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Market Participation, Information and Volatility

Juan Dubra
Universidad de Montevideo

and

Helios Herrera
Instituto Tecnológico Autónomo de México

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Av. Camino a Santa Teresa # 930
Col. Héroes de Padierna
México, D.F. 10700
M E X I C O

Market Participation, Information and Volatility

Juan Dubra[*] Helios Herrera[†]

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Abstract

We analyze how the entry of less informed participants in a market for a risky asset affects the volatility of the price of the asset. In an endogenous participation model, we show that in equilibrium the new market entrants are less informed than the rest of the participants. We study how volatility depends on market participation and on the level of information of the participants. The condition that guarantees that new market participation leads to increased asset price volatility, is that all investors are sufficiently risk-averse. In the increasing volatility case, a higher volatility is associated with a higher welfare for the new entrants.

(JEL: G12, D40, C70)

Keywords: endogenous participation, volatility, information heterogeneity.

1 Introduction

During the past years, an increase in the number and in the diversity of traders has generated changes in financial asset markets. For instance, US stock market participation has increased consistently since the fifties and this increase has been most dramatic in the eighties and nineties. The number of shareholders in the United States increased by more than 60% from 1989 to

[*]Universidad de Montevideo, e-mail: dubraj@um.edu.uy

[†]ITAM-Centro de Investigacion Economica, e-mail: helios@itam.mx

1998, when it reached approximately 84 million individuals.[1] Since 1995, a great number of new investors began purchasing assets in financial markets worldwide especially through the Internet.[2]

It is interesting to understand the effects of this change in participation on prices of financial assets and in particular on the volatility of those prices. One explanation for this increase in participation in the stock market is a decrease in the cost of entry in the market. Whereas in the past higher transaction and information costs kept certain types of investors out of risky asset markets, nowadays the easier access has attracted new types of investors into financial markets. As a result of information technology and telecommunications improvements, there is no doubt that the cost of acquiring information on assets declined dramatically in the past years. However, while there may be a consensus that the lowering of the pecuniary (brokerage) and non-pecuniary (information and setup) costs of participation has encouraged the entry of new investor-types into asset markets, whether these new market participants have *increased or decreased* the asset price volatility is a question that still needs theoretical and empirical exploration.

In the quest to study under what conditions asset price volatility increases with more market participation and what are the relevant factors that affect the volatility in one way or the other, we focus in this paper on how the heterogeneity in information of market participants affects price volatility. We are interested in this question because it is not obvious what happens to volatility as new less informed but *rational* participants enter the market. For instance, you may think that traders with less information have noisier beliefs which induce noisier demands and hence a higher randomness to the aggregate demand. Less informed traders if they are Bayesian though, must also rely less on their less precise information and more on the public information. So their demands will not necessarily be noisier or more volatile.[3]

The present work is one step in the direction of exploring the possible "origins" of price volatility of financial assets. We use an endogenous market participation model to show how volatility emerges from the self-selection of potential investors. As we show, this source of volatility is an immediate consequence of the change in the overall composition of the information

[1] According to the Survey of Consumer Finances (1998).

[2] From 1995 through mid-2000, investors opened 12.5 million on-line brokerage accounts, as Barber and Odean document (JEP 2001).

[3] For the same reason, it is also not obvious for instance that the lower the precision of the information of the new market participants, the higher the price volatility becomes.

precision of investors. The change in the composition of market participants follows from the easier access to the stock market allowing the entry of new investors. We analyze the different participation equilibria that arise from a market entry game as the cost of entry in the asset market is lowered. Different participation equilibria imply different price volatilities, depending on what types of investors enter the asset market.[4] First, we find that the more informed types enjoy a *higher* benefit than the less informed types from participating in a risky asset market. This happens because the market is not perfectly revealing. Even after observing the partially revealing price, the information of the less informed remains less precise than the information of the more informed investors. As a consequence, for a high cost of market participation, the more informed are the only types to enter the market. For low entry costs, all investors enter the market. However for intermediate entry costs, there are multiple participation equilibria: low information types may or may not decide to join the high information types in the market. Second, we characterize the conditions under which the entry of new (less informed) rational investors increases price volatility. More participation increases volatility provided that the market is sufficiently risk-averse and provided that in the market there are enough noise traders. We also find that, ceteris paribus, volatility is more "likely" to increase the lower is the information precision of the new market entrants. Lastly, when multiple participation equilibria arise the full participation equilibrium is Pareto dominant, implying that if volatility increases, higher welfare is associated with higher volatility.

