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Bidding in First-Price Auctions:  
A Laboratory Study**

**Alexander Elbittar**  
**Instituto Tecnológico Autónomo de México**

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# Impact of Valuation Ranking Information on Bidding in First-Price Auctions: A Laboratory Study\*

A. Alexander Elbittar<sup>†</sup>  
*Instituto Tecnológico Autónomo de México*

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## Abstract

Landsberger, *et al.* (2001) have identified optimal bidder behavior in first-price private-value auctions when the ranking of valuations is common knowledge, and derived comparative-statics predictions regarding the auctioneer's expected revenue and the efficiency of the allocation. The experiment reported here tests the behavioral components of these comparative-statics predictions using the dual-market bidding procedure, which permits very powerful tests. The results support the predictions that buyers are inclined to bid more aggressively when they learn they have the low value. Contrary to theory, buyers are inclined to bid less when they learn they have the high value. Once information is revealed, bidders tend to move toward better responses, exploiting new economic opportunities. Consistent with theory, the overall proportion of efficient allocations is lower than in the first-price auction before information is revealed. But as a result of high-value bidders decreasing their bids, the expected revenue does not increase on a regular basis, contrary to the theory's predictions.

*Keywords:* Asymmetric auctions, laboratory experiments, affiliation and economics of information.

*Journal of Economic Literature* Classification Numbers: C92, D44, D82.

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<sup>†</sup>Instituto Tecnológico Autónomo de México, Centro de Investigación Económica, Camino a Santa Teresa 930, México, D.F., México, 10700. E-mail: elbittar@itam.mx.

# 1 Introduction

Traditional auction models assume that agents do not know how their valuations stand relative to those of their rivals, with knowledge limited to the underlying distribution from which values are drawn. In real-life situations, this is often an unrealistic assumption since agents know more than this. Such would be the case in art auctions where bidders might revise their bidding strategies based on the participation of a wealthy art collector who is known to have a high valuation. Likewise, in procurement auctions, due to experience accumulated, bidders learn who is the strongest bidder. There is also a general perception that, in privatizations, firms already in the market or, in takeovers, firms with related activities have a greater valuation than potential entrants or competing firms.<sup>1</sup>

The relevance of information revelation on bidding behavior is a common factor to all of these examples although some of them use different auction formats. Therefore, there is currently a need to determine the impact of information revelation on bidding behavior in auction markets, on seller's revenue, and on the efficiency of the auction outcome.<sup>2</sup>

This paper reports an auction experiment in which two bidders are required to bid for a single item following a first-price sealed-bid allocation rule and under two different information conditions. While each bidder knows the value of the item to herself, the revelation of the valuation ranking induces a particular *asymmetric affiliation* between bidders' valuations.<sup>3</sup>

Landsberger, Rubinstein, Wolfstetter and Zamir (2001) – hereafter referred to as LRWZ – developed a theoretical model in which two bidders draw their values from the same distribution, but the ranking of these valuations is common knowledge. Thus, the auction is ex-ante symmetric since bidders' valuations are drawn from a single density distribution. However, the revelation of the rankings makes it asymmetric since the subsequent conditional distributions differ.

The experiment reported here evaluates the key predictions of LRWZ's model regarding the

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<sup>1</sup>For instance, in the recent privatization of airwaves in the US, there was a general perception that the Pacific Telephone Company had a greater valuation for the Los Angeles area than did the other potential bidders. Another example would be the auctioning of the third generation mobile phone license in the UK, where firms already in the market were to have greater value for the new licenses being sold than potential entrants. Also, in the selling process of Wellcome, a drug company, it was commonly known that Wellcome and Glaxo had particular synergies that made Wellcome worth more to Glaxo than to any other competing firms (Klemperer, 2000).

<sup>2</sup>For a description of the importance of the issue of information revelation in auction markets, see Klemperer (1999) and Wolfstetter (1996, 1999) for recent surveys in theoretical research, and Kagel (1995) for a recent survey in experimental research.

<sup>3</sup>In auctions with *affiliated* private-values, values remain private for bidders, but the higher (lower) the value is for one bidder, the more likely the value will be higher (lower) for other bidders as well (Milgrom and Weber, 1982). While for Milgrom and Weber (1982) the affiliation is among *symmetric* distributions of signals, this paper considers a model in which the affiliation is among *asymmetric* distribution of signals.

auctioneer’s expected revenue and the efficiency of the first-price auction (FPA) institution in cases where the ranking of valuations is common knowledge to two bidders: (i) The bidder with the lower reservation value will bid more aggressively than the high-value bidder. Thus, the proportion of efficient allocations should be lower than when the bidders lack information about the ranking of valuations. (ii) Bidders are expected to bid more aggressively than in the standard FPA model, in which information regarding the ranking of valuations is not available. (iii) For a number of distributions, including the uniform distribution studied here, the expected revenue of sellers should be higher when rankings are known than when they are unknown.

For the low-value bidder, the more aggressive bidding behavior is an immediate consequence of knowing her position. Meanwhile, for the high-value bidder, the more aggressive bidding behavior is a consequence of incorporating her expectations about the low-value bidder response into her strategy. Notice that this behavior presumes a higher level of strategic thinking for the high-value bidder. If the high-value bidder does not take the low-value bidder’s more aggressive bidding into consideration, she might be tempted to keep bidding the same, or even less, in a misguided attempt to take advantage of her higher valuation. Hence, the experiment tests these behavioral components of the comparative-static predictions.

The experimental results support the prediction that low-value bidders are inclined to bid more aggressively once information about the ranking of valuations has been revealed. Contrary to theory, high-value bidders are inclined to bid less aggressively. However, in both cases bidders improve their average surplus per auction. That is, high-value bidders are closer to best respond with information revelation than without it. There are two (non mutually exclusive) ways to interpret high-value bidders response after information is released: (i) A failure to take into account low-value bidders’ responses to the revelation of the rankings. (ii) A reduction on bidders’ effort to win the item regardless of their valuations. Consistent with the theory, the overall proportion of efficient allocations is lower than in the first-price auction before information is revealed. But as a result of high-value bidders decreasing their bids, the expected revenue does not increase consistently. Further, to the extent these results for high-value bidders go beyond the lab, they have strategic implications for seller revenue when information about rankings is known.

This paper is organized as follows. Section 2 shows the related auction literature. Section 3 describes the experimental design. Section 4 specifies the Risk Neutral Nash Equilibrium (RNNE) bidding strategies for the auction models and states the comparative-statics predictions for the

FPA when the information about the ranking of valuations is revealed. Section 5 evaluates the experimental results regarding these auctions settings. Section 6 summarizes the research results.

