# Dollarization and the Integration of International Capital Markets: A Contribution to the Theory of Optimal Currency Areas*

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November 1999

JEL Classification: E5, F4
Keywords: dollarization, indeterminacy, excess volatility

**Abstract**

We consider the following questions: (1) Does the adoption of a common currency either reduce or enhance the scope for endogenously generated volatility to emerge? (2) Does the adoption of a common currency reduce or enhance the scope for indeterminacies to arise? (3) Is there a welfare justification for the adoption of a single currency? (4) What are the fiscal consequences of a move to a single currency? We address these issues in a two-country model with both a transactions, and a store of value role for currency. Moreover, banks arise endogenously in each country. In this context we compare the determination and characteristics of an equilibrium in each of two situations: one where each country issues its own currency, and one where one of the countries adopts the currency of the other country. The consequences of this "dollarization" depend very much on the degree of integration of the capital markets of the two countries. When their credit markets are poorly integrated, a regime with two currencies displays a unique stationary equilibrium. Dollarization, under very weak conditions, gives rise to a continuum of equilibrium paths. These may exhibit oscillation. Hence, when capital markets are segmented, dollarization may be a source of indeterminacy and endogenously arising volatility. In addition, the welfare justifications for dollarizing are weak, and dollarization may have adverse fiscal consequences. When credit markets are fully integrated internationally, the results are substantially different. In that case, both regimes display a unique equilibrium path. Hence, in the presence of unrestricted international capital flows, the adoption of a common currency does not affect the scope for indeterminacy. However, dollarization may still – albeit under relatively extreme conditions – allow "excess volatility" to be observed.

*We thank the ITAM project on "Optimal Monetary Institutions for Mexico" for generous financial support, and Ross Levine, Jack Kareken, Larry Christiano, and participants of a seminar at Iowa State University for very helpful comments on an earlier draft of the paper.
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# 1 Introduction

When Mundell (1961) wrote \A Theory of Optimum Currency Areas," he argued[1] that \it hardly appears within the realm of political feasibility that national currencies would ever be abandoned in favor of any other arrangement." However, in recent years other arrangements are burgeoning. The European Monetary Union will create a single currency area. Moreover, many economies have either adopted or { at least { seriously considered the adoption of a currency board. And now some countries are discussing \dollarization". Dollarization amounts to the unilateral creation of a monetary union which results when one country abandons its currency altogether in favor of another currency, such as the dollar.

Why are so many countries now so willing to consider abandoning { or virtually abandoning, as in the case of a currency board { their own currencies? Giving up a national currency clearly does involve some costs, including the loss of seigniorage revenue and the loss of monetary policy independence. However, there also appear to be several candidate reasons for why a policy like dollarization may be attractive. First, it is a device for committing to a regime of low in°ation. Second, it reduces the costs and uncertainties that arise when international transactions require currency conversion and involve the risk of exchange rate movements. And, third, dollarization presumably limits or eliminates the scope for speculation against a domestic currency. To the extent that currency speculation increases volatility in the domestic economy, this would be a bene¯t.

While many advocate dollarization as a means of promoting price stability, in our view only the second and third reasons constitute potentially compelling arguments for dollarization. Other devices exist for obtaining and committing to low rates of in°ation.[2] And, in Latin America { where

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[1] See Mundell (1961), p.657.

[2] For instance, even communist regimes with no commitment to market mechanisms were able to stabilize hyperin-

much of the discussion of dollarization is occurring { the second reason has limited force as well. As Calvo (1999) has noted, dollars are already in widespread use { and transacting in dollars is already widely feasible { throughout most of Latin America. Thus we focus our attention on the role of dollarization in limiting (or enhancing) the scope for the indeterminacy of equilibrium and for economic °uctuations.

It must also be the case that the issue of primary concern is \endogenously arising" volatility. Indeed, if volatility stems from the conduct of domestic monetary policy, this conduct can be made more \stable." And, if volatility stems from other fundamentals, it must be re°ected somewhere in the economy. Therefore, it seems to us that the volatility of concern must be \market generated" volatility.

In this paper we pose several questions. The two most prominent ones are: (1) does the adoption of a single currency (dollarizing) either reduce or enhance the scope for endogenously generated volatility to emerge? (2) Does the adoption of a common currency reduce or enhance the scope for indeterminacies to arise? The ¯rst issue is obviously important, as it directly addresses one of the primary arguments in favor of dollarization. And, the second issue is signi¯cant because, in the case where indeterminacies arise in the presence of a common currency, there is no well-de¯ned answer to the question of how monetary or other shocks in, say, the United States impact on Mexico. Finally, taken together, we view these two questions as of paramount importance in the analysis of any monetary arrangement. As Friedman (1960, p.23) has argued,

> the central problem [of monetary policy] is not to construct a highly sensitive instrument that can continuously o®set instability introduced by other factors, but rather to prevent monetary arrangements from themselves becoming a primary source of instability.

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°ations in Russia and China essentially by establishing a gold standard. This was accomplished without abandoning a national currency.

Having analyzed these two questions, we then move on to two others: (3) what is the welfare justification for the unilateral adoption of a common currency (one that does not involve an agreement on policy "cooperation"), and (4) what are the fiscal consequences of dollarization?

Despite the large literature on currency boards, and on the creation of monetary unions, there is very little in the way of theoretical frameworks for thinking about their consequences. In this paper we develop a theoretical framework in which the implications of currency boards or dollarization for the presence of indeterminacy and endogenous volatility can be addressed. We consider a two country model which has a transactions as well as a store of value role for currency. It also has a role for a banking system in each country. In this context we compare the determination and characteristics of an equilibrium in each of two situations: one where each country issues its own currency, and one where one of the countries adopts the currency of the other country. We also consider two scenarios with respect to the integration of the financial systems of the two countries: one where the two countries have well integrated capital markets, and one where they do not. Finally, to fix ideas, when there are two currencies we confine attention to a regime of flexible exchange rates. However, our results apply virtually unaltered to a comparison of dollarization with a regime of two currencies under fixed exchange rates. And, our results also apply intact to a comparison of a two currency, fixed or flexible exchange rate regime with the adoption of a currency board.

With respect to our first two questions, our results indicate that the determination and characteristics of equilibrium depend very much on the extent to which credit markets are integrated internationally. When there are no capital flows between the two countries, a regime with two currencies displays a unique stationary equilibrium, while "dollarization", under very weak conditions, gives rise to a continuum of equilibrium paths. These paths may display oscillation around the steady state equilibrium. Thus dollarization is not a remedy for indeterminacies or excess volatility; when domestic capital markets are poorly integrated with world capital markets it is a formula for generating them. On the other hand, when capital markets are fully integrated internationally, our economy always displays a unique equilibrium path. When the world is dollarized, that equilibrium path may display oscillation. Thus, when domestic and international capital markets are well integrated, the adoption of a common currency does not a®ect the scope for indeterminacies to arise. And, while endogenously generated volatility is still possible under dollarization, the conditions required for it to emerge are much more restrictive when domestic and world capital markets are well integrated than when they are not. Finally, while our analysis focuses on the scope for indeterminacies and endogenously arising volatility, our results also have strong implications for the answer to the question of how an economy responds to various shocks under the di®erent monetary regimes. In particular, this question has an unambiguous answer under a regime of two national currencies, and in a dollarized world with integrated capital markets. However, in a dollarized world with segmented capital markets there is a continuum of possible reactions to any unanticipated shock, either permanent or temporary.

Why does dollarization promote indeterminacies of equilibrium and endogenous volatility when international capital markets are poorly integrated? The reason is simple: with two currencies there is a well-de¯ned supply of, and demand for the currency of each country. Along with domestic goods market clearing conditions, the necessity of achieving an equilibrium in money markets determines the price level in each country, as well as the real and nominal exchange rate. On the other hand, when there is only a single currency, the distribution of this currency across countries matters for the domestic price levels. What determines this distribution? With limited mobility of capital, the distribution is primarily determined by agents beliefs about prices. If agents believe that prices in

Mexico are high, they will bring a large quantity of dollars to Mexico to make purchases. And, this validates the expectation of high Mexican prices. Moreover, if cross-border currency °ows can be relatively large in each direction, dollar in°ows to a country today are readily reversed tomorrow. Thus endogenously arising volatility can easily be observed as well. In short, dollarization becomes a device for making beliefs matter.

This situation changes as capital market transactions become less costly. When it is cheap for agents to borrow or lend internationally, in°ows of dollars for goods purchases can be o®set by corresponding ¯nancial transactions. Thus as domestic and world capital markets become better integrated, the scope for intedeterminacy and endogenous volatility in a dollarized economy are correspondingly reduced.[3]

What about the ¯scal consequences of dollarization? We show that when international capital markets are segmented, dollarization always implies a reduction in steady state world seigniorage revenue when the country that continues to issue a ¯at currency allows the stock of that currency to grow either at a very low rate, or at a very high rate. Hence, in those cases, some government must raise taxes to keep world government expenditures unchanged.[4]

Finally, the scope for improving steady state welfare as a consequence of dollarization is limited in our economy. Indeed, when international capital markets are segmented, the welfare of domestic borrowers always decreases with dollarization, while the only potential sources of an increase in welfare for lenders are a decrease in the rate of in°ation, and an appreciation of the real exchange

[3] This result contrasts with that obtained in cash-in-advance models. For example, in King, Wallace and Weber (1992), ease of transacting in all markets and all currencies promotes the indeterminacy of equilibrium.

[4] In other words, under appropriate conditions, having the U.S. share the seigniorage revenue it gains when others dollarize is not enough to mitigate the ¯scal consequences of dollarization.

rate. But, in our view, a reduction in the rate of inflation is a very weak rationale for dollarizing. An identical reduction in inflation could be achieved by having the dollarizing economy retain its own currency, and simply reduce its rate of money creation. And, under weak conditions, dollarization will not cause the terms of trade to move in favor of the dollarizing economy. We regard all these results as calling into question the desirability of the adoption of a single currency if the capital markets of the countries in question are not well integrated.

Clearly, our analysis provides some new criteria for defining and characterizing what constitutes an "optimum currency area." We have argued that it is reasonable for two countries to become a common currency area only if their capital markets are well integrated. Or, put otherwise, two regions constitute an optimal currency area only if capital is reasonably mobile between them. Of course all theories of optimal currency areas emphasize mobility of some type. Mundell (1961) defined an optimum currency area as a region with high internal capital and labor mobility: "an essential ingredient of a common currency, or a single currency area, is a high degree of factor mobility" (Mundell, 1961, p. 661). Mc Kinnon (1963) argued that an optimum currency area is one with substantial mobility of goods. And, while Mundell's original paper specifically mentioned both capital and labor mobility as essential ingredients of an optimum currency area, later contributions focused on labor market integration as the single most important criterion for assessing the desirability of a common currency. Interestingly, Calvo (1999, p.3) points to the fact that the optimum currency area literature has had little to say about financial issues. In contrast, our paper focuses attention on the importance of capital mobility in defining an optimum currency area.

Moreover, our criterion regarding capital mobility has empirical content, since it is possible to assess how well integrated various capital markets are with world markets. For example, both Bekaert and Harvey (1995) and Korajczyk (1996) question the integration of both Argentinian and

Mexican ¯nancial markets with world capital markets. In our analysis this suggests that they are not good candidates for dollarization. Interestingly, Centeno and Mello (1999) report that even for European countries, the evidence points to considerable market segmentation in bank lending. This result calls into question whether Europe is an optimum currency area. However, one drawback of all the empirical evidence mentioned is that it does not take into account developments in the second half of the 1990s. Finally James' (1978) results on the lack of integration of postbellum American regional ¯nancial markets suggest that even the U.S. was not an optimal currency area in the late 19th century.

The remainder of the paper is structured as follows. Section 2 presents the model environment. Section 3 describes equilibrium when both countries issue a national ¯at currency. We ¯rst describe the case of segmented international capital markets, and then turn our attention to the case of international capital mobility. Section 4 describes the behavior of a dollarized economy. Again we ¯rst present the case of international capital market segmentation, followed by the case of international capital market integration. Section 5 discusses the ¯scal consequences of dollarizing, while results on the welfare implications of dollarization are presented in Section 6. Concluding remarks are o®ered in Section 7.

# 2 The Environment

## 2.1 Description

We consider a two country, single good, pure exchange economy.[5] Each country is inhabited by

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[5] This model is a two country version of Champ, Smith and Williamson (1996), and a two country, pure exchange version of Bhattacharya, et al. (1997), and Schreft and Smith (1997, 1998). Its closest analog is Betts and Smith

an infinite sequence of two-period-lived overlapping generations and an infinitely lived government. The consumption good is perishable and cannot be transported between countries or locations.