To explore the effect on volatility of information heterogeneity, we formulate the problem in the following way. We assume that the characteristics of the market (the volatility and the utility of participants) stay the same if more investors with the *same* information as the incumbents enter the market (we call this assumption *scale invariance).* We study what happens to volatility and utility of the participants if *less*-informed traders decide to enter the market, instead than investors with the same information. To find the equilibrium price and study its volatility we use a rational expectations equilibrium (REE) concept. Hence, on top of having more informed and less informed investors or potential market participants, we need to add some form of noise to generate trade and to maintain an imperfectly revealing

[4] For a similar analysis based on heterogeneity on risk-aversion rather than heterogeneity on information see Herrera (2002).

equlibrium price. We adopt the noise trader modelling strategy. A model with a fixed number of noise traders is inconvenient because the noise per agent is decreasing on the number of market participants, if the number of noise traders remains constant. In order to maintain the market scale invariant, we assume that the number of noise traders is the same as the number of rational investors. This can be visualized as if every investors (more or less informed) upon entering the market brings in the market one noise trader as well. This way we eliminate one channel of distortions: the gains from entering of an additional trader depend only on his information precision. To eliminate another channel of distortions induced by the change in the number of participants, we assume that the asset is in zero net supply on average, so that the per-investor supply of the asset remains constant as participation changes.

Related papers are Allen and Gale (1994) and Pagano (1989). These papers also study the link between volatility and participation treating the latter endogenously. They focus though on heterogeneity in liquidity needs, risk aversion and hedging needs and abstract from any analysis of information. To the best of our knowledge no previous work has explored the effects of information heterogeneity on asset prices and volatility in an endogenous participation setting.

Orosel (1998) analyzes a model in which participation is determined endogenously and fluctuates over time. He shows that there is a positive link between movements in prices and fluctuations in participation. In his model the reason why prices fluctuate are changes in endogenous participation, triggered by innovation in dividends that follow a Markov process. The endogenous fluctuations of market participation lead to increased volatility of the share price. In our model instead, volatility is not related to endogenous fluctuations in market participation. Although different across equilibria, market participation is fixed within each equilibrium. The reason being that we want to explore what forces affect volatility as a consequence of a change in the composition of market participants.

Most of the other work in the limited participation literature takes a different approach, see for instance Merton (1987), Basak and Cuoco (1998) and Shapiro (2002). These papers do not treat participation endogenously, but assume that certain exogenously chosen agents are prevented from investing in some given financial assets.

The rest of the paper is organized as follows. Section 2 sets up the formal model and finds the link between price volatility and the composition of the market participants. Section 3 takes the composition of market participants as endogenous and finds the participation equilibria. Section 4 concludes.

2 The Model

There are three types of investors: informed, uninformed (or less informed) and noise traders. The noise traders trade for reasons exogenous to the model and have a random net demand q exogenously given by:

$$q \sim N\left(0, Q^{-1}\right)$$

We assume there are a continuum $\gamma \in [0, \Gamma]$ of informed investor-participants, a continuum $\mu \in [0, M]$ of uniformed investors. As was discussed before, we will assume that there is a noise trader per rational investor, and hence $\gamma + \mu \in [0, \Gamma + M]$ noise traders. Assuming that the number of noise traders per "rational" investors is 1 is without loss of generality, since the parameter Q can be adjusted in order to have more or less noise per rational trader. The letters γ and μ indicate the number of participants of every kind, whereas Γ and M indicate the total number of *potential* participants. We assume that the informed traders know all the available information about the return of the risky asset. The price (as information aggregation) does not reveal to them any additional information. Formally, we assume that all informed traders receive the same signal s^I, which is the asset return x plus a noise term f :

$$\begin{aligned} s^I &= x + f \\ x &\sim N\left(0, X^{-1}\right) \\ f &\sim N(0,1) \end{aligned}$$

We assume that the uninformed investors receive a signal that is always less informative than the informed signal. We model this by adding a noise term g to the informed signal:

$$\begin{aligned} s^U &= x + f + g \\ g &\sim N\left(0, G^{-1} - 1\right) \\ 0 &\leq G \leq 1 \end{aligned}$$

The smaller the G, the higher the information asymmetry.