## 2 Related Auction Literature

Experimental research on auctions has focused its effort on testing important implications coming from theory. These studies started by evaluating the predictions derived from the Independent Private Value (IPV) auction model (Myerson, 1982; Riley and Samuelson, 1981): a single unit auction under the assumptions of bidders' risk neutrality, *independence* of bidders' reservation values and *symmetry* of bidders' beliefs (Coppinger, Smith and Titus, 1980 and Cox, Robertson and Smith, 1982). Closer to our research, Kagel, Harstad and Levin (1987) tested some predictions derived by Milgrom and Weber (1982), regarding the impact of public information in the revenue ranking among different auctions institutions with *affiliated* private-values. In this case, bidders' private values are no longer independent, but are still symmetric. Conversely, Pezanis (2000) and Güth, Ivanova-Stenzel and Wolfstetter (2001) concentrated on checking some of the revenue ranking propositions between the second-price and first-price auctions derived respectively by Maskin and Riley (2000) and Plum (1992) with IPV's, but with values drawn from different distributions. We concentrate on evaluating the comparative statics predictions derived from an auction model that relaxes simultaneously the assumptions of independence and symmetry (LRWZ, 2001). In particular, this model considers an auction with ex-ante independent valuations from a single density, but with ex-post *affiliated* valuations from *different* conditional distributions once the ranking of valuations is common knowledge.

## 3 Experimental Design

The experimental design used directly measures the information impact of the ranking of valuations on bidding behavior in first-price private-value auctions. The particulars of the experimental design follow.

### 3.1 Structure of the Auction

Subjects in this experiment are required to bid for a single item following a first-price sealed-bid allocation rule under two different information conditions. In the first-price sealed-bid auction,

the high bidder earns a surplus equal to her value of the item less the high bid, while the other subject earns nothing. In one information condition (hereafter referred to as *symmetric condition*), two private values, one for each bidder, are independently drawn from a commonly known uniform distribution. In the symmetric condition bidders have no information about the rank order of valuations. In the second information condition (hereafter referred to as *asymmetric condition*), both bidders are informed about the rank order of their valuations before bidding, whether they have the highest or lowest valuation (but not informed about the size of the differences in valuations.)

### 3.2 Dual-Market Bidding Procedure

A dual-market bidding was employed. In the dual-market, bidders just submit a bid under the symmetric condition. Then, once these bids are collected, but before they are posted, information about the ranking of valuations is released and subjects are asked to then submit a second bid. The winner in each market is determined after the second bid is submitted. Payoffs are determined based on just one of the two markets (chosen with equal probability). Participants' positions as high-value or low-value bidder are determined randomly in each period.

The dual-market bidding procedure, involving the same two bidders bidding for the same item under two different information conditions, has the advantage of directly controlling for between-subject variability in bidding.<sup>4</sup> The rule of flipping a fair coin to determine which market to pay in ensures that, under the expected utility hypothesis, the optimal strategy in the private information market should be unaffected by bids made after the ranking is revealed, and vice-versa.

In order to determine whether the dual-market bidding procedure actually affects the way bidders bid, and to familiarize subjects with the auction conditions so as to make it easier for them to bid in dual-auction markets, each session began with bidding in single-auction markets. For the single-auction markets, each bidder submitted one bid in each period. Table 1 briefly summarizes the experimental treatments. Sessions 1 and 2 began with ten periods of single-auction markets with the asymmetric condition followed by twenty periods of dual-auction markets. During these ten initial periods, each bidder maintained her role as a high-value or low-value bidder. Sessions 3 and 4 began with ten periods of single-auction markets with the symmetric condition followed by twenty periods of dual-auction markets.

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<sup>4</sup>A similar procedure has been used in previous auction experiments by Kagel and Levin (1986) and Kagel, Harstad and Levin (1987).

Table 1: Experimental Design

Experimental Treatment	Sessions	Auction Periods	Markets	Information Condition
Asym. Condition and Dual-Market Bidding	1 & 2	1 to 10	Single-Auction	Asymmetric
		11 to 30	Dual-Auction	Sym./Asym.
Sym. Condition and Dual-Market Bidding	3 & 4	1 to 10	Single-Auction	Symmetric
		11 to 30	Dual-Auction	Sym./Asym.

Note: In session 1, during the dual-auction markets, only 18 trading periods (out of 20) were actually run since the network system broke down at that stage.

### 3.3 Design Parameters

This section characterizes the basic parameters and the general procedure of the experiment.

*Values' Probability Distributions.* All bidders were informed that their valuations were drawn randomly from a uniform distribution in the interval \$0.00 to \$6.00.

*Participation Fee.* Each group of players received an initial cash balance of zero. Since bidders' roles were not changed during the initial ten auction periods of sessions 1 and 2, the low-value bidders received an additional \$5 at the end of each session. This was a fixed amount that was not expected to impact behavior.

*Matching Procedure.* Bidders were informed that participants would be randomly matched in every period, but that no pair of bidders would be matched twice in two consecutive periods. Bidders were also informed that, in the dual-auction markets section, each of them would face the same opponent (with the same valuation) in both markets.

*Information Feedback.* During the single-auction markets with the asymmetric condition, each subject received feedback about her earnings and the competing bidder's valuation and bid. During the single-auction markets with the symmetric condition, additional information about the ex-post ranking of valuations was posted on the computer screens. For the dual-auction markets, each subject received complete feedback about who the high bidder was in each market, as well as her potential (and actual) earnings in each market, what the other bidder's value and bids were in each market, and which market was randomly selected to pay off in.

*Dry Runs.* In order to familiarize subjects with the auction procedures, two practice periods occurred at the beginning of the single-auction markets, and one practice period occurred just

before the dual-auction markets began.

*Subjects.* For each session, the subjects were drawn from a wide cross-section of students (mostly undergrads) at the University of Pittsburgh and Carnegie Mellon University. There were 18 subjects for session 1 and 20 subjects for sessions 2, 3 and 4. Subjects participated in only one session. The experiment was run in the economics lab at the University of Pittsburgh using computers.<sup>5</sup>

## 4 Theoretical Predictions<sup>6</sup>

LRWZ consider the specific situation where a single object is auctioned between two bidders. Both bidders have risk-neutral utility functions and independently drawn private valuations. These valuations  $V_1$  and  $V_2$  are ex-ante identically distributed according to a continuous probability density,  $g(v)$ , of valuation within the strictly positive support  $[0, c]$ .  $G(v)$  is a differentiable cumulative distribution function of  $g(v)$ .<sup>7</sup>  $G$  is common knowledge to bidders. Each bidder  $i$  knows whether she has the higher or the lower valuation but does not know her opponent's exact valuation.

