Financial Market Imperfections, Real Exchange Rates, and Capital Flows*

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August 1999

Abstract

We explore the role of domestic financial market frictions in explaining sharp movements in real and nominal exchange rates, capital flows, and output for a small open economy. Financial intermediaries arise endogenously to insulate depositors from the consequences of liquidity shocks and stochastic investment project returns, and to provide intermediation for efficient capital accumulation in the presence of a costly state verification problem. An increase in the world interest rate may provoke an increase in the fraction of credit-rationed entrepreneurs, a decrease in the steady state capital stock, and – when the elasticity of substitution between labor and capital is low – a depreciation of the real exchange rate and an outflow of capital. Hence we can account qualitatively for the recent experience of several emerging market economies in a model where all prices, including exchange rates, are perfectly flexible.

JEL Classification: E5, F4

*We thank Bruce D. Smith for encouraging this line of research and for many valuable discussions and suggestions. Betts acknowledges the financial support of the Haynes Foundation. Huybens acknowledges funding provided by CONACyT and by the Asociación Mexicana de Cultura. We would like to thank Eduardo Camero Godínez, Luis Sanchez Mier, and Juan Vargas Hernández for excellent research assistance. All remaining errors are ours.

1 Introduction

After opening their markets to international trade in goods and assets, emerging economies have experienced large and volatile capital flows combined with substantial real exchange rate volatility. In particular, episodes of real exchange rate appreciation, capital inflows and increases in real investment and output have been have been followed by periods of real depreciation, capital outflows and sharp reductions in real activity. Such reversals have often been accompanied by international and domestic financial crises. This has led to calls for restrictions on international capital flows, as well as to proposals for improved monitoring and regulation of domestic banking systems. Such policies, it is implied, could potentially stabilize international capital flows, and help economies avoid the financial crises that are followed by sudden sharp downturns in investment and output. Despite such policy prescriptions, there have been very few attempts at formalizing the origins of swings in capital flows and real exchange rates. Much less have models addressed the interaction between domestic financial institutions, international capital flows and exchange rate behavior, which has been argued to be at the root of the recent crises.

Our goal in this paper is to develop a theoretical model that explores the role of domestic financial market frictions in explaining sharp movements in real and nominal exchange rates, capital flows, and output. To achieve this, we marry two models of banking and capital accumulation presented earlier in the literature. The first, developed by Schreft and Smith (1997) and extended to an open economy by Betts and Smith (1997), models banks as providers of insurance for agents who confront individual uncertainty regarding future liquidity needs. In our economy, "workers" are subject to individual liquidity shocks. These are represented by stochastic relocations of workers, both domestically and to the rest of the world. Workers can transport only currency between locations, and inter-location exchange requires the currency of the location in which the seller resides.[1] This feature provides a liquidity motivation for holding ¯at moneys. In addition, it implies that only a fraction of locally produced goods are traded, and trade requires cash in advance. The remainder of goods are nontraded and consumed in their location of origin, where either cash or local credit can be used as a means of payment. Hence the law of one price fails generically and the model embodies a non-trivial theory of real exchange rate determination. Moreover, the possibility of stochastic relocation, coupled with the role of currency in inter-location exchange, plays a role similar to a \liquidity preference shock" in the Diamond-Dybvig (1983) model, and hence banks arise endogenously to insure workers against their random, currency-speci¯c liquidity needs. In order to provide this insurance, ¯nancial intermediaries hold reserves of both the foreign and domestic currency, and in addition they invest in interest bearing assets. These consist of loans to domestic \entrepreneurs", and a risk-free one-period asset which earns the world interest rate.

The second model of banking, introduced by Boyd and Smith (1997), focuses on the allocation of credit for capital accumulation. In our economy, \entrepreneurs", need external ¯nancing to invest in a capital production project which has an uncertain return and is subject to a costly state veri¯cation (CSV) problem.[2] This feature creates a role for ¯nancial intermediaries in eliminating the duplication of monitoring costs that would otherwise be incurred by individual lenders. It also allows for the existence of some unful¯lled demand for credit, and we will focus on equilibria in which such credit-rationing obtains.

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[1] Along these dimensions, our model resembles Townsend (1987), Mitsui and Watanabe (1989), Hornstein and Krusell (1993) and, most speci¯cally, Champ, Smith and Williamson (1996), Greenwood and Smith (1996), Schreft and Smith (1997, 1998), Betts and Smith (1997) and Bhattacharya, Guzman, Huybens and Smith (1997).

[2] The CSV problem we introduce is of the type considered by Townsend (1979), Gale and Hellwig (1985), Williamson (1986, 1987), and most speci¯cally, Boyd and Smith (1997, 1998), and Huybens and Smith (1998, 1999).

Hence the financial intermediaries that we analyze undertake several functions that are identified by authors such as McKinnon (1973) and Shaw (1973) as being important for developing economies. They insulate depositors from the consequences of liquidity shocks and stochastic investment project returns, while providing intermediation for efficient capital accumulation. In other words, the frictions we consider render financial intermediation the efficient arrangement for allocating both liquidity and credit. This is an important aspect of our model because the discussion on adequate regulation of the banking sector in emerging economies has called for measures to be taken both on the liability and the asset side.

In this environment we obtain the following results. First, we describe technical conditions under which there are at most two steady state equilibria with an excess demand for credit. One steady state has a relatively low capital stock, while the other one has a relatively high level of capital. We discuss the properties of equilibrium dynamics in a neighborhood of any steady state and illustrate by example that only the low capital stock steady state can be approached. We then investigate how in a long-run equilibrium, the capital stock, real exchange rate and net foreign holdings of domestic assets are influenced by a change in the real world interest rate. We focus on the low-capital-stock steady state since that is the attainable long-run equilibrium. We show how an increase in the world interest rate always provokes an increase in the fraction of credit-rationed entrepreneurs, and a decrease in the steady state capital stock for the low-capital-stock steady state. At the same time, the effect of an increase in the world interest rate on the real exchange rate depends on the degree of substitutability between labor and capital in production. When the elasticity of substitution between labor and capital is low, we demonstrate by way of an example that an increase in the world interest rate may provoke a depreciation of the real exchange rate in the low-capital-stock steady state, as well as a reduction in the net foreign holdings of domestic assets.

These results recall the recent experience of Mexico. Indeed, over the period 1987-1993, following the liberalization of its external sector, Mexico experienced large capital in°ows, a rising capital stock and level of output, and an appreciating real exchange rate. During 1994, world real and nominal interest rates rose due to a tightening of monetary policy in the US and Europe. This is commonly believed to have been only a minor factor leading to the reversal of capital °ows and the ¯nancial crisis that ensued. Here we demonstrate, in a model driven by ¯nancial market frictions, that an increase in the world interest rate can account for a depreciated real exchange rate, a decline in output and investment, and capital out°ows - phenomena that were all experienced by Mexico during the 1994-1996 period. Furthermore, our economy displays aggravated credit rationing as the world interest rate increases. This could, loosely speaking, be associated with the di±culties experienced by the Mexican banking sector in 1995 and 1996.

Many commentators have argued that swings in capital °ows and real exchange rate levels occur because countries peg their nominal exchange rate at a value which is \too high". When prices are sticky, this results in an \overvalued" real exchange rate, and growing current account de¯cits. At some point these de¯cits are considered \unsustainable" by foreign investors who rapidly withdraw liquid investment funds. In contrast to this story, our paper provides an example of an economy in which exchange rate misalignment and price rigidities are absent and yet the qualitative features of the recent data for emerging economies can be accounted for. Our work might thus be viewed as part of a very recent literature that does not rely on nominal rigidities or ¯xed exchange rates to explain real exchange rate °uctuations and capital °ows for open economies. Examples of this literature include Antinol¯ and Huybens (1998), Betts and Kehoe (1999), and Fernandez de Cordoba and Kehoe (1999). Our paper di®ers from previous work in its emphasis on ¯nancial market frictions in explaining both the determination of output and the real exchange rate.[3]

The remainder of the paper proceeds as follows. Section 2 describes the environment, while Section 3 discusses financial intermediation. Section 4 analyzes general equilibrium, focusing on the existence of steady state equilibria, local dynamics, and comparative statics. Section 5 concludes.

2 The Model

We consider a small open economy in the tradition of Diamond's (1965) neoclassical growth model. The economy is inhabited by an infinite sequence of two-period-lived overlapping generations and an infinitely lived government. The economy produces a single final consumption good. The world price of this good as well as the world interest rate, are taken as given by domestic agents.

2.1 Production

The consumption good is produced using a constant returns production technology with capital and labor as inputs. Neither the good, nor the factors of production are mobile and we assume that capital depreciates completely in production. Let K_t denote the date t capital input and L_t the date t labor input of a representative firm. Then its final output is $F(K_t; L_t)$, where F is a CES production function, that is $F(K;L) = \left[b_1 K^{\beta} + b_2 L^{\beta}\right]^{\frac{1}{\beta}}$; with $-1 < \beta < 1$. In addition, if $k = K/L$ denotes the capital labor ratio, then $f(k) \equiv F(k;1) = \left[b_1 k^{\beta} + b_2\right]^{\frac{1}{\beta}}$ denotes the intensive

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[3] The model presented in Antinolfi and Huybens (1998) is most similar to ours. However, they consider an informational friction in credit markets only. Hence their environment is not suited for evaluating policies directed to banks' liability structure. Moreover, the mechanism they introduce for real exchange rate determination is different from ours.

production function. Frequently, we will make use of the following properties of CES technology.

Lemma 1.

a) Let $f(k) = \left[b_1 k^{\beta} + b_2\right]^{\frac{1}{\beta}}$; with $-\infty < \beta < 1$, then

$\lim_{k \to 0} f(k) = b_2^{\frac{1}{\beta}}$ iff $0 < \beta < 1$; $\lim_{k \to 0} f(k) = 0$ iff $-\infty < \beta \le 0$; $f'(k) > 0 > f''(k)$ holds $\forall k$;

$\lim_{k \to 0} f'(k) = \infty$ iff $0 < \beta \le 1$; $\lim_{k \to 0} f'(k) = b_1^{\frac{1}{\beta}}$ iff $-\infty < \beta < 0$;

b) let $w(k) \equiv f(k) - kf'(k)$, then

$\lim_{k \to 0} w(k) = b_2^{\frac{1}{\beta}}$ iff $0 < \beta < 1$; $\lim_{k \to 0} w(k) = 0$ iff $-\infty < \beta \le 0$; $w'(k) > 0 \ \forall$ finite $k > 0$;

c) let $\Omega(k) \equiv \frac{k}{w(k)}$, then

$\lim_{k \to 0} \Omega(k) = 0$ iff $0 \le \beta < 1$; $\lim_{k \to 0} \Omega(k) = \infty$ iff $-\infty < \beta < 0$;

$\Omega'(k) > 0$ if $0 \le \beta < 1$ or if $-\infty < \beta < 0$ and $k > \left[\frac{1}{-\beta}\frac{b_2}{b_1}\right]^{\frac{1}{\beta}}$;

$\Omega'(k) < 0$ iff $-\infty < \beta < 0$ and $k < \left[\frac{1}{-\beta}\frac{b_2}{b_1}\right]^{\frac{1}{\beta}}$:

Proof. The proof of Lemma 1 is straightforward. ■

2.2 Agents, Preferences and Endowments

There are two symmetric locations in the domestic economy, and at each date t a continuum of young agents of measure unity is assigned to each location. Each generation is identical in size and comprises two types of agent. A fraction α of young agents in each location are risk neutral "potential entrepreneurs" (potential borrowers), while the remaining fraction $(1-\alpha)$ are risk averse "workers" (lenders). An initial old generation with unit mass inhabits each location at date 0. All agents are assumed to care only about old age consumption so that all young period income is saved. In particular, we assume that generation t workers have the lifetime expected utility function $U_t = E_t \ln c_{t+1}$ while generation t borrowers derive lifetime expected utility as $U_t = E_t c_{t+1}$.

All young agents are endowed with a single unit of labor which is supplied inelastically to production, and they are retired when old. The initial old generation is endowed with the initial capital stock $K_0 > 0$, and holds the initial money stock of the economy, which we denote by $M^s_{-1} > 0$. Finally, we assume that there is a given initial stock of net real international assets, $a_{-1} = 0$. Young agents are differentiated by their endowments of entrepreneurial ability as we now describe.

2.2.1 Entrepreneurs

Only young potential entrepreneurs have access to a stochastic linear technology for converting date t final goods into date $t + 1$ capital. Thus all capital is produced by entrepreneurs.

The capital investment technology is indivisible: in particular, each potential entrepreneur is endowed with one investment project which must be operated at scale q. When $q > 0$ units of the final good are invested at t; they yield zq units of capital at date $t + 1$, where z is an iid (across entrepreneurs and periods) random variable which is realized at date $t + 1$. We let G denote the probability distribution function of z, and assume that G has a differentiable density function g with support $[0; \bar{z}]$. Then $\hat{z} \equiv \int_0^{\bar{z}} zg(z)dz$ is the expected value of z.