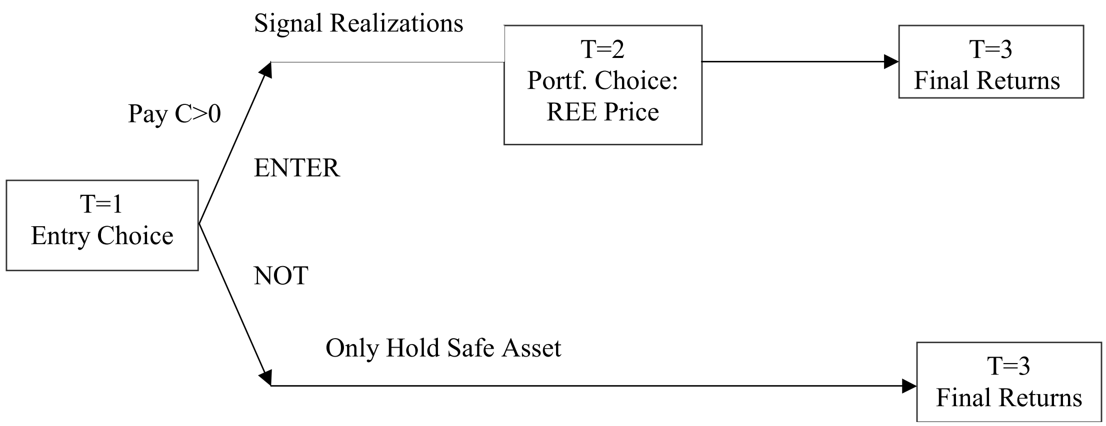


Figure 1: Timing of Decisions

Our framework has three periods and entails two sequential decisions made by investors: in the first period, an entry decision in a risky asset market; in the second period (only in the case of entry) a portfolio allocation decision involving the risky asset and a safe asset. In the third period all uncertain returns are realized. As customary in the endogenous participation literature (see for instance, Pagano 1989 or Allen and Gale 1994), the entry decision is separated and precedes the portfolio decision. This entry cost represents the cost of gathering information about how the stock market works in general (setup cost) and about the returns of the stocks in particular (information cost). Only after an investor has gathered the necessary information (entered the market), is he able to decide how much to invest in the asset.

Since the problem has two stages, we solve backwards. Starting with the second stage decision, we find every investor's posterior beliefs, their demand for the risky asset and aggregate these demands, taking as given the number of market participants, which is derived later when we solve the first stage problem. Solving for the REE equilibrium, we obtain the equilibrium price of the asset and the indirect utility from entry of every market participant as a function of the number of participants. Comparing the utility from entry minus the entry cost to the utility of not entering, we obtain the entry condition which determines the first stage entry decision. Finally, we look for the Subgame Perfect Nash equilibria of the entry game and find the equilibrium number of participants. Different equilibrium levels of participation imply different volatilities of the asset price, which was previously derived in the second stage.

The random elements of the model are $X = (x, f, g, q)^T \sim N(0, S)$ (T denotes transpose) where the variance-covariance matrix is:

$$S = \begin{pmatrix} X^{-1} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & G^{-1}-1 & 0 \\ 0 & 0 & 0 & Q^{-1} \end{pmatrix}$$

As customary in a REE , we assume a linear price conjecture. Agents conjecture that the price has a linear form, that depends on their signal and on the noise traders' actions:

$$p = Bs^I + Cs^U + Dq \tag{1}$$

where B, C, D are constants to be determined. To evaluate the demands, we need to calculate the posterior expectations of the market participants.

2.1 Posterior Moments of the Informed

We have that, since $\left(x, s^I, p\right) = A\,(x, f, g, q)^T$ with:

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ B+C & B+C & C & D \end{pmatrix},$$

the distribution of $\left(x, s^I, p\right)$ is joint normal with mean $(0,0,0)$ and variance matrix: $V = ASA^T$. Letting Σ be the variance covariance matrix of $\left(s^I, p\right)$,

$$\Sigma = \begin{pmatrix} X^{-1}+1 & \left(X^{-1}+1\right)(B+C) \\ \left(X^{-1}+1\right)(B+C) & \left(X^{-1}+1\right)(B+C)^2 + \left(G^{-1}-1\right)C^2 + Q^{-1}D^2 \end{pmatrix}$$

we have that the expected value of x conditional on p and s^I is:

$$E\left(x \mid s^I, p\right) = X^{-1} \begin{pmatrix} 1 & B+C \end{pmatrix} \Sigma^{-1} \begin{pmatrix} s^I \\ p \end{pmatrix} = \frac{1}{1+X} s^I$$

The conditional variance of the return x is given by:

$$Var\left(x \mid s^I, p\right) = X^{-1} - X^{-2} \begin{pmatrix} 1 & B+C \end{pmatrix} \Sigma^{-1} \begin{pmatrix} 1 \\ B+C \end{pmatrix} = \frac{1}{1+X}$$

2.2 Posterior Moments of the Un-informed

We must first find the joint distribution of $\left(x, s^U, p\right)$. Since $\left(x, s^U, p\right) = Z\left(x, f, g, q\right)^T$ for

$$Z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ B+C & B+C & C & D \end{pmatrix}$$

its distribution is joint normal with mean $(0,0,0)$ and variance matrix $V_u = ZSZ^T$, so letting Ω be the variance covariance matrix of $\left(s^U, p\right)$,

$$\Omega = \begin{pmatrix} X^{-1} + G^{-1} & \left(X^{-1}+1\right)(B+C) + \left(G^{-1}-1\right)C \\ \left(X^{-1}+1\right)(B+C) + \left(G^{-1}-1\right)C & \left(X^{-1}+1\right)(B+C)^2 + (G^{-1}-1)C^2 + Q^{-1}D^2 \end{pmatrix}$$

we obtain that the expected value of x conditional on p and s^U is:

$$\begin{aligned} E\left(x \mid s^U, p\right) &= X^{-1} \begin{pmatrix} 1 & B+C \end{pmatrix} \Omega^{-1} \begin{pmatrix} s^U \\ p \end{pmatrix} \\ &= \frac{\left(D^2 G - QBC\left(1-G\right)\right) s^U + \left(1-G\right) BQp}{XQG\left|\Omega\right|} \end{aligned}$$