Let  $H$  be the bidder with higher valuation,  $v_H = \max\{v_1, v_2\}$ , and  $L$  be the bidder with lower valuation,  $v_L = \min\{v_1, v_2\}$ . LRWZ proved that there exists a unique equilibrium in pure strategies that is strictly increasing with regard to the valuation. It should be noted that LRWZ's analysis is not trivial and works with a non-standard ordinary differential equation system.<sup>8</sup>

Under the assumption that the underlying distribution of valuations is uniform and that  $c = 1$ , both agents maximization problems can be reduced to the following differential equation system by the first order necessary conditions:

$$l'(x) = \frac{l(x)}{h(x) - x} \tag{1}$$

$$h'(x) = \frac{h(x) - l(x)}{l(x) - x} \tag{2}$$

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<sup>5</sup>The site <http://ciep.itam.mx/~elbittar/instructions/lrwz.pdf> contains the set of instructions given to subjects.

<sup>6</sup>This model presentation follows the results of LRWZ's paper very closely.

<sup>7</sup>LRWZ also assume, as a technical requirement, that  $G$  has a Taylor expansion around zero (i.e.,  $G(x) = \alpha x + \beta x^2 + \dots$ , with  $\alpha > 0$ )

<sup>8</sup>In this model,  $V_L$  and  $V_H$  each has a strictly positive marginal density in the interval  $[0, c]$  such that the joint density (with a triangular support) is  $f(v_H, v_L) = 2g(v_H)g(v_L)$  for  $v_H \geq v_L$  and  $(v_H, v_L) \in [0, c] \times [0, c]$ . LRWZ pointed out that since values are stochastically dependent,  $V_H$  and  $V_L$  can be viewed as *affiliated à la* Milgrom and Weber (1982). While for Milgrom and Weber the affiliation is between symmetric distributions of signals, LRWZ's model can be seen as a special affiliated private-value model that considers a specific asymmetric distribution of signals (triangular).

Table 2: Numerical Results from the Equilibrium Bid Functions

Auction Model	Sellers Revenue	Typical Buyer		High-Value		Low-Value		Proportion of POA <sup>a</sup>	Proportion of ME <sup>b</sup>
		Surplus	Pr[Win]	Surplus	Pr[Win]	Surplus	Pr[Win]		
Symmetric*	2.0000	1.0000	0.5000	(-)	(-)	(-)	(-)	1.0000	1.0000
Asymmetric	2.2192	(-)	(-)	1.3606	0.7103	0.2625	0.2897	0.7103	0.9565

Note: Valuations are drawn from the uniform interval [0,6]. Mean is computed by Monte Carlo using 10,000 samples of 1,000 drawings.

\*: Calculations are made analytically.

<sup>a</sup>: Pareto-Optimal Allocations: *POA*

<sup>b</sup>: Mean Efficiency:  $ME = \frac{v_{min}}{v_h}$

$$h(0) = l(0) = 0 \tag{3}$$

$$\exists t^* \in [0, 1] \text{ such that } h(t^*) = l(t^*) = 1 \tag{4}$$

where  $l(\cdot)$  and  $h(\cdot)$  are the inverse bid functions of  $b_L$  and  $b_H$ , respectively. By l'Hôpital's Rule,  $h'(0) = 2$  and  $l'(0) = \frac{4}{3}$ . Notice that in equilibrium,  $v_L = l(x)$  when  $L$  bids  $x$ , and  $v_H = h(x)$  when  $H$  bids  $x$ .

A solution to (1), (2), (3) and (4) characterizes the equilibrium for the FPA, where the optimal bid functions take values in the interval  $[0, 1]$ . This system of differential equations lacks an analytical solution. For a numerical approximation of the above system of equations, we follow a general method implemented by Elbittar and Ünver (2001).

The bidding response for the symmetric and independent private-value FPA model,  $b_S$ , can be derived analytically. For the case of  $n$  risk neutral bidders with a uniform distribution, the unique symmetric Nash Equilibrium bid function is (Vickrey, 1961):

$$b_S(v) = \frac{(n-1)}{n}v$$

In Figure 1, the equilibrium bidding response for the low-value bidder and for the high-value bidder are represented by  $b_L$  and  $b_H$ , respectively. In the same graphic, the equilibrium bidding response for the symmetric and independent private-value model when  $n = 2$  is represented by  $b_S$ .

#### 4.1 Comparative-Statics Predictions

The comparative-statics predictions of these two auction models define the theoretical benchmark for measuring the possible impact of valuation ranking revelation on bidding behavior. These predictions also provide us with precise statements about the changes in the seller's revenue and the

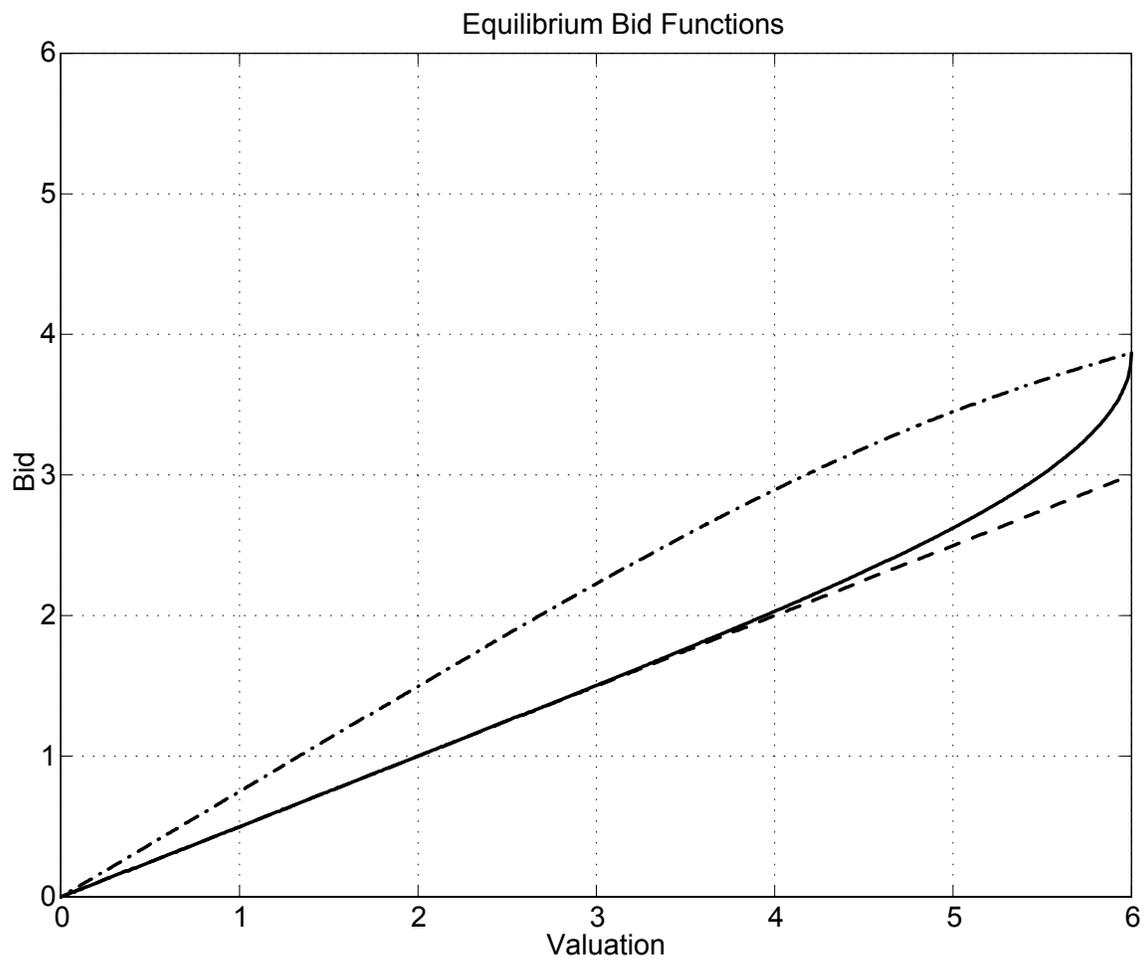


Figure 1: Equilibrium Bid Functions:  $b_L(v)$  [-·-],  $b_H(v)$  [—] and  $b_S(v)$  [- -]