There are two symmetric locations in each country, and at each date $t = 0, 1, \ldots,$ a continuum of young agents of measure one is born in each location. Half of these agents are "lenders" and the remaining half are "borrowers".[6] Borrowers in the domestic (foreign) country receive no goods endowment when young, and are endowed with $w^d$ $(w^f) > 0$ units of the consumption good when old. Moreover, borrowers in either country care only about youthful consumption, and we will assume their lifetime utility is given by $u(c_1, c_2) = c_1$, where $c_j \in \Re_+$ denotes consumption in period $j$ of life. Lenders in the domestic (foreign) country have an endowment $y^d$ $(y^f) > 0$ when young, and no endowment when old. They care only about old age consumption and their lifetime utility is given by $u(c_1, c_2) = \ln c_2$.

In addition, lenders, and lenders alone, face the possibility of random relocation.[7] Specifically, all young domestic (foreign) lenders face the probability $\pi_d^d$ $(\pi_f^f)$ of being relocated to the "other" location within the country in which they are born, and the probability $\pi_f^d$ $(\pi_d^f)$ of moving abroad. Young domestic (foreign) lenders remain in their initial location with probability $1 - \pi_d^d - \pi_f^d > 0$ $(1 - \pi_f^f - \pi_d^f > 0)$. Relocation probabilities are constant across time, known by all inhabitants, symmetric across locations within the same country, and relocation realizations are iid across young lenders. Thus, despite the presence of individual uncertainty for young lenders, there is no aggregate uncertainty in the economy.

(1997).

[6] Since we allow their endowments to differ, this equal division of the population between borrowers and lenders entails no loss of generality. The same remark applies to the equality of the populations accross countries.

[7] We do not allow borrowers to be relocated, as this would raise issues about how they repay their loans.

We now describe the timing of events in this world. At the beginning of a period, there is no communication across locations nor any movement of agents.[8] Young lenders and old borrowers receive their endowments, goods market trade takes place in each location, and consumption of goods by old lenders and young borrowers follows immediately. Following consumption, domestic and international asset markets open. Now agents can communicate without restriction across locations, and trade freely in all assets. These include the currencies issued by both countries, as well as loans to borrowers in both countries. Once asset markets close, all cross-location communication ceases, and relocation shocks are realized. At this time, there is no opportunity for portfolio adjustment or liquidations of assets. In particular, relocated agents can not now interact with other lenders or with borrowers. Therefore, relocated agents - of which there are a measure $¼_d^d + ¼_f^d$ ($¼_f^f + ¼_d^f$) in the domestic (foreign) country - contact their bank, withdraw the required currency, and move. This timing of transactions and trade is depicted in Figure 1.

These assumptions on the timing of trade and cross-location communication prevent the veri¯-cation, and hence the use of cheques, or private credit instruments, outside their location of issue. Currency, on the other hand, is universally recognizable, non-counterfeitable and thus acceptable in inter-location exchange. In addition, we assume that all goods acquired with cash must be purchased using the currency that is legal tender in the location of the seller.[9]

In this environment, relocation shocks are also liquidity shocks, which require lenders to liquidate higher yielding assets in exchange for currency. Our assumptions provide a well-de¯ned motive for holding various currencies irrespective of their relative rates of return. We will focus on equilibria

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[8] As in Townsend (1987), spatial separation and limited communication create a transactions role for currency. See also Mitsui and Watanabe (1989), and Hornstein and Krusell (1993).

[9] This assumption allows us to avoid the indeterminacy of exchange rates discussed by Kareken and Wallace (1981).

in which currencies are strictly dominated in rate of return by interest-earning assets (loans). In particular, young lenders will have an incentive to diversify their asset portfolios by holding both types of currency, as well as interest earning assets. Moreover, as in Diamond and Dybvig (1983), banks will arise endogenously in this setting to insure lenders against random liquidity (or relocation) shocks.

We now describe the behavior of individuals and banks, and analyze perfect foresight equilibria. We start with the case where both countries print a fiat currency and accept as legal tender only the currency that is issued locally. We then turn to the case of a "dollarized" world, where only the foreign country issues a fiat currency, while the domestic country imposes as legal tender the currency issued by the foreign country.

## 3 Each Country Issues its Own Currency

When both countries issue their own fiat money, we assume that the domestic (foreign) government selects, for all time, a fixed rate of money creation, which we denote by $\sigma^d \left(\sigma^f\right)$. Thus, $M_t^d = \sigma^d M_{t-1}^d \left(M_t^f = \sigma^f M_{t-1}^f\right) \forall t \geq 0$, with $M_0^d \left(M_0^f\right)$ given as initial conditions, and where $M_t^d \left(M_t^f\right)$ denotes the outstanding stock of nominal domestic (foreign) balances per domestic (foreign) lender at time t. To fix ideas, when two currencies are issued, we assume that there is a flexible exchange rate regime in place. However, all of our results also obtain under a regime of fixed exchange rates between the two currencies.

The creation of fiat currency generates seigniorage revenue which is used to finance an endogenously determined stream of government expenditures.[10] We assume that individuals derive no

[10] The analysis is slightly – but only slightly – more complicated if there is an exogenously given stream of government expenditures in each country. All of our results apply in a virtually unaltered form to this case as well.

utility from government expenditures. We let $g_t^d$ $\left(g_t^f\right)$ denote domestic (foreign) government consumption of final goods at time t; per domestic (foreign) lender, while $p_t^d$ $\left(p_t^f\right)$ denotes the time t domestic (foreign) price level. Then $m_t^d \equiv \frac{M_t^d}{p_t^d}$ $\left(m_t^f \equiv \frac{M_t^f}{p_t^f}\right)$ will denote domestic (foreign) real balances, per domestic (foreign) lender. Thus, the government budget constraints of the domestic and foreign government imply that

$$g_t^j = \frac{M_t^j - M_{t-1}^j}{p_t^j} = m_t^j \left( \frac{\sigma^j - 1}{\sigma^j} \right) \quad t \geq 0; \; j = d; f. \tag{1}$$

## 3.1 Behavior of Agents and Banks

Let $R_t^d$ denote the gross real rate of interest on loans in the domestic country between time t and t + 1. A young domestic borrower will then choose a loan quantity $b_t^d$ to maximize $c_{1t}^d$ subject to $c_{1t}^d \leq b_t^d$, and $0 \leq w^d - R_t^d b_t^d$. Clearly, the solution to this problem sets $b_t^d = \frac{w^d}{R_t^d}$. Similarly, for the foreign country, $b_t^f = \frac{w^f}{R_t^f}$.

For young lenders, matters are substantially more complex. These agents face the risk of relocation, and as a consequence it can be shown that it is not optimal for lenders to hold assets directly [Greenwood and Smith (1997)]. They will prefer to have their savings intermediated by banks. Banks take deposits and use them to hold primary assets - money and loans - directly. Domestic banks promise to pay lenders who are relocated domestically $r_{dt}^d$ units of the domestic good at t + 1 per unit of domestic good saved. Domestic depositors who are relocated abroad are promised a real return of $r_{ft}^d$ units of the foreign good at t + 1 per unit of domestic good deposited, and $r_t^d$ is the real return paid at t + 1 to lenders who remain in their original location.

We assume that there is free entry into banking and that banks behave competitively in the sense that they take the real returns on assets as given. On the deposit side we assume that intermediaries are Nash competitors. That is, banks announce deposit return schedules $(r^d_{dt}, r^d_{ft}, r^d_t)$, taking the announced return schedules of other banks as given. These announced return schedules must satisfy a set of constraints, which we now describe.

A young domestic lender, caring only for old age consumption, will deposit his entire savings, $y^d$, with a bank. Per young depositor, the bank acquires an amount $m^d_{dt}$ of domestic real balances, and an amount $m^f_{dt}$ of foreign real balances. The former is measured in units of domestic goods, while the latter is measured in units of foreign goods. Let $e_t$ denote the domestic currency price of a unit of foreign exchange at t, and let $x_t \equiv \frac{e_t p^f_t}{p^d_t}$ denote the real exchange rate, or - in other words - the time t price of foreign goods in units of domestic goods. Hence the domestic goods value of the bank's holdings of foreign real balances will be $x_t m^f_{dt}$. In addition, the bank holds domestic loans of $l^d_t$ per depositor, and an amount $i^d_t$ of loans issued to foreign banks.[11] Thus the bank's balance sheet requires that

$$m^d_{dt} + x_t m^f_{dt} + l^d_t + i^d_t \leq y^d, \; t \geq 0. \tag{2}$$

In addition, announced deposit returns must satisfy the following constraints. First, domestically relocated agents, of whom there are $\pi^d_d$ per domestic depositor, must be given domestic currency, since that is the only asset which will allow these agents to consume in their new location. This is accomplished by using the bank's holdings of domestic real balances. Since $p^d_t$ denotes the time t domestic price level, the real return on these balances is $p^d_t/p^d_{t+1}$ between t and t + 1. Thus

$$\pi^d_d r^d_{dt} y^d \leq m^d_{dt} \frac{p^d_t}{p^d_{t+1}}, \; t \geq 0 \tag{3}$$

[11] Since goods are not transportable across locations, these loans - and their repayments - will be made with currency.

must hold. By the same token, a fraction $\pi_f^d$ of domestic depositors is relocated abroad. These lenders must be given the currency which is legal tender in the foreign country. Thus payments made to these agents are constrained by the bank's holdings of foreign real balances, $m_{dt}^f$. The real return, in units of foreign goods per unit of domestic good, on these balances is $p_t^d/e_t p_{t+1}^f$ between $t$ and $t+1$. Therefore,

$$\pi_f^d r_{ft}^d y^d \le x_t m_{dt}^f \frac{p_t^d}{e_t p_{t+1}^f};\ t \ge 0 \tag{4}$$

must be satisfied.

Furthermore, we will only be interested in a world where interest-bearing assets (loans) dominate currencies in rate of return. Under this condition, it is easy to show that a bank will never choose to carry money balances between $t$ and $t+1$. "Non-movers", of whom there are a fraction $1-\pi_d^d-\pi_f^d$, will therefore be paid out of the return on the bank's holdings of loans to domestic borrowers and to foreign banks. Let $R_{ft}^d$ be the gross real rate of return on loans by a domestic bank to a foreign bank. Then, when $R_t^d > p_t^d/p_{t+1}^d$ and $R_{ft}^d > p_t^d/e_t p_{t+1}^f$, $r_t^d$ must satisfy

$$\left(1-\pi_d^d-\pi_f^d\right) r_t^d y^d \le R_t^d l_t^d + R_{ft}^d i_t^d;\ t \ge 0. \tag{5}$$

An absence of arbitrage opportunities will require that $R_t^d = R_{ft}^d$. Noting that (2) will hold with equality, we can then transform the preceding equations as follows. Let $\gamma_{dt}^d \equiv \frac{m_{dt}^d}{y^d}$ be the ratio of domestic reserves to deposits, and let $\gamma_{ft}^d \equiv \frac{x_t m_{dt}^f}{y^d}$ be the ratio of foreign reserves to deposits. Then $1-\gamma_{dt}^d-\gamma_{ft}^d \equiv \frac{l_t^d+i_t^d}{y^d}$ is the ratio of interest-bearing assets to deposits, and the constraints (3)-(5) can be written as

$$r_{dt}^d \le \frac{\gamma_{dt}^d}{\pi_d^d}\frac{p_t^d}{p_{t+1}^d};\ t \ge 0 \tag{6}$$

$$r^d_{ft} \le \left(\frac{\gamma^d_{ft}}{\pi^d_f}\right)\frac{p^d_t}{e_t p^f_{t+1}}; \; t \ge 0 \tag{7}$$

$$r^d_t \le \frac{\left(1 - \gamma^d_{dt} - \gamma^d_{ft}\right)}{1 - \pi^d_d - \pi^d_f} R^d_t; \; t \ge 0. \tag{8}$$

In addition, holdings of currencies are constrained to be non-negative. Hence $\gamma^d_{dt} \ge 0$ and $\gamma^d_{ft} \ge 0$ must hold, $\forall t \ge 0$.

Competition for depositors among banks will, in equilibrium, force banks to choose return schedules and portfolio allocations so as to maximize the expected utility of a representative depositor, subject to the constraints we have described. Thus, in equilibrium, banks choose values for $r^d_{dt}$, $r^d_{ft}$, $r^d_t$, $\gamma^d_{dt}$, and $\gamma^d_{ft}$ to solve the problem

$$\max \pi^d_d \ln\left(r^d_{dt} y^d\right) + \pi^d_f \ln\left(r^d_{ft} y^d\right) + \left(1 - \pi^d_d - \pi^d_f\right) \ln\left(r^d_t y^d\right)$$

subject to (6)-(8), $\gamma^d_{dt} \ge 0$ and $\gamma^d_{ft} \ge 0$. The solution to this problem sets $\gamma^d_{dt} = \pi^d_d$, $\gamma^d_{ft} = \pi^d_f$, $r^d_{dt} = \frac{p^d_t}{p^d_{t+1}}$, $r^d_{ft} = \frac{p^d_t}{e_t p^f_{t+1}}$, and $r^d_t = R^d_t$. The problem faced by foreign banks is completely analogous to that of domestic banks. Hence the solution to that problem sets $\gamma^f_{ft} = \pi^f_f$, $\gamma^f_{dt} = \pi^f_d$, $r^f_{ft} = \frac{p^f_t}{p^f_{t+1}}$, $r^f_{dt} = \frac{e_t p^f_t}{p^d_{t+1}}$, and $r^f_t = R^f_t$. We now turn to the analysis of perfect foresight equilibria in this environment.

