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How many sorting equilibria are there (generically)?

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M E X I C O

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Abstract

It is shown that in a generic two-jurisdiction model of the type introduced by Caplin and Nalebuff (1997), the number of sorting equilibria (with jurisdictions providing distinct policies) is finite and even.

1 Introduction

Caplin and Nalebuff (1997) introduce a general class of models with multiple jurisdictions that can be used to generate equilibrium population sorting under free mobility. They also provide a sorting equilibrium existence result when there are two jurisdictions, but only for the case when jurisdictional policies are even-dimensional. Gomberg (2002) provides a similar result for the odd-dimensional case.

Until now the issue of local uniqueness and the number of sorting equilibria in these models has remained unexplored. At the same time, much of the applied work in this area involved comparative statics analysis, which implicitly assumes at least local uniqueness of equilibrium. The present paper intends to feel in this gap in the literature.

The key difficulty involved in establishing both the existence results of the earlier work and those of this paper lies in the fact that in addition to sorting equilibria these model also typically possess equilibria in which all jurisdictions provide identical policies and the, consequently indifferent, population is distributed to support such outcomes. Such equilibria, indeed, are often easy to explicitly construct. Unfortunately, this means that usual (i.e., Brouwer-like) fixed point results are not suitable to establish results about sorting equilibria. Fortunately, in the two-jurisdiction case the problem may be reduced to a

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simple mathematical issue of existence of fixed points of maps on spheres. An interesting implication of this is that sorting equilibria in models of this type are almost never unique; in fact, the number of equilibria is, generically, even.

The apparently unusual result is easier to understand if one recalls that I only consider the number of *sorting* equilibria - that is the number of equilibria with distinct jurisdictional policies and fully sorting population partitions. One may therefore conjecture that the number of equilibria with identical jurisdictional policies supported by the appropriate partitioning of the indifferent population is odd (at least, as long as it is finite).

One lesson of this paper is that index theory has a potential for establishing results on the existence and number of different types of equilibria (in this case, this would be the sorting and the pooling equilibria). This suggests, that similar techniques could be useful in other models with distinct equilibrium types, such as, for instance, models asymmetric information. In that case it could be possible to achieve non-constructive results for existence and number of pooling and separating equilibria. This is, indeed, related to the approach of Gale (1992) in his study of existence of separating equilibria in markets with adverse selection.

Aside from the difficulties arising from the fact that the underlying space is non-contractible, the mathematical tools employed in this paper have been standard in economics since Debreu (1970). Most of them can be found in Dierker (1974). For references on the Lefschetz fixed point Theorem see McLennan (1989) and Brown (1974).

The rest of the paper is organized as follows. Section 2 introduces the model, section 3 formulates the main results and section 4 discusses their intuition and indicates possible extensions.

2 The model

I shall first outline the model first introduced by Caplin and Nalebuff (1997).

Let $M = \{1, 2\}$ be the set of jurisdictions;

$X_j \subset \mathbb{R}^n$ - the set of policies available to community $j = 1, 2$; for simplicity, I shall assume that X_j is *open*.

$X \equiv X_1 \times X_2 \subset \mathbb{R}^{2n}$ - the set of all possible policy profiles;

$\mathcal{T} = (A, \mathcal{B}, \mu)$ - the measure space of agents where $\mathcal{B}$ is the (Borel) σ-algebra over A, μ is a probability measure on A, and $u(.;\alpha) : X \to \mathbb{R}$ stands for the utility function of agent type $\alpha \in A$.

Assume that each individual must choose to join exactly one jurisdiction, resulting in a population partition.

Definition 1 *A pair of measures $\sigma = \{\mu_1, \mu_2\}$ over $(A, \mathcal{B})$ is called an admissible* ***population partition*** *if for any $C \in \mathcal{B}$ $\mu(C) = \mu_1(C) + \mu_2(C)$*

Given a population partition σ, I shall denote the support of jurisdiction j population measure μ_j as S_j. The set of all admissible population partitions shall be denoted as $\hat{\Sigma}$.