The conditional variance of x is given by:

$$\begin{aligned} Var\left(x \mid s^U, p\right) &= X^{-1} - X^{-2} \begin{pmatrix} 1 & B+C \end{pmatrix} \Omega^{-1} \begin{pmatrix} 1 \\ B+C \end{pmatrix} \\ &= \frac{QB^2\left(1-G\right) + D^2}{XQG\left|\Omega\right|} \end{aligned}$$

2.3 Equilibrium

We now derive the demands of each market participant. Their final wealth is given by:

$$W\left(x, p, Z^j\right) = (w-c) + (x-p)\, Z^j$$

where w is the wealth before entering the market, c is the cost of entering, x is the (random) return of the risky asset, p is the unit price of the risky asset and Z^j is the quantity demanded. Upon receiving their signals and observing the price, investors choose Z^j to maximize the mean-variance objective:

$$U_j = E\left(W \mid s^j, p\right) - \frac{b}{2} Var\left(W \mid s^j, p\right)$$

where b is a risk-aversion parameter. That is, they must choose between the security and cash (safe asset). The demand of the market participant is:

$$Z\left(s^j, p\right) = \frac{E\left(x \mid s^j, p\right) - p}{bVar\left(x \mid s^j, p\right)}, \quad j = I, U$$

We have that optimal demands are

$$\begin{aligned} Z_I &= \frac{s^I - (1+X)\,p}{b} \\ Z_U &= \frac{\left(D^2 G - QBC\,(1-G)\right) s^U + \left((1-G)\,(1-(1+X)\,B)\,BQ - D^2\,(X+G)\right) p}{b\left(QB^2\,(1-G) + D^2\right)} \end{aligned}$$

so that the demand-supply equality condition yields:

$$(\gamma + \mu)\, q + \mu Z_U + \gamma Z_I = 0$$

Using the equations for Z_I and Z_U in this equation and defining $r = \frac{\mu}{\gamma}$ we obtain an equation for p, which equated term by term with the price conjecture (1) and using $k \equiv \frac{Q}{b^2}$, yields the following coefficients (using $X = x - 1$ and $G = g + 1$):

$$\begin{aligned} C &= \frac{r\,(1+r)\,(g+1)}{(1+r)\,(x + r\,(x+g)) - kxg} \\ B &= \frac{1 + r - kg}{(1+r)\,(x + r\,(x+g)) - kxg} \\ D &= (1+r)\, bB \\ E &= B + C \end{aligned}$$

The resulting volatility in full participation is:

$$Var\,(p)_F = \left(1 + X^{-1}\right) E^2 + \left(G^{-1} - 1\right) C^2 + Q^{-1} D^2.$$

In order to calculate the variance of the price under limited participation, one only needs to set $G = 1$ to obtain:

$$Var\,(p)_L = \frac{1 + \left(1 + X^{-1}\right) k}{(1+X)^2\, k}$$

3 Volatility Analysis

We compare the volatility in full participation to volatility in limited participation. The following proposition is one of the main results of the paper. It characterizes the conditions under which the entry of less informed investors increases the volatility of the asset price. It also allows us to separate the dependance on noise and risk aversion from all the other characteristics of the market, such as the composition of its participants r, their overall precision of information X and the differential in information between the informed and the uninformed G.

Proposition 1 *Volatility increases with the entry of less informed investors, if and only if.:*

$$k \equiv \frac{Q}{b^2} < K$$

where:

$$K = \frac{1+r}{2xgr}\left(r\left(1+r\right)\left(g+x\right)+2g-\sqrt{\left(1+r\right)\left(g-rx\right)4g+\left(r\left(1+r\right)\right)^2\left(x+g\right)^2}\right)$$

Proof. (see Appendix). ■

High noise trading (Q^{-1}) and high risk-aversion (b^2) guarantee that volatility increases with more participation. This happens for the following reasons. As new less informed participants enter the market, the impact of noise traders on the price increases, because the market is overall less informed and can distinguish less effectively noise from real information. Moreover, a more risk averse market can arbitrage away less the price misalignements caused by noise traders. This happens because more risk averse agents are willing to take less risky positions and engage in smaller trades, hence allowing larger mismatches between price and real information. To see this effect, take the extreme case of a risk-neutral market. With risk-neutral agents noise has no influence on the price. The demand is perfectly elastic and the price responds to news one to one: $p = E(x|s^I)$. The price misalignements (caused by noise) are arbitraged away by the larger trades taken by highly risk-tolerant agents (higher volume), i.e., the riskier positions that the agents are willing to take. If there is small noise and/or high risk-aversion price volatility can increase with more participation. The reason is that as less informed participants enter the market a volatility-reducing effect is also present. The aggregate response of the market to “relevant” news decreases, because the less informed respond less to their information. The lower signal-reaction of the less informed makes the market overall less responsive to information, because the less informed participants rely more on their prior beliefs. The stronger average weight assigned to the common prior tends to lower the unconditional volatility when the new less informed participants enter the market. We call this the *common prior effect.*

It is interesting to study how the upper bound K, which determines the volatility-increasing condition, depends on the parameters. The lower it is, the “more likely” is volatility to increase with more participation. The following Proposition gives the comparative statics for K with respect to r, x and g.