While the distribution function of z is common knowledge, the actual amount of capital produced by any project is private information and only the project owner can costlessly observe z_{t+1}. Other agents may verify the realized value of z on a project only by incurring a fixed cost of $\gamma > 0$ units of capital.[4]

We impose two additional assumptions on the parameters of the investment technology.

[4] In assuming that capital is used up in verification, we follow Bernanke and Gertler (1989), Boyd and Smith (1997) and many others.

Assumption 1. $g(z) + \left(\frac{\gamma}{q}\right) g' \geq 0; \quad \forall z \in [0; \bar{z}]$.

Assumption 2. $q > \gamma g(0)$.

These assumptions will impose a simple structure on the expected return function of any lender and thus greatly simplify our analysis.

2.2.2 Workers

While young workers have no access to the stochastic investment technology, they do confront individual uncertainty regarding their future liquidity needs. Specifically, all young workers in each location face an exogenously given, fixed and known probability π of being relocated to the second location within the economy in which they were born. In addition, every young worker in either domestic location faces a probability π^* of being relocated to the rest of the world, and a probability $(1 - \pi - \pi^*) > 0$ of remaining in his initial location. Likewise, all agents in the rest of the world confront a probability ξ of being relocated to the domestic economy. These probabilities of relocation are iid across young workers and time periods.

Relocation shocks are realized at the end of date t, following the closure of all date t markets. They therefore represent liquidity shocks. In particular, immediately following the realization of the relocation shocks, young workers subject to relocation must move. We assume that relocated workers can carry with them only currency for the purpose of purchasing goods in a new domestic (foreign) location, and that all other assets have zero value in exchange for movers. Indeed, our assumptions on the timing of trade and cross-location communication (described below) prevent the verification and hence the redemption of interest bearing assets outside their location of issue. Currency, on the other hand, is universally recognizable, non-counterfeitable and thus acceptable in spatial exchange. In addition, we assume that all goods must be purchased in the currency of the seller. Thus, agents relocated to the second domestic location (to the rest of the world) must take with them domestic (foreign) currency in order to achieve consumption when old.[5]

These assumptions provide a well-defined motive for holding (multiple) currencies. Currency provides liquidity in interlocation exchange, irrespective of its relative rate of return. As we describe below, we will focus on equilibria in which currency is strictly dominated in rate of return by any interest-earning asset. As a result, all young workers will have an incentive to diversify their asset portfolios by holding both types of currency, as well as interest earning assets.

Finally, it bears emphasis that, despite the presence of individual uncertainty for all young agents, there is no aggregate uncertainty in the economy.

2.3 Government Activity

The government prints fiat currency to finance an endogenously determined stream of final consumption. Individuals derive no utility from this government consumption. We let g_t denote per capita government consumption of final goods at date t.

We assume that the government selects, for all time, a fixed rate of money growth which we denote by $\sigma > 1$. Thus, $\frac{M_t^s}{M_{t-1}^s} = \sigma \quad \forall t \geq 0$, where M_t^s denotes the outstanding per capita stock of currency at the end of date t. Let $m_t^s \equiv M_t^s/p_t$ denote the outstanding per capita real balances in the economy, where p_t denotes the domestic currency price of a unit of the final good at t. Then, the government's consumption spending must satisfy

$$g_t = \frac{M_t^s - M_{t-1}^s}{p_t} \equiv m_t^s - m_{t-1}^s(p_{t-1}/p_t) \quad \forall t \geq 1. \tag{1}$$

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[5] Hence, due to the presence of spatial seperation, some fraction of the purchases of the single good must be made with cash. Consequently, an alternative interpretation of these assumptions is that, in the aggregate, there are some cash goods and some credit goods.

Hence, using the fact that $(p_{t-1}/p_t) \equiv (m_t^s/m_{t-1}^s)(M_{t-1}^s/M_t^s) \equiv (m_t^s/m_{t-1}^s)(1/\sigma)$,

$$g_t = m_t^s \left(\frac{\sigma - 1}{\sigma} \right) \quad \forall t \geq 1. \tag{2}$$

2.4 Trade

2.4.1 Factor and goods markets

At the beginning of a period, there is no communication across locations nor any movement of goods or agents. Firms hire capital and labor in competitive factor markets, and factor market trade is conducted autarkically in each location. Thus, capital and labor earn their marginal products. Denoting by w_t the date t real wage rate and by ρ_t the date t capital rental rate, we obtain

$$\rho_t = f'(k_t) \tag{3}$$

$$w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t). \tag{4}$$

Once factor markets clear, production occurs and goods market trade takes place in each location. Consumption of goods follows immediately.

2.4.2 Asset markets

Following consumption, domestic and international asset markets open. Now agents can communicate without restriction across locations, and trade freely in all assets. The stores of value in the economy are as follows.

Government issued domestic currency, as well as foreign currency may be held between periods. Domestic currency obviously pays the gross real rate of return between dates t and $t+1$ of p_t/p_{t+1}, measured in domestic goods. Letting p_t^* denote the foreign currency price of a good produced in the rest of the world, we normalize $p_t^* = 1$. In addition, let e_t denote the domestic currency price of a unit of foreign exchange. Then foreign currency held by a domestic agent between t and $t+1$

earns the domestic real return $(p_t^*/p_{t+1}^*)(e_{t+1}/e_t)(p_t/p_{t+1}) = (e_{t+1}/e_t)(p_t/p_{t+1}) \equiv (x_{t+1}/x_t)$; where $x_t \equiv e_t p_t^*/p_t = e_t/p_t$ denotes the real exchange rate of the small open economy.

In addition, agents may hold interest-earning assets issued abroad. These assets are assumed to be of one period maturity, default-risk free and earn the world gross real interest rate r^* - measured in foreign goods at date $t+1$ per unit invested at t. In domestic units, this return is $r^*(x_{t+1}/x_t)$.

Finally, agents may hold domestically issued private debt by lending to domestic entrepreneurs. The determination of the rate of return to those domestic loans is discussed below.

We will focus throughout the paper on equilibria in which net nominal interest rates are strictly positive, so that both domestic and foreign currency is strictly dominated in rate of return (in a common unit) by all interest-earning assets, whether they are domestically issued loans or risk-free assets issued abroad.

Once asset markets close, all cross-location communication ceases, and relocation shocks are realized. At this time, there is no opportunity for portfolio adjustments or liquidations of interest-earning assets. Relocated agents - of which there are a measure $(1-\alpha)(\pi+\pi^*)$ - move, taking with them only the currency of the country of their ultimate location. All such movers would prefer - were asset markets open or communication between locations feasible - to prematurely liquidate or trade other assets held in their portfolio. However, in absence of such trading possibilities, young workers are unable to insure themselves against liquidity shocks.

This timing of transactions and trade is depicted in Figure 1.

3 Financial Intermediaries

The economy that we have just described displays two sources of inefficiencies motivating the emergence of financial intermediaries that attract a positive measure of depositors and borrowers. First, the CSV problem provides a role for banks in the allocation of credit. Indeed, banks may pool savings to provide loans for entrepreneurs. They can therefore avoid the duplication of verification costs which would arise if savers engaged directly in lending to entrepreneurs. Second, banks may insure young workers against the random liquidity shocks associated with stochastic relocation. We now describe financial intermediaries which assume both these roles.

3.1 The Bank's Problem: Borrowers

Potential entrepreneurs have young period income w_t, and we will only consider equilibria in which they require external financing to run a project, i.e. for which $q > w(k_t)$. Clearly, a necessary condition for such equilibria to exist is $q > w(0)$. Hence we will maintain the following assumption.

Assumption 3. $q > b_2^{\frac{1}{\beta}}$ for $0 < \beta < 1$; $q > 0$ for $-1 < \beta \leq 0$.

We define $l_t = q - w(k_t)$ as the amount of a project that needs to be funded by borrowing. All such borrowing is intermediated in the manner described by Williamson (1986). That is, intermediation with respect to borrowers is motivated because it allows the economy to avoid the duplication of verification costs which would arise if loans were held by individual lenders. Intermediaries behave competitively with respect to the asset side of their balance sheets, taking as given the rates of return to currencies, and risk-free assets issued abroad, and simply choosing whether to accept or reject the loan contract terms offered by potential entrepreneurs who seek external funding for their projects. In equilibrium, intermediaries will be perfectly diversified, and earn a nonstochastic return on their lending portfolio.

It is well known that in the CSV environment, the optimal loan contract offered by potential borrowers takes the form of a standard debt contract.[6] In particular, a funded entrepreneur either

[6] For a formal derivation of this contract, as well as the other results described in this section, see Boyd and Smith

repays a non state-contingent gross loan interest rate v_t or he defaults. In the latter case, the intermediary monitors the project outcome, and retains all proceeds net of monitoring costs. When Assumptions 1 and 2 hold, the expected return to lending, $\pi(v_t)$; implied by these contracts is a strictly concave function of the loan interest rate v_t, which attains its maximum at $\hat{v}_t$ such as depicted in Figure 2. For relatively low loan interest rates, expected returns rise as v_t increases, because the rise in gross repayments of principal plus interest, $v_t l_t$; outweighs the rise in costs due to an increased number of bankruptcies. However, beyond $\hat{v}_t$, a further increase in the loan interest rate produces a decrease in the expected return to lending. Indeed, for loan interest rates higher than $\hat{v}_t$; the fraction of projects going bankrupt becomes so high, that the increased costs associated with monitoring and with the fact that bankrupt firms cannot fully repay principal plus interest, dominate the increase in gross repayments which one would otherwise expect.

This environment is also characterized by the fact that unfulfilled demand for credit or "credit rationing" may arise.[7] When credit rationing obtains, all potential entrepreneurs offer the loan interest rate $\hat{v}_t$ that maximizes the expected return for a prospective lender. Unfunded entrepreneurs cannot then alter loan contract terms in order to obtain credit since this would simply reduce the expected return to any potential lender. Hence credit rationing may be an equilibrium outcome,[8] and we henceforth focus on economies in which equilibrium credit rationing arises at all dates. This seems justified given the prevalence of this feature in the credit markets of the emerging economies that we intend this model to represent. When all potential entrepreneurs offer the gross loan interest

(1997, 1998). For completeness, we have reproduced these derivations in a referee's Appendix.

[7] Gale and Hellwig (1985) and Williamson (1986, 1987) first noted this feature of the CSV environment.

[8] As we will discuss in Section 4, this will be true regardless of the fact that we will allow for unrestricted access to international capital.

rate $\hat{r}_t$ that maximizes a potential lender's expected return, the critical project return at which a borrower's project income exactly covers loan principal plus interest is independent of the level of the capital stock, and we denote it by η. Thus, at all dates project verification occurs iff $z \in [0; \eta)$.

When credit is rationed, the expected return to lending at time t can be expressed as

$$r_t = \pi(\hat{r}_t) = \frac{\rho_{t+1} q}{l_t}\left[\eta - \frac{\gamma}{q}G(\eta) - \int_0^{\eta} G(z)dz\right]: \tag{5}$$

Equivalently, the expected income to the intermediary from lending l_t units at time t; is

$$r_t l_t = \pi(\hat{r}_t) l_t = \rho_{t+1} q \eta - \rho_{t+1}\gamma G(\eta) - \rho_{t+1} q \int_0^{\eta} G(z)dz:$$

Here the first term represents gross expected repayments of principal plus interest, $\rho_{t+1} q \eta = \hat{r}_t l_t$, while the last two terms represent the intermediary's costs due to bankruptcies. The second term denotes expected monitoring costs, while the third term stands for expected losses due to bankrupt firms' inability to fully repay principal plus interest.

Defining $\psi \equiv q\left[\eta - \frac{\gamma}{q}G(\eta) - \int_0^{\eta} G(z)dz\right]$; equation (5) implies that the expected return received by a lender at t; r_t, is equal to $\psi(\rho_{t+1}/l_t)$ when credit is rationed. In other words, under credit rationing, the expected return on funds loaned in private credit markets is proportional to the ratio $\frac{\rho_{t+1}}{l_t}$, and hence depends only on the small open economy's capital stock at t and t + 1.

It is also straightforward to derive the expected income (utility) for a potential borrower under credit rationing as

$$\rho_{t+1} q \left[\hat{z} - \frac{\gamma}{q}G(\eta)\right] - r_t l_t: \tag{6}$$

Defining $\phi \equiv \hat{z} - \frac{\gamma}{q}G(\eta) > 0$; the expected payoff of a funded entrepreneur can be written compactly as $\phi\rho_{t+1} q - r_t l_t$: Here the first term is expected project income, while the second term represents expected loan repayments. For future reference we observe that the parameter ϕ represents the expected amount of capital produced per unit invested, net of monitoring costs, when credit is rationed.