Every jurisdiction is assumed to have a well-defined policy rule $P_j : \hat{\Sigma} \to X_j$, which shall be called its **constitution** (in the terminology of Caplin and Nalebuff (1997) such constitutions are called *membership-based);* let $P \equiv \{P_1, P_2\}$ denote the profile of constitutions.

A membership-based multi-community economy is then defined as the list

$$E \equiv \left(M, X, P, \mathcal{T}, \{u(.;\alpha)\}_{\alpha \in A}\right)$$

An equilibrium is a population partition/policy profile such that everyone resides in the jurisdiction he or she prefers and policies are set according to constitutions. I shall concentrate on sorting equilibria, in which jurisdictions provide distinct policies and individuals of different types separate:

Definition 2 *A policy profile - admissible population partition pair* $(x, S) \in X \times \hat{\Sigma}$ *is said to be a* ***sorting equilibrium*** *(thereafter, just* ***equilibrium)*** *of* E *if:*

(i) $S_j \subset \{\alpha \in A : u(x_j; \alpha) \geq u(x_k; \alpha),\ k \neq j\}$ *for every* $j = 1, 2$; *(free mobility);*

(ii) $P_j(S) = x_j$, *for every* $j = 1, 2$; *(constitutionality);*

(iii) $x_1 \neq x_2$ *(distinct policies).*

I impose a number of assumptions on agent preferences and jurisdictional constitutions, mostly following Caplin and Nalebuff (1997) and Gomberg (2002). The first assumption, introduced mainly to simplify the analysis, restricts the preferences to be linear (up to a transformation of the policy space, this includes, for instance, CES and Euclidean preferences):

Assumption 1:

(i) (parameter space) $A = \mathbb{R}^n$

(ii) (linear preferences) an individual residing in town j with the policy vector $x_j \subset X_j$ receives utility

$$u(x_j; \alpha) = \alpha \cdot x_j$$

The second assumption ensures that the agents' distribution does not have a concentration along any hyperplane and that any policy direction is viewed as "ideal" by some agents (this second part of the assumption could be easily relaxed).

Assumption 2 (no concentration of agents and full support):

(i)μ is hyperdiffuse[1] over A.

(ii) the support of μ includes an open neighborhood of the origin.

It is straightforward to see that under assumptions 1 and 2, given a policy profile such that $x_1 \neq x_2$, the set of indifferent agents will be a (zero-measure) hyperplane, while on each side of the hyperplane there will be a positive mass

[1] I.e., no hyperplane in A of dimension less then n may contain a positive measure of agents

of agents strictly preferring one jurisdiction's policy over another. Therefore, if I identify together all population partitions that differ only by the jurisdiction choice of a zero measure of agents, the space $\Sigma \subset \hat{\Sigma}$ of population partitions (S_1, S_2) that can be induced by such a policy profile can be parametrized on a unit sphere $\mathbb{S}^{n-1} = \{x \in \mathbb{R}^n : ||x|| = 1\}$ by the unit vector τ^S orthogonal to the hyperplane of indifferent agents.

Assuming constitutions P_j disregard movements of zero-measure coalitions of agents, P is well-defined when viewed as a function on Σ. Given the previous parametrization, one can write $P : \mathbb{S}^{n-1} \to X$. I furthermore assume that given a partition with populations on the opposite sides of a hyperplane, the jurisdictions indeed choose different policies:

Assumption 3: (distinct policy outcomes) for any $S \in \Sigma$, $P_1(\tau^S) \neq P_2(\tau^S)$.

It remains to define a topology on the space Σ. Fortunately, it turns out that, given assumption 2 simply taking the usual Euclidean topology on the parameter space $\mathbb{S}^{n-1}$ is rather natural, since it has the virtue of defining as "nearby" partitions that differ in actions of a small measure of agents.[2] With this topology continuity of constitutions simply means that small population movements result in small policy changes.

Assumption 4: (continuity) P is continuous on $\mathbb{S}^{n-1}$.