proportion of optimal allocations (LRWZ, 2001). First, the high-value bidder should bid less aggressively than the low-value bidder for any given value ( $b_L(v) > b_H(v)$ ). Since the low-value bidder can obtain the item with positive probability, the proportion of efficient allocations should decrease. Second, for a number of different distributions, including the case of uniform distribution studied here, revealing the ranking should induce both bidders to bid higher than when this information is not known ( $b_i(v) > b_S(v)$ ,  $i = L, H$ ).<sup>9</sup> Therefore, the seller’s expected revenue should increase.

These changes are associated with two behavioral components, both producing positive effects on the seller’s expected revenue (Maskin and Riley, 2000). One component is the “sure thing effect”: The high-value bidder must take into consideration that the low-value bidder is willing to increase her bid in order to increase her probability of winning the object. Note that the high-value bidder must think more deeply to formulate the right response compared to the low-value bidder. However, if the high-value bidder does not take the low-value bidder’s more aggressive bidding into consideration, she could be tempted to keep bidding the same, or even less, in a misguided attempt to take advantage of her higher valuation. The second behavioral component would be the “weight effect”: Since the low-value bidder is expected to bid more aggressively than the high-value bidder, the higher bid is weighted more in the expected revenue due to the higher probability of winning.

Table 2 summarizes some important theoretical predictions for both auction models using the equilibrium bid functions. The seller’s expected revenue, the proportion of Pareto-Optimal Allocations (POA), the proportion of the maximal surplus – or Mean Efficiency (ME) –, and each type of bidder’s unconditional expected surplus and probability of winning the item are numerically estimated, drawing valuations from the uniform distribution in the interval  $[0,6]$ . The values used in these results indicate that the theoretical impact of the valuation ranking revelation would increase the seller’s expected revenue from \$2.00 to around \$2.22, with the percentages of POA and of ME decreasing from 100.00% to 71.03% and 95.65%, respectively.

## 5 Experimental Results

This section compares the experimental results for the dual-auction markets in order to directly measure how revealing the ranking of valuations affects bidding behavior.<sup>10</sup>

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<sup>9</sup>This will always be true for the low-value bidder. However, for the high-value bidder, LRWZ showed that there exist probability distributions for which  $b_S(v) > b_H(v)$  for some values of  $v$ .

<sup>10</sup>As we will see in each of the following sections, the results of single-auction markets are consistent with the results for the dual-auction markets and indicate that the dual-market bidding procedure does not to affect the

Table 3: Observed Bid Factors

Statistics	Bid Factor* Symmetric Condition			Bid Factor Asymmetric Condition	
	$\delta_L^S(v)$	$\delta_H^S(v)$	$\delta^S(v)$	$\delta_L^A(v)$	$\delta_H^A(v)$
Median	0.2959	0.3417	0.3228	0.1731	0.3919
Mean	[0.1689]	[0.3538]	[0.2614]	[0.2365]	[0.3976]
Std. Error	(0.1314)	(0.0058)	(0.0658)	(0.0087)	(0.0060)

Note 1:  $\delta^S$  is the aggregation of  $\delta_L^S$  and  $\delta_H^S$ .

\*: Bid Factor:  $\delta_i^j = \frac{v-b(v)}{v}$ ,  $i = L, H$  and  $j = S, A$ .

Table 4: Matched Pairs Wilcoxon Test Statistic for Comparison of Bid Factors

Auction Period	${}^a W_L$	${}^b W_H$	Auction Period	$W_L$	$W_H$
11	2.5813*	-0.6919	21	3.6463*	-1.9848 <sup>+</sup>
12	0.4422	-2.0758 <sup>+</sup>	22	2.5283*	-0.9917
13	2.5196*	-1.4295	23	4.0828*	-2.8466 <sup>+</sup>
14	2.8830*	-1.7762 <sup>+</sup>	24	3.6434*	-3.0750 <sup>+</sup>
15	2.7150*	-2.5276 <sup>+</sup>	25	2.3679*	-1.6268
16	3.9388*	-1.3133	26	4.0760*	-3.6548 <sup>+</sup>
17	1.5333	-3.1508 <sup>+</sup>	27	2.7678*	-1.4697
18	2.8415*	-3.4032 <sup>+</sup>	28	4.4288*	-2.4496 <sup>+</sup>
19	1.7637*	-1.5098	29	2.7557*	-0.3346
20	3.2189*	-2.6063 <sup>+</sup>	30	2.7758*	-2.8590 <sup>+</sup>

Note: Periods 11-28 count with 39 observations each, while periods 29 and 30 count with 30 observations each.

<sup>a</sup>: Matched Pairs Wilcoxon test.  $H_0 : \delta_i^S \leq \delta_i^A$  against  $H_1 : \delta_i^S > \delta_i^A$ .

\*: Reject  $H_0 : \delta_i^S \leq \delta_i^A$  against  $H_1$  at  $p < 0.05$ .

<sup>b</sup>: Matched Pairs Wilcoxon test.  $H_0 : \delta_i^S \geq \delta_i^A$  against  $H_1 : \delta_i^S < \delta_i^A$ .

<sup>+</sup>: Reject  $H_0 : \delta_i^S \geq \delta_i^A$  against  $H_1$  at  $p < 0.05$ .

## 5.1 Bidding Behavior

Table 3 reports the observed bid factor,  $\delta_i^j(v)$ , for each type of bidder, and each information condition. The bid factor, or relative difference between subjects' valuations and bids, tells us how much a bidder shaves off her own reserve value when bidding for an item.

LRWZ predict that both types of bidders bid more aggressively under the asymmetric condition than under the symmetric condition ( $\delta_i^S(v) > \delta_i^A(v)$ ,  $i = L, H$ ).