Since any potential entrepreneur always has the option of foregoing his project, saving his wage income, and earning the return on deposits offered by banks or the world real interest rate on directly held foreign assets, it is necessary to verify that potential borrowers will prefer to borrow, rather than lend, in equilibrium. We denote the deposit return which banks offer to entrepreneurs by R_t^b. A borrower therefore prefers borrowing to lending under credit rationing iff

$$\phi\rho_{t+1}q - r_t l_t \geq \max\left[R_t^b; r^*(x_{t+1}/x_t)\right] w(k_t). \tag{7}$$

In Section 4 we will specify conditions under which (7) is satisfied. We henceforth analyze equilibria in which credit is rationed and potential entrepreneurs prefer borrowing over lending. Hence all potential entrepreneurs would prefer to run their investment projects, but only some of them will actually receive the external funding needed to do so.

3.2 The Bank's Problem: Depositors

While banks are competitive on the asset side of their balance sheet, we assume that on the deposit side they act as Nash competitors, announcing deposit contract terms and accepting all deposits forthcoming at these terms. We now consider the behavior of banks which seek to obtain the deposits of both workers and credit rationed entrepreneurs.

To attract the deposits of any credit rationed entrepreneurs, banks will offer them at least the world real interest rate, hence

$$R_t^b \geq r^*(x_{t+1}/x_t). \tag{8}$$

To attract the deposits of young workers, intermediaries will choose deposit returns dependent on the relocation status of the agent, such as to maximize worker's expected utility, subject to a balance sheet constraint, a set of budget constraints, and subject to (8).

Thus, banks hold reserves of both domestic and foreign currency to meet early withdrawal demand by movers, as well as interest-earning assets. The balance sheet constraint faced by a representative bank is:

$$[(1 - \alpha) + \alpha(1 - \mu_t)]w(k_t) = [1 - \alpha\mu_t]w(k_t) \geq m_t + m_t^{*}x_t + l_t^{a} + a_t x_t. \tag{9}$$

Here $\mu_t < 1$ denotes the fraction of potential entrepreneurs that actually obtains credit, $(1 - \alpha\mu_t)w(k_t)$ is per capita deposits, m_t denotes per capita domestic real balances held by a representative domestic bank, m_t^{*} denotes per capita foreign real balances held by a domestic bank, $l_t^{a} \equiv \alpha\mu_t l_t$ is per capita loans made to domestic entrepreneurs, and a_t denotes per capita net foreign assets held by the bank.

The bank also faces four budget constraints. First, the bank knows that, at the end of date t, $\pi(1 - \alpha)$ depositors will withdraw their deposits and require domestic currency to consume at $t + 1$. Each of these domestic movers deposit $w(k_t)$, and the bank guarantees them the gross real rate of return R_t^m. Then, the bank's per capita holdings of domestic real balances must satisfy

$$(1 - \alpha)\pi w(k_t)R_t^m \leq m_t(p_t/p_{t+1}). \tag{10}$$

Similarly, foreign real balances held by the bank satisfy

$$(1 - \alpha)\pi^{*}w(k_t)R_t^{m*} \leq x_t m_t^{*}(p_t/p_{t+1})(e_{t+1}/e_t), \tag{11}$$

where R_t^{m*} is the gross real rate of return promised by a bank to a depositor relocated to the rest of the world.

In addition, the bank confronts two types of potential depositors that do not move or require currency in order to consume. Since we focus on equilibria in which currency is strictly dominated in rate of return by interest-earning assets, currency reserves are held exclusively for the purpose of making payments to movers. Thus, against the deposits of all other agents, profit maximizing banks hold only interest-earning loans to domestic entrepreneurs and net foreign assets. For foreign assets and loans to both be held in equilibrium, the following no arbitrage condition must hold at each date:

$$r^*(x_{t+1}/x_t) = \frac{\psi f'(k_{t+1})}{[q - w(k_t)]}. \tag{12}$$

At the beginning of date $t+1$, $(1 - \varepsilon - \varepsilon^*)(1 - \alpha)$ workers that have not been relocated contact their local bank and withdraw their deposits. In addition, $(1 - \mu_t)\alpha$ credit rationed entrepreneurs will withdraw deposits at this time. The budget constraint which a bank confronts with respect to non-moving workers is therefore

$$(1 - \varepsilon - \varepsilon^*)(1 - \alpha)w(k_t)R_t^l \le \delta_t(a_t x_t + l_t^a)r^*(x_{t+1}/x_t) \tag{13}$$

where R_t^l denotes the gross real rate of return the bank promises to such depositors and $\delta_t \le 1$ denotes the fraction of the bank's interest-earning asset portfolio assigned to back the deposits of these workers. Finally, the bank's budget constraint with respect to credit rationed entrepreneurs is[9]

$$(1 - \mu_t)\alpha w(k_t)R_t^b \le (1 - \delta_t)(a_t x_t + l_t^a)r^*(x_{t+1}/x_t). \tag{14}$$

We now define $\lambda_t \equiv m_t/[(1 - \alpha\mu_t)w(k_t)]$, and $\lambda_t^* \equiv m_t^* x_t/[(1 - \alpha\mu_t)w(k_t)]$ as the portfolio weights assigned to domestic and foreign currency respectively, expressed as a fraction of per capita

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[9] Unfunded entrepreneurs could conceivably hold foreign assets directly - since they do not face the probability of relocation. Here we assume that all holdings of foreign assets in the domestic economy are intermediated. This is not unrealistic: Corsetti et al. (1998) present evidence suggesting that a very large fraction of capital inflows in emerging economies is intermediated by domestic banks.

deposits. Then $1 - \lambda_t - \lambda_t^* \equiv (a_t x_t + l_t^a)/[(1 - \alpha\mu_t)w(k_t)]$ denotes the portfolio weight of interest-earning assets.

Rewriting the budget constraints confronted by a representative bank then yields

$$
\begin{aligned}
R_t^m &\leq \frac{\lambda_t(1-\alpha\mu_t)}{\varepsilon(1-\alpha)}(p_t/p_{t+1}) && (15)\\
R_t^{m*} &\leq \frac{\lambda_t^*(1-\alpha\mu_t)}{\varepsilon^*(1-\alpha)}(x_{t+1}/x_t) && (16)\\
R_t^l &\leq \frac{\delta_t(1-\lambda_t-\lambda_t^*)(1-\alpha\mu_t)}{(1-\varepsilon-\varepsilon^*)(1-\alpha)}r^*(x_{t+1}/x_t) && (17)\\
R_t^b &\leq \frac{(1-\delta_t)(1-\lambda_t-\lambda_t^*)(1-\alpha\mu_t)}{\alpha(1-\mu_t)}r^*(x_{t+1}/x_t). && (18)
\end{aligned}
$$

The problem of a bank which seeks to attract the deposits of workers and credit rationed entrepreneurs is, therefore, to

$$\max E(U) = \varepsilon \ln[w(k_t)R_t^m] + \varepsilon^* \ln[w(k_t)R_t^{m*}] + (1-\varepsilon-\varepsilon^*)\ln[w(k_t)R_t^l]$$

by choice of R_t^m, R_t^{m*}, R_t^l, $\lambda_t \in (0,1)$, $\lambda_t^* \in (0,1)$ and $\delta_t \in (0,1)$ subject to (15)-(18), and (8).

The solution to this problem satisfies (15)-(18) with equality,

$$R_t^b = r^*(x_{t+1}/x_t); \tag{19}$$

and the bank sets

$$
\begin{aligned}
\lambda_t &= \frac{\varepsilon(1-\alpha)}{1-\alpha\mu_t} && (20)\\
\lambda_t^* &= \frac{\varepsilon^*(1-\alpha)}{1-\alpha\mu_t} && (21)\\
(1-\lambda_t-\lambda_t^*) &= \frac{(1-\alpha)(1-\varepsilon-\varepsilon^*)+\alpha(1-\mu_t)}{1-\alpha\mu_t} && (22)\\
\delta_t &= \frac{(1-\alpha)(1-\varepsilon-\varepsilon^*)}{(1-\alpha)(1-\varepsilon-\varepsilon^*)+\alpha(1-\mu_t)}. && (23)
\end{aligned}
$$

We now turn to a description of the general equilibrium of our economy.

4 General Equilibrium

In equilibrium, an international no arbitrage condition holds, the capital, labor, money and goods markets clear, the government satisfies its intertemporal budget constraint, and domestic capital inflows exactly offset any net excess of domestic investment over domestic savings. We consider only equilibria in which credit rationing obtains and for which entrepreneurs prefer borrowing over lending. Finally, we focus exclusively on equilibria in which domestic and foreign currency are strictly dominated in rate of return by all interest-bearing assets.

The international no arbitrage condition equates the real return on foreign assets to the return on domestic investment projects, expressed in domestic units. Thus (12) must hold at all $t \geq 0$, and consequently, the domestic gross nominal interest rate on all interest-bearing assets is given by $I_t \equiv r^*(x_{t+1}/x_t)(p_{t+1}/p_t) = r^*(e_{t+1}/e_t)$, while the foreign gross nominal interest rate is $I^* \equiv r^*(p^*_{t+1}/p^*_t) = r^*$. Then, in equilibria in which domestic and foreign currency are strictly dominated in rate of return, $r^*(x_{t+1}/x_t) > (p_t/p_{t+1}) \Rightarrow r^*(e_{t+1}/e_t) \equiv I_t > 1$ and $r^*(x_{t+1}/x_t) > (x_{t+1}/x_t) \Rightarrow r^* \equiv I^* > 1$ hold. Thus,

$$r^* > \max\left(1, \frac{e_t}{e_{t+1}}\right) \tag{24}$$

must be satisfied at all dates.

Equilibrium factor prices at $t \geq 0$ are determined by (3) and (4). In addition, the date $t+1$ supply of capital is given by the aggregate demand for investment goods at date t, $\alpha\mu_t q$, times the expected yield of capital per unit invested, net of monitoring costs, ϕ. Then,

$$k_{t+1} = \phi\alpha\mu_t q \quad \forall t \geq 0. \tag{25}$$

At each date, the domestic money market clears. The per capita supply of real balances is m^s_t. These balances are held by domestic banks and by foreign agents relocating to the domestic country.

Normalizing the population in the rest of the world as well as their per capita young period income to 1, the per capita foreign demand for domestic currency measured in domestic goods is ξx_t. The per capita domestic demand for domestic real balances is $\lambda_t w(k_t)(1 - \alpha\mu_t) = \varepsilon \left(\frac{1-\alpha}{1-\alpha\mu_t}\right) w(k_t)(1 - \alpha\mu_t)$. Thus, the domestic money market clearing condition takes the form

$$m_t^s = \varepsilon(1 - \alpha)w(k_t) + \xi x_t; \quad \forall t \geq 0. \tag{26}$$

At all dates $t \geq 1$, the demand for domestic goods in any location includes the consumption of all the old agents that reside in the domestic country at t. First, there are $\varepsilon(1 - \alpha)$ old domestic workers who were relocated domestically and a fraction ξ old foreigners who were relocated from abroad. Together these agents hold the entire per capita stock of date $t - 1$ domestic real balances, which earn the gross real rate of return p_{t-1}/p_t. In addition, there are $(1 - \varepsilon - \varepsilon^*)(1 - \alpha)$ old workers and $\alpha(1 - \mu_{t-1})$ credit rationed old entrepreneurs who consume the proceeds of their intermediated holdings of assets, which earn the world real interest rate. Moreover, there are $\alpha\mu_{t-1}$ old entrepreneurs who consume the return on their projects, which comprises their capital rental income $f'(k_t)\phi q$ less their repayments of debt to banks $r^*(x_t/x_{t-1})[q - w(k_{t-1})]$. Young entrepreneurs invest goods of total per capita value $\alpha\mu_t q = k_{t+1}/\phi$. In addition, g_t units of the final good per capita are consumed by the government. The domestic goods market clearing condition therefore takes the form

$$\begin{aligned} f(k_t) &= m_{t-1}^s(p_{t-1}/p_t) + [(1-\alpha)(1-\varepsilon-\varepsilon^*) + \alpha(1-\mu_{t-1})]w(k_{t-1})r^*(x_t/x_{t-1}) \\ &+ \alpha\mu_{t-1}\left\{f'(k_t)q\phi - r^*(x_t/x_{t-1})[q - w(k_{t-1})]\right\} + \alpha\mu_t q + g_t; \qquad \forall t \geq 1. \end{aligned} \tag{27}$$

We use (25), and the fact that $g_t + m_{t-1}^s(p_{t-1}/p_t) \equiv g_t + m_t^s/\sigma = m_t^s$ - which follows from (2) - to express this condition as

$$f(k_t) = (1-\alpha)\varepsilon w(k_t) + \xi x_t + [(1-\alpha)(1-\varepsilon-\varepsilon^*) + \alpha]w(k_{t-1})r^*(x_t/x_{t-1}) \tag{28}$$

$$+k_t f'(k_t) - \frac{k_t}{\phi} r^*(x_t/x_{t-1}) + \frac{k_{t+1}}{\phi}.$$

Finally, sources and uses of funds in the home country must be equal in equilibrium. In other words,

$$w(k_t) = (\epsilon + \epsilon^*)(1 - \alpha)w(k_t) + \alpha\mu_t q + x_t a_t. \tag{29}$$

This condition is equivalent to the bank's balance sheet constraint, or to a balance of payments condition.

