicity and comparability. To assess the importance of my decision, I include simulations assuming that the "earnings test" is the implicit tax on continuing to work at the age of entitlement computed by Duval. Therefore it becomes a continuous variable between 0 and 1. The quality of predictions of my model remains nonetheless, as I show in the Appendix. In the real world, however, incentives on continuing to work are not constant after early retirement and they would require detailed modeling of the normal entitlement age and entitlement decisions, or an age dependent tax on social security that captured incentives with some accuracy. I leave this as a future extension as I am already performing relatively well and making such change would increase the computational burden without clear advantages. Figure 11 shows the model simulations under two different assumptions: all countries with "earning test" set to 0 and vice versa. For most of the countries, it is important to make an appropriate choice, and this is why a careful modeling of social security for each country is necessary. The message that we get from these experiments is that the three key features of social security that I chose are able to explain a substantial portion of retirement behavior. From the sensitivity of the results to the measurement of the "earnings test" we can also conclude that this is a very important variable, as observed in the previous work by French (2005).

![Figure 11. Sensitivity of employment at ages 60-64](image_url)
5.2.2 Employment to Population

Differences in social security also account for substantial differences in cross national employment, given the relative size of older population today. These differences are accounted for through retirement behavior, as there is not much action in the employment decisions before age 50. My model is not intended to capture total employment rates or labor supply, but this result suggests that using this class of model, fed with a rich of institutional details, may be a promising endeavor to understand many dimensions of labor supply across nations. Figure 12 shows the fit of the employment to population ratio relative to the US. There are many factors that may affect employment behavior during a lifetime, so it is remarkable that social security is able to account for such a large portion. It is also worth noting the role of the "earnings test". Figure 13 is analogous to Figure 11 and shows how sensitive the employment to population ratio is to assuming that every country limits collecting social security by those who work. My model misses non-European countries like Turkey and Mexico, Eastern European countries like Poland, Hungary and from Slovak Republic to Belgium, Italy, France and Germany. An extension that included differences in income taxation independent of social security would fill part of what is missing regarding continental European countries. Turkey, Mexico, Greece and Italy have female populations that are not as integrated into the labor force as men are, and when I look at employment to population of males relative to the US I get a different picture. Figure 14 shows the fit of the model when I am restricting to males. Note that the picture for retirement will not change significantly, as retirement decisions are usually coordinated. Still, retirement decisions of couples are interesting in themselves and how different treatment of social security with respect to spouses may matter for individual and joint retirement choices.

![Figure 12. Employment to population](image)

(a) Males and Females  
(b) Females
5.2.3 What features of social security are most important?

Using an eyeball measure it is possible to tell that the "earnings test" is an important feature of social security in generating retirement and employment variability. However, to pin down rigorously what the most important contributors into accounting for employment variability are, I run some counter-factual simulations. First, I will focus on individual roles of individual features of social security: the replacement rate, entitlement age and "earnings test". I let one feature remain active at a time and set other features to US levels. My measure of variability is the coefficient of variation of data and the model. I compute standard deviation and mean of employment relative to the US for different ages. The ratio of standard deviation to mean gives a unit-less measure of variability. I do the same for my model simulations of OECD countries. Figure 15 (a) shows that the most important features individually are "earnings tests" and replacement rates, whereas entitlement ages are not important in accounting for the measure of variability I use. As there are potentially important interactions, I allow two features to be active at a time. I find that the replacement rate and restrictions on collecting social security while working account for almost all variability after age 60, and that the entitlement age is not an important contributor.