## 3.2 General Equilibrium: Segmented Capital Markets

In this section we assume there are no international capital flows, so that national credit markets are segmented. Hence $i^d_t = i^f_t = 0$. We will later relax this assumption, which will allow us to evaluate how the integration of international capital markets affects our results.

In the absence of international capital flows, a perfect foresight equilibrium is a set of sequences $\left\{R^j_t\right\}, \left\{p^j_t\right\}, \left\{m^j_t\right\}; j = d, f$; and $\{x_t\}$ which satisfies six conditions. First loan markets must clear in both countries. Thus

$$l_t^d \;=\; b_t^d \quad \text{or} \quad \left(1-\gamma_{dt}^d-\gamma_{ft}^d\right)y^d = \left(1-\pi_d^d-\pi_f^d\right)y^d = \frac{w^d}{R_t^d} \tag{9}$$

$$l_t^f \;=\; b_t^f \quad \text{or} \quad \left(1-\gamma_{ft}^f-\gamma_{dt}^f\right)y^f = \left(1-\pi_f^f-\pi_d^f\right)y^f = \frac{w^f}{R_t^f} \tag{10}$$

must hold. Clearly, equations (9) and (10) imply that

$$R_t^d \;=\; \frac{w^d}{y^d\left(1-\pi_d^d-\pi_f^d\right)}\ \forall t; \tag{11}$$

$$R_t^f \;=\; \frac{w^f}{y^f\left(1-\pi_f^f-\pi_d^f\right)}\ \forall t. \tag{12}$$

Second, the domestic and foreign goods markets must clear at all dates. The supply of goods in the domestic country is simply the sum of the endowments received by young lenders and old borrowers. The demand for domestic goods at date t includes the consumption of young domestic borrowers, who each consume $\frac{w^d}{R_t^d}$. In addition, there are $\pi_d^d$ old domestic lenders who were relocated domestically and $\pi_d^f$ old foreign lenders who were relocated from abroad. Together, these agents hold the entire outstanding stock of date $t-1$ domestic nominal balances. Moreover, there are $\left(1-\pi_d^d-\pi_f^d\right)$ old domestic lenders who were not relocated at all, and who consume the proceeds of their intermediated savings. Finally, the domestic government consumes $g_t^d$ units of the final good, per young lender. Taking into account equation (1), the domestic goods market clearing condition thus takes the form

$$y^d + w^d = \frac{w^d}{R_t^d} + \frac{M_{t-1}^d}{p_t^d} + \left(1-\pi_d^d-\pi_f^d\right)r_{t-1}^d y^d + \left(\frac{\sigma^d-1}{\sigma^d}\right)\frac{M_t^d}{p_t^d}\ \forall t \geq 1. \tag{13}$$

Using the equilibrium solution to the bank's problem, and the fact that $\frac{M_t^d}{\sigma^d p_t^d} = \frac{M_{t-1}^d}{p_t^d}$, the domestic goods market clearing condition can be rewritten as

$$y^d + w^d = \frac{w^d}{R_t^d} + \left(1-\pi_d^d-\pi_f^d\right)R_{t-1}^d y^d + \frac{M_t^d}{p_t^d}\ \forall t \geq 1. \tag{14}$$

Moreover, after substituting equation (11) into (14), we obtain

$$m_t^d = \left(\pi_d^d + \pi_f^d\right) y^d \quad \forall t \geq 1. \tag{15}$$

Hence the goods market clearing condition allows us to determine equilibrium domestic real balances at every date. The same reasoning for the foreign country establishes

$$m_t^f = \left(\pi_f^f + \pi_d^f\right) y^f \quad \forall t \geq 1. \tag{16}$$

Finally, the money market clearing conditions have to be satisfied for both countries at all dates. Thus, for the domestic country, we have

$$m_t^d = \gamma_{dt}^d y^d + \gamma_{dt}^f y^f x_t \quad \forall t \geq 0. \tag{17}$$

Making use of the solution to the bank's problem, we can rewrite equation (17) to yield

$$m_t^d = \pi_d^d y^d + \pi_d^f y^f x_t \quad \forall t \geq 0. \tag{18}$$

Combined, equations (15) and (18) determine the equilibrium real exchange rate, $x_t$, at all dates,

$$x_t = \frac{\pi_f^d y^d}{\pi_d^f y^f} \quad \forall t \geq 1. \tag{19}$$

By Walras' law, the money market clearing condition for the foreign country delivers the same result.

Clearly, when each country prints its own flat money and no international capital flows are allowed, there is a unique equilibrium path: the economy is at all dates in the steady state equilibrium characterized by equations (11), (12), (15), (16), and (19). Of course, we have to verify that the domestic (foreign) flat currency is dominated in rate of return by domestic (foreign) loans at all dates. Clearly, that will be the case if and only if $\frac{1}{y^d}\left(\frac{w^d}{1-\pi_d^d-\pi_f^d}\right) > \frac{1}{\sigma^d}$ $\left(\frac{1}{y^f}\left(\frac{w^f}{1-\pi_f^f-\pi_d^f}\right) > \frac{1}{\sigma^f}\right)$. Thus, when these conditions are satisfied, there exists a unique equilibrium with both currencies being dominated in rate of return.

To conclude this section, we provide an example of the kind of economy that we just described.

**Example 1** Let

$y^d = 2$; $w^d = 1.75$; $\pi_d^d = 0.1$; $\pi_f^d = 0.1$; and $\sigma^d = 1.1$; while

$y^f = 10$; $w^f = 8.5$; $\pi_f^f = 0.1$; $\pi_d^f = 0.1$; and $\sigma^f = 1.05$.

Then $R^d = 1.0937$; $R^f = 1.0625$; $x = 0.2$; $m^d = 0.4$; $m^f = 2$; $g^d = 0.036$; $g^f = 0.095$.

## 3.3 General Equilibrium: Integrated Capital Markets

We will now allow for international capital flows between the two economies, so that $i_t^d \neq 0$, and $i_t^f \neq 0$ may hold. In this case, the domestic and foreign loan market clearing conditions become

$$\left(1-\pi_d^d-\pi_f^d\right)y^d - i_t^d = \frac{w^d}{R_t^d}\ \forall t \geq 0 \tag{20}$$

$$\left(1-\pi_f^f-\pi_d^f\right)y^f - i_t^f = \frac{w^f}{R_t^f}\ \forall t \geq 0. \tag{21}$$

Moreover, the market for international loans has to clear. Hence

$$i_t^d + x_t i_t^f = 0\ \forall t \geq 0; \tag{22}$$

has to be satisfied at all dates. In addition, since $R^{d}_{ft} = R^{f}_{t}\frac{x_{t+1}}{x_t}$ must be satisfied, international loans will only be held in equilibrium when the following uncovered interest parity condition holds at all dates:

$$R^{d}_{t} = R^{f}_{t}\frac{x_{t+1}}{x_t} \quad \forall t \geq 0. \tag{23}$$

The domestic goods market and money market clearing conditions continue to be given by equations (14) and (18). Upon substituting equation (18) into equation (14), rearranging terms, and bringing the result forward one period, we obtain

$$R^{d}_{t+1} = \frac{w^{d}}{\left(1-\pi^{d}_{d}\right)y^{d} + w^{d} - \pi^{f}_{d}y^{f}x_{t+1} - \left(1-\pi^{d}_{d}-\pi^{d}_{f}\right)y^{d}R^{d}_{t}} \quad \forall t \geq 1. \tag{24}$$

On the other hand, equations (20)-(22) imply that

$$x_t = -\frac{i^{d}_{t}}{i^{f}_{t}} = \frac{\left(1-\pi^{d}_{d}-\pi^{d}_{f}\right)y^{d} - \frac{w^{d}}{R^{d}_{t}}}{\frac{w^{f}}{R^{f}_{t}} - \left(1-\pi^{f}_{f}-\pi^{f}_{d}\right)y^{f}} \quad \forall t \geq 0. \tag{25}$$

Rearranging terms in equation (25), and taking into account the uncovered interest parity condition presented in (23), yields

$$x_{t+1} = \left[\left(1-\pi^{f}_{f}-\pi^{f}_{d}\right)y^{f}x_t + \left(1-\pi^{d}_{d}-\pi^{d}_{f}\right)y^{d}\right]\frac{R^{d}_{t}}{w^{f}} - \frac{w^{d}}{w^{f}} \quad \forall t \geq 0. \tag{26}$$

Equations (24) and (26) describe the evolution of the equilibrium sequences $\left\{R^{d}_{t}\right\}$ and $\{x_t\}$.

### 3.3.1 Steady State Equilibria

In a steady state equilibrium, equation (26) becomes

$$x = \frac{\left(1 - \pi^d_d - \pi^d_f\right) y^d R^d - w^d}{w^f - \left(1 - \pi^f_f - \pi^f_d\right) y^f R^d}; \tag{27}$$

where we now omit time subscripts. Similarly, equation (24) reduces to

$$x = \frac{-\left(R^d\right)^2 \left(1 - \pi^d_d - \pi^d_f\right) y^d + R^d \left[\left(1 - \pi^d_d\right) y^d + w^d\right] - w^d}{\pi^f_d y^f R^d}. \tag{28}$$

A steady state equilibrium is a pair of values $R^d > 0$ and $x > 0$ which solve equations (27) and (28), and which have $\sigma^d R^d > 1$. We proceed to establish conditions under which our economy displays a unique steady state equilibrium, as depicted in Figure 2.

**Proposition 1** When both the domestic and the foreign country issue a flat currency, and when loan markets are fully integrated internationally, the economy exhibits a unique steady state equilibrium if

(a) $\frac{w^d}{y^d\left(1 - \pi^d_d - \pi^d_f\right)} > \frac{w^f}{y^f\left(1 - \pi^f_f - \pi^f_d\right)} > \max\left\{\frac{1}{\sigma^d}; \frac{1}{\sigma^f}\right\}$, or

(b) $\frac{w^f}{y^f\left(1 - \pi^f_f - \pi^f_d\right)} > \frac{w^d}{y^d\left(1 - \pi^d_d - \pi^d_f\right)} > \max\left\{\frac{1}{\sigma^d}; \frac{1}{\sigma^f}\right\}$.

The proof of Proposition 1 appears in Appendix A.

### 3.3.2 Local Dynamics

We now characterize local dynamics in a neighborhood of the unique steady state equilibrium. Equations (24) and (26) constitute a system of two first order difference equations. We approximate this dynamical system by

$$\left(R^d_{t+1} - R^d; \; x_{t+1} - x\right)' = J\left(R^d_t - R^d; \; x_t - x\right)'$$

where $J$ is the Jacobian matrix

$$J = \begin{bmatrix} \frac{\partial R^d_{t+1}}{\partial R^d_t} & \frac{\partial R^d_{t+1}}{\partial x_t} \\ \frac{\partial x_{t+1}}{\partial R^d_t} & \frac{\partial x_{t+1}}{\partial x_t} \end{bmatrix};$$

with all partial derivatives evaluated at the unique steady state. From (24), (26), (27) and (28) we find that

$$\frac{\partial R^d_{t+1}}{\partial R^d_t} = \left(\frac{R^d}{w^d}\right)^2 \left(1 - \pi^d_d - \pi^d_f\right) y^d + \left(\frac{R^d}{w^d}\right)^2 \pi^f_d y^f \frac{\partial x_{t+1}}{\partial R^d_t} \tag{29}$$

$$\frac{\partial R^d_{t+1}}{\partial x_t} = \left(\frac{R^d}{w^d}\right)^2 \pi^f_d y^f \frac{\partial x_{t+1}}{\partial x_t} \tag{30}$$

$$\frac{\partial x_{t+1}}{\partial R^d_t} = \frac{x w^f + w^d}{w^f R^d} \tag{31}$$

$$\frac{\partial x_{t+1}}{\partial x_t} = \left(1 - \pi^f_f - \pi^f_d\right) \frac{y^f}{w^f} R^d. \tag{32}$$

We can now state the following proposition.

**Proposition 2** When $R^d \geq 1$ holds, the unique steady state is a saddle. There is also a unique dynamical equilibrium path approaching it. Along this path, all variables exhibit monotone dynamics.