Proposition 2 *The comparative statics of K with respect to its variables are:* $\frac{\partial K}{\partial x}, \frac{dK}{dg} < 0$,

Proof. (see Appendix). ■

The derivative with respect to x says that if the information of traders is relatively reliable (that is, if x is low, which means high variance of asset returns, and hence placing a high weight on new information, relative to the prior), then K increases. Therefore, when information is reliable, volatility is more likely to increase with the entry of uninformed participants. To see this, note that if the signals are very noisy and the information of the participants is completely unreliable, then all investors stick to their common priors, the price is independent from the signals and the unconditional price volatility is zero.

The derivative with respect to g says that the lower the quality of information of the new (uninformed) participants (lower g) with respect to the old (informed) participants, the higher the upper bound K and, ceteris paribus, the less likely is volatility to increase. As before, when g decreases, the aggregate response of the market to news decreases, because the less informed respond less to their information. Once again, we have the common prior effect: the stronger average weight that the market puts on the common prior tends to lower volatility.

4 Indirect Utility

In order to study what the entry decisions of the I and U types will be, we need to calculate the equilibrium utilities of being in the market for both types.

4.0.1 Payoffs to informed types

For the informed the indirect utility is (see appendix):

$$U_I = \frac{b}{2} Var\left(x \,|\, s^I, p\right) E\left[Z_I^2\right]$$

so using $Var\left(x \mid s^I, p\right) = x^{-1}$ and

$$E\left[Z_I^2\right] = \frac{V\left(s^I\right)}{b^2} - \frac{2x}{b^2} Cov\left(s^I, p\right) + \frac{x^2}{b^2} V\left(p\right)$$

we obtain, from V

$$U_I = (1+r)^2 \frac{k^2 g^2 x - kxr^2 g - kg^2 r^2 - 2kxg\left(1+r\right) + x\left(1+r\right)^2}{2bk\left(kxg - x\left(1+r\right)^2 - rg\left(1+r\right)\right)^2}$$

Proposition 3 *The benefit from entering of the informed increases in the number of uninformed participants and in the variance of the uninformed participants' information. That is,* $\frac{dU_I}{dr}, \frac{dU_I}{dg} < 0$.

Proof. (see Appendix). ■

This tells us that the informed entrant is better off the more uninformed are in the market and the less informed are in the market. The more precise the information of the uniformed the more they will exploit the noise traders leaving the informed less scope to do that.

4.0.2 Payoffs to uninformed types

For the uninformed the indirect utility is (see appendix):

$$\begin{aligned} U_U &= \frac{b}{2} Var\left(x \,|\, s^U, p\right) E\left[Z_U^2\right] \\ &= \frac{\left(\left(1+r\right)^2 - gk\right)\left(x+g\right)\left(1+r\right)^2}{2bk\left(x\left(1+r\right)^2 + g\left(r + r^2 - kx\right)\right)^2} \end{aligned}$$

Proposition 4 *The informed types are always better-off by being in the market than the uninformed:*

$$U_I \geq U_U$$

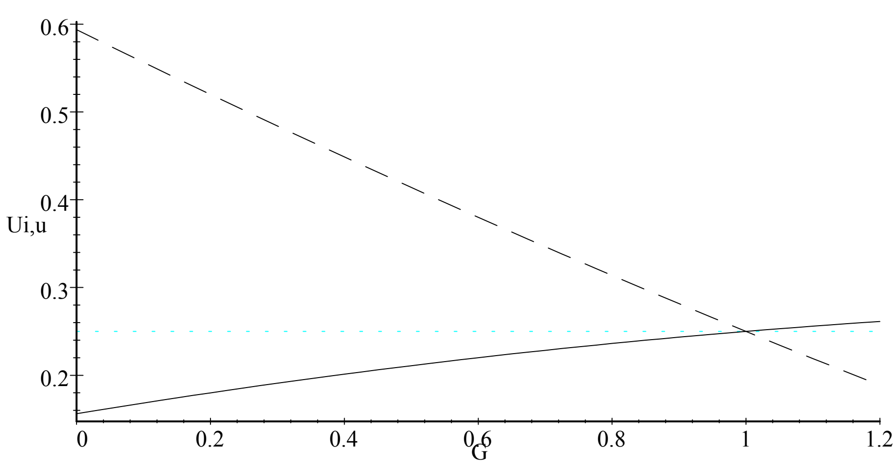
Proof. (see Appendix). ■

This tell us that if in the market there are some uninformed investors then all the informed must be participating too.