Analysis of the median of actual bid factors in Table 3 indicates that, contrary to expectations,  $\delta_H^S(v) < \delta_H^A(v)$ .<sup>11</sup> For the pooled data, the high-value bidders shave off 5.0 percentage points more after information is released. In particular, bidders with valuations less than 4 shave off 7.0 percentage points more after information is released, while bidders with valuations greater than 4 shave off 3.0 percentage points.<sup>12</sup> Thus, *high-value bidders tend to bid lower under the asymmetric condition*. By contrast, low-value bidders tend to discount less after information about the ranking of valuations is released. For the pooled data, the same bidders shave off 12.3 percentage points less after information is released. As expected, *low-value bidders tend to bid higher under the asymmetric condition*.<sup>13</sup>

Table 4 displays, for every auction period, a one-tailed Matched Pairs Wilcoxon ( $W_i$ ) test statistic for each group of players type  $i$ . The null hypothesis that the bid factor for low-value bidders under the symmetric condition is larger than or equal to that under the asymmetric condition is rejected in all but two auction periods. For high-value bidders, however, the same null hypothesis holds for all auction periods. In fact, in twelve of the auction periods, high-value bidders tend to reduce their bid (or increase their bid factor) after information is released.<sup>14</sup>

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bidding behavior.

<sup>11</sup>Since the mean is significantly affected for the extreme values, we concentrate the analysis of the experimental data using the median instead.

<sup>12</sup>Here we check whether bids for high-value bidders increase when ranking information is revealed. According to Figure 1, bids do not begin to increase measurably until valuations exceed 4. However, we observe bidding reduction in the whole range of valuations.

<sup>13</sup>Looking at the median for the single-auction markets, we find results similar to those for the dual-auction markets – i.e., low (high) value bidders under the asymmetric condition tend to bid more (less) aggressively than bidders under the symmetric condition. Using a one-tailed Mann-Whitney test statistic, the null hypothesis that the bid factor for the low-value bidders under the symmetric condition is greater than or equal to this factor under the asymmetric condition is rejected. However, for the high-value bidders, the same null hypothesis is not rejected.

<sup>14</sup>The null hypothesis that the bid factor for high-value bidders in the asymmetric environment is smaller than or equal to that under the symmetric environment is rejected in twelve of the auction periods. For the other eight auction periods, high-value bidders also reduced their bids although not significantly.

Based on the second of the comparative-statics predictions, it is expected that high-value bidders bid less aggressively than low-value bidders for any given value under the asymmetric condition ( $b_L(v) > b_H(v)$ ). In order to better assess this prediction, it is necessary to estimate and compare subjects' actual bid functions for the same valuation.

Analysis of the experimental data indicates a significant pattern of heteroscedasticity with respect to valuations,  $v$ , and heterogeneity of individual responses. Estimation of the relative deviation with respect to the equilibrium response,  $d_{it} = \frac{b(v) - b^*(v)}{v}$ , allows us to correct for heteroscedasticity with respect to the size of valuations and then recover the actual bid function. We consider the following fixed-effect model with groupwise heteroscedasticity:

$$d_{it} = \mu + \alpha_i + \beta_1 v_{it} + \beta_2 v_{it}^2 + \varepsilon_{it}$$

$$s.t. \quad \begin{aligned} \sum_{i=1}^I \alpha_i &= 0 \\ E(\varepsilon_{it}) &= 0 \\ E(\varepsilon_{it}^2) &= \sigma_i^2 \end{aligned}$$

The parameters of this model were estimated using an iterative feasible GLS procedure. The estimated bid functions under the asymmetric condition for low-value and high-value bidders are denoted  $\hat{b}_L$  and  $\hat{b}_H$ , respectively;  $\hat{b}_S$  corresponds to the estimated bid function under the symmetric condition. Figure 2 graphs each of the estimated bid functions.

Table 5 reports the mean of the difference in bid factors evaluated for the same set of random draws.<sup>15</sup> The mean of the difference in bid factors between low and high-value bidders,  $\hat{\delta}_H^A(v) - \hat{\delta}_L^A(v)$ , represents a significant difference in bidding behavior.<sup>16</sup> Specifically, high-value bidders shave off their reserve value approximately 15.6 percentage points higher than low-value bidders. Thus, for the same set of random draws, low-value bidders, on average, bid 41¢ more than high-value bidders. This difference can be broken down into two components. The first component is the mean of the difference in bid factors between low-value bidders under both conditions,  $\hat{\delta}^S(v) - \hat{\delta}_L^A(v)$ . In particular, low-value bidders shave off their reserve value around 8.2 percentage points less under the asymmetric condition than under the symmetric one, thus representing an additional bidding of approximately 27¢. The second component is the mean of the difference between the bid factors

<sup>15</sup>Valuations were drawn from the common support of the bid functions. Mean and standard error were computed by Monte Carlo using 10,000 samples of 1,000 drawings.

<sup>16</sup>In order to assess the accuracy of the estimated bid functions, we compare the estimated bid factors using the same set of valuations drawn in the experiment and the direct estimated bid factors available from the experiment (See Table 3). The null hypothesis that the estimated bid factors for high-value and low-value bidders in the asymmetric environment are each equal to those estimated in the experiment can not be rejected at  $p < 0.05$ .

Table 5: Difference of Estimated Bid Factors

Difference			
Statistics	$\widehat{\delta}_H^A(v) - \widehat{\delta}_L^A(v)$	$\widehat{\delta}^S(v) - \widehat{\delta}_L^A(v)$	$\widehat{\delta}^S(v) - \widehat{\delta}_H^A(v)$
Mean	0.1559	0.0816	-0.0743
Std. Error	$(0.1155 \times 10^{-4})$	$(0.0558 \times 10^{-4})$	$(0.1651 \times 10^{-4})$

Note:  $\delta_i^j = (v - b(v))/v$ ,  $i = L, H$  and  $j = S, A$ .

of high-value bidders in both conditions,  $\widehat{\delta}^S(v) - \widehat{\delta}_H^A(v)$ . High-value bidders shave off their reserve value around 7.4 percentage points *more* under the asymmetric condition than under the symmetric one, resulting in a lower bidding of approximately 14¢.

In conclusion, estimates of bid functions based on the experimental data indicate that *revealing the ranking of valuations induces low-value bidders to bid more than high-value bidders with the same valuation*. This result is consistent with the comparative-static prediction of LRWZ. It also supports our previous finding that low-value bidders tend to bid more aggressively under the asymmetric condition and high-value bidders are inclined to bid less aggressively.

In addition to these behavioral results, which are directly related to the comparative-statics predictions, there is a clear indication of a larger bidding variation in the response of high-value bidders than in that of low-value bidders. Namely, the standard deviation of the bidding regression for high-value bidders ( $\widehat{\sigma}_{b_H} = 0.5394$ ) is significantly larger than that for low-value bidders ( $\widehat{\sigma}_{b_L} = 0.3643$ ).<sup>17</sup> This result could be attributed to the greater complexity of bidding strategies for high-value bidders once information is released. For low-value bidders, the obvious response to such information is to increase their bids in order to increase the probability of obtaining the object. For high-value bidders, the response is less obvious: Once high-value bidders are aware of their strong position, they might either be reluctant to take on the risk of submitting lower bids because of low-value bidders' more aggressive bidding behavior or be willing to take such a risk in hopes of increasing their average surplus.