The proof of Proposition 3 appears in Appendix B.

Proposition 3 states that, when steady state real interest rates are positive, the long-run equilibrium is a saddle, with a positive stable root. Moreover, there is a single free initial condition in this economy. Hence, when world capital markets are fully integrated, and when there are two currencies, there is a unique equilibrium path displaying monotonic dynamics.

To conclude this section, we present a numerical example that illustrates these stability properties.

**Example 2** For the parameters of example 1, $R^d = 1.078$; $R^f = 1.078$; and $x = 0.202$ constitute a steady state equilibrium. The eigenvalues of $J$ are $\lambda_1 = 0.64$; and $\lambda_2 = 1.69$.

# 4 A Dollarized World

We now turn to the analysis of a "dollarized world", one where the domestic economy adopts as legal tender the currency issued by the foreign economy. The foreign country continues to print flat money at a constant rate $\sigma^f$; accepting as legal tender only the currency it issues. Hence the foreign government's budget constraint continues to be given by equation (13). The domestic country however, no longer prints flat currency. To make up for the loss in seigniorage revenue, the domestic government imposes a lump-sum tax $\tau$ on all endowments so that the total revenue collected from this tax exactly equals the total steady state seigniorage revenue collected when the country prints its own currency. Hence, when capital markets are segmented,

$$\tau = \left(\frac{\sigma^d - 1}{2\sigma^d}\right)\left(\pi_d^d + \pi_f^d\right) y^d. \tag{33}$$

## 4.1 Behavior of Agents and Banks

Independent of the monetary system in place, the solution to the borrower's problem continues to set $b_t^f = \frac{w^f}{R_t^f}$, while it is easy to see that for the domestic country, $b_t^d = \frac{w^d - \tau}{R_t^d}$ will hold.

Domestic banks will now hold foreign balances to satisfy the liquidity needs of all relocated agents. Indeed, both lenders who relocate within the domestic country and those who are relocated to the foreign economy will need foreign country currency to use in transactions in their new location. Hence, the domestic bank's balance sheet constraint now becomes

$$x_t m_{dt}^f + l_t^d + i_t^d \leq y^d - \tau; \quad t \geq 0 \tag{34}$$

where now $x_t = \frac{p_t^f}{p_t^d}$ since $e_t = 1$.

In addition, announced deposit returns must now satisfy the following constraints. First, domestically and internationally relocated agents, of whom there are $\pi_d^d + \pi_f^d$ per domestic depositor, must be given foreign currency, since that is the only asset which will allow these agents to consume in their new location. This is accomplished by using the bank's holdings of foreign real balances which have a real return of $p_t^d/p_{t+1}^d$ in units of domestic goods obtained at $t+1$ per unit of domestic good foregone at t, and a real return of $p_t^d/p_{t+1}^f$ in units of foreign goods obtained at $t+1$ per unit of domestic good foregone at $t$. Therefore,

$$\pi_d^d r_{dt}^d \left(y^d - \tau\right) p_{t+1}^d + \pi_f^d r_{ft}^d \left(y^d - \tau\right) p_{t+1}^f \le x_t m_{dt}^f p_t^d; \quad t \ge 0 \tag{35}$$

must be satisfied. For non-movers, the bank' resource constraint becomes

$$\left(1 - \pi_d^d - \pi_f^d\right) r_t^d \left(y^d - \tau\right) \le R_t^d l_t^d + R_{ft}^d i_t^d; \quad t \ge 0. \tag{36}$$

Clearly, (34) will hold with equality. Then, taking into account the no-arbitrage condition between domestic and international loans, we can again transform the domestic bank's resource constraints as follows. Let $\gamma_t^d \equiv \frac{x_t m_{dt}^f}{y^d - \tau}$ be the ratio of (foreign) reserves to deposits, so that $1 - \gamma_t^d \equiv \frac{l_t^d + i_t^d}{y^d - \tau}$ is the ratio of interest-bearing assets to deposits. Then constraints (35) and (36) can be written as

$$\pi_d^d r_{dt}^d \frac{p_{t+1}^d}{p_t^d} + \pi_f^d r_{ft}^d \frac{p_{t+1}^f}{p_t^d} \le \gamma_t^d; \qquad t \ge 0 \tag{37}$$

$$\left(1 - \pi_d^d - \pi_f^d\right) r_t^d \le \left(1 - \gamma_t^d\right) R_t^d; \qquad t \ge 0. \tag{38}$$

In addition, holdings of currency remain constrained to be non-negative. Hence $\gamma_t^d \geq 0$ must hold, $\forall t \geq 0$.

In a dollarized economy domestic banks must now choose values for $r_{dt}^d$, $r_{ft}^d$, $r_t^d$, and $\gamma_t^d$, to solve the problem

$$\max \pi_d^d \ln\left[r_{dt}^d\left(y^d-\tau\right)\right]+\pi_f^d \ln\left[r_{ft}^d\left(y^d-\tau\right)\right]+\left(1-\pi_d^d-\pi_f^d\right)\ln\left[r_t^d\left(y^d-\tau\right)\right]$$

subject to (37)-(38), and $\gamma_t^d \geq 0$. The solution to this problem sets $\gamma_t^d = \pi_d^d+\pi_f^d$, $r_{dt}^d = \frac{p_t^d}{p_{t+1}^d}$, $r_{ft}^d = \frac{p_t^d}{p_{t+1}^f}$, and $r_t^d = R_t^d$. The problem faced by foreign banks is completely analogous to that of domestic banks.[12] Therefore, the solution to that problem sets $\gamma_t^f = \pi_f^f+\pi_d^f$, $r_{ft}^f = \frac{p_t^f}{p_{t+1}^f}$, $r_{dt}^f = \frac{p_t^f}{p_{t+1}^d}$, and $r_t^f = R_t^f$. We now turn to the analysis of perfect foresight equilibria in this environment.

## 4.2 General Equilibrium: Segmented Capital Markets

Again, we start with the case where no international capital flows are allowed, so $i_t^d = i_t^f = 0$. Clearly, the loan market clearing condition for the foreign country continues to be given by (10), while for the domestic country

$$\left(1-\pi_d^d-\pi_f^d\right)\left(y^d-\tau\right) = \frac{\left(w^d-\tau\right)}{R_t^d} \quad \forall t \geq 0 \tag{39}$$

must hold. Hence, equations (39) and (10) determine the loan interest rates for both countries,

$$R_t^d = \frac{\left(w^d-\tau\right)}{(y^d-\tau)\left(1-\pi_d^d-\pi_f^d\right)} \quad \forall t; \tag{40}$$

$$R_t^f = \frac{w^f}{y^f\left(1-\pi_f^f-\pi_d^f\right)} \quad \forall t. \tag{41}$$

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[12] With the exception of the fact that no tax is levied on any endowments in the foreign country, where the government continues to earn seigniorage revenue.

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Second, the single money market has to clear, which implies

$$m_t^f = \frac{(\pi_d^d + \pi_f^d)\left(y^d - \tau\right)}{x_t} + (\pi_f^f + \pi_d^f)y^f \quad \forall t \geq 0. \tag{42}$$

Clearly, equation (42) allows us to write the equilibrium real exchange rate, $x_t$, as a function of foreign real balances, $m_t^f$,

$$x_t = \frac{(\pi_d^d + \pi_f^d)\left(y^d - \tau\right)}{m_t^f - (\pi_f^f + \pi_d^f)y^f} \quad \forall t \geq 0. \tag{43}$$

Finally, the domestic and foreign goods markets must clear at all dates. The supply of goods in the domestic country is simply the sum of the endowments received by young lenders and old borrowers. The demand for domestic goods at date t includes the consumption of young domestic borrowers, $\frac{w^d - \tau}{R_t^d}$. In addition, there are $\pi_d^d$ old domestic lenders who were relocated domestically, $\pi_d^f$ old foreign lenders who were relocated from abroad, and there are $\left(1 - \pi_d^d - \pi_f^d\right)$ old domestic lenders who were not relocated at all. All these agents consume the proceeds of their intermediated savings. Finally, the domestic government consumes $g_t^d = 2\tau$ units of the final good per lender. Thus the domestic goods market clearing condition takes the form

$$y^d + w^d = \tag{44}$$

$$\frac{w^d - \tau}{R_t^d} + \pi_d^d r_{dt-1}^d \left(y^d - \tau\right) + \pi_d^f r_{dt-1}^f y^f + \left(1 - \pi_d^d - \pi_f^d\right) r_{t-1}^d \left(y^d - \tau\right) + 2\tau \quad \forall t \geq 1.$$

Taking into account the solution to the bank's problem, we can rewrite the domestic goods market clearing condition as follows

$$y^d + w^d = \tag{45}$$

$$\frac{w^d-\tau}{R^d_t}+\pi^d_d\frac{p^d_{t-1}}{p^d_t}\left(y^d-\tau\right)+\pi^f_d\frac{p^f_{t-1}}{p^d_t}y^f+\left(1-\pi^d_d-\pi^d_f\right)R^d_{t-1}\left(y^d-\tau\right)+2\tau \quad \forall t\geq 1.$$

After substituting (40) into (45) and rearranging terms we obtain

$$\left(\pi^d_d+\pi^d_f\right)\left(y^d-\tau\right)=\pi^d_d\frac{p^d_{t-1}}{p^d_t}\left(y^d-\tau\right)+\pi^f_d\frac{p^f_{t-1}}{p^d_t}y^f \quad \forall t\geq 1. \tag{46}$$

Notice that the right hand side of (46) represents the real value of dollars circulating in the domestic country at the beginning of time $t$. Now, since $m^f_t \equiv \frac{M^f_t}{p^f_t}$, it is also the case that $p^f_t \equiv \frac{M^f_t}{m^f_t}$, and thus $\frac{p^f_{t-1}}{p^f_t}=\frac{m^f_t}{\sigma^f m^f_{t-1}}$. Further, note that $\frac{p^f_{t-1}}{p^d_t}=\frac{p^f_t}{p^d_t}\frac{p^f_{t-1}}{p^f_t}=x_t\frac{m^f_t}{\sigma^f m^f_{t-1}}$, while $\frac{p^d_{t-1}}{p^d_t}=\frac{p^f_{t-1}}{p^d_t}\frac{p^d_{t-1}}{p^f_{t-1}}=\frac{x_t}{x_{t-1}}\frac{m^f_t}{\sigma^f m^f_{t-1}}$. Thus, we can transform the goods market clearing condition to

$$\left(\pi^d_d+\pi^d_f\right)\left(y^d-\tau\right)=\pi^d_d\frac{x_t}{x_{t-1}}\frac{m^f_t}{\sigma^f m^f_{t-1}}\left(y^d-\tau\right)+\pi^f_d x_t\frac{m^f_t}{\sigma^f m^f_{t-1}}y^f \quad \forall t\geq 1. \tag{47}$$

Finally, substituting equation (43) into equation (47), and rearranging terms, we obtain,

$$\frac{1}{m^f_t}=\frac{1}{\sigma^f m^f_{t-1}}\left[\frac{\pi^d_d}{\left(\pi^d_d+\pi^d_f\right)}-\frac{\pi^f_d}{\left(\pi^f_f+\pi^f_d\right)}\right]+\frac{\pi^d_f+\pi^d_d\left(\frac{\sigma^f-1}{\sigma^f}\right)}{\left(\pi^d_d+\pi^d_f\right)\left(\pi^f_f+\pi^f_d\right)y^f} \quad \forall t\geq 1. \tag{48}$$

Equation (48) describes the evolution of the equilibrium sequence $\left\{m^f_t\right\}$.

### 4.2.1 Steady State Equilibria

We now provide a characterization of the steady state for this case. In a steady state, equation (48) reduces to

$$\frac{1}{m^f}=\frac{1}{\sigma^f m^f}\left[\frac{\pi^d_d}{\left(\pi^d_d+\pi^d_f\right)}-\frac{\pi^f_d}{\left(\pi^f_f+\pi^f_d\right)}\right]+\frac{\pi^d_f+\pi^d_d\left(\frac{\sigma^f-1}{\sigma^f}\right)}{\left(\pi^d_d+\pi^d_f\right)\left(\pi^f_f+\pi^f_d\right)y^f}. \tag{49}$$

Hence it is easy to verify that the unique long run equilibrium is characterized by

$$m^f = \frac{\left[\sigma^f \pi^d_f + \pi^d_d \left(\sigma^f - 1\right)\right]\left(\pi^f_f + \pi^f_d\right) y^f + \pi^f_d \left(\pi^d_d + \pi^d_f\right) y^f}{\sigma^f \pi^d_f + \pi^d_d \left(\sigma^f - 1\right)}. \tag{50}$$

It is obvious from equation (50) that $\sigma^f \geq 1$ is a sufficient condition for steady state real balances to be positive. From (43), it is clear that this condition also implies that the steady state real exchange rate is positive. In addition, we need to verify that the return on loans in each country dominates the return on real balances in a steady state equilibrium. The following proposition states sufficient conditions for the unique solution to (50), (40), (41), and (43) to satisfy all these requirements.