6 Conclusion and Future Research

In this paper I have employed a life cycle general equilibrium model of retirement to study the role of three key features of social security: the replacement rate, entitlement age and "earnings tests". A key feature of my model was its ability to match heterogeneity in retirement decisions in the US observed in the data. When I use my model to study cross national variation in retirement decisions, it is able to explain roughly 90% of the differences in the employment to population ratio.
for ages 60-64. The graphs included in the Appendix show that the model produces quite accurate employment profiles for people older than 50. The most important factor in these differences in retirement are the replacement rate and "earnings tests". There is still a feature of social security that I have not examined which is worth considering. Countries design social security to have different levels of progresivity. It turns out that when I define progresivity as the difference between the replacement rate of individual earnings below $0.5 \times AW$ and $2 \times AW$, there are large differences in progresivity across the OECD. For example, in the Slovak Republic the replacement rate of individuals who earn 2 times or more than AW is 10 percentage points above the replacement rate of individuals who earn 0.5 times or less than AW, whereas in Denmark this relationship is reversed, with lower earners having a replacement rate 60 percentage points above highest earners. Figure 16 illustrates the large differences in progresivity. In a recent paper, Guvenen et al. (2009) study the role of progresivity of the tax code in accounting for different evolution of wage inequality between Continental Europe and the US. Using a parsimonious representation of income tax they are able to separate the effects of progressivity from those of generosity. An suitable extension to my work would be to use a parsimonious representation of social security benefits to decompose average generosity from redistribution of social security systems. This is left as future research as I would introduce human capital, making the computational exercise more challenging. My model can also be extended in some interesting dimensions. First, Wallenius (2009) studies the role of social security on differences in hours of work. It would be interesting to extend her model with heterogeneity to determine whether there are interactions between the idiosyncratic labor income risk, human capital accumulation and social security. Guvenen et al. (2009) find that differences in the tax code are able to account for 50% of differences in hours per employee.
As they use a model that shares the human capital accumulation feature of Wallenius, and the presence of labor income risk as in my model, it would be interesting to investigate to what extent these features account for differences in the extensive and intensive margins. For example, why do some countries like Sweden and Norway have employment to population ratios that are above US levels and hours per employee below US levels, whereas countries like the Czech Republic, Hungary, Greece and Ireland have employment to population ratios below the US and hours per employee above the US.

Finally, all the space in this paper has been devoted to social security, which is the biggest tax and transfer program across the OECD. Health insurance programs are of the same order of magnitude in the GDP and they may also play an important role into shaping employment over the life cycle. Understanding how different health insurance programs affect employment across the OECD will require introducing health explicitly and it will provide an interesting insight on the current policy debate about health reform ongoing in the US.

References


# Appendices

## 7.1 Appendix A: OECD Social Security Data

### Table 2

OECD Social Security Relative to the US

<table>
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<th>Country</th>
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<th>Replacement Net</th>
<th>Replacement Public</th>
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<th>Wealth Net</th>
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<th>Eligibility ERA</th>
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Source: "Pensions at Glance", OECD 2009 and Duval, 2004

ERA: Early Retirement Age

WSS: Working and Collecting Social Security
7.2 Appendix B: Alternative Calibration

In the benchmark calibration of the model I have used data for white males from the CPS and then run simulations under different configurations of social security. However, the simulations are compared to data from the OECD, which abstracts from the selection I choose. I recalibrate the model to US employment to population at ages 50-54, 55-59, 60-64, 65-69 and 70-74 and find that the parameters that match the aggregate moments and the employment distribution are roughly similar but with a higher value of leisure in the utility function ($\lambda$) to compensate for an employment distribution that is shifted down when women and ethnic groups are included. I keep the same relative wages by age as there are no big differences by sex. The results under the new calibration are similar and available upon request.

\begin{table}
\centering
\caption{Calibrated Parameters}
\begin{tabular}{cccccccc}
\hline
$A$ & $n$ & $\sigma$ & $\lambda$ & $\beta$ & $\alpha$ & $\delta$ & $\rho$ & $\sigma^2$ \\
\hline
75  & 0.012 & 2.85 & 6.00 & 0.958 & 0.36 & 0.066 & 0.977 & 0.0141 \\
\hline
\end{tabular}
\end{table}
7.3 Appendix C: Numerical Methods

The algorithm used to compute the equilibrium of the model is similar to Hugget & Ventura (1999). The following steps describe the salient features of computation:

1. Choose an initial value of aggregate capital ($K_0$), aggregate labor in efficiency units ($L_0$), accidental bequests ($B_0$) and payroll tax ($\tau_0$).