Proposition 5 *The uninformed are better off by entering the more they are participating and the less the informed investors are participating.*

Proof. (see Appendix). ■

There are two reasons for this result. First, the more the uninformed are the less they are exploited in per capita terms by the informed. Second, the less informed agents are in the market, the more scope have the uninformed to take advantage of the noise traders. This tells us that in any stable participation equilibrium either all the uninformed participate or none of them participates.



Utility for Informed (dashed) and Uninformed (continuous), with X=Q=b=1.

Proposition 6 *There is a full participation equilibrium in which all investors enter and a limited participation in which all the informed enter and the uninformed stay out. For some range of entry cost these equilibria coexist and the former Pareto dominates the latter.*

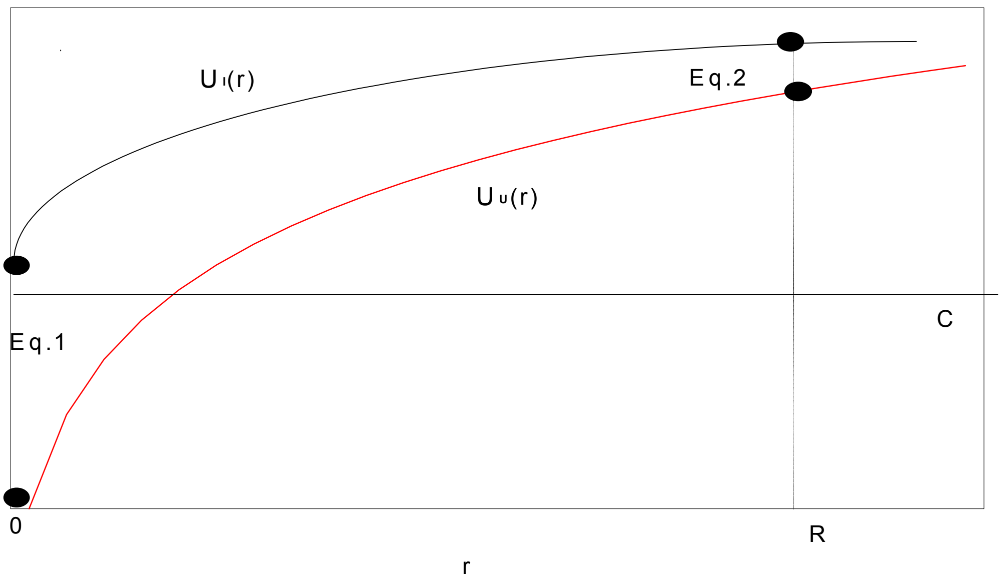


Figure 2: Utility of Informed and Uninformed

Proof. (see Appendix). ■

The picture below shows the two stable equilibria for given entry cost c. Where the U_U crosses the cost line there is an unstable equilibrium to be found, which we disregard.[5]

5 Comments and Summary

We have shown two main results: First, the more informed I-types have a higher expected gain from entering than the less informed U-types, so, in equilibrium, they are the first ones to enter. Second, as the uninformed enter the market (full participation) the volatility is higher than when the informed are the only market participants (limited participation), provided that the market is sufficiently risk averse or has sufficient noise trading.

There are many factors that affect volatility in financial asset markets

[5] In the unstable equilibrium less informed investors are indifferent weather to enter or not. It is an equilibrium only if the right number of uninformed decides to enter, but a small perturbation on that number leads to disequilibrium.

in one direction or the other. This paper identifies one of them. Using an endogenous market participation model we showed how volatility emerges from a change in the composition of information of investors. If the potential market participants differ in their information precision, we tried to illustrate what market variables are crucial in determining volatility and, more precisely, under what conditions more participation can increase volatility. When looking for the possible sources of volatility, we cannot disregard the diversity of investor characteristics that arises from self-selection of potential investor-participants. While we have explored the effect of heterogeneity in information, it would be interesting to explore the effect of other kinds of heterogeneity, the most important of these being, perhaps, wealth. This task is left to further research.[6]

6 Appendix

6.1 Payoffs from Entering the Market

We calculate the indirect utility from entering the market for any type k. The objective investors are maximizing upon entry is:

$$U(s_k, p) = E_x(W \mid s_k, p) - \frac{b}{2} Var_x(W \mid s_k, p)$$

The expectations are calculated after the signal and price are realized, i.e. conditional on them. To find the expected utility from entering, we need to take the expectation of $U(s_k, p)$ over all the possible signals and prices:

$$E_{s_k,p}[U(s_k, p)] = E_{s_k,p}\left[E_x(W \mid s_k, p) - \frac{b}{2} Var_x(W \mid s_k, p)\right].$$

We have that, substituting Z for the optimal demand Z_k,

$$\begin{aligned}
U(s_k, p) &= E_x(W \mid s_k, p) - \frac{b}{2} Var_x(W \mid s_k, p) \\
&= w - c + Z_k [E_x(x \mid s_k, p) - p] - \frac{b}{2} Var_x(W \mid s_k, p) \\
&= w - c + Z_k [E_x(x \mid s_k, p) - p] - \frac{b}{2} Z_k^2 Var_x(x \mid s_k, p).
\end{aligned}$$

[6] For the effects of heterogeneity in risk-aversion on volatility see Herrera (2002).