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<sup>17</sup>The standard deviation of the bidding regression for the symmetric condition is  $\widehat{\sigma}_{b_S} = 0.4944$ . The  $R$ -squared for the recovered bid function are  $\widehat{R}_{b_H}^2 = 0.7548$ ,  $\widehat{R}_{b_L}^2 = 0.9159$ ,  $\widehat{R}_{b_S}^2 = 0.8312$ .

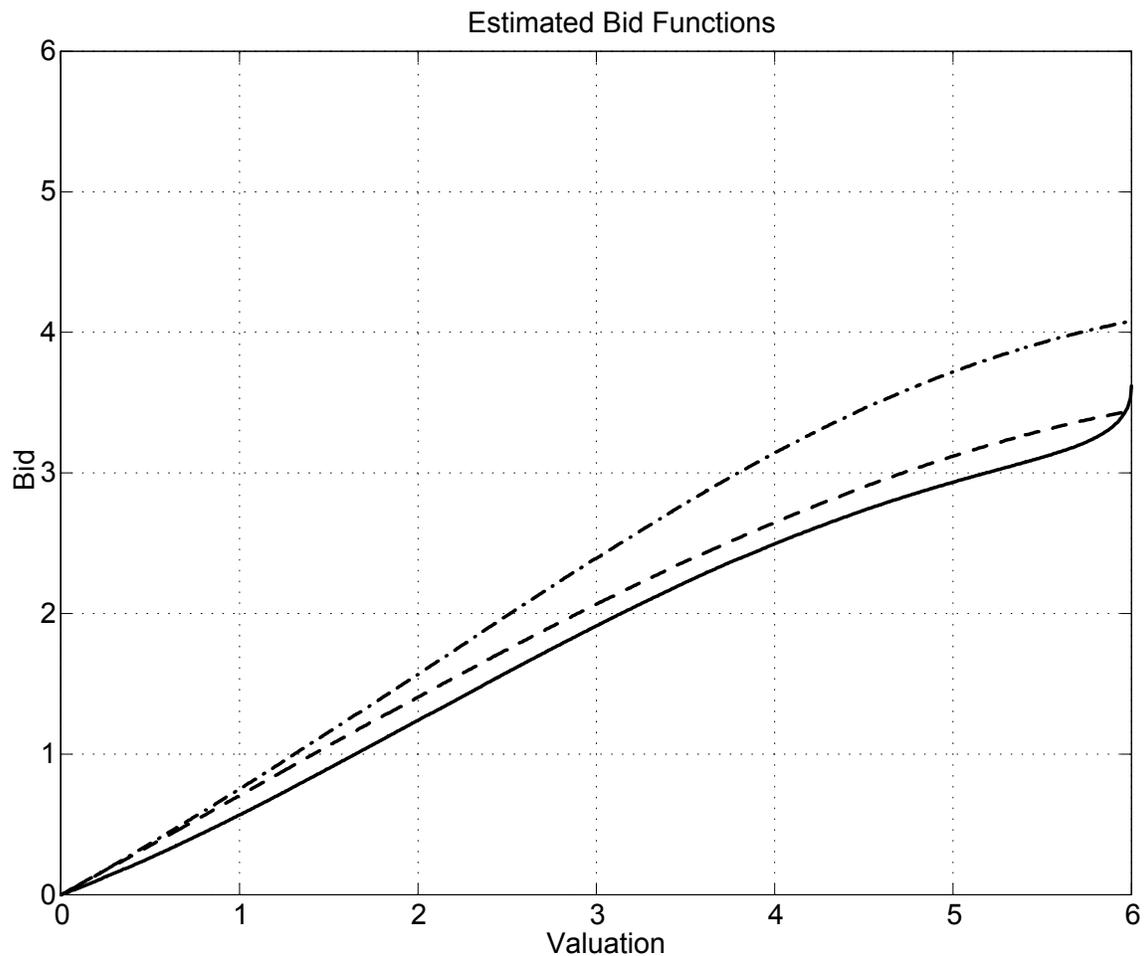


Figure 2: Estimated Bid Functions:  $\hat{b}_L(v)$  [-·],  $\hat{b}_H(v)$  [—] and  $\hat{b}_S(v)$  [—]

Table 6: Estimated Bid Functions Comparison: Numerical Results

Bid Functions Comparison	Statistics	Seller Revenue	High-Value		Low-Value	
			Surplus	Pr[Win]	Surplus	Pr[Win]
$\hat{b}_H(v)$ vs $\hat{b}_L(v)$	Mean Std. Error	2.5451 ( $0.3244 \times 10^{-3}$ )	1.1729 ( $0.2997 \times 10^{-3}$ )	0.7766	0.1000 ( $0.1104 \times 10^{-3}$ )	0.2234
$\hat{b}_S(v)$ vs $\hat{b}_L(v)$	Mean Std. Error	2.7407 ( $0.4029 \times 10^{-3}$ )	0.9088 ( $0.3330 \times 10^{-3}$ )	0.7763	0.1112 ( $0.1180 \times 10^{-3}$ )	0.2237
$\hat{b}_H(v)$ vs $\hat{b}_S(v)$	Mean Std. Error	2.5117 ( $0.3200 \times 10^{-3}$ )	1.2655 ( $0.3012 \times 10^{-3}$ )	0.8234	0.0783 ( $0.1097 \times 10^{-3}$ )	0.1766

Note: Valuations were drawn from the common support of the bid functions. Mean and standard error were computed by Monte Carlo using 10,000 samples of 1,000 drawings.

## 5.2 Do bidders profitably deviate?

The baseline of the comparative-static predictions in LRWZ is what risk-neutral, fully rational bidders bid under the symmetric condition. Well known pattern of bidding above RNNE in first-price IPV auctions (Kagel, 1995) is replicated here, as well as for both types of asymmetric conditions. For instance, using a one-tailed Kolmogorov-Smirnov ( $D_i$ ) test statistic on the observed bids compared with the theoretical bids, the null hypothesis that the observed distribution of bids,  $G(b)$ , is similar to or larger than the theoretical distribution of bids,  $G(b^*)$ , is rejected for all auction markets within each information condition at  $p < 0.05$ .<sup>18</sup>

Hence, the question to be addressed next is whether there exist monetary incentives for high-value bidders to decrease their bids rather than increase them (or at least keep them approximately the same). In order to check whether such incentives exist, we examine whether high-value bidders increase their average surplus by bidding based on the estimated bid function,  $\hat{b}_H(v)$ , rather than continuing to use the estimated bid function previous to the revelation of the ranking,  $\hat{b}_S(v)$ .