**Proposition 3** In a dollarized world with fully segmented international credit markets, the economy exhibits a unique steady state equilibrium with positive nominal interest rates if $\sigma^f \geq 1$; $\frac{(w^d - \tau)}{(y^d - \tau)\left[1 - \pi^d_d - \pi^d_f\right]} > \frac{1}{\sigma^f}$; and $\frac{w^f}{y^f\left[1 - \pi^f_f - \pi^f_d\right]} > \frac{1}{\sigma^f}$ all hold.

To conclude this section, we provide an example of such a steady state equilibrium.

**Example 3** The example is identical to example 1 except that the economy is dollarized. Then $R^d = 1.0923$; $R^f = 1.0625$; $x = 0.22$; $m^f = 3.81$; $g^d = 0.036$; and $g^f = 0.182$.

### 4.2.2 Dynamics

In order to analyze dynamical equilibria, we define $z_t = \frac{1}{m^f_t}$, and rewrite equation (48) to obtain

$$z_t = \frac{z_{t-1}}{\sigma^f}\left[\frac{\pi^d_d}{\left(\pi^d_d + \pi^d_f\right)} - \frac{\pi^f_d}{\left(\pi^f_f + \pi^f_d\right)}\right] + \frac{\pi^d_f + \pi^d_d\left(\frac{\sigma^f - 1}{\sigma^f}\right)}{\left(\pi^d_d + \pi^d_f\right)\left(\pi^f_f + \pi^f_d\right) y^f}. \tag{51}$$

Clearly, equation (51) represents a linear law of motion, whose properties are stated in the following proposition.

**Proposition 4** (a) The unique steady state is stable and the equilibrium path is indeterminate if $\sigma^f \geq 1$. (b) Equilibrium paths display endogenous volatility if $\sigma^f \geq 1$ and $\pi^d_d \pi^f_f < \pi^d_f \pi^f_d$.

**Proof.** (a) The unique steady state is stable iff $\left|\left(\frac{\pi_d^d}{\pi_d^d+\pi_f^d}\right) - \left(\frac{\pi_d^f}{\pi_f^f+\pi_d^f}\right)\right| \le \sigma^f$. Hence, $\sigma^f \ge 1$ is a sufficient condition for the unique steady state to be stable. Moreover, it is straightforward to show that $m_0$ and therefore $z_0$ is not predetermined; hence an indeterminacy of equilibrium exists. Indeed, given $p_0^f$, we know $m_0$ and thus $z_0$, and the money market clearing condition (43) then determines $x_0$ and $p_0^d = \frac{x_0}{p_0^f}$. However, $p_0^f$ is not determined by any equilibrium condition. Hence the value of $z_0$ is not predetermined and the economy displays a continuum of non- stationary equilibrium paths.

(b) Non-stationary equilibrium paths - which exist when $\sigma^f \ge 1$ - display oscillation when the slope of the law of motion is negative, that is, when $\left(\frac{\pi_d^d}{\pi_d^d+\pi_f^d}\right) - \left(\frac{\pi_d^f}{\pi_f^f+\pi_d^f}\right) < 0$. Clearly, this is the case iff $\pi_d^d \pi_f^f < \pi_f^d \pi_d^f$. ■

As Proposition 4 demonstrates, the steady state equilibrium is asymptotically stable. Moreover, there is one free initial condition in this economy. Hence, there is a continuum of equilibrium paths. And, under some conditions, each of these equilibrium paths exhibits oscillation around the long run equilibrium. Proposition 4 thus states a very important result. First, it implies that there are very weak conditions under which "dollarization", in the presence of fully segmented credit markets, gives rise to an indeterminacy of equilibrium. Second, under stronger conditions, endogenously arising volatility in real money balances, real exchange rates, and consumption bundles may also be observed.

#### 4.2.3 Discussion

Why is it that an indeterminacy of equilibrium may arise in a dollarized world? The answer is that, when both countries are using the same currency, nothing pins down what fraction of the total dollar supply circulates in either the domestic or the foreign economy, respectively. Suppose, for instance, that agents expect future prices to be high in the domestic economy. Then the $\pi_d^d$ domestic lenders who move domestically, and the $\pi_d^f$ foreign lenders who move to the domestic economy, will receive a relatively large fraction of the supply of dollars when relocating. And, at the beginning of the next period, a relatively large fraction of all dollars will circulate in the domestic economy. But then it follows from the domestic goods market clearing condition (46) that next period's domestic price level will indeed be high. Hence indeterminacy of equilibrium can arise as the result of a self-fulfilling prophecy. Such self-fulfilling behavior can not occur when both countries issue a flat currency because in that case two separate money market clearing conditions need to be satisfied.

In short, in a dollarized world with no international capital markets, the money supply of the domestic economy is at the mercy of cross-border currency flows. Under the conditions we have described, these flows can also be quite volatile. This will allow the price levels and interest rates of each country to fluctuate, despite any shocks to "fundamentals".

The conditions under which endogenously arising volatility is possible, however, in a dollarized world, might appear to be quite strong. In particular $\pi_d^d \pi_f^f < \pi_f^d \pi_d^f$ requires – loosely speaking – that international flows of currency exceed domestic flows of currency. However, in our view it should be regarded as impressive that endogenously generated volatility is possible at all, under our assumptions. Indeed, the assumption of logarithmic utility for lenders implies the absence of any interest elasticity in asset supplies. This absence works strongly here – as it does in the conventional overlapping generations model – against finding indeterminacy or volatility. Thus we regard the finding that any volatility is possible at all – under dollarization – in this economy as quite a striking result.

If domestic (and only domestic) lenders are given the utility function $u(c_1; c_2) = \frac{c_2^{1-\rho}}{1-\rho}$; with $\rho > 0$, then it is possible to show that endogenously arising volatility will be observed if $\rho < 1$ holds, and if $w^d$ is sufficiently large. Under these same conditions, there will be a unique equilibrium (the steady state) if there are two currencies and no flows of capital internationally. The condition $\pi_d^d \pi_f^f < \pi_f^d \pi_d^f$ is not required for endogenous volatility to be observed under dollarization.

If it were possible to borrow and lend freely in international capital markets, it would be feasible (although not necessarily optimal) to offset the currency flows that generate indeterminacy and market-generated volatility in this economy. We now consider this situation.

### 4.3 General Equilibrium: Integrated Capital Markets

We will now relax the restriction on international credit flows between the two economies, and consider the case of a "dollarized economy" where $i_t^d \neq 0$, and $i_t^f \neq 0$ may hold. For this economy, the foreign loan market clearing condition continues to be given by (21), while the domestic market clearing condition for loans becomes

$$\left(1-\pi_d^d-\pi_f^d\right)\left(y^d-\tau\right)-i_t^d=\frac{w^d-\tau}{R_t^d} \quad \forall t \geq 0. \tag{52}$$

In addition, the international loan market clearing condition (22) has to be satisfied in this world. Together with the domestic and foreign loan market clearing conditions, this implies

$$x_t=-\frac{i_t^d}{i_t^f}=\frac{\left(1-\pi_d^d-\pi_f^d\right)\left(y^d-\tau\right)-\frac{w^d-\tau}{R_t^d}}{\frac{w^f}{R_t^f}-\left(1-\pi_f^f-\pi_d^f\right)y^f} \quad \forall t \geq 0. \tag{53}$$

Moreover, the uncovered interest parity condition (23) continues to hold. Substituting (23) into (53) and solving for $x_{t+1}$ yields

$$x_{t+1}=\left\{\left(1-\pi_f^f-\pi_d^f\right)y^f x_t+\left(1-\pi_d^d-\pi_f^d\right)\left(y^d-\tau\right)\right\}\frac{R_t^d}{w^f}-\frac{w^d-\tau}{w^f} \quad \forall t \geq 0. \tag{54}$$

And, the domestic goods market clearing condition, equation (45), can be rewritten as

$$y^d + w^d = \tag{55}$$

$$\frac{w^d-\tau}{R^d_t} + \pi^d_d \frac{x_t}{x_{t-1}} \frac{m^f_t}{\sigma^f m^f_{t-1}} \left(y^d-\tau\right) + \pi^f_d x_t \frac{m^f_t}{\sigma^f m^f_{t-1}} y^f + \left(1-\pi^d_d-\pi^d_f\right) R^d_{t-1}\left(y^d-\tau\right) + 2\tau \quad \forall t \geq 1.$$

Bringing equation (55) forward one period, and solving for $R^d_{t+1}$ yields

$$R^d_{t+1} = \frac{w^d-\tau}{(w^d-\tau)+(y^d-\tau)\left[1-\left(1-\pi^d_d-\pi^d_f\right)R^d_t - \left[\pi^f_d y^f x_{t+1} + \pi^d_d(y^d-\tau)\frac{x_{t+1}}{x_t}\right]\frac{m^f_{t+1}}{\sigma^f m^f_t}\right]} \quad \forall t \geq 1. \tag{56}$$

Finally, the money market clearing condition continues to be given by equation (42). Substituting (42) into (56) yields

$$R^d_{t+1} = \frac{w^d-\tau}{(w^d-\tau)+(y^d-\tau)\left[1-\left(1-\pi^d_d-\pi^d_f\right)R^d_t - \left(\frac{\pi^f_d y^f x_t+\pi^d_d(y^d-\tau)}{\sigma^f}\right)\frac{(\pi^d_d+\pi^d_f)(y^d-\tau)+(\pi^f_f+\pi^f_d)y^f x_{t+1}}{(\pi^d_d+\pi^d_f)(y^d-\tau)+(\pi^f_f+\pi^f_d)y^f x_t}\right]}$$

$$\forall t \geq 1. \tag{57}$$

Equations (54) and (57) describe the evolution of the equilibrium sequences $\left\{R^d_t\right\}$ and $\{x_t\}$.

### 4.3.1 Steady State Equilibria

In a steady state, after rearranging terms, equation (57) reduces to

$$x = \frac{\left(1-\pi^d_d-\pi^d_f\right)\left(y^d-\tau\right)R^d - \left(w^d-\tau\right)}{w^f - \left(1-\pi^f_f-\pi^f_d\right)y^f R^d}; \tag{58}$$

where we now omit time subscripts. Similarly, equation (54) becomes

$$x = \frac{-\left(R^d\right)^2\left(1-\pi^d_d-\pi^d_f\right)\left(y^d-\tau\right) + R^d\left[\left(1-\frac{\pi^d_d}{\sigma^f}\right)\left(y^d-\tau\right)+\left(w^d-\tau\right)\right] - \left(w^d-\tau\right)}{R^d\frac{\pi^f_d y^f x}{\sigma^f}}. \tag{59}$$

A steady state equilibrium is a pair of values $R^d > \frac{1}{\sigma^f}$ and $x > 0$ which solve equations (58) and (59). These equations define loci which are depicted in Figure 3. We proceed to establish conditions under which our economy displays a unique steady state equilibrium.

**Proposition 5** In a dollarized world with fully integrated international capital markets, the economy exhibits a unique steady state equilibrium if

(a) $\frac{w^d-\tau}{(y^d-\tau)\left(1-\pi_d^d-\pi_f^d\right)} > \frac{w^f}{y^f\left(1-\pi_f^f-\pi_d^f\right)} > \frac{1}{\sigma^f}$, or if

(b) $\frac{w^f}{y^f\left(1-\pi_f^f-\pi_d^f\right)} > \frac{w^d-\tau}{(y^d-\tau)\left(1-\pi_d^d-\pi_f^d\right)} > \frac{1}{\sigma^f}$.

The proof of Proposition 5 is essentially identical to the proof of Proposition 1, and is omitted here.