Of course for entrepreneurs to be willing to borrow, we need to check that (7) is satisfied at all dates. Given (19) and (12), that will clearly be the case when the following condition holds,

$$\phi[q - w(k_t)] \geq \psi. \tag{30}$$

Finally, equilibria need to satisfy $\mu_t < 1\ \forall t$, for credit to be rationed in all periods. Parenthetically, what accounts for the existence of credit rationing in a small open economy; one which presumably can absorb large quantities of funds without disrupting world capital markets? The answer is related to the CSV problem as reflected in the no-arbitrage condition (12). At date t, the domestic economy has an inherited capital stock of k_t. Given this capital stock, the implied real wage rate w_t, and the equilibrium sequence of real exchange rates, x_t, domestic entrepreneurs must offer intermediaries an expected return equal to the world rate of interest. The highest expected return they can offer is $\psi f'(k_{t+1})/[q - w(k_t)]$; at t this is obviously determined by k_{t+1}. Hence equation (12) implies that world capital markets fund the largest quantity of domestic capital investment that is consistent with domestic borrowers being able to offer the required market rate of return. Any further capital investments would lower the marginal product of capital such that domestic borrowers could no longer compete in world markets. This is the source of credit rationing in the domestic economy.

4.1 The Initial Period

In the initial period, the domestic goods market equilibrium condition, and the government budget constraint, take a different form than at any other date. At $t = 0$, the predetermined supply of goods is $f(k_0)$ per capita. The demand for goods derives from old agents who earn the rental rate on their holdings of capital, $f'(k_0)k_0$, and the value of initial real balances, M^s_{-1}/p_0. In addition, young entrepreneurs invest wage income in capital projects, where gross investment at $t = 0$ is $k_1/\psi = \alpha\mu_0 q$, and the government consumes g_0. Thus, initial goods market clearing requires that

$$f(k_0) = \frac{M^s_{-1}}{p_0} + f'(k_0)k_0 + \alpha\mu_0 q + g_0 \tag{31}$$

holds. The initial period government budget constraint is $g_0 = \frac{M^s_0 - M^s_{-1}}{p_0}.$ We can then rewrite the goods market clearing condition at date 0 as

$$f(k_0) = \frac{M^s_0}{p_0} + f'(k_0)k_0 + \frac{k_1}{\psi}. \tag{32}$$

Hence, given an initial domestic price level, p_0, the initial per capita capital stock, k_0, and the date zero nominal money supply $M^s_0 = \sigma M^s_{-1}$, date 0 real balances are immediately determined while the date 1 capital stock is derived from (32). Once k_1 is determined, the equilibrium value of $\mu_0 = \left[\frac{k_1}{\alpha q \psi}\right]$ is given. In addition, $x_0 \equiv e_0/p_0$ and hence e_0 follows from the money market clearing condition (26), while government consumption is read from the government's date 0 budget constraint. The date 0 net foreign asset position can be derived from the balance of payment condition (29). Finally, x_1 is determined by the no-arbitrage condition (12), p_1 by the date 1 money market clearing condition, and k_2 by date 1 goods market clearing (28).

The entire equilibrium sequence of future real money balances, prices, interest rates, capital stocks, real exchange rates, net holdings of foreign assets and values of μ_t can thus be determined given an initial domestic price level, p_0. However, the value of p_0 is not determined by the equilibrium conditions and is thus a free initial condition. The properties of dynamic equilibrium paths are analyzed below. We first turn our attention to stationary equilibria.

4.2 Steady State Equilibria

In a steady state equilibrium of the economy, the no-arbitrage condition,

$$r^* = \frac{\psi f'(k)}{q - w(k)}; \tag{33}$$

must hold and this pins down the steady state capital stock. From goods market clearing, the steady state real exchange rate, x; is simply

$$x = \frac{1}{\xi}\left\{ w(k)\,\{1 - (1-\alpha)\varepsilon - [1 - (1-\alpha)(\varepsilon + \varepsilon^*)]\, r^*\} + \frac{k}{\psi}(r^* - 1) \right\}: \tag{34}$$

Steady state real balances are then given by

$$m^s = (1-\alpha)\varepsilon w(k) + \xi x: \tag{35}$$

The steady state fraction of entrepreneurs whose projects are funded, μ, is determined by

$$\mu = \frac{k}{\phi\alpha q}: \tag{36}$$

Steady state government consumption is derived from

$$g = m^s\left(\frac{\sigma - 1}{\sigma}\right): \tag{37}$$

Finally, per capita net holdings of foreign assets in steady state follow from

$$a = \frac{1}{x}\left[w(k)[1 - (\varepsilon + \varepsilon^*)(1-\alpha)] - \frac{k}{\psi} \right]: \tag{38}$$

Clearly, for any solution to (33)-(38) to constitute an equilibrium steady state requires that a number of restrictions are satisfied. These are presented in the following Definition.

Definition 1. A pair of stationary values $(k; x)$ is a steady state equilibrium if

a) k solves the no arbitrage condition (33) and the pair $(k; x)$ solves the goods market clearing condition (34); b) $k \in (0, \infty)$; c) $x \in (0, \infty)$; d) borrowers prefer borrowing to lending or $\phi[q - w(k)] \geq \psi$ is satisfied; e) credit rationing obtains or $\mu < 1$; f) currencies are return dominated or (24) holds.

We now develop sufficient conditions for the existence of a steady state equilibrium. We collect these conditions in Proposition 3. We start by defining $H(k) \equiv \psi f'(k)/[q - w(k)]$. Then

$$r^* = H(k) \tag{39}$$

is an alternative representation of (33). Clearly the function $H(k)$ plays a key role in determining long run equilibria. We now establish some of its properties.

Lemma 2. The function $H(k)$ satisfies

a) $\lim_{k \to 0} H(k) = \infty$ iff $0 \leq \beta < 1$; $\lim_{k \to 0} H(k) = \frac{\psi}{q} b_1^{\frac{1}{\beta}}$ iff $-1 < \beta < 0$;

b) $\lim_{k \to \hat{k}} H(k) = \infty$ where $\hat{k} \equiv w^{-1}(q)$ and $\infty > w^{-1}(q) > 0$

c) $H'(k) > (\leq) 0$ as $k > (\leq) f^{-1}(q)$.

Proof. Part a) of the Lemma is immediate from Lemma 1. Part b) is also obvious. Part c) follows straightforwardly from differentiation of $H(k)$, which yields $H'(k) = -\psi f''(k) \frac{[f(k) - q]}{[q - w(k)]^2}$. ■

Lemma 2 implies that $H(k)$ has the configuration depicted in Figures 3 and 4, and Proposition 1 follows immediately.

Proposition 1. Suppose that $H[f^{-1}(q)] < r^* < \lim_{k \to 0} H(k)$. Then there are exactly two values of $k \in (0, \infty)$ that satisfy (39), which we denote by k_1 and k_2; where $k_2 > k_1$.

Hence if the world interest rate satisfies the conditions spelled out in Proposition 1, there are exactly two candidate steady state capital stocks, k_1 and k_2, for which credit is rationed. Each of these is associated with a distinct stationary real exchange rate - which we denote by x_i, $i = 1,2$ - where x_i is determined by (34). Given k_i and x_i, the associated values of μ_i, m_i^s, g_i, and a_i, $i = 1,2$, follow immediately.

We now establish conditions under which both candidate steady state equilibrium capital stocks also satisfy $0 < x_i < \infty$.

First, we define

$$\Gamma(r^*) \equiv 1 - (1-\alpha)\varepsilon - [1 - (1-\alpha)(\varepsilon + \varepsilon^*)]r^*; \tag{40}$$

$$\Pi(r^*) \equiv \frac{\phi\Gamma(r^*)}{1 - r^*}; \tag{41}$$

and

$$\hat{r}^* \equiv \frac{1 - (1-\alpha)\varepsilon}{[1 - (1-\alpha)(\varepsilon + \varepsilon^*)]}. \tag{42}$$

It is then easy to prove the following Lemma.

Lemma 3. Let $\Gamma(r^*)$, $\Pi(r^*)$ and $\hat{r}^*$ be given by (40), (41), and (42), respectively. Then,

a) $\Gamma'(r^*) = -[1 - (1-\alpha)(\varepsilon + \varepsilon^*)] < 0$ and $\Gamma(r^*) < 0$ iff $r^* > \hat{r}^*$;

b) $\Pi'(r^*) = \phi\left\{\frac{(1-\alpha)\varepsilon^*}{(1-r^*)^2}\right\} > 0$ and $\Pi(r^*) < 0$ iff $r^* \in (1, \hat{r}^*)$.

We are now ready to state the following proposition which is illustrated in Figures 3 and 4.

Proposition 2. If $r^* \in (1, \hat{r}^*)$ then the real exchange rates x_i associated with the solutions k_i to (39) satisfy $0 < x_i < \infty$.

Proof. Equation (34) implies that $0 < x_i < \infty \Leftrightarrow \Omega(k_i) \equiv \frac{k_i}{w(k_i)} > \Pi(r^*)$. But from Lemma 3, we know that $\Pi(r^*) < 0$ for $r^* \in (1, \hat{r}^*)$. Hence $r^* \in (1, \hat{r}^*)$ is a sufficient condition for $0 < x_i < \infty$. ■

We now develop a set of conditions on r^* that are sufficient to guarantee the existence of exactly two solutions to (39), both of which satisfy all the conditions for a steady state equilibrium listed in Definition 1.

Proposition 3. Suppose that $\min\{w^{-1}(q - \psi/\phi), \phi\alpha q\} > f^{-1}(q)$,
$\min\{H[w^{-1}(q - \psi/\phi)], \lim_{k \to 0} H(k), \hat{r}^*\} > \max\left\{H\left(f^{-1}(q)\right), 1, \frac{1}{\sigma}\right\}$; and
let $r^* \in \left(\max\left\{H\left(f^{-1}(q)\right), 1, \frac{1}{\sigma}\right\}, \min\{H[w^{-1}(q - \psi/\phi)], H[\phi\alpha q], \lim_{k \to 0} H(k), \hat{r}^*\}\right)$. Then there exist two steady state equilibria for which credit is rationed.

Proof. For there to exist exactly two solutions to (39) requires that $H[f^{-1}(q)] < r^* < \lim_{k \to 0} H(k)$. We also know that a necessary and sufficient condition for credit rationing to arise at any stationary capital stock k is $\mu < 1$, or equivalently from (36), $k < \phi\alpha q$. Hence $\phi\alpha q > f^{-1}(q)$ is necessary for the two solutions to (39) to exhibit credit rationing, while $r^* < H[\phi\alpha q]$ is sufficient for credit to be rationed in both solutions. In addition, from (30), we know that $\phi[q - w(k)] > \psi$, or equivalently $k < w^{-1}(q - \psi/\phi)$, is necessary and sufficient for potential borrowers to have an incentive to borrow rather than lend. Hence $w^{-1}(q - \psi/\phi) > f^{-1}(q)$ is necessary for the two solutions to (39) to be incentive compatible with borrowing, while $r^* < H[w^{-1}(q - \psi/\phi)]$ is sufficient for entrepreneurs in both solutions to prefer borrowing over lending. For both solutions to (39) to be associated with positive and finite real exchange rates, it is sufficient that $r^* \in (1, \hat{r}^*)$. Finally, $r^* > \max\left\{1, \frac{1}{\sigma}\right\}$ is necessary for both currencies to be return dominated in steady state. Indeed, in a steady state equilibrium, the inflation rate is given by $p_{t+1}/p_t = (m_t/m_{t+1})(M_{t+1}/M_t) = (M_{t+1}/M_t) = \sigma$, and the domestic nominal interest rate is $I = r^*(x_{t+1}/x_t)(p_{t+1}/p_t) = r^*\sigma$, while the foreign nominal rate remains $I^* = r^*$. Then, any steady state equilibrium in which both currencies are return dominated and (24) holds must satisfy $I > 1$, and $I^* > 1$, or $r^* > \max(1, 1/\sigma)$. ■

The financial market frictions that we have introduced in this economy clearly play a major role in the determination of long run equilibrium. Indeed, due to the CSV problem in credit markets, our economy potentially displays two steady states in which credit is rationed. The parameter ψ, which embodies the fixed cost of verification γ, determines the severity of credit rationing and hence the steady state level of the capital stock. Moreover, the steady state real exchange rate depends crucially on the liquidity shocks, ε and ε^*. In particular, the greater the fraction of workers who are subject to cash in advance constraints, the more depreciated is the real exchange rate. Indeed, for a given level of the capital stock and world real interest rate, the domestic consumption of non-relocated agents declines for higher levels of ε and ε^*, and this will be offset by a depreciation of the real exchange rate, which raises the purchasing power of foreign agents relocated to the domestic economy.