In Table 6, the first row displays the results of a Monte Carlo experiment in which high-value bidders bid against low-value bidders, with both groups bidding based on the estimated response functions  $\hat{b}_H(v)$  and  $\hat{b}_L(v)$ , respectively. The second row indicates results of a similar numerical experiment in which high-value bidders based their bids on the estimated response before the ranking was revealed,  $\hat{b}_S(v)$ . Table 6 displays, for each comparison, the seller's expected revenue, the bidder's expected surplus and the probability of winning the item.

The numerical results of Table 6 indicate that, if high-value bidders were using the bid function previous to the revelation of the ranking,  $\hat{b}_S(v)$ , the average surplus would also fall (by around 26¢ per auction); however, the probability of winning the item would almost be the same. As result of this trade-off, their average surplus, conditional on winning, would drop from \$1.51 to \$1.16 per auction. Therefore, it is better for high-value bidders to drop their bids, which in fact is what they do.

For low-value bidders, the question is whether it is better to increase their bids based on the estimated bid function,  $\hat{b}_L(v)$ , rather than continuing to use the estimated bid function prior the revelation of the ranking,  $\hat{b}_S(v)$ . The third row of Table 6 reports results of a numerical experiment

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<sup>18</sup>This consistent pattern of overbidding behavior would not be consistent in our case with the idea of risk averse bidders (Cox, Smith and Walker, 1988) since risk aversion would induce a more aggressive bidding behavior of both bidders after information revelation for a given parameter of risk aversion.

in which low-value bidders bid based on the estimated response before the revelation of the ranking,  $\widehat{b}_S(v)$ , but high-value bidders based their bids on the estimated bid function after information is revealed,  $\widehat{b}_H(v)$ . Comparing the numerical results of the first and the third rows in Table 6, it is clear that, if low-value bidders were bidding using the bid function previous to the revelation of the ranking,  $\widehat{b}_S(v)$ , they would suffer a reduction in both their average surplus (of around 2¢ per auction) and their probability of their winning the item (of 5.0 percentage points).

As general conclusion, it can be claimed that *each group of bidders improves the average surplus per auction by properly deviating after information is released*. In particular, the revelation of the ranking provides incentives for high-value bidders to decrease their bids and to exploit new economic opportunities. The reason of this result comes from the fact that each group seems to deviate in the direction of a better response, which include both the equilibrium and the optimal response.<sup>19</sup> Furthermore, a pattern of adjustment in bidders' decision to change their bids after information was released is observed. The proportion of high-value bidders reducing their bids moves up 7.7 percentage points from the first four auction rounds (48.1%) to the last four auctions rounds (55.8%). The proportion of low-value bidders increasing their bids moves up 9.3 percentage points from the first four auction rounds (54.5%) to the last four auctions rounds (63.4%).

There are still two (non mutually exclusive) ways to interpret high-value bidders response after information is released: (i) A failure to take into account low-value bidders' response to the revelation of the rankings. This would be consistent with most of the answers provided by subjects about their bidding strategies once they learned their positions as a high-value bidder.<sup>20</sup> (ii) A reduction on bidders' effort to win the item regardless of their valuations. This would be consistent

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<sup>19</sup>Usually, the baseline of comparison has been the equilibrium bid response. However, this might not be the best response for a particular group of bidders once their opponents have bid out of the equilibrium. Using the estimated response function of bidders' opponents, we calculate numerically the optimal bidding response for each group of bidders. The question we address is whether there exist significant incentives for each group of bidders to deviate towards the optimal response when their opponents have bid away from the equilibrium. If low-value bidders were using the optimal bid function, their average surplus, conditional on winning, would increase from \$0.45 to \$0.85. If high-value bidders were bidding using their optimal best response, their average profit, conditional on winning, would increase from \$1.51 to \$2.08 per auction. Therefore, we confirm that in fact *there exist incentives for each group of bidders (in particular, for the high-value bidders) to move toward better responses, exploiting new economic opportunities*. However, it is uncertain whether these bidders would be able to recognize and be motivated by a small amount of additional expected surplus per auction, especially since such additional gains might diminish the bidder's frequency of winning (Slonim and Roth, 1998).

<sup>20</sup>At the end of the sessions, subjects were asked to write down what their bidding strategies were once they learned their positions as either high-value bidder or low-value bidder. Most of answers were as follow: "If I was a low I raised my bid. While, if I was a high I lowered my bid." Just a few participants indicated an apparently more sophisticated reasoning: "Once I knew I was a high bidder, I usually lowered my bid. Sometimes I highered (sic) it because I knew the other person knew they were a low bidder, so they might also increase their bid."

Table 7: Effect of Ranking Information on Revenue

Statistics	Symmetric Condition	Asymmetric Condition
	$p_S$	$p_A$
Mean	2.6102	2.5722
Std. Error	(0.0365)	(0.0378)
Median	[2.6550]	[2.6200]
$^a t$	-2.0487	
$^b W$	-2.3777	

$^a$ : Matched Pairs t-test.  $H_0: p_A(b) \leq p_S(b)$  against  $H_1: p_A(b) > p_S(b)$ .

$^b$ : Matched Pairs Wilcoxon test.  $H_0: p_A(b) \leq p_S(b)$  against  $H_1: p_A(b) > p_S(b)$ .

\*: Reject  $H_0$  against  $H_1$  at  $p < 0.05$ .

with the persistent behavior of bidding above the RNNE in symmetric auctions as an effort by bidders to win the item regardless of their valuations (Kagel, 1995).

### 5.3 Expected Revenue

With the high-value bidders decreasing their bids, the enhanced revenue possibilities of the FPA with the ranking of valuations revealed no longer necessarily holds; rather it becomes an empirical question of whether high-value bidders' decreasing bids dominate low-value bidders' increasing bids after information is released.

Table 7 reports the mean, standard error and median of the observed price realization for each information condition. Each average price realization represents the observed seller's expected revenue for a particular information condition  $j$ :  $p_j$ ,  $j = S, A$ . According to LRWZ, seller's revenue expected to be higher under the asymmetric condition than under the symmetric condition ( $p_A > p_S$ ).

As noted in the statistics in Table 7, however, the mean of the selling prices under the symmetric condition (\$2.61) is slightly higher than under the asymmetric condition (\$2.57). This observation is confirmed by a one-tailed Matched Pairs t-test statistic ( $t$ ), the result of which is displayed at the bottom of the same table. The alternative hypothesis that the price realization under the

Table 8: Effect of Ranking Information on Efficiency

Concepts	Symmetric Condition	Asymmetric Condition
<sup>a</sup> ME	97.22	94.84
<sup>b</sup> POA	[87.14]	[74.80]
<sup>c</sup> Z <sub>ME</sub>	2.376*	
<sup>d</sup> Z <sub>POA</sub>	6.134*	

<sup>a</sup>: Mean Efficiency:  $ME = \frac{v_{winner}}{v_h} \times 100$ .