### 4.3.2 Local Dynamics

We now proceed to characterize local dynamics in a neighborhood of the unique steady state equilibrium. Equations (54) and (57) constitute a system of two first order difference equations. We approximate this dynamical system by

$$(R^d_{t+1}-R^d,\ x_{t+1}-x)' = J(R^d_t-R^d,\ x_t-x)'$$

where $J$ is the Jacobian matrix

$$J = \begin{bmatrix} \frac{\partial R^d_{t+1}}{\partial R^d_t} & \frac{\partial R^d_{t+1}}{\partial x_t} \\ \frac{\partial x_{t+1}}{\partial R^d_t} & \frac{\partial x_{t+1}}{\partial x_t} \end{bmatrix};$$

with all partial derivatives evaluated at the unique steady state. From (54), (57), (58), and (59) it is easy to show that

$$\frac{\partial R^d_{t+1}}{\partial R^d_t} = \frac{\left(1-\pi_d^d-\pi_f^d\right)\left(y^d-\tau\right)\left(R^d\right)^2}{(w^d-\tau)} + \frac{\partial x_{t+1}}{\partial R^d_t}\frac{\left(R^d\right)^2}{\sigma^f(w^d-\tau)}\frac{\left[\pi_d^f y^f x+\pi_d^d\left(y^d-\tau\right)(\pi_f^f+\pi_d^f)y^f\right]}{\left[\left(\pi_d^d+\pi_f^d\right)(y^d-\tau)+\left(\pi_f^f+\pi_d^f\right)y^f x\right]} \tag{60}$$

$$\frac{\partial R^d_{t+1}}{\partial x_t} = \frac{\left(R^d\right)^2\pi_d^f y^f}{\sigma^f(w^d-\tau)} - \frac{\left(R^d\right)^2}{\sigma^f(w^d-\tau)}\frac{\left[\pi_d^f y^f x+\pi_d^d\left(y^d-\tau\right)(\pi_f^f+\pi_d^f)y^f\right]}{\left[\left(\pi_d^d+\pi_f^d\right)(y^d-\tau)+\left(\pi_f^f+\pi_d^f\right)y^f x\right]} + \frac{\partial x_{t+1}}{\partial x_t}\frac{\left(R^d\right)^2}{\sigma^f(w^d-\tau)}\frac{\left[\pi_d^f y^f x+\pi_d^d\left(y^d-\tau\right)(\pi_f^f+\pi_d^f)y^f\right]}{\left[\left(\pi_d^d+\pi_f^d\right)(y^d-\tau)+\left(\pi_f^f+\pi_d^f\right)y^f x\right]} \tag{61}$$

$$\frac{\partial x_{t+1}}{\partial R^d_t} = \frac{xw^f+\left(w^d-\tau\right)}{w^f R^d} \tag{62}$$

$$\frac{\partial x_{t+1}}{\partial x_t} = \left(1-\pi_f^f-\pi_d^f\right)\frac{y^f}{w^f}R^d. \tag{63}$$

We can now state the following proposition.

**Proposition 6** (a) The steady state is a saddle and local dynamics are monotone if $R^d \geq 1$; $\left(1-\pi_f^f-\pi_d^f\right)\left(\pi_d^d+\pi_f^d\right)+\pi_d^d\pi_f^f \geq \pi_f^d\pi_d^f$, and $\left(1-\pi_d^d-\pi_f^d\right)\left(\pi_f^f+\pi_d^f\right)+\pi_d^d\pi_f^f \geq \pi_f^d\pi_d^f$ all hold.
(b) The steady state is a saddle and local dynamics are oscillatory if $R^d \geq 1$, $\left(\pi_d^d+\pi_f^d\right)\frac{w^f}{y^f}\sigma^f + \pi_d^d\pi_f^f < \pi_f^d\pi_d^f$, $\left[\frac{1-\pi_d^d-\pi_f^d}{1-\pi_f^f-\pi_d^f}\right]\left(\pi_f^f+\pi_d^f\right)\frac{w^f}{y^f}\sigma^f+\pi_d^d\pi_f^f < \pi_f^d\pi_d^f$, $\pi_d^d+\pi_f^d \geq 0.5$, $\pi_f^f+\pi_d^f \geq 0.5$, and $\sigma^f\left(1-\pi_f^f-\pi_d^f\right)^2 \geq \frac{y^f}{w^f}\left(\frac{y^d-\tau}{w^d-\tau}\right)\left(\pi_d^f\pi_f^d-\pi_d^d\pi_f^f\right)$ all hold.
(c) A unique equilibrium path exists both in case (a) and case (b).

The proof of Proposition 6 is presented in Appendix C.

The conditions stated in Proposition 6 (a) imply that the steady state is a saddle with monotone local dynamics if $\pi_d^d\pi_f^f \geq \pi_f^d\pi_d^f$. Notice that that is exactly the condition under which local dynamics are monotone in a dollarized world with segmented capital markets. Hence, for a dollarized world, monotone dynamics in the absence of capital flows implies monotone dynamics with international capital flows.

We now present a numerical example that illustrates these stability properties.

**Example 4** The example is identical to example 3, except for the presence of international capital flows. Then $R^d = 1.073$; $R^f = 1.073$; $x = 0.372$; $m^f = 3.06$; $g^d = 0.036$; and $g^f = 0.146$; while the eigenvalues of J are $\lambda_1 = 0.62$; $\lambda_2 = 2.07$.

### 4.3.3 Discussion

A comparison of propositions 4 and 6 yields two interesting observations. First, in a single currency world, indeterminacies arise under weak conditions if international capital markets are poorly integrated. There are no indeterminacies, under weak conditions, if international capital markets are well-integrated. This happens because, when international ¯nancial markets are integrated, any °ows of dollars that are associated with goods purchases, can be undone by international borrowing and lending. Thus, if indeterminacies are undesirable { as Friedman (1960) and others have argued { dollarization is more attractive when international ¯nancial markets are integrated than when they are not.

Second, while dollarization allows endogenous volatility to arise under some conditions, these conditions become more stringent as capital markets become better integrated. Or, in other words, endogenous volatility is likely to be of greater concern with poorly integrated than with well integrated international capital markets.

Intuitively, when capital markets are integrated, why is endogenous volatility possible at all? Why don't agents use these markets to fully undo the e®ects of currency °ows? The answer is that, while it is feasible to undo these °ows in international capital markets, this is not enough. Agents must also perceive an incentive to do so. Proposition 6 indicates that agents will perceive such an incentive, except under relatively extreme assumptions on parameter values.

# 5 The Fiscal Consequences of Dollarization

Dollarization has implications for the overall seigniorage revenue collected in the world. This is important because, if the total revenues from the in°ation tax fall after dollarization, then some country must raise taxes to keep world government expenditures unchanged.[13] We now establish conditions under which world seigniorage revenue falls when the domestic country dollarizes. For

[13] This is obviously true even if the foreign country shares any seigniorage revenue it gains as a result of dollarization by the domestic economy.

simplicity we focus on the case where international capital markets are fully segmented, and we consider only steady state equilibria.

When there are two currencies, it follows from equations (15) and (16) that world steady state seigniorage revenue is

$$\left(\frac{\sigma^d-1}{\sigma^d}\right)\left(\pi^d_d+\pi^d_f\right)y^d+\left(\frac{\sigma^f-1}{\sigma^f}\right)\left(\pi^f_f+\pi^f_d\right)y^f. \tag{64}$$

On the other hand, when the domestic country dollarizes, equation (50) implies that total world steady state seigniorage revenue is given by

$$\left(\frac{\sigma^f-1}{\sigma^f}\right)\frac{\left[\sigma^f\pi^d_f+\pi^d_d\left(\sigma^f-1\right)\right]\left(\pi^f_f+\pi^f_d\right)y^f+\pi^f_d\left(\pi^d_d+\pi^d_f\right)y^f}{\left[\sigma^f\pi^d_f+\pi^d_d\left(\sigma^f-1\right)\right]}. \tag{65}$$

We can now state the following proposition

**Proposition 7** When capital markets are segmented, internationally, dollarization implies a reduction in total world steady state seigniorage revenue if $\sigma^f \geq 1$ and

(a) $\pi^f_d y^f \leq \left(\frac{\sigma^d-1}{\sigma^d}\right)\left(\pi^d_d+\pi^d_f\right)y^d$; or

(b) $\pi^f_d y^f > \left(\frac{\sigma^d-1}{\sigma^d}\right)\left(\pi^d_d+\pi^d_f\right)y^d$ and $\pi^f_d y^f \geq \frac{\left[y^d\left(\frac{\sigma^d-1}{\sigma^d}\right)\pi^d_d+\pi^f_d y^f\right]^2}{4y^d\left(\frac{\sigma^d-1}{\sigma^d}\right)\left[\pi^d_d+\pi^f_d\right]}$; or

(c) $\pi^f_d y^f > \left(\frac{\sigma^d-1}{\sigma^d}\right)\left(\pi^d_d+\pi^d_f\right)y^d$, $\pi^f_d y^f < \frac{\left[y^d\left(\frac{\sigma^d-1}{\sigma^d}\right)\pi^d_d+\pi^f_d y^f\right]^2}{4y^d\left(\frac{\sigma^d-1}{\sigma^d}\right)\left[\pi^d_d+\pi^f_d\right]}$; and $\left(\sigma^f-1\right)$ is sufficiently small or $\sigma^f$ is sufficiently large.

The proof of proposition 7 is presented in Appendix D.

Proposition 7 states a very important result. It indicates that in a world where international capital markets are segmented, dollarization always implies a reduction in world seigniorage revenue when the country that continues to issue a flat currency allows the stock of that currency to grow either at a very low rate, or at a very high rate. And, a reduction in world seigniorage revenue has the fiscal implications that we noted previously.

# 6 The Welfare Implications of Dollarization

We now briefly discuss the welfare implications of dollarization when capital markets are segmented internationally. We start by analyzing the effect of dollarization on steady state interest rates, and on the real exchange rate.

From equations (12) and (41), it is clear that the interest rate in the foreign country is unaffected by the domestic country's decision to dollarize. On the other hand, dollarization does affect the interest rate of the country that abandons its fiat currency. Indeed, when the domestic country issues a fiat currency, its interest rate is given by $R^d = \frac{w^d}{y^d\left(1 - \pi_d^d - \pi_f^d\right)}$: Under dollarization, the interest rate for the domestic country is $R^d = \frac{(w^d - \tau)}{(y^d - \tau)\left(1 - \pi_d^d - \pi_f^d\right)}$. Note that

$$\frac{1}{R^d}\frac{\partial R^d}{\partial \tau} = \frac{w^d - y^d}{(w^d - \tau)(y^d - \tau)}:$$

Hence we can state the following result.

**Lemma 1** $\frac{\partial R^d}{\partial \tau} > 0$ iff $w^d > y^d$:

Lemma 1 states that dollarization will be associated with an increase in the loan interest rate in the country that abandons its fiat currency if the country is a classical case economy, in Gale's (1973) sense. If the country is a Samuelson case economy, dollarization will lead to a decrease in the gross real interest rate.

We now turn our attention to the effect of dollarization on the real exchange rate. From equation (19), we know that the steady state real exchange rate is given by $x = \frac{\pi_f^d y^d}{\pi_d^f y^f}$ when both countries issue a fiat currency. With one currency, equation (47) implies that the steady state real exchange rate is $x = \tilde{x} = \frac{\sigma^f (y^d - \tau)}{\pi_d^f y^f}\left[\pi_f^d + \pi_d^d\left(\frac{\sigma^f - 1}{\sigma^f}\right)\right]$: We now summarize the effect of dollarization on the real exchange rate in Lemma 2.

**Lemma 2** Dollarization implies a depreciation of the real exchange rate for the domestic country iff

$$\sigma^d\left(1 - \pi_d^d - \pi_f^d\right) + \pi_d^d + \pi_f^d > \sigma^d\left[\frac{\left(2 - \sigma^f\right)\pi_f^d - \left(\sigma^f - 1\right)\pi_d^d}{\left(\sigma^f - 1\right)\pi_d^d + \sigma^f\pi_f^d}\right] \tag{66}$$

holds: Dollarization will be associated with an appreciation of the real exchange rate if $\sigma^f - 1$ is sufficiently small, and with a depreciation of the real exchange rate if $\left|\sigma^f - \sigma^d\right|$ is sufficiently small.

**Proof.** The condition (66) follows immediately from the definitions of $\hat{x}$ and $x$, and from (33). Moreover, notice that (66) fails to hold if $\sigma^d > \sigma^f = 1$. And, for $\sigma^d = \sigma^f$, (66) is satisfied. Hence the remainder of the lemma follows from continuity. ■

Lemma 2 states that dollarization is associated with an appreciation of the real exchange rate for the domestic country when $\sigma^f$ is close to one. On the other hand, the dollarizing country will experience a depreciation of its steady state real exchange rate when its original rate of money creation is close to the rate at which the foreign country prints fiat currency.

We can now evaluate the effect of dollarization on steady state welfare when international credit markets are segmented. Clearly, the steady state welfare of foreign agents is unaffected by dollarization.