Given a partition $S \in \Sigma$, the profile of policy functions P induces a position profile $x \in X$ ($x_1 \neq x_2$ by assumption 3) which, in turn, induces a new partition S'. Thus, one can define the **mobility map** $\phi : \mathbb{S}^{n-1} \to \mathbb{S}^{n-1}$ from the space of possible partitions to itself, given by

$$\phi(\tau^S) = \frac{P_1(\tau^S) - P_2(\tau^S)}{||P_1(\tau^S) - P_2(\tau^S)||}$$

where $\phi(\tau^S) \in \mathbb{S}^{n-1}$ is the unit vector normal to the partition hyperplane induced by the policy profile $P(\tau^S)$. If assumption 4 holds, this map is continuous. It is not hard to see that if τ^Sbelongs to the set of fixed points of ϕ, $\mathcal{F}(\phi)$, the constitutional policy profile $P(\tau^S)$ induces back the population partition under free mobility S^*, i.e. $\left(S^*, P(\tau^{S^*})\right)$ is an equilibrium. Conversely, if $\tau^S \notin \mathcal{F}(\phi)$, S cannot be an equilibrium partition.

Proposition 3 *Under assumptions 1-3, 4 the pair* $(x^*, S^*) \in X \times \Sigma$ *is a sorting equilibrium if and only if* $\tau^{S^*} \in \mathcal{F}(\phi)$ *and* $x^* = P(\tau^{S^*})$.

[2] In general, if we were to require a topology on a larger subspace of the partition space $\hat{\Sigma}$, where such simple parametrization is not available, we might consider topologies explored in the study of information partitions, such as the topology induced by the variation semimetric d_{vn}, defined by $d_{vn}(\sigma, \sigma') = \sup_{C \in \mathcal{B}} \sqrt{\sum_{j \in M} \left(\mu_j(C) - \mu'_j(C)\right)^2}$ (see, for instance, Stinchcombe 1990).

The final assumption used in Caplin and Nalebuff (1997) and Gomberg (2002), which can often be motivated by the jurisdictions choosing policies optimally, given their own populations, says that once the policies are set in a constitutional fashion, it will never be the case that the populations of towns simply reverse. In its simplest form it can be stated as:

Assumption 5 (weak Pareto condition)**:** for every $S \in \Sigma$ $\phi\left(\tau^S\right) \neq -\tau^S$

3 The number of equilibria.

In this section it shall be shown that for "most" differentiable constitution profiles of two-jurisdiction economies there exists at most a finite number of equilibria. Furthermore, somewhat unusually, this number is guaranteed to be even.

The basic approach of this section is to define the class of "regular" smooth policy profiles for which the number of equilibria can easily be seen to be finite and then to show that this class, in fact, contains "almost all" smooth policy profiles.

I shall denote as $K \subset C^1(\mathbb{S}^{n-1}, X)$ the set of all continuously differentiable constitutions such that assumption 5 holds. I next assume that policy spaces of jurisdictions, in fact, have a non-empty interior, so that the actual policies can be perturbed in any direction and that policy rules are in fact smooth maps into the interior of X.

Assumption 6. For $j = 1, 2$, $X_j \neq \emptyset$ and is open, $P \in K$ and $P_j\left(\tau^S\right) \in int\left(X_j\right)$, $j = 1, 2$.

It can be shown that if assumption 6 holds $\phi \in C^1(\mathbb{S}^{n-1}, \mathbb{S}^{n-1})$.

For any $f \in C^1(\mathbb{S}^{n-1}, \mathbb{S}^{n-1})$, $Df\left(x\right)$ denotes the derivative of f at x.

Definition 4 *An equilibrium $\left(S^*, P\left(\tau^{S^*}\right)\right)$ in a differentiable two-jurisdiction model is called **regular** if the matrix $Id_{\mathbb{R}^n} - D\phi\left(\tau^S\right)$ is non-singular.*

For regular fixed points $\tau \in \mathcal{F}\left(\phi\right)$, $ind_\tau\left(\phi\right) = sgn|Id_{\mathbb{R}^n} - D\phi\left(\tau\right)| = \pm 1$. It is immediate that any regular fixed point of ϕ corresponds to an *isolated* equilibrium of the model that is also robust to small perturbations of ϕ.