2. For these values I solve iterating backward, starting from $V(x, A) = 0$, Bellman’s equation of the individual at each point of the individual state space ($k, z^w, \bar{e}$). As a result I get policy functions $c(x, a), k'(x, a), h(x, a)$ for every $a = 1, \ldots, A$.

3. I compute distributions over the individual’s state space ($\Psi_a(a)$) using Monte Carlo’s simulations. I assume that individuals start with capital equal to accidental bequests, average earnings of zero and an initial draw of productivity belonging to the stationary distribution of $z^w$.

4. I update $K_0, L_0, B_0$ and $\tau_0$ aggregating over simulated distributions to $K_1, L_1, B_1$ and $\tau_1$.

5. If aggregate variables in the previous point are close enough and product markets clear, I stop iterations. Otherwise I continue until convergence.

I choose 90 points for individual capital, 30 points for idiosyncratic shock and 4 points for average earnings. I have to be careful in computations as the problem is non-standard, since there is a non-convexity in labor choice. This is probably not a problem in theory, as I integrate the value function over continuous distribution with no mass points. Nevertheless, in numerical computations I am on a grid and this can be a problem. As I do not attempt to prove that the objective function is concave and differentiable, I use golden section search at each point of the individual state for each employment status ($0$ or $\bar{h}$) and then choose the maximum between these two numbers. Note that golden section search just requires that the objective be single peaked on an interval that one chooses and it does not use any derivative at all. There is a trade off between reliability and computational efficiency that makes this type of problems time consuming. For example, solving for the stationary equilibrium of the model may take between 30 min to 3 hours. Calibration may take from a few days to weeks.
7.4 Appendix D: Sensitivity analysis

### Table 4
Correlation between data and simulations (full sample)

<table>
<thead>
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<th>Age</th>
<th>Net Replacement</th>
<th>Gross Replacement</th>
<th>Public Replacement</th>
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<td>55-59</td>
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<td>0.20 0.21</td>
<td>0.31 0.30</td>
<td>0.13 0.08</td>
</tr>
<tr>
<td>60-64</td>
<td>0.67 0.71</td>
<td>0.48 0.51</td>
<td>0.62 0.67</td>
<td>0.64 0.69</td>
</tr>
<tr>
<td>65-69</td>
<td>0.65 0.64</td>
<td>0.31 0.31</td>
<td>0.66 0.65</td>
<td>0.64 0.63</td>
</tr>
<tr>
<td>70-74</td>
<td>0.65 0.64</td>
<td>0.37 0.35</td>
<td>0.45 0.51</td>
<td>0.60 0.58</td>
</tr>
</tbody>
</table>

Full sample includes all the OECD

Benchmark simulations are under the assumption of a 0-1 earnings tests

### Table 5
Correlation between data and simulations (full sample)

<table>
<thead>
<tr>
<th>Age</th>
<th>Net Replacement</th>
<th>Gross Replacement</th>
<th>Public Replacement</th>
<th>Net Wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Males</td>
<td>All Males</td>
<td>All Males</td>
<td>All Males</td>
</tr>
<tr>
<td>20-75</td>
<td>0.64 0.64</td>
<td>0.58 0.57</td>
<td>0.62 0.59</td>
<td>0.59 0.58</td>
</tr>
<tr>
<td>50-54</td>
<td>0.31 0.28</td>
<td>0.14 0.01</td>
<td>0.31 0.13</td>
<td>0.12 -0.03</td>
</tr>
<tr>
<td>55-59</td>
<td>0.20 0.22</td>
<td>0.18 0.19</td>
<td>0.26 0.28</td>
<td>0.15 0.16</td>
</tr>
<tr>
<td>60-64</td>
<td>0.59 0.60</td>
<td>0.56 0.57</td>
<td>0.54 0.55</td>
<td>0.56 0.56</td>
</tr>
<tr>
<td>65-69</td>
<td>0.64 0.63</td>
<td>0.58 0.56</td>
<td>0.64 0.63</td>
<td>0.61 0.60</td>
</tr>
<tr>
<td>70-74</td>
<td>0.62 0.60</td>
<td>0.56 0.54</td>
<td>0.42 0.48</td>
<td>0.56 0.55</td>
</tr>
</tbody>
</table>