To conclude this section, we provide an example of an economy that satisfies all the conditions listed in Proposition 3, and hence displays two long run equilibria.

Example 1. Let $f(k) = \left[0.8\, k^{-0.1} + 0.35\right]^{-10}$, and assume that $q = 1.05$, $g(z) = 1/\bar{z}$ with $\bar{z} = 20.67$, and $\gamma = 10.55$, which implies that $\phi = 15.5$ and $\psi = 8.6$. Assume further that $\xi = 1$, $\alpha = 0.9$, $\varepsilon = 0.45$, $\varepsilon^* = 0.45$, $\sigma = 1.01$ and $r^* = 1.015$. For these parameter values, $\phi\alpha q = 14.65$; $\hat{k} = 38.76$; $w^{-1}(q - \psi/\phi) = 13.01$; $f^{-1}(q) = 8.59$; $H\left[f^{-1}(q)\right] = 1.0002$; $H\left[w^{-1}(q - \psi/\phi)\right] = 1.043$; $H[\phi\alpha q] = 1.078$; $\lim_{k\to 0} H(k) = 76$, and $\hat{r}^* = 1.049$. Hence Proposition 3 applies, and two steady state equilibria exist. Their values are, $k_1 = 6.39$; $\frac{k_1 f'(k_1)}{f(k_1)} = 0.65$; $x_1 = 0.015$; $a_1 = -9.1$; and

$k_2 = 11.11, \frac{k_2 f'(k_2)}{f(k_2)} = 0.64, x_2 = 0.025, a_2 = -12.73.$

It is worth noting that the shares of GDP earned by capital in our example are high by the standards of developed economies, but quite realistic for emerging economies.

4.3 Local Dynamics

We now analyze the properties of dynamical equilibria to determine which steady state(s) can be attained. We use (12) and (28) to reduce the system of general equilibrium conditions to two second order difference equations in x_t and k_t. Specifically,

$$
\begin{aligned}
x_{t+1} &= \left(\frac{\psi}{r^*}x_t\right)^{\rho}\left(\frac{f'(k_{t+1})}{q-w(k_t)}\right)^{\sigma} && (43)\\
k_{t+2} &= \phi\{f(k_{t+1}) - (1-\alpha)^2 w(k_{t+1})\} - \frac{\xi\psi\phi}{r^*}x_t\left(\frac{f'(k_{t+1})}{q-w(k_t)}\right)^{\sigma} && (44)\\
&\quad - \phi[(1-\alpha)(1-\varepsilon-\varepsilon^*)+\alpha]w(k_t)\psi^{\rho}\left(\frac{f'(k_{t+1})}{q-w(k_t)}\right)^{\sigma}\\
&\quad - \phi k_{t+1}f'(k_{t+1}) + k_{t+1}\psi^{\rho}\left(\frac{f'(k_{t+1})}{q-w(k_t)}\right)^{\sigma}.
\end{aligned}
$$

To simplify matters, we transform these two second order equations into a system of three first order difference equations.

Let $y_t = k_{t+1}$. Then, we can express (43)-(44) as

$$
\begin{aligned}
k_{t+1} &= y_t && (45)\\
x_{t+1} &= \left(\frac{\psi}{r^*}x_t\right)^{\rho}\left(\frac{f'(y_t)}{q-w(k_t)}\right)^{\sigma} && (46)\\
y_{t+1} &= \phi\{f(y_t) - (1-\alpha)^2 w(y_t)\} - \frac{\xi\psi\phi}{r^*}x_t\left(\frac{f'(y_t)}{q-w(k_t)}\right)^{\sigma} && (47)\\
&\quad - \phi[(1-\alpha)(1-\varepsilon-\varepsilon^*)+\alpha]w(k_t)\psi^{\rho}\left(\frac{f'(y_t)}{q-w(k_t)}\right)^{\sigma}\\
&\quad - \phi y_t f'(y_t) + y_t\psi^{\rho}\left(\frac{f'(y_t)}{q-w(k_t)}\right)^{\sigma}.
\end{aligned}
$$

We then linearize this system in the neighborhood of any steady state equilibrium (k, x, y) as $(k_{t+1} - k, x_{t+1} - x, y_{t+1} - y) = J\,(k_t - k, x_t - x, y_t - y)'$, where J is the Jacobian matrix of partial derivatives given by

$$J = \begin{bmatrix} \frac{\partial k_{t+1}}{\partial k_t} = 0 & \frac{\partial k_{t+1}}{\partial x_t} = 0 & \frac{\partial k_{t+1}}{\partial y_t} = 1 \\ \frac{\partial x_{t+1}}{\partial k_t} & \frac{\partial x_{t+1}}{\partial x_t} & \frac{\partial x_{t+1}}{\partial y_t} \\ \frac{\partial y_{t+1}}{\partial k_t} & \frac{\partial y_{t+1}}{\partial x_t} & \frac{\partial y_{t+1}}{\partial y_t} \end{bmatrix};$$

with all partial derivatives evaluated at the appropriate steady state. From (45)-(47), we find that

$$\begin{aligned} \frac{\partial x_{t+1}}{\partial k_t} &= -\frac{xkf''(k)}{q - w(k)}; \\ \frac{\partial x_{t+1}}{\partial x_t} &= 1; \\ \frac{\partial x_{t+1}}{\partial y_t} &= \frac{\psi}{r^*}\frac{xf''(k)}{[q - w(k)]}. \end{aligned}$$

Finally, the partial derivatives of y_{t+1} are

$$\begin{aligned} \frac{\partial y_{t+1}}{\partial k_t} &= \phi r^* \left\{ \frac{kf''(k)}{q - w(k)} \right\} \left\{ \frac{\xi x}{r^*} + [(1-\alpha)(1-\varepsilon-\varepsilon^*) + \alpha] q - \frac{k}{\phi} \right\}; \\ \frac{\partial y_{t+1}}{\partial x_t} &= -\phi\xi; \\ \frac{\partial y_{t+1}}{\partial y_t} &= \phi \left\{ \frac{r^*}{\phi} - (1-\alpha)\varepsilon w'(k) - kf''(k) \right\} + \frac{\phi r^* f''(k)}{f'(k)} \left\{ \frac{k}{\phi} - \frac{\xi x}{r^*} - [(1-\alpha)(1-\varepsilon-\varepsilon^*) + \alpha] w(k) \right\} \end{aligned}$$

The characteristic equation for J takes the form

$$p(\Lambda) = \Lambda^3 - \Lambda^2 \left[1 + \frac{\partial y_{t+1}}{\partial y_t} \right] + \Lambda \left[\frac{\partial y_{t+1}}{\partial y_t} + \phi\xi \frac{\partial x_{t+1}}{\partial y_t} - \frac{\partial y_{t+1}}{\partial k_t} \right] + \frac{\partial x_{t+1}}{\partial k_t}\phi\xi + \frac{\partial y_{t+1}}{\partial k_t} = 0. \qquad (48)$$

While it is difficult to provide a general characterization of the local stability properties of steady state equilibria, we have calculated a series of numerical examples. For each of these, the low-capital-stock steady state is locally a saddle point with two positive eigenvalues smaller than one, while the high-capital-stock steady state is locally a saddle point with one eigenvalue smaller than 1 in absolute value. Given that our system has two predetermined state variables, only the low-capital-stock steady state can be approached for all these examples, and the equilibrium path leading to the low-capital-stock steady state is unique.

We proceed to present a numerical example which illustrates these stability properties. We use the same parameters as in our earlier Example 1.

Example 2.

At k_1; $\lambda_1 = 0{:}38$; $\lambda_2 = 0{:}93$; and $\lambda_3 = 1{:}08$;

At k_2; $\lambda_1 = 0{:}15$; and $\mathrm{mod}(\lambda_2) = \mathrm{mod}(\lambda_3) = 1{:}05$:

For this example, the equilibrium path towards the low capital stock steady state is monotonic in the neighborhood of the steady state since the stable roots for k_1 are real and positive.

4.4 Comparative Statics

We now discuss the consequences of changes in the world real interest rate for the steady state equilibrium capital stocks and real exchange rates. We focus on the effects for the low-capital-stock steady state, since our analysis of dynamic properties indicates that is the only attainable steady state.

From Figures 5 and 6, it is evident that an increase in r^* raises the high-activity steady state capital stock, k_2, while it lowers the low activity steady state capital stock, k_1. We summarize this result in the following proposition.

Proposition 4.

a) $\frac{dk}{dr^*}\big|_{k_1} < 0$; $\frac{dk}{dr^*}\big|_{k_2} > 0$;

b) $\lim_{r^* \to H[f^{-1}(q)]} \frac{dk}{dr^*}\big|_{k_1} = -\infty$; $\lim_{r^* \to H[f^{-1}(q)]} \frac{dk}{dr^*}\big|_{k_2} = +\infty$:

Proof. Note that (39) implies that $dr^* = H'(k)\,dk$, and thus $\frac{dk}{dr^*} = \frac{1}{H'(k)}$: The results then follow from Lemma 2, taking into account that $k_1 < f^{-1}(q) < k_2$. ■

The intuition underlying these effects is the following. An increase in r^* must be met by an increase in the steady state equilibrium value of the real return to lending. In the low-capital-stock steady state it is the behavior of the marginal product of capital, $f'(k)$, that determines how the real return to lending varies. Hence an increase in the real interest rate requires a corresponding increase in the rental rate of capital, and thus a decrease in the capital stock. In the high-capital-stock steady state, adjustments in the amount of internal finance, $w(k)$, govern the behavior of the real return to lending, so the level of internal finance must increase to raise the return to lending. This requires a corresponding increase in the real wage, which is associated with an increase in the steady state level of the capital stock.

The impact of an increase in r^* on the steady state equilibrium real exchange rate is ambiguous, and depends critically on the elasticity of substitution between capital and labor in production. From equation (34) it follows that

$$\frac{dx}{dr^*} = \frac{\partial x}{\partial r^*} + \frac{\partial x}{\partial k}\frac{dk}{dr^*}; \tag{49}$$

where

$$\frac{\partial x}{\partial r^*} = \frac{1}{\xi}\left[-{}'(r^*)\, w(k) + \frac{k}{\phi} \right]; \tag{50}$$

$$\frac{\partial x}{\partial k} = \frac{1}{\xi}\left[-(r^*)\, w'(k) + \frac{r^* - 1}{\phi} \right]: \tag{51}$$

It is then straightforward to prove the Lemma 4.

Lemma 4.

a) Let $\frac{\partial x}{\partial r^*}$ be given by (50), then $\frac{\partial x}{\partial r^*} > 0$ iff $-(k) > -\phi -'(r^*)$;

b) Let $\frac{\partial x}{\partial k}$ be given by (51), then $\frac{\partial x}{\partial k} > 0$ iff $r^* \in (1; \hat{r}^*)$;

Proof. Part a) is obvious from the definition of $\frac{\partial x}{\partial r^*}$: Part b) is also straightforward since $\frac{\partial x}{\partial k} > 0$ iff $\frac{1}{w'(k)} > \frac{\phi - (r^*)}{[r^* - 1]} \equiv \Pi(r^*)$: The result then follows from Lemma 3. ■

From equations (49)-(51), Proposition 4 and Lemma 4 it is clear that the effect of an increase in the world interest rate on the real exchange rate is ambiguous and may cause an appreciation or a depreciation of the real exchange rate in either steady state equilibrium. The ambiguity of the effect of an increase in the world interest rate can best be explained by referring to the goods market condition (27). Clearly, abstracting from its effect on the capital stock, a rise in r^* implies an increase in the steady state income of domestic non-relocated depositors, but an increase in loan repayments and hence a decrease in steady state income for domestic entrepreneurs. The net effect of these two forces depends critically on the value of the capital stock k; relative to the value of the wage income $w(k)$. When k is high relative to $w(k)$, which implies that $\Omega(k)$ is high, then the decrease in the income of domestic borrowers outweighs the increase in the income of domestic depositors, and the ensuing loss in domestic consumption will be offset by a depreciation of the real exchange rate which increases the purchasing power of foreign agents, hence $\frac{\partial x}{\partial r^*} > 0$. On the other hand, when k is low relative to $w(k)$, which implies that $\Omega(k)$ is low, then the reverse obtains, and the increase in domestic consumption will be offset by an appreciation of the real exchange rate which reduces the purchasing power of foreign agents, hence $\frac{\partial x}{\partial r^*} < 0$: Thus, the effect of an increase in the world interest rate on the real exchange rate depends crucially on the elasticity of substitution between labor and capital in production, as is reflected in Figures 5 and 6. As is obvious from these figures, a depreciation of the real exchange rate in the low capital stock steady state induced by an increase in the world interest rate, is much more likely to occur in the case where production exhibits relatively low substitutability between capital and labor ($\beta < 0$).