Definition 5 *A two-jurisdiction economy $E = \left(M, X, P, \mathcal{T}, \{u(.;\alpha)\}_{\alpha \in A}\right)$ is called **regular** if all fixed points of the mobility map ϕ are regular.*

It can be shown that regular economies are, in the usual sense, generic:

Proposition 6 *Under assumptions 1-6, the set $\mathcal{R} \subset K$ of constitutions P such that the economy is regular is open and dense in K.*

Proof. *Part I. Density.*

I shall consider two separate cases: n - even and n - odd.

Case I. n - even.

Take an arbitrary $P \in K$. For every θ in a small neighborhood of $\frac{\pi}{2}$ consider the $n \times n$ square matrix

$$B_n(\theta) = \begin{bmatrix} \sin\theta & \cos\theta & 0 & 0 & ... & ... & 0 & 0 \\ -\cos\theta & \sin\theta & 0 & 0 & ... & ... & 0 & 0 \\ 0 & 0 & \sin\theta & \cos\theta & ... & ... & 0 & 0 \\ 0 & 0 & -\cos\theta & \sin\theta & ... & ... & 0 & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... \\ ... & ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & 0 & ... & ... & \sin\theta & \cos\theta \\ 0 & 0 & 0 & 0 & ... & ... & -\cos\theta & \sin\theta \end{bmatrix}$$

Let

$$P_1(\tau^S;\theta) = B_n(\theta) P_1(\tau^S) + (I - B_n(\theta)) P_2(\tau^S).$$

Notice that $P_1(\tau^S;\frac{\pi}{2}) = P_1(\tau^S)$ and for θ sufficiently close to $\frac{\pi}{2}, P_1(\tau^S;\theta)$ is close to $P_1(\tau^S)$. Therefore, since I have assumed that X_1 is open, $P_1(\tau^S;\theta)$ is a well-defined constitution for such θ.

Consider the constitution profile

$$P(\tau^S;\theta) = (P_1(\tau^S;\theta), P_2(\tau^S))$$

It can be easily shown that, one can approximate $P(\tau^S)$ by $P(\tau^S,\theta)$ for θ sufficiently close to $\frac{\pi}{2}$, i.e. for any $\varepsilon > 0$ there exists $\delta > 0$ such that if $|\theta - \frac{\pi}{2}| < \delta$ implies $||P(\tau^S;\theta) - P(\tau^S)||_{C^1} < \varepsilon$ (where for $f \in C^1(X,Y), ||f||_{C^1} = \sup_{x\in X}|f(x)| + \sup_{x\in X; h=1,2; k=1,2...n} |\frac{\partial f_h}{\partial x_k}(x)|$).

Assumption 3 is sufficient to ensure that $P_1(\tau^S;\theta) \neq P_2(\tau^S)$ for θ sufficiently close to $\frac{\pi}{2}$. Therefore, the corresponding mobility map is well defined. Indeed,

$$\phi(\tau^S;\theta) = B_n(\theta)\phi(\tau^S)$$

is continuously differentiable in τ^S and assumption 5 is easily satisfied for θ around $\frac{\pi}{2}$.

One can use a small neighborhood U of $\frac{\pi}{2}$ as a parameter space for the class of mobility functions. For any $S \in \Sigma$, $\phi(.;\theta)$ is a submersion of U into $\mathbb{S}^{n-1}$ and therefore $\phi(\tau^S;\theta) - \tau^S$ is transversal to zero. Applying the transversality theorem (theorem 9.1 in Dierker 1974) one gets that the set of θ with only regular fixed points is dense in U and that therefore the set of constitutions with only regular equilibria is dense in K.

Case II. n - odd.

Consider the $n \times n$ matrix

$$B_n(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \dots & \dots & 0 & 0 \\ 0 & \sin\theta & \cos\theta & 0 & 0 & \dots & \dots & 0 & 0 \\ 0 & -\cos\theta & \sin\theta & 0 & 0 & \dots & \dots & 0 & 0 \\ 0 & 0 & 0 & \sin\theta & \cos\theta & \dots & \dots & 0 & 0 \\ 0 & 0 & 0 & -\cos\theta & \sin\theta & \dots & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots & \dots & \sin\theta & \cos\theta \\ 0 & 0 & 0 & 0 & 0 & \dots & \dots & -\cos\theta & \sin\theta \end{bmatrix} As$$

before, $P(\tau^S)$ may be approximated by

$$P\left(\tau^S;\theta\right) = (B_n(\theta)P_1(\tau^S), P_2(\tau^S))$$

If $\phi = Id_{\mathbb{S}^{n-1}}$, then for all θ in some open neighborhood U of $\frac{\pi}{2}$ there exist only two fixed points of $\phi\left(\tau^S;\theta\right)$: $(1,0,...,0)$ and $(-1,0,...,0)$. In fact, by direct computation, both are easily seen to be regular.