Full sample includes all the OECD
### Table 6

Correlation between data and simulations (full sample)

<table>
<thead>
<tr>
<th>Age</th>
<th>Net Replacement</th>
<th>Gross Replacement</th>
<th>Public Replacement</th>
<th>Net Wealth</th>
</tr>
</thead>
<tbody>
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<td>All Males</td>
<td>All Males</td>
<td>All Males</td>
<td>All Males</td>
</tr>
<tr>
<td>20-75</td>
<td>0.67 0.67</td>
<td>0.63 0.60</td>
<td>0.53 0.48</td>
<td>0.60 0.56</td>
</tr>
<tr>
<td>50-54</td>
<td>0.40 0.42</td>
<td>0.25 0.17</td>
<td>0.24 0.13</td>
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</tr>
<tr>
<td>55-59</td>
<td>0.22 0.24</td>
<td>0.21 0.22</td>
<td>0.23 0.25</td>
<td>0.16 0.16</td>
</tr>
<tr>
<td>60-64</td>
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<td>0.52 0.52</td>
<td>0.46 0.46</td>
<td>0.51 0.50</td>
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<td>0.54 0.53</td>
<td>0.64 0.61</td>
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<tr>
<td>70-74</td>
<td>0.62 0.61</td>
<td>0.55 0.54</td>
<td>0.39 0.42</td>
<td>0.55 0.54</td>
</tr>
</tbody>
</table>

Restricted sample excludes Luxembourg and Iceland

Benchmark simulations are under the assumption of a 0-1 earnings tests

### Table 7

Correlation between data and simulations (restricted sample)

Using Duval’s (2004) definition of implicit taxes

<table>
<thead>
<tr>
<th>Age</th>
<th>Net Replacement</th>
<th>Gross Replacement</th>
<th>Public Replacement</th>
<th>Net Wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Males</td>
<td>All Males</td>
<td>All Males</td>
<td>All Males</td>
</tr>
<tr>
<td>20-75</td>
<td>0.71 0.82</td>
<td>0.50 0.53</td>
<td>0.52 0.60</td>
<td>0.59 0.74</td>
</tr>
<tr>
<td>50-54</td>
<td>0.42 0.34</td>
<td>0.33 0.27</td>
<td>0.29 0.12</td>
<td>0.28 0.13</td>
</tr>
<tr>
<td>55-59</td>
<td>0.24 0.22</td>
<td>0.21 0.22</td>
<td>0.21 0.21</td>
<td>0.14 0.08</td>
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<tr>
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<td>0.46 0.50</td>
<td>0.56 0.61</td>
<td>0.63 0.67</td>
</tr>
<tr>
<td>65-69</td>
<td>0.75 0.72</td>
<td>0.51 0.49</td>
<td>0.58 0.58</td>
<td>0.70 0.68</td>
</tr>
<tr>
<td>70-74</td>
<td>0.66 0.65</td>
<td>0.42 0.42</td>
<td>0.41 0.45</td>
<td>0.59 0.59</td>
</tr>
</tbody>
</table>

Restricted sample excludes Luxembourg and Iceland

### 7.5 Appendix F: country employment profiles
Figure 16. Australia

Figure 17. Austria
Figure 18. Belgium

Figure 19. Canada
Figure 20. Czech Republic

Figure 21. Denmark
Figure 22. Finland

Figure 23. France
Figure 24. Germany

Figure 25. Greece
Figure 26. Hungary

Figure 27. Iceland
Figure 28. Ireland

Figure 29. Italy
Figure 30. Japan

Figure 31. Korea
Figure 32. Luxembourg

Figure 33. Mexico
Figure 34. Netherlands

Figure 35. New Zealand
Figure 36. Norway

Figure 37. Poland
Figure 38. Portugal

Figure 39. Slovak Republic
Figure 40. Spain

Figure 41. Sweeden
Figure 42. Turkey

Figure 43. United Kingdom