The demand of agent with signal k is:

$$Z_k(p) = \frac{[E_x(x \mid s_k, p) - p]}{bVar_x(x \mid s_k, p)}$$

when p is the equilibrium price X_k is the quantity demanded in equilibrium by agent-k. Substituting the square bracket yields:

$$U(s_k, p) = w - c + \frac{b}{2} Z_k^2 Var_x(x \mid s_k, p)$$

Since the posterior variance of the return x does not depend on the particular signal realization, the ex-ante utility from entering is:

$$E_{s_k,p}[U(s_k, p)] = w - c + \frac{b}{2} E_{s_k,p}\left[Z_k^2\right] Var_x(x \mid s_k, p)$$

The gain from entering or entry condition is:

$$\frac{b}{2} E_{s_k,p}\left[Z_k^2\right] Var_x(x \mid s_k, p) > c$$

6.2 A conjecture

Conjecture 7 *The utility of the uninformed is always increasing in G (their information). True for $r \leq 1$.*

Proof. Not always true: the sign of the derivative is ambiguous:

$$\frac{\partial U_U(G)}{\partial G} = \frac{b}{4Q}\left(\frac{1+r}{2}\right)^2 \frac{\begin{pmatrix} (1-G)\left(k^2(1+X)^2 + (1+r)k(1+X)\right) + \\ +(1+r)^3((1+X) - r(X+G)) + (1+r)^2 k(1+X)^2 \end{pmatrix}}{\left(k\frac{1+X}{2}(1-G) + \frac{1+r}{2}(1+(1+r)X + rG)\right)^3}$$

$$\frac{\partial U_U(G)}{\partial G}|_{r=1} = \frac{b}{4Q} \frac{(1-G)\left(k^2(1+X)^2 + 2k(1+X) + 8\right) + 4k(1+X)^2}{\left(k\frac{1+X}{2}(1-G) + (1+2X+G)\right)^3} > 0$$

■

The utility of the informed in limited participation (when they are the only ones in the market) is always smaller than their utility in full participation and always larger than the uninformed utility (in full participation, of course). That is, for every $G^{-1} > 1$:

$$U_U(G) < U_I^L = U_I(1) < U_I(G)$$

6.3 Proofs

Proof of Proposition 1. The following inequality

$$Var\left(p\right)_F - Var\left(p\right)_L \geq 0$$

is a polynomial of degree 2 in k. Solving for its roots we find that the only positive root is

$$K = \frac{1+r}{2xgr}\left(r\left(1+r\right)\left(g+x\right)+2g-\sqrt{\left(1+r\right)\left(g-rx\right)4g+\left(r\left(1+r\right)\right)^2\left(x+g\right)^2}\right)$$

The condition for volatility to increase with more participation is:

$$k < K$$

■

Proof of Proposition 2. We have that

$$\frac{\partial K}{\partial x} = \left(1+r\right)\frac{p-\sqrt{qs}}{2x^2r\sqrt{s}}$$

for

$$\begin{aligned} p &= \left(x+g\right)r^4+2\left(x+g\right)r^3+\left(g-x\right)r^2+2\left(2g-x\right)r+4g \\ q &= r^4+2r^3+5r^2+4r+4 \\ s &= \left(x+g\right)^2r^4+2\left(x+g\right)^2r^3+\left(g-x\right)^2r^2+4g\left(g-x\right)r+4g^2. \end{aligned}$$

Note that the last three terms in p are negative, so we only need to show that $\sqrt{qs} \geq \left(x+g\right)r^4+2\left(x+g\right)r^3$. To establish this inequality, notice that all terms in both q and s are positive, so qs is greater or equal than the products of their first two terms. That is, letting $x+g=z$,

$$qs \geq \left(r^4+2r^3\right)z^2\left(r^4+2r^3\right) = \left[z\left(r^4+2r^3\right)\right]^2$$

as was to be show.

Turn now to $\frac{dK}{dg} < 0$. We have that

$$\begin{aligned} \frac{dK}{dg} &= \left(1+r\right)\frac{t-\sqrt{\left(1+r\right)^2 s}}{2g^2\sqrt{s}} \\ t &= \left(x+g\right)r^3+2\left(x+g\right)r^2+\left(x-g\right)r-2g \end{aligned}$$

and since $t^2 - (1+r)^2 s = -(2+5r+4r^2+r^3)\,4g^2 r$ the result follows. ■

Proof of Proposition 3. We have that

$$\frac{dU_I}{dr} = gx(1+r)\frac{\begin{pmatrix} (-r^2+r+kx+1)\,k^2g^2 - \left(2(1+r)^2 + kx(2+2r+r^2)\right)kg \\ +(1+r)^3 + kx(1+r)^2 \end{pmatrix}}{bk\left(kxg - (x + r(g+x))(1+r)\right)^3}$$

and since the numerator is greater than

$$-r^2k^2g^2 - k^2xr^2g \geq 0 \Leftrightarrow -g \leq x$$

we know that it is positive. The denominator is negative since $g + x = G + X \geq 0$.