Let $\phi \neq Id_{\mathbb{S}^{n-1}}$. By continuity of ϕ there exists an open set $V \subset \mathbb{S}^{n-1}$ such that $V \cap \mathcal{F}(f) = \emptyset$. Without loss of generality, there exists $\tau^S \in V$ such that $-\tau^S \in V$ (otherwise, one can diffeomorphically distort coordinates on the sphere to achieve this). Give τ^S coordinates $(1,0,...,0)$ and apply the above approximation. For any $\tau^S \in \mathcal{F}(f)$, therefore, $\phi(.;\theta)$ is a submersion of U into $\mathbb{S}^{n-1}$ and the rest of the argument above holds.

Part II. $\mathcal{R}$ is open.

The set of all $\phi \in C^1$ which have only regular fixed points is open in the C^1 topology of uniform convergence. Indeed, suppose otherwise. Then there exists a sequence $\phi_k \to \phi \in \mathcal{R}$ in $C^1(\mathbb{S}^{n-1}, \mathbb{S}^{n-1})$, such that for every k $\det(Id_{\mathbb{S}^{n-1}} - D\phi_k) = 0$ for some $y_k \in \mathcal{F}(\phi_k)$. Since $\mathbb{S}^{n-1}$ is compact one can find a convergent subsequence $y_{k_l} \to y \in \mathbb{S}^{n-1}$. The uniform convergence of $\phi_{k_l} \to \phi \in C^1(\mathbb{S}^{n-1}, \mathbb{S}^{n-1})$ implies that $\phi(y) = y$ and $\det(Id_{\mathbb{S}^{n-1}} - D\phi) = 0$ at y - contradiction.

It is therefore sufficient to show that for any $\varepsilon > 0$ there exists $\delta > 0$ such that $||f-P||_{C^1} < \delta$ implies $||\phi - \frac{f_1-f_2}{||f_1-f_2||}||_{C^1} < \varepsilon$.

In fact, this is going to be true, as long as both ϕ and its derivative are uniformly bounded. But that is clearly the case since, by assumption A3 for any $S \in \Sigma$, $P_1(\tau^S) \neq P_2(\tau^S)$. Together with continuity of P_j's (assumption A4) this implies that there exists $\eta > 0$ such that for any $S \in \Sigma$, $||P_1(\tau^S) - P_2(\tau^S)|| \geq \eta$. Since, furthermore, the domain of ϕ is compact, the uniform boundedness follows. I shall take an open neighborhood $V \subset C^1(\mathbb{S}^{n-1}, X)$ of P defined by $V = \{f \in C^1(\mathbb{S}^{n-1}, X) : ||f-P||_{C^1} < \delta\}$. It is not hard to show that as $\delta \to 0$ $||f_1(\tau^S) - f_2(\tau^S)||$ has to likewise be bounded away from zero, which indeed ensures convergence of $\frac{f_1-f_2}{||f_1-f_2||}$ to ϕ in C^1 topology. ∎

It has thus been shown that one is justified in concentrating, primarily, on regular economies. For these, the following general result is true.