So far we have abstracted from the effect of a rise in the world interest rate on the steady state capital stock. However, this effect also directly affects the goods market and hence the equilibrium real exchange rate. When k falls, which is the case for the low-capital-stock steady state, the number of domestic depositors increases, but their income decreases. On the other hand, the number of domestic lenders decreases, their rental income from capital increases but their loan repayments increase as well. As long as $r^* \in (1, \hat{r}^*)$ this induces a decrease in domestic consumption which is compensated for by an appreciation of the real exchange rate which raises the purchasing power of foreigners, that is $\frac{\partial x}{\partial k}\frac{dk}{dr^*} < 0$. When k increases, which is the case for the high-capital-stock steady state, exactly the reverse holds and $\frac{\partial x}{\partial k}\frac{dk}{dr^*} > 0$.

All in all, the effect of an increase in the world interest rate is complex and only under very specific conditions can we state unambiguous results, which we collect in the following proposition.

Proposition 5. Assume the conditions stated in Proposition 3 hold. Then,

a) $\lim_{r^* \to H[f^{-1}(q)]} \frac{dx}{dr^*}\big|_{k_1} = -\infty$; $\lim_{r^* \to H[f^{-1}(q)]} \frac{dx}{dr^*}\big|_{k_2} = +\infty$;

b) if $0 < \beta < 1$ and $\Omega\left[f^{-1}(q)\right] > -\phi - {}'(r^*)$; then $\frac{dx}{dr^*}\big|_{k_2} > 0$;

Proof. Part a) is obvious from equation (49) and Proposition 4. For part b) notice from Lemma 1 that $\Omega'(k) > 0$ for $0 < \beta < 1$. Hence if $\Omega\left[f^{-1}(q)\right] > -\phi - {}'(r^*)$, then $\Omega(k_2) > -\phi - {}'(r^*)$ as well since $k_2 > f^{-1}(q)$. Consequently, from Lemma 4 a) we know that $\frac{\partial x}{\partial r^*}\big|_{k_2} > 0$. Moreover, if the conditions of Proposition 3 are satisfied then $r^* \in (1, \hat{r}^*)$, thus Lemma 4 b) applies and $\frac{\partial x}{\partial k}\big|_{k_2} > 0$ as well. Finally from Proposition 4 we know $\frac{dk}{dr^*}\big|_{k_2} > 0$ always holds. Hence, all the terms in (49) are positive and thus $\frac{dx}{dr^*}\big|_{k_2} > 0$. ■

We conclude this section with two examples that illustrate our comparative statics results. First, we present an example for which the parameter values are the same as in Example 1. In particular, $\beta = -0.1 < 0$; and the substitutability between labor and capital in production is relatively low. We report only the values for the low-capital-stock steady state since that is the only long-run equilibrium that can be approached.

Example 3.

a) For $r^* = 1.015$, $\mu_\infty = 0.44$; $k_\infty = 6.39$; $x_\infty = 0.01556$; and $a_\infty = -9.1$.

b) For $r^* = 1.020$, $\mu_\infty = 0.42$; $k_\infty = 6.08$; $x_\infty = 0.01558$; and $a_\infty = -8.4$.

c) For $r^* = 1.025$, $\mu_\infty = 0.40$; $k_\infty = 5.82$; $x_\infty = 0.01561$; and $a_\infty = -7.8$.

d) For $r^* = 1.030$, $\mu_\infty = 0.38$; $k_\infty = 5.59$; $x_\infty = 0.01562$; and $a_\infty = -7.3$.

Clearly, for this example, as the world interest rate increases, the fraction of potential entrepreneurs awarded credit decreases in the low-capital-stock steady state, hence the capital stock decreases. At the same time, the real exchange rate depreciates, and net foreign holdings of domestic assets decrease. These results recall the recent experience of Mexico. Indeed, over the period 1987-1993, following the liberalization of its external sector, Mexico had experienced large capital inflows, a rising capital stock and level of output, and an appreciating real exchange rate. During 1994, world real and nominal interest rates rose due to a tightening of monetary policy in the US and Europe. This is commonly believed to have been only a minor factor leading to the reversal of capital inflows and the financial crisis that ensued. Here we demonstrate, in a model driven by financial market frictions, that an increase in the world interest rate can account for a depreciated real exchange rate, a decline in output and investment, and capital outflows - phenomena that were all experienced by Mexico during the 1994-1996 period. Furthermore, our economy displays aggravated credit rationing as the world interest rate increases. This could, loosely speaking, be associated with the difficulties experienced by the Mexican banking sector in 1995 and 1996. It is worth underlining that we obtain these results despite the fact that all prices, including the nominal exchange rate, are perfectly flexible.

We now present a contrasting example for an economy with relatively high substitutability between labor and capital in production ($\beta = 0.1$):

Example 4. Let $f(k) = \left[0.8\, k^{0.1} + 0.35\right]^{10}$; and assume that $q = 1.05$, $g(z) = 1/\bar{z}$ with $\bar{z} = 20.56$, and $\gamma = 20.22$; which implies that $\phi = 11.5$ and $\psi = 0.13$. Assume further that $\xi = 1$; $\alpha = 0.9$, $\varepsilon = 0.45$, $\varepsilon^* = 0.45$; and $\sigma = 1.01$.

a) For $r^* = 1.015$, $\mu_1 = 0.0065$; $k_1 = 0.070$; $x_1 = 0.0079$; and $a_1 = 28$.

b) For $r^* = 1.020$, $\mu_1 = 0.0062$; $k_1 = 0.068$; $x_1 = 0.0067$; and $a_1 = 32$.

c) For $r^* = 1.025$, $\mu_1 = 0.0061$; $k_1 = 0.066$; $x_1 = 0.0055$; and $a_1 = 39$.

d) For $r^* = 1.030$, $\mu_1 = 0.0059$; $k_1 = 0.064$; $x_1 = 0.0044$; and $a_1 = 48$.

For this example, as the world interest rate increases, the fraction of potential entrepreneurs awarded credit decreases in the low-capital-stock steady state, hence the capital stock decreases, while net foreign holdings of domestic assets decrease. However, for this example, these phenomena are accompanied by an appreciating real exchange rate.

5 Conclusion

We have examined a simple model of a small open economy in which goods market frictions allow for permanent deviations from the law of one price, and in which financial market frictions render financial intermediation the efficient arrangement for allocating both credit and liquidity.

Entrepreneurs face a CSV problem in investment projects, which creates a role for banks in efficient capital production. Workers face the possibility of stochastic relation in which only currency can be redeemed, hence banks also insure lenders against random liquidity needs.

We have used this economy to analyze how the operation of financial markets impacts on capital accumulation and exchange rate behavior. We have described conditions under which there are at most two steady states in this economy (in which credit is rationed). One has a low capital stock, a low level of internal project finance, and hence a relatively severe CSV problem. The other has a high capital stock, a high level of internal finance contributed by entrepreneurs, and thus a relatively mild CSV problem. However, we have also established by example that only the low capital stock steady state can be approached in this economy.

We then study the effect of an increase in the world interest rate, and focus on the low-capital-stock steady state. In this steady state, an increase in the world interest rate produces a decrease in the capital stock and output, while the effect on the real exchange rate depends on the elasticity of substitution between labor and capital in production. When that elasticity is relatively low, an increase in the world interest rate may induce a depreciation of the real exchange rate and a reduction in the net holdings of domestic assets by foreigners. These results are reminiscent of the recent experience of emerging economies. Moreover, it is worth underlining that we obtain these results in a model where all prices, including the nominal exchange rate, are perfectly flexible.

Clearly, our set-up can be used to examine the effect of several policy interventions on investment and capital accumulation. Obvious candidates are exchange rate policy, restrictions on international capital flows, and measures with respect to the domestic banking sector, such as the introduction of reserve requirements on domestic and foreign currency. Of particular interest is the question as to whether any of these policies can change the stability properties of the long-run equilibria and can induce the high-capital-stock steady state to be approachable. We could also verify whether any of these interventions can restore the potency of monetary policy in this economy. Indeed, in the framework we discussed, domestic monetary policy is entirely impotent: neither the determination of long-run equilibria nor the stability properties of these equilibria depend on the rate at which the domestic economy issues currency. Finally, after the introduction of aggregate randomness, our economy could be used to evaluate the e®ect of a deposit insurance scheme on investment and output. We leave all these topics for future research.

REFEREE'S APPENDIX

Following Williamson (1986, 1987), a loan contract consists of three objects. First, there is a specified set of project return realizations, A_t, for which verification (monitoring) of the project return occurs at date t. Verification of project returns does not occur if $z \in B_t \equiv [0; \bar{z}] - A_t$. Second, if $z \in A_t$, then the contractual debt repayment can be made contingent on z in a meaningful way, and we denote the real value of such a payment by $R_t(z)$. Finally, if $z \in B_t$, then no such state contingency is possible for debt repayment terms, and the only incentive compatible contract offers a non-conditional real payment of v_t per unit borrowed. Announced contracts are either accepted or rejected by intermediaries. If accepted, the intermediary commits to monitoring project returns whenever an entrepreneur reports $z \in A_t$.

Intermediaries make loans, taking as given the expected gross market return on loans between t and $t + 1$ - r_t. If a loan contract is to have any prospect of acceptance, it must satisfy the expected return constraint

$$\int_{A_t} [R_t(z)I_t - \rho_{t+1}\gamma]\, g(z)dz + v_t I_t \int_{B_t} g(z)dz \geq r_t I_t. \tag{52}$$

Note that the expected monitoring cost, in units of date t goods, depends on the marginal product of capital, ρ_{t+1}, since γ units of capital are expended at $t + 1$ when project returns are verified.

In addition, since only project owners observe project returns directly, an equilibrium contract must have the feature that it is consistent with the incentive for entrepreneurs to correctly reveal when a monitoring state has occurred. The appropriate incentive constraint is:

$$R_t(z) \leq v_t; \quad z \in A_t. \tag{53}$$

Finally, the repayments specified in a contract must be feasible for a borrower so that they never exceed the real value of the capital yielded by an investment project, which in state z is just $\rho_{t+1} zq$.

Thus

$$R_t(z) \leq \frac{\rho_{t+1} z q}{l_t}; \quad z \in A_t \tag{54}$$

$$v_t \leq \inf_{z \in B_t} \frac{\rho_{t+1} z q}{l_t} \tag{55}$$

must both hold.

Announced loan contract terms at date t are therefore selected to maximize the expected utility of a potential entrepreneur

$$\rho_{t+1} q \hat{z} - l_t \int_{A_t} R_t(z) g(z) dz - v_t l_t \int_{B_t} g(z) dz$$

subject to (52)-(55).