We have that

$$\frac{dU_I}{dg} = (1+r)^2 rx \frac{\begin{pmatrix} -kg(2kx + kxr + 2 + 2r) + kx(2+5r) + \\ +kr^3(g+x) + kr^2(g+4x) + 2(1+r^3) + 6r(1+r) \end{pmatrix}}{2bk\left(kxg - (x + r(g+x))(1+r)\right)^3}$$

which is negative because the numerator is positive and the denominator is negative. ■

Proof of Proposition 4. Compare the numerators in the two utilities, you obtain:

$$U_U - U_I = g(1+r)^2 \frac{(kx+1)\left((1+r)^2 - kg\right) + kgr^2}{2bk\left(-kxg + x + 2rx + xr^2 + rg + r^2g\right)^2} \leq 0.$$

The inequality follows because $g \leq 0$, so that the numerator must be positive, which obtains since $(kx+1)\left((1+r)^2 - kg\right) \geq kxr^2$, and hence the numerator is greater than $kxr^2 + kgr^2 = kr^2(X+G)$. ■

Proof of Proposition 5.

$$0 < \frac{\partial U_U(G)}{\partial r} =$$

$$\frac{b}{4Q}\frac{1+r}{2}\frac{(X+G)(1-G)\left((1+r)^3 + k(1+r)^2(1-G+1+X) + k^2(1-G)(1+X)\right)}{\left(k\frac{1+X}{2}(1-G) + \frac{1+r}{2}(1+(1+r)X + rG)\right)^3}$$

■

Proof of Proposition 6. The indirect utility from entry of the informed is always higher than the utility of the uniformed, so in any equilibrium, if some uninformed enter the market than all the informed enter too. In any equilibrium, the uninformed on their part either all enter or all stay out. In the former case all informed enter too and we have a full participation equilibrium. In the latter case, the utility of the uninformed investor is independent on how many informed are in the market (scale invariance). For some range of entry cost (not too high) they will all enter. The full participation equilibrium is welfare improving with respect to the limited participation equilibrium because the utility of both types of agents is increasing in the number of uninformed entrants. ■

Proposition 8 $\frac{\partial K}{\partial r} < 0$ *if* $r \leq 1$

Proof. $\frac{\partial K}{\partial r} =$

$$-\frac{(1-G)+r^2(1+r)(X+G)}{(1+X)(1-G)r^2}$$
$$+\frac{(1+r)\left(r^3(1+r)(X+G)^2+(1-G)\left(2r^2(1+X)-r(X+G)-2(1-G)\right)\right)}{(1+X)(1-G)r^2\sqrt{\left((1+r)\left(r^2(1+r)(X+G)^2+4r(1+X)(1-G)+4(1-G)^2\right)\right)}}$$

The sign is negative, ie:

$$0 > -\left((1-G)+r^2(1+r)(X+G)\right) \tag{2}$$
$$\frac{(1+r)\left(r^3(1+r)(X+G)^2+(1-G)\left(2r^2(1+X)-r(X+G)-2(1-G)\right)\right)}{\sqrt{\left((1+r)\left(r^2(1+r)(X+G)^2+4r(1+X)(1-G)+4(1-G)^2\right)\right)}}$$

if:

$$r^3(1+r)(X+G)^2+2r^2(1-G)(1+X) < (1-G)\left(r(X+G)+2(1-G)\right)$$

(which happens e.g. for small r). If the converse is true , ie:

$$r^3(1+r)(X+G)^2+2r^2(1-G)(1+X) > (1-G)\left(r(X+G)+2(1-G)\right)$$

(which happens e.g. for $r \geq 1$), then the derivative is negative provided that the following inequality is satisfied:

$$\left(r^2 (r+2)(X+G)^2 + r(X+G)(2-G+X) + (1-G)\left(G+2+3X-r^2(1+X)\right)\right) > 0 \tag{3}$$

which is true if $r \leq 1$. So, in the range $r \leq 1$ the derivative is negative (for high r the derivative is also negative). The inequality (3) is obtained by squaring both sides of (2), obtaining:

$$\begin{aligned} & (1+r)\left(r^3(1+r)(X+G)^2 + (1-G)\left(2r^2(1+X) - r(X+G) - 2(1-G)\right)\right)^2 \\ > \; & \left(r^2(1+r)(X+G)^2 + 4r(1+X)(1-G) + 4(1-G)^2\right)\left((1-G) + r^2(1+r)(X+G)\right)^2 \end{aligned}$$

and finally simplifying the algebra. ■

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