Theorem 7 *Under assumptions 1-6 in regular two-jurisdiction economies the* *number of sorting equilibria is finite and even (and this number can be guaranteed to be non-zero if n is odd).*[3]

Proof. The proof of finiteness of the number of regular fixed points is standard (see Dierker 1974). The unusual part is the evenness, which follows almost immediately from the fact that $\mathbb{S}^{n-1}$ is a compact polyhedron with the Euler characteristic $\chi\left(\mathbb{S}^{n-1}\right) = \begin{cases} 2, & n \text{ - odd} \\ 0, & n \text{ -even} \end{cases}$. Furthermore, from the weak Pareto condition (A5) I know that $\phi(\tau^S) \neq -\tau^S$. Consequently, for any $t \in [0,1]$ $||t\tau^S + (1-t)\phi\left(\tau^S\right)|| \neq 0$. Therefore, one can construct a homotopy map

$$H(S,t) = \frac{t\tau^S + (1-t)\phi\left(\tau^S\right)}{||t\tau^S + (1-t)\phi\left(\tau^S\right)||}$$

But this implies that the Lefschetz number of the mobility map $\Lambda(\phi) = \chi\left(\mathbb{S}^{n-1}\right)$ is even. But $\Lambda(\phi) = \sum_{\tau\in\mathcal{F}(\phi)} ind_\tau \phi$ which implies that for any regular ϕ the number of equilibria is indeed even. ■

4 Discussion and further research

In this paper it is shown that subject to a generic regularity condition on policy functions, there is a finite and even number of sorting equilibria in the models of Caplin and Nalebuff (1997) and Gomberg (2002).

A further intriguing intuition for the result can be obtained by considering the *diagonal* Δ of the policy profile space (i.e., the set of policy profiles such that $x_1 = x_2$). In Caplin and Nalebuff (1997) and Gomberg (2002) approach to the problem one "excises" this diagonal by assuming that once the jurisdictions adopt distinct ("off-diagonal") policies they will never be mapped into the diagonal. In fact, it is due to this excision from the policy profile space (the dual of Σ) that Σ has a whole of the dimension equal to the dimension of Δ.

Admittedly, since when the policies are identical the entire population is indifferent as to the location choice, any population partition in the larger space $\hat{\Sigma}$ can be sustained under free mobility by any policy profile on the diagonal. Consequently, one can extend the mobility mapping to Δ by defining $\phi(\sigma) = \hat{\Sigma}$ for any $\sigma \in \hat{\Sigma}$ such that $P(\sigma) \in \Delta$ (of course, now ϕ is a correspondence, rather than a function). With this extension, every equilibrium (including the pooling ones) corresponds to the fixed points of ϕ.

Unfortunately, unlike Σ, the space $\hat{\Sigma}$ cannot be viewed as a subset of the Euclidean space. The arguments become more transparent if one instead considers the mapping $\chi : X \to X$ defined by

$$\chi(x) = P\left(\frac{x_1 - x_2}{||x_1 - x_2||}\right), x \notin \Delta$$
$$\chi(x) = P\left(\hat{\Sigma}\right), x \in \Delta$$

[3] To ensure actual sorting equilibrium existence when n is even additional assumptions are needed (see Caplin and Nalebuff 1997).

As is the case for ϕ, it is not hard to see that every equilibrium corresponds to a fixed point of χ. Clearly, χ is upper hemi-continuous; suppose, further, it is contractible-valued. Since X is a subset of $\mathbb{R}^{2n}$ it is possible to extend the notion of the fixed point index for χ (see McLennan (1989) for defining the index for correspondences). Furthermore, if X is contractible the sum of the indices over all fixed points should be equal to 1. The main result of this paper suggests that, under the assumptions above, this number for the diagonal would indeed be equal to ± 1 - that is, if the number of pooling equilibria is finite one would expect it to be odd.

Reversing the argument of the previous paragraph suggests that by studying the action of the mobility mapping near the diagonal of the policy profile space it would be possible to obtain an alternative argument for the number of sorting equilibria. Indeed, given the complexity of the space of sorting population partitions when the number of jurisdictions is greater than two, this approach may be preferable for extending the two-jurisdiction results in the literature to the general multi-jurisdiction case.

As noted in the introduction, the index theory approach may be useful in studying existence of different types of equilibria in models of asymmetric information, as in Gale (1992). In that paper, however, the separating equilibrium existence has been achieved by showing that pooling equilibria, in that setting, are *inessential*, in the sense that they are destroyed under small perturbations of the model (see McLennan 1989 for the definition of essential fixed points). Since the index of inessential fixed points has to be equal to zero (proposition 4.6 in McLennan 1989), the results of this paper are valid even when all types of equilibria are robust to small changes in the model.

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