The solution to the entrepreneur's problem is to offer a standard debt contract (modified for the presence of internal finance). In particular, the borrower either repays $v_t l_t$ (principal plus interest) or defaults. In the latter case, the lender monitors the project and retains the proceeds net of monitoring costs. Formally,

Proposition 6. Suppose $q > l_t$. Then the optimal loan contract terms satisfy

$$R_t(z) = \frac{\rho_{t+1} z q}{l_t}; \quad z \in A_t \tag{56}$$

$$A_t = \left(0; \frac{v_t l_t}{q \rho_{t+1}}\right) \tag{57}$$

$$\int_{A_t} \left[R_t(z) - \frac{\rho_{t+1} \gamma}{l_t} \right] g(z) dz + v_t \int_{B_t} g(z) dz = r_t. \tag{58}$$

The proof of Proposition 6 is standard and we omit it here.[10]

We can derive a simple expression for the expected return to a financial intermediary by substituting (56) and (57) into (58) to obtain

[10] See Gale and Hellwig (1985) and Williamson (1986, 1987).

$$
\begin{aligned}
r_t &\equiv \int_{A_t} \left[R_t(z) - \frac{\rho_{t+1}\gamma}{l_t} \right] g(z)dz + v_t \int_{B_t} g(z)dz \\
&= v_t \left[1 - G\left(\frac{v_t l_t}{\rho_{t+1} q} \right) \right] - \left(\frac{\rho_{t+1}\gamma}{l_t} \right) G\left(\frac{v_t l_t}{\rho_{t+1} q} \right) + \int_0^{\frac{v_t l_t}{(\rho_{t+1} q)}} \frac{\rho_{t+1} q}{l_t} z g(z)dz \\
&= v_t - \left(\frac{\rho_{t+1}\gamma}{l_t} \right) G\left(\frac{v_t l_t}{\rho_{t+1} q} \right) - \frac{\rho_{t+1} q}{l_t} \int_0^{\frac{v_t l_t}{(\rho_{t+1} q)}} G(z)dz \\
&= \pi \left[v_t, \frac{l_t}{\rho_{t+1}} \right].
\end{aligned} \tag{59}
$$

Assumption 1 implies that $\pi_{11} < 0$. Then if both Assumption 1 and 2 hold, π has the configuration depicted in Figure 2. Therefore, given $\frac{l_t}{\rho_{t+1}}$, there is a unique value of v_t that maximizes the expected return to any financial intermediary. We denote this value by $\psi\left(\frac{l_t}{\rho_{t+1}}\right)$, where the function ψ is defined implicitly by

$$
\pi_1 \left[\psi\left(\frac{l_t}{\rho_{t+1}} \right), \frac{l_t}{\rho_{t+1}} \right] \equiv 1 - \frac{\gamma}{q} g\left[\psi\left(\frac{l_t}{\rho_{t+1}} \right) \frac{l_t}{\rho_{t+1} q} \right] - G\left[\psi\left(\frac{l_t}{\rho_{t+1}} \right) \frac{l_t}{\rho_{t+1} q} \right] \equiv 0. \tag{60}
$$

We now define

$$
\eta \equiv \psi\left(\frac{l_t}{\rho_{t+1}} \right) \left(\frac{l_t}{\rho_{t+1} q} \right). \tag{61}
$$

From Assumption 2, η is a positive constant satisfying $1 - \left(\frac{\gamma}{q}\right) g(\eta) - G(\eta) \equiv 0$. When all potential entrepreneurs offer the gross interest rate ψ that maximizes a potential lender's expected return, η is the critical project return at which a borrower's project income exactly covers loan principal plus interest. Thus, project verification occurs iff $z \in [0, \eta)$.

A feature of the environment that we have just described is that it may be characterized by credit rationing. When this obtains, $v_t = \psi\left(\frac{l_t}{\rho_{t+1}}\right)$ must hold, which implies that all potential entrepreneurs offer contracts that maximize the expected return for a prospective lender. Unfunded entrepreneurs cannot then alter loan contract terms in order to obtain credit since this would simply reduce the expected return to any potential lender and credit rationing may be an equilibrium outcome.

When credit rationing occurs, the expected return to financial intermediaries is given by

$$
\begin{aligned}
r_t &= \pi\left[\phi\left(\frac{l_t}{\rho_{t+1}}\right); \frac{l_t}{\rho_{t+1}}\right] \\
&\equiv \frac{\rho_{t+1}q}{l_t}\left[\phi\left(\frac{l_t}{\rho_{t+1}}\right)\frac{l_t}{\rho_{t+1}q} - \frac{\gamma}{q}G\left[\phi\left(\frac{l_t}{\rho_{t+1}}\right)\frac{l_t}{\rho_{t+1}q}\right] - \int_0^{\frac{\phi l_t}{(\rho_{t+1}q)}} G(z)dz\right] \\
&\equiv \frac{\rho_{t+1}q}{l_t}\left[\varepsilon - \frac{\gamma}{q}G(\varepsilon) - \int_0^{\varepsilon} G(z)dz\right];
\end{aligned}
$$

the last line of which is equation (5).

We can derive also the expected income (utility) for a potential borrower under credit rationing as

$$
\rho_{t+1}q\hat{z} - r_t l_t - \rho_{t+1}\gamma G\left[\phi\left(\frac{l_t}{\rho_{t+1}}\right)\frac{l_t}{\rho_{t+1}q}\right] \equiv \rho_{t+1}q\left[\hat{z} - \frac{\gamma}{q}G(\varepsilon)\right] - r_t l_t;
$$

which is equation (6).

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Figure 1:
Timing of Trade

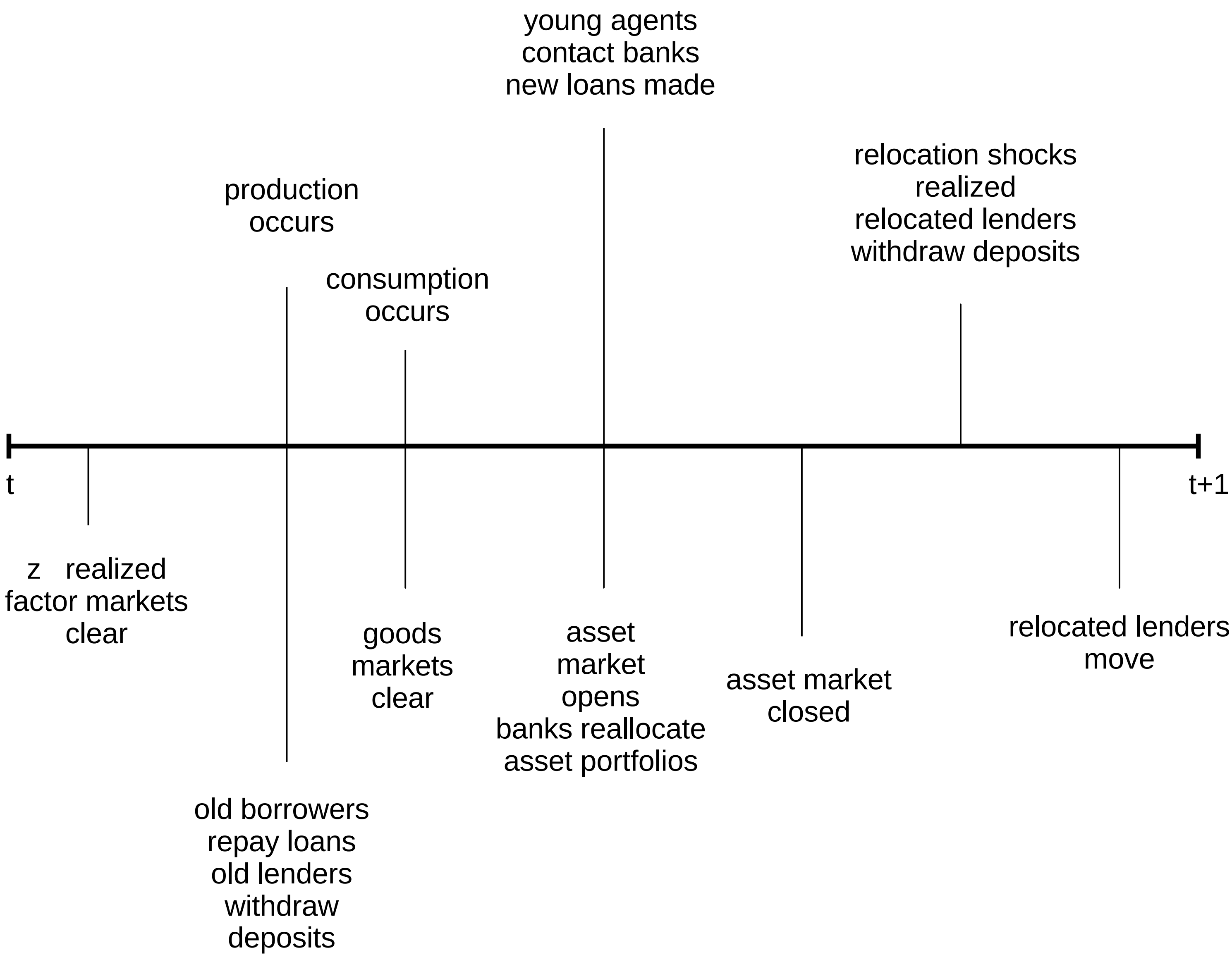


Figure 2:
The Expected Return to Loans

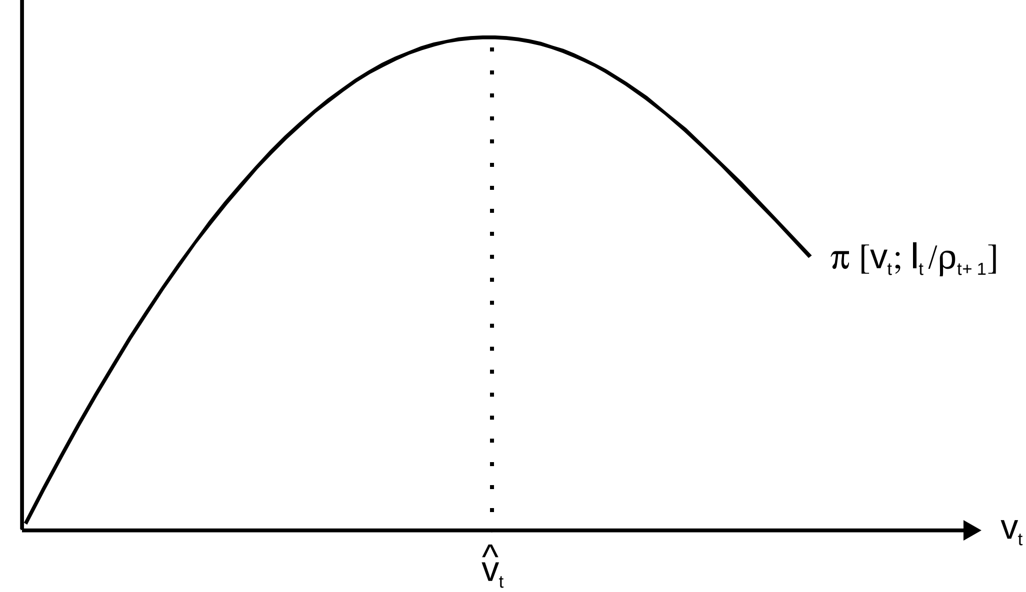
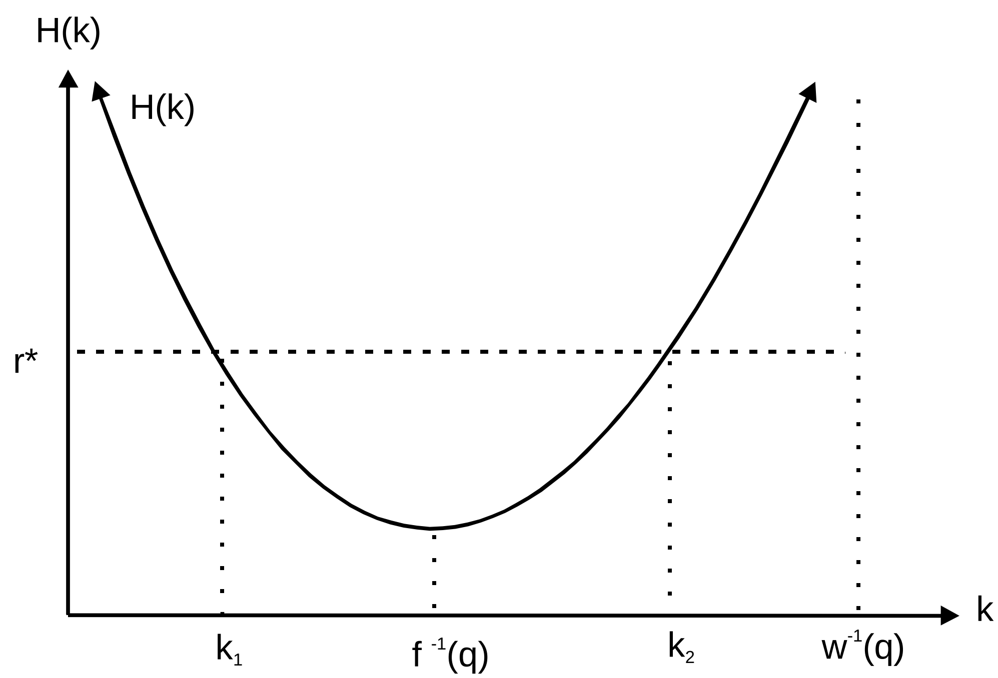