<sup>b</sup>: Percentage of Pareto-Optimal Allocations: POA.

<sup>c</sup>: Population Proportion test.  $H_0: ME_S \leq ME_A$   
against  $H_1: ME_S(b) > ME_A$ .

<sup>d</sup>: Population Proportion test.  $H_0: POA_S \leq POA_A$   
against  $H_1: POA_S(b) > POA_A$ .

\*: Reject  $H_0$  against  $H_1$  at  $p < 0.05$ .

symmetric condition is lower than under the asymmetric condition could not be accepted, showing that the seller’s expected revenue does not always increases after information is released.<sup>21</sup>

In conclusion, *the experimental data do not support the prediction that the seller’s revenue increases after information about the ranking is released.*<sup>22</sup> As discussed, this is due to a significant reduction of the high-value bidders’ response under the asymmetric condition.<sup>23</sup>

## 5.4 Efficiency

Table 8 reports the following two measures of optimal allocation and economic efficiency: *i*) Pareto-Optimal Allocations (POA), reported as the percentage of objects given to the high-value bidder; and *ii*) Mean Efficiency (ME), reported as the percentage of the maximal surplus that was generated

<sup>21</sup>Testing for every auction period, the alternative hypothesis that the price realization under the symmetric condition is lower than under the asymmetric condition,  $H_1 : p_A > p_S$ , could be accepted in just one of the twenty auction periods at  $p < 0.05$ .

<sup>22</sup>For the single-auction markets, the average price realization under the asymmetric condition is not higher than under the symmetric condition. This observation is confirmed by a one-tailed t-test. The alternative hypothesis, that the price realization under the symmetric condition is lower than the price realization under the asymmetric condition, could not be accepted.

<sup>23</sup>Notice that due to the significant pattern of overbidding behavior, the average revenue for each information condition was significantly higher than the expected revenue at the risk-neutral equilibrium. For the dual-auction markets, the average price realization for the asymmetric (symmetric) condition was \$2.57 (\$2.61). Meanwhile, the expected revenue at equilibrium for the asymmetric (symmetric) condition was \$2.22 (\$2.00).

as a result of the auction.<sup>24</sup> Since low-value bidders generally have a positive probability of obtaining the item, the percentage of efficient allocations was expected to be lower under the asymmetric condition than under the symmetric condition.

As seen in the calculations displayed in Table 8, the percentage of POA and ME are consistently higher under the symmetric condition than under the asymmetric condition. This result is due to the more aggressive bidding of the low-value bidders and to the less aggressive bidding of the high-value bidders under the asymmetric condition. In the same table, a one-tailed Population Proportion ( $Z$ ) test statistic for each measure of optimal allocation is reported. The null hypothesis is rejected in favor of the alternative hypothesis that the efficiency under the symmetric condition is higher than under the asymmetric condition.<sup>25</sup>

In conclusion, *the FPA is, in the aggregate, more efficient in the absence of information.*<sup>26</sup> This result confirms the prediction about the reduction of the proportion of optimal allocations.<sup>27</sup>

## 6 Conclusion

A first-price private-value auction experiment was conducted in which the same two bidders had to bid for a single item in two markets and under two different information conditions. The purpose of this experiment was to examine the information impact of ranking of valuations on bidding behavior in first-price private-value auctions.

Experimental results indicated that, after information about the ranking of valuations was released, the two groups of bidders responded differently: As theory predicts, low-value bidders were inclined to bid more aggressively. Contrary to the predictions of the theory, high-value bidders tended to bid less aggressively. By properly deviating after information was revealed, each group of bidders improved their average surplus per auction. However, while the probability of low-value

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<sup>24</sup>A level below 100 characterizes unrealized gains from trade.

<sup>25</sup>Testing for every auction period, the null hypothesis is rejected in favor of the alternative hypothesis that the efficiency under the symmetric condition is higher than under the asymmetric condition in nineteen of the twenty auction periods at  $p < 0.05$ .

<sup>26</sup>A result similar to the one obtained with the dual-auction markets was observed for the FPA efficiency under each information condition in the single-auction markets. The percentage of POA and ME are consistently higher under the symmetric condition than under the asymmetric condition. These results are consistent with the more aggressive behavior of low-value bidders and the less aggressive behavior of high-value bidders under the asymmetric condition as compared to the symmetric condition.

<sup>27</sup>Notice that for the dual-auction markets, the actual proportion of optimal allocations for the asymmetric condition (74.80%) was higher than at the risk-neutral equilibrium (71.02%), but not for the symmetric, when the proportion of optimal allocations (87.14%) was significantly lower than at equilibrium (100%).

bidders winning the item increased, the probability of high-value bidders winning it decreased. Therefore, as expected, the proportion of efficient allocations decreased with respect to the FPA under the symmetric condition. Contrary to the predictions, seller's expected revenue did not increase on a regular basis because high-value bidders decreased their bids once information was released.

Based on theoretical predictions, it was expected that both groups of bidders would increase their bids once information is released. This expectation presumes strategic thinking from high-value bidders: i.e., once they are aware that low-value bidders need to increase their bids in order to obtain the item, high-value bidders will be reluctant to take on the risk of submitting lower bids. The baseline of this prediction is that risk-neutral fully rational bidders bid under the symmetric condition. In the laboratory, however, bidders were observed bidding well above the risk-neutral equilibrium under the symmetric condition. Based on this, high-value bidders were willing to take on the risk of submitting lower bids and increase their expected surplus once information about the ranking was released. This behavior seemed to be reinforced by the fact that, by decreasing their bids and moving in the direction of the optimal responses, bidders could still improve their average surplus conditional on winning.

From the standpoint of a seller, these behavioral results have strategic implications.<sup>28</sup> Contrary to expectations, the revelation of the ranking of valuations might not produce higher revenues. Therefore, if the seller were interested in collecting higher revenues and she had a device to communicate the ranking to certain group of bidders, she should reveal this information only to low-value bidders.<sup>29</sup> In the absence of such a device, however, the seller should not make the information public.

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<sup>28</sup>In a recent work, Kaplan and Zamir (2000) study theoretically the strategic use of seller information using LRWZ as a baseline. In their work, the seller can increase his expected revenue by committing to a signaling strategy to both bidders.

<sup>29</sup>In order to prevent a possible predatory activity in ascending auctions from the bidder who is thought to be stronger than potential rivals, Klemperer (2000) proposed the implementation of the Anglo-Dutch auction. In this auction format the auctioneer begins running an ascending price auction until all but two bidders have dropped out. Then, the remaining bidders are each required to bid in a final sealed-bid first-price offer no lower than the current asking price. It is the case, for example, that in this auction format the seller might consider to reveal whether the bidder who is thought to have the stronger position is participating in the final sealed-bid first-price auction.

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