For domestic lenders, steady state welfare in an economy with two currencies is

$$\ln y^d + \pi_d^d \ln\left(\frac{1}{\sigma^d}\right) + \pi_f^d \ln\left(\frac{1}{\sigma^f x}\right) + \left(1 - \pi_d^d - \pi_f^d\right)\ln R^d.$$

Using (12), this becomes

$$\ln\left[\frac{w^d}{\left(1 - \pi_d^d - \pi_f^d\right)}\right] + \pi_d^d \ln\left(\frac{1}{\sigma^d}\right) + \pi_f^d \ln\left(\frac{1}{\sigma^f x}\right) - \left(\pi_d^d + \pi_f^d\right)\ln\left[\frac{w^d}{\left(1 - \pi_d^d - \pi_f^d\right)y^d}\right]. \tag{67}$$

Similarly, with one currency, the steady state welfare of domestic lenders is

$$\ln\left[\frac{w^d - \tau}{\left(1 - \pi_d^d - \pi_f^d\right)}\right] + \pi_d^d \ln\left(\frac{1}{\sigma^f}\right) + \pi_f^d \ln\left(\frac{1}{\sigma^f \hat{x}}\right) - \left(\pi_d^d + \pi_f^d\right)\ln\left[\frac{w^d - \tau}{\left(1 - \pi_d^d - \pi_f^d\right)\left(y^d - \tau\right)}\right]. \tag{68}$$

For domestic borrowers, welfare is $\frac{w^d}{R^d} = y^d \left(1 - \pi_d^d - \pi_f^d\right)$ when there are two currencies. When the economy is dollarized, the steady state welfare of domestic borrowers is $\frac{(w^d - \tau)}{R^d} = \left(y^d - \tau\right)\left(1 - \pi_d^d - \pi_f^d\right).$

We can now state the following proposition.

**Proposition 8** When credit markets are segmented internationally,

(a) the welfare of domestic borrowers is always lower in a dollarized economy, compared to an economy with two currencies;

(b) the welfare of domestic lenders is lower in a dollarized economy, compared to an economy with two currencies, if $\sigma^f$ is sufficiently close to $\sigma^d$.

**Proof.** The proof of part (a) is obvious, given that $\tau > 0$. For part (b), note that (67) and (68) imply that dollarization raises the steady state welfare of domestic lenders iff

$$\left(1 - \pi_d^d - \pi_f^d\right) \ln\left[\frac{w^d - \tau}{w^d}\right] + \left(\pi_d^d + \pi_f^d\right) \ln\left[\frac{y^d - \tau}{y^d}\right] + \pi_d^d \ln\left(\frac{\sigma^d}{\sigma^f}\right) + \pi_f^d \ln\left(\frac{x}{\bar{x}}\right) > 0 \qquad (69)$$

holds. When $\left|\sigma^d - \sigma^f\right|$ is sufficiently small, we know from Lemma 2 that $\bar{x} > x$, and hence all terms of (69) are negative, except $\ln\left(\frac{\sigma^d}{\sigma^f}\right)$. Moreover, given (33), $\pi_d^d \left\{\ln\left[\frac{y^d - \tau}{y^d}\right] + \ln\left(\frac{\sigma^d}{\sigma^f}\right)\right\} = \pi_d^d \ln\left[\frac{\sigma^d}{\sigma^f} - \frac{\sigma^d - 1}{2\sigma^f}\left(\pi_d^d + \pi_f^d\right)\right] \leq 0$ holds iff $\sigma^d - \sigma^f \leq \frac{\sigma^d - 1}{2}\left(\pi_d^d + \pi_f^d\right)$. Thus (69) is negative when $\left|\sigma^d - \sigma^f\right|$ is sufficiently small. ■

Intuitively, when the domestic country dollarizes, domestic residents must be taxed to make up for the loss of seigniorage revenue.[14] This is one source of reduced welfare. Thus the only possible benefit of dollarization (if $\left|\sigma^d - \sigma^f\right|$ is "small") is the implied reduction in steady state inflation if $\sigma^d > \sigma^f$. And, if $\left|\sigma^d - \sigma^f\right|$ is "small", this benefit cannot offset the other costs of dollarizing.

[14] We do not consider the possibility of a deficit reduction, as this could be accomplished without a change in the domestic country's monetary arrangement.

Moreover, in our view, a reduction in the rate of in°ation is a very weak rationale for dollarizing. Obviously the same in°ation reduction could be obtained by having the domestic country retain its own currency, and simply reduce its rate of money creation.

## 7 Conclusion

Is it desirable for an economy to adopt as legal tender the currency of another country, and hence unilaterally enter into a de facto monetary union? In this paper we have tried to answer that question. Following Friedman, we consider a policy desirable when it leaves relatively little scope for indeterminacy and \excessive" economic volatility. Using this criterion, we have compared the characteristics of an equilibrium in each of two situations: one where each country issues its own currency, and one where one of the countries adopts the currency of the other country. Our results depend very much on the degree of integration of capital markets between the two countries.

When national ¯nancial markets are poorly integrated, and when there are two currencies, there is a unique (stationary) equilibrium. With a common currency there exists a continuum of equilibrium paths, as long as the country which issues the currency has a positive rate of money creation. Moreover, under some conditions, these equilibrium paths will display oscillation. Hence, when international capital markets are segmented, \dollarization" may be a source of indeterminacy and \excessive volatility", rather than a remedy for them.

Matters are substantially di®erent when credit markets are integrated internationally. In that case, there is a unique equilibrium path, regardless of the monetary regime in place. In other words, in contrast to the situation with segmented credit markets, the adoption of a common currency does not a®ect the scope for indeterminacy when credit markets are fully integrated. Endogenously arising volatility can be observed when one economy dollarizes, whereas it cannot when there are two currencies. However, the conditions under which such volatility will emerge { in a dollarized economy { are quite restrictive. Thus the case for dollarization is stronger the more integrated are domestic and world capital markets.

Finally, our analysis suggests that the welfare justi¯cations for dollarization may be weak { particularly when capital markets are segmented. Overall, then, we ¯nd strong arguments against dollarization for economies whose ¯nancial markets are not well integrated with world markets. These arguments are weakened { but do not disappear entirely { when ¯nancial market integration exists.

While we have focused on dollarization, our framework can also be used to analyze the formation of a monetary union or the adoption of a currency board. And, the issues raised apply whether there is a ¯xed or a °exible exchange rate regime, in the two currency case.

Finally, the framework we have developed in this paper is also well suited to address other questions of importance in evaluating the desirability of dollarizing, adopting a currency board, or forming a monetary union. In particular we could investigate how exogenous shocks in either country are propagated under a common currency arrangement, and we could contrast this with the situation where each country issues its own currency. Moreover, in a world with more than two countries, one could ask whether one country's decision regarding dollarization might depend on whether or not other economies are dollarized, either explicitly, or implicitly through the mechanism of a currency board. Thus, for example, one could ask whether the adoption of a currency board by Argentina a®ects the desirability of dollarization in Mexico?

Questions of how bank regulation might a®ect the consequences of dollarization would also be easy to address using our framework. So would issues concerning the nature of the transition from two currencies to a common currency.

When one country adopts the currency of another country as legal tender, its central bank can no longer provide lender of last resort services by printing money. Our framework may be used to evaluate alternative arrangements under which the government might function as a lender of last resort. These arrangements could include the use of taxation or the issuance of government debt to obtain foreign currency reserves, which might then be lent to the banking system.

Finally, we have taken the degree of integration of international financial markets as given. However, an interesting possibility is that dollarization may affect the degree of financial integration between the countries involved. All of these would be interesting topics for future investigation.

## Appendix A: Proof of Proposition 1

We present the proof for the case where $\frac{w^d}{y^d\left(1-\pi_d^d-\pi_f^d\right)} > \frac{w^f}{y^f\left(1-\pi_f^f-\pi_d^f\right)}$. When $\frac{w^d}{y^d\left(1-\pi_d^d-\pi_f^d\right)} < \frac{w^f}{y^f\left(1-\pi_f^f-\pi_d^f\right)}$; the proof is analogous.

(a) From equation (27) we have,

$$x = 0 \text{ for } R^d = \frac{w^d}{y^d\left(1-\pi_d^d-\pi_f^d\right)};$$

$$\text{as } R^d \to \frac{w^f}{y^f\left(1-\pi_f^f-\pi_d^f\right)},\ x \to \infty;$$

$$\left.\frac{\partial x}{\partial R^d}\right|_{(27)} = \frac{w^f y^d\left(1-\pi_d^d-\pi_f^d\right) - w^d y^f\left(1-\pi_f^f-\pi_d^f\right)}{\left[w^f - y^f\left(1-\pi_f^f-\pi_d^f\right)R^d\right]^2};$$

$$\left.\frac{\partial^2 x}{(\partial R^d)^2}\right|_{(27)} = \frac{2y^f\left(1-\pi_f^f-\pi_d^f\right)\left[w^f y^d\left(1-\pi_d^d-\pi_f^d\right) - w^d y^f\left(1-\pi_f^f-\pi_d^f\right)\right]}{\left[w^f - y^f\left(1-\pi_f^f-\pi_d^f\right)R^d\right]^3}.$$

Moreover, equation (27) delivers a value of $x > 0$ iff $R^d \in \left(\frac{w^f}{y^f\left(1-\pi_f^f-\pi_d^f\right)}, \frac{w^d}{y^d\left(1-\pi_d^d-\pi_f^d\right)}\right) = A$. Clearly, equation (27) is downward sloping and convex $\forall R^d \in A$, and hence equation (27) defines a locus with the shape depicted in Figure 2. Moreover, it is clear from Figure 2 that any steady state equilibrium with $x > 0$ will have $R^d \in A$.

(b) Differentiating equation (28) yields,

$$\left.\frac{\partial x}{\partial R^d}\right|_{(28)} = \frac{1}{\pi_d^f y^f}\left[-y^d\left(1-\pi_d^d-\pi_f^d\right) + \frac{w^d}{(R^d)^2}\right];$$

$$\left.\frac{\partial^2 x}{(\partial R^d)^2}\right|_{(28)} = -\frac{2w^d}{\pi_d^f y^f (R^d)^3} < 0 \text{ for } R^d > 0.$$

Thus, (28) defines a concave locus. Moreover, it is easy to show that

$$x|_{(28)} = 0 \text{ iff } R^d = \frac{y^d\left(1-\pi_d^d\right)+w^d}{2y^d\left(1-\pi_d^d-\pi_f^d\right)} \pm \left\{\left[\frac{y^d\left(1-\pi_d^d\right)+w^d}{2y^d\left(1-\pi_d^d-\pi_f^d\right)}\right]^2 - \frac{w^d}{y^d\left(1-\pi_d^d-\pi_f^d\right)}\right\}^{0.5}$$

$$= \frac{y^d\left(1-\pi_d^d\right)+w^d \pm \left\{\left[y^d\left(1-\pi_d^d\right)-w^d\right]^2 + 4y^d w^d \pi_f^d\right\}^{0.5}}{2y^d\left(1-\pi_d^d-\pi_f^d\right)}.$$

Clearly, both these roots are real and positive. Hence (28) has two positive intersections with the horizontal axis in Figure 2. We now proceed to show that the smaller (larger) intersection is less (greater) than $\left(\frac{w^d}{y^d\left(1-\pi_d^d-\pi_f^d\right)}\right)$. Then (28) defines a locus as depicted in Figure 2, and the existence of a unique intersection of (27) and (28) satisfying $x > 0$ and $R^d > 0$ follows from the convexity of (27) and the concavity of (28).

(c) It now remains to show that

$$\frac{w^d}{y^d\left(1-\pi_d^d-\pi_f^d\right)} > \frac{y^d\left(1-\pi_d^d\right)+w^d-\left\{\left[y^d\left(1-\pi_d^d\right)-w^d\right]^2+4y^dw^d\pi_f^d\right\}^{0.5}}{2y^d\left(1-\pi_d^d-\pi_f^d\right)} \quad \text{and} \tag{70}$$

$$\frac{w^d}{y^d\left(1-\pi_d^d-\pi_f^d\right)} < \frac{y^d\left(1-\pi_d^d\right)+w^d+\left\{\left[y^d\left(1-\pi_d^d\right)-w^d\right]^2+4y^dw^d\pi_f^d\right\}^{0.5}}{2y^d\left(1-\pi_d^d-\pi_f^d\right)} \tag{71}$$

hold. Clearly, (70) will be satisfied iff

$$w^d-y^d\left(1-\pi_d^d\right) > -\left\{\left[y^d\left(1-\pi_d^d\right)-w^d\right]^2+4y^dw^d\pi_f^d\right\}^{0.5}. \tag{72}$$

If $w^d > y^d\left(1-\pi_d^d\right)$, (70) is trivially satisfied. On the other hand, if $w^d < y^d\left(1-\pi_d^d\right)$ then (72) is equivalent to

$$\left[w^d-y^d\left(1-\pi_d^d\right)\right]^2 < \left[y^d\left(1-\pi_d^d\right)-w^d\right]^2+4y^dw^d\pi_f^d;$$

which obviously holds.

Equation (71) is equivalent to

$$\left[w^d-y^d\left(1-\pi_d^d\right)\right]^2 < \left[y^d\left(1-\pi_d^d\right)-w^d\right]^2+4y^dw^d\pi_f^d;$$

which obviously holds.