Figure 3:
Equilibrium
Case a: $0 < \beta < 1$



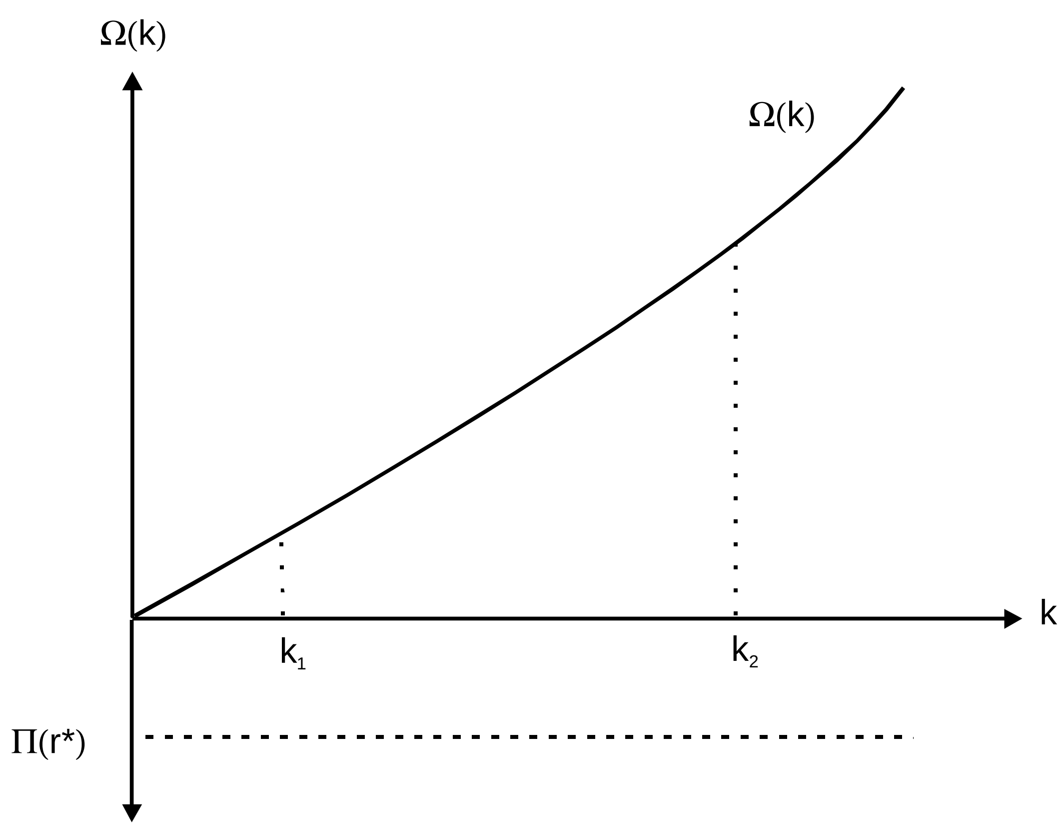
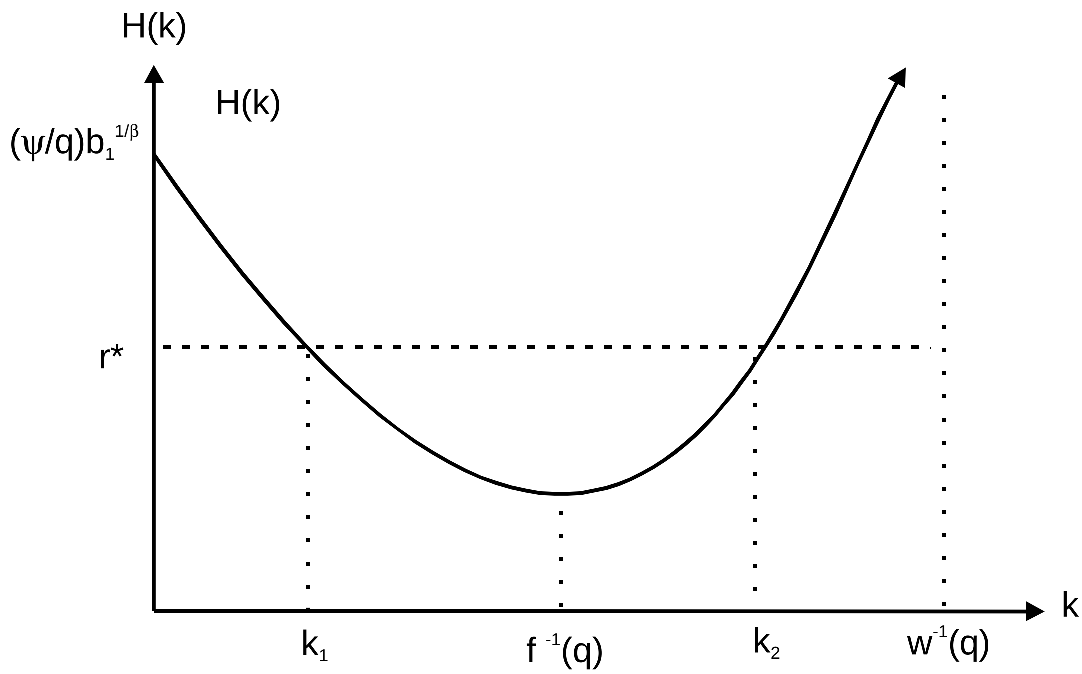


Figure 4:
Equilibrium
Case b: $-\infty < \beta < 0$



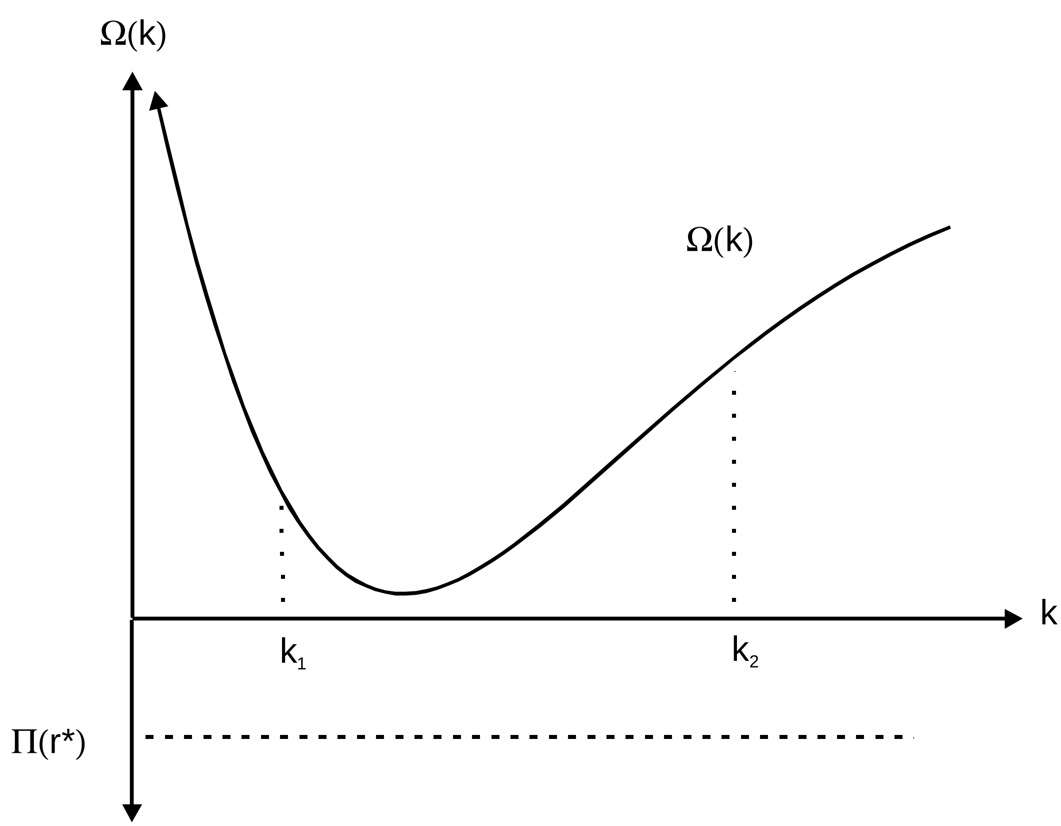
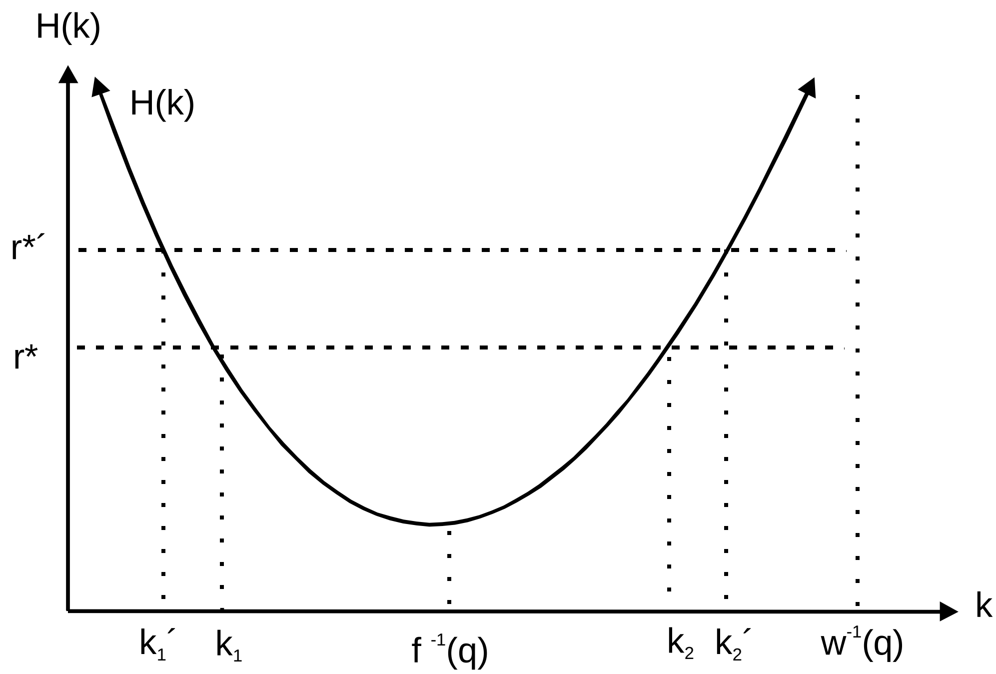


Figure 5:
The Effect of an Increase in r*
Case a: 0 < β < 1



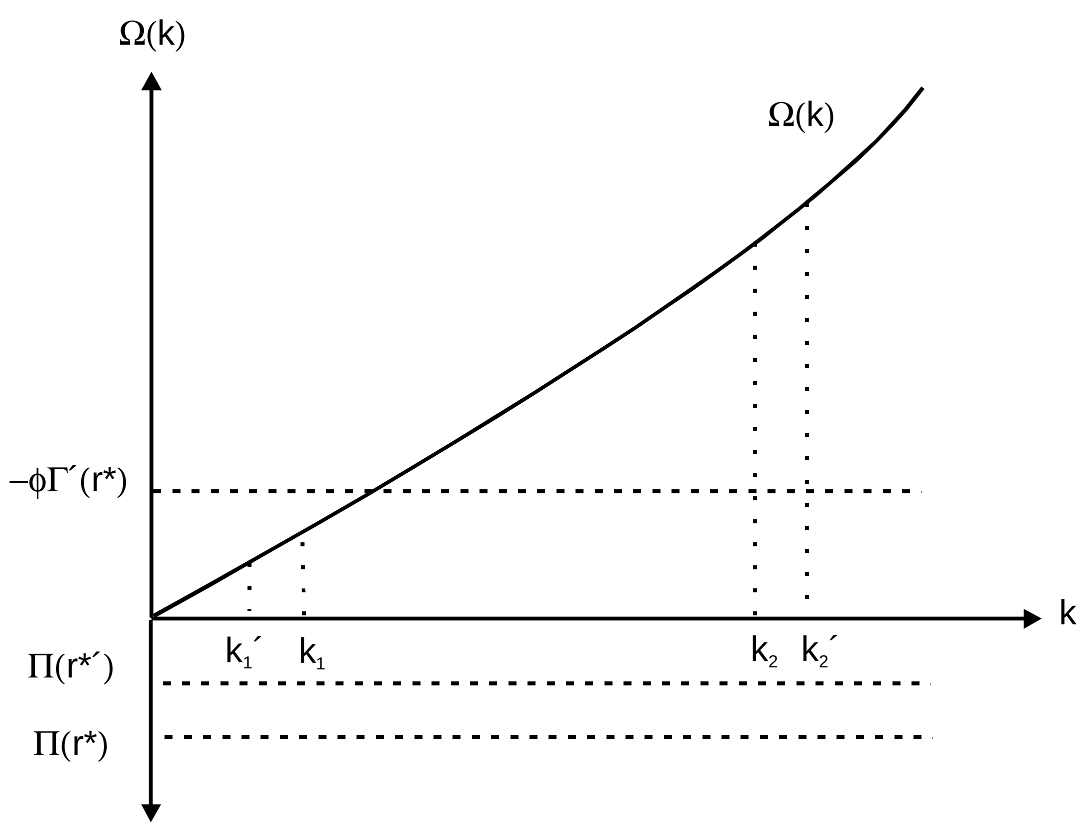


Figure 6:
The Effect of an Increase in r*
Case b: $-\infty < \beta < 0$