So far we have shown that (27) and (28) have a unique intersection with $x > 0$ and $R^d > 0$. It remains to be shown that this yields a steady state equilibrium with positive nominal rates of interest. This requires that $R^d\sigma^d > 1$ and $R^d\sigma^f > 1$. Since $R^d \in A$, clearly this will be the case if

$\frac{w^f}{y^f\left(1-\pi^f_f-\pi^f_d\right)}\sigma^d > 1$ and $\frac{w^f}{y^f\left(1-\pi^f_f-\pi^f_d\right)}\sigma^f > 1$ hold. These conditions are satisfied (with $\sigma^d > 1$ and $\sigma^f > 1$), if the underlying foreign economy is not "too strongly Samuelson case". ■

## Appendix B: Proof of Proposition 2

The proof is for the case $\frac{w^d}{y^d\left(1-\pi^d_d-\pi^d_f\right)} < \frac{w^f}{y^f\left(1-\pi^f_f-\pi^f_d\right)}$. Here $R^d \in \left[\frac{w^d}{y^d\left(1-\pi^d_d-\pi^d_f\right)}, \frac{w^f}{y^f\left(1-\pi^f_f-\pi^f_d\right)}\right] = B$. The case where $\frac{w^f}{y^f\left(1-\pi^f_f-\pi^f_d\right)} < \frac{w^d}{y^d\left(1-\pi^d_d-\pi^d_f\right)}$ is left to the reader.

When Trace J > 0 and Det J > 0, the steady state is a saddle with monotone dynamics if and only if Trace J > 1 + Det J holds (see, for example, Azariadis, 1993). From equations (29) - (32) we have

$$\text{Trace } J = \frac{\left(R^d\right)^2}{w^d}\left(1-\pi^d_d-\pi^d_f\right)y^d + \frac{R^d}{w^d}\pi^f_d y^f\left[x + \frac{w^d}{w^f}\right] + \left(1-\pi^f_f-\pi^f_d\right)\frac{y^f}{w^f}R^d > 0. \tag{73}$$

Moreover, using equation (28), we know that

$$\frac{\left(R^d\right)^2}{w^d}\left(1-\pi^d_d-\pi^d_f\right)y^d + \frac{R^d}{w^d}\pi^f_d y^f x = R^d\left[1 + \left(1-\pi^d_d\right)\frac{y^d}{w^d}\right] - 1.$$

Hence, we can rewrite (73) as

$$\text{Trace } J = R^d - 1 + R^d\left[\left(1-\pi^d_d\right)\frac{y^d}{w^d} + \left(1-\pi^f_f\right)\frac{y^f}{w^f}\right].$$

Furthermore, equations (29) - (32) also imply that

$$\text{Det } J = \frac{\left(R^d\right)^3}{w^d w^f}\left(1-\pi^d_d-\pi^d_f\right)\left(1-\pi^f_f-\pi^f_d\right)y^d y^f > 0. \tag{74}$$

Therefore, from (73) and (74),

$$\text{Trace } J = \frac{\text{Det } J}{\left(1-\pi^f_f-\pi^f_d\right)R^d}\frac{w^f}{y^f} + \frac{R^d}{w^d}\pi^f_d y^f x + \left(1-\pi^f_f\right)\frac{y^f}{w^f}R^d.$$

Thus, Trace J > 1 + Det J holds if and only if

$$\frac{R^d}{w^d}\pi^f_d y^f x > \text{Det } J\left[1 - \frac{w^f}{\left(1-\pi^f_f-\pi^f_d\right)y^f R^d}\right] + 1 - \left(1-\pi^f_f\right)\frac{y^f}{w^f}R^d. \tag{75}$$

A sufficient condition for (75) to hold is

$$\frac{R^d}{w^d}\pi_d^f y^f x \geq \text{Det}\,J\left[1-\frac{w^f}{\left(1-\pi_f^f-\pi_d^f\right)y^f R^d}\right]+1-\left(1-\pi_f^f-\pi_d^f\right)\frac{y^f}{w^f}R^d$$

$$=\left[1-\left(1-\pi_f^f-\pi_d^f\right)\frac{y^f}{w^f}R^d\right]\left[1-\frac{\text{Det}\,J\;w^f}{\left(1-\pi_f^f-\pi_d^f\right)y^f R^d}\right]$$

$$=\left[1-\left(1-\pi_f^f-\pi_d^f\right)\frac{y^f}{w^f}R^d\right]\left[1-\frac{\left(R^d\right)^2}{w^d}\left(1-\pi_d^d-\pi_f^d\right)y^d\right].$$

But $R^d < \frac{w^f}{\left(1-\pi_f^f-\pi_d^f\right)y^f}$, so $1 > \left(1-\pi_f^f-\pi_d^f\right)\frac{y^f}{w^f}R^d$ holds. Moreover $R^d > \frac{w^d}{y^d\left(1-\pi_d^d-\pi_f^d\right)}$. Thus if $R^d \geq 1$, then $1 < \left(1-\pi_d^d-\pi_f^d\right)\frac{y^d}{w^d}\left(R^d\right)^2$ holds as well. Therefore, (75) necessarily holds if $R^d \geq 1$, and the steady state is a saddle with monotone local dynamics.

It remains to be shown that there is a unique equilibrium path approaching the steady state. Given $R_0^d$, the domestic market clearing condition, equation (13), determines $p_0^d$. This value can be used in the domestic loan market clearing condition for period 0, equation (20), to obtain $i_0^d$. Moreover, using $p_0^d$ in the period 0 domestic money market clearing condition, equation (18), allows us to obtain $x_0$. With $x_0$ and $i_0^d$ determined, the market clearing condition for international loans, equation (22), gives $i_0^f$. The initial value of the loan interest rate for the foreign country, $R_0^f$, can then be obtained from (21). Finally, using $x_0$ and $R_0^d$ in equations (24) and (26), yields the equilibrium sequence $\left\{x_t, R_t^d\right\}$. Clearly, $R_0^d$ is not determined by any equilibrium condition, hence one state variable is not predetermined. Since the dimension of the stable manifold is one, this implies that there is a unique equilibrium path approaching the long run equilibrium. ■

## Appendix C: Proof of Proposition 6

We consider the case where $\frac{w^d-\tau}{\left(y^d-\tau\right)\left(1-\pi_d^d-\pi_f^d\right)} < \frac{w^f}{y^f\left(1-\pi_f^f-\pi_d^f\right)}$, so that $R^d \in \left(\frac{w^d-\tau}{\left(y^d-\tau\right)\left(1-\pi_d^d-\pi_f^d\right)}, \frac{w^f}{y^f\left(1-\pi_f^f-\pi_d^f\right)}\right)$. The proof for the opposite case is left to the reader.

From equations (60) - (63), it is obvious that

$$\begin{aligned} \text{Trace } J &= \frac{\left(1-\pi^d_d-\pi^d_f\right)\left(y^d-\tau\right)}{(w^d-\tau)}\left(R^d\right)^2+\left(1-\pi^f_f-\pi^f_d\right)\frac{y^f}{w^f}R^d \\ &+ \left[\frac{w^f x+\left(w^d-\tau\right)}{\sigma^f w^f\,(w^d-\tau)}\right]\left\{\frac{\left[\pi^f_d y^f x+\pi^d_d\left(y^d-\tau\right)\right]-\left(\pi^f_f+\pi^f_d\right)y^f}{\left(\pi^d_d+\pi^d_f\right)(y^d-\tau)+\left(\pi^f_f+\pi^f_d\right)y^f x}\right\}R^d>0; \end{aligned} \tag{76}$$

while

$$\begin{aligned} \text{Det } J &= \left(1-\pi^d_d-\pi^d_f\right)\left(1-\pi^f_f-\pi^f_d\right)\left(\frac{y^d-\tau\;\,y^f}{(w^d-\tau)\,w^f}\right)\left(R^d\right)^3 \\ &+ \left[\frac{w^f x+\left(w^d-\tau\right)}{\sigma^f w^f\,(w^d-\tau)}\right]\left\{\frac{\left(y^d-\tau\right)y^f\left(\pi^d_d\pi^f_f-\pi^d_f\pi^f_d\right)}{\left(\pi^d_d+\pi^d_f\right)(y^d-\tau)+\left(\pi^f_f+\pi^f_d\right)y^f x}\right\}R^d. \end{aligned} \tag{77}$$

We now establish conditions for which $\text{Det } J \geq 0$. Equation (77) implies that $\text{Det } J \geq 0$ holds iff[15]

$$\left(1-\pi^d_d-\pi^d_f\right)\left(1-\pi^f_f-\pi^f_d\right)R^d \geq -\left[\frac{w^f x+\left(w^d-\tau\right)}{\sigma^f R^d}\right]\frac{\left(\pi^d_d\pi^f_f-\pi^d_f\pi^f_d\right)}{\left(\pi^d_d+\pi^d_f\right)(y^d-\tau)+\left(\pi^f_f+\pi^f_d\right)y^f x};$$

which is equivalent to the condition

$$\begin{aligned} &\left(1-\pi^d_d-\pi^d_f\right)\left(y^d-\tau\right)\left\{\left(1-\pi^f_f-\pi^f_d\right)\left(\pi^d_d+\pi^d_f\right)\sigma^f R^d+\pi^d_d\pi^f_f-\pi^d_f\pi^f_d\right\}+ \\ &\left(1-\pi^f_f-\pi^f_d\right)y^f x\left\{\left(1-\pi^d_d-\pi^d_f\right)\left(\pi^f_f+\pi^f_d\right)\sigma^f R^d+\pi^d_d\pi^f_f-\pi^d_f\pi^f_d\right\}\geq 0. \end{aligned}$$

$\text{Det } J > 0$ holds when this inequality is strict.

Since $\sigma^f R^d > 1$, sufficient conditions for $\text{Det } J > 0$ are

$$\left(1-\pi^f_f-\pi^f_d\right)\left(\pi^d_d+\pi^d_f\right)+\pi^d_d\pi^f_f \geq \pi^d_f\pi^f_d, \text{ and} \tag{78}$$

$$\left(1-\pi^d_d-\pi^d_f\right)\left(\pi^f_f+\pi^f_d\right)+\pi^d_d\pi^f_f \geq \pi^d_f\pi^f_d. \tag{79}$$

And, since $\left(1-\pi^f_f-\pi^f_d\right)R^d < \frac{w^f}{y^f}$, sufficient conditions for $\text{Det } J < 0$ are

$$\left(\pi^d_d+\pi^d_f\right)\frac{w^f}{y^f}\sigma^f+\pi^d_d\pi^f_f \quad < \quad \pi^d_f\pi^f_d \tag{80}$$

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[15] Note that $\text{Det } J > 0$ necessarily holds if $\pi^d_d\pi^f_f \geq \pi^d_f\pi^f_d$.

$$\frac{\left(1-\pi_d^d-\pi_f^d\right)}{\left(1-\pi_f^f-\pi_d^f\right)}\left(\pi_f^f+\pi_d^f\frac{w^f}{y^f}\sigma^f+\pi_d^d\pi_f^f\right) < \pi_f^d\pi_d^f. \tag{81}$$

(a) We first consider the case where $\text{Det}\, J > 0$. When $\text{Trace}\, J > 0$ and $\text{Det}\, J > 0$, we know that the steady state is a saddle and local dynamics are monotone if and only if $\text{Trace}\, J > 1 + \text{Det}\, J$ holds (see, for example, Azariadis, 1993). Note that (76) and (77) imply

$$\begin{aligned}\text{Trace}\, J &= \text{Det}\, J + \left(1-\pi_f^f-\pi_d^f\right)\frac{y^f}{w^f}\left\{1-\left(1-\pi_d^d-\pi_f^d\right)\left(\frac{y^d-\tau}{(w^d-\tau)}\right)\left(R^d\right)^2\right\}R^d \\ &\quad + \left(1-\pi_d^d-\pi_f^d\right)\left(\frac{y^d-\tau}{(w^d-\tau)}\right)\left(R^d\right)^2 + \left[\frac{w^f x+\left(w^d-\tau\right)}{\sigma^f w^f\,(w^d-\tau)}\right]\pi_d^f y^f R^d.\end{aligned} \tag{82}$$

Thus, $\text{Trace}\, J > 1 + \text{Det}\, J$ holds if

$$\left[\left(1-\pi_f^f-\pi_d^f\right)\frac{y^f}{w^f}R^d-1\right]\left\{1-\left(1-\pi_d^d-\pi_f^d\right)\left(\frac{y^d-\tau}{(w^d-\tau)}\right)\left(R^d\right)^2\right\}\geq 0. \tag{83}$$

Since $R^d < \left(\frac{w^f}{1-\pi_f^f-\pi_d^f\; y^f}\right)$, and $R^d > \left(\frac{(w^d-\tau)}{1-\pi_d^d-\pi_f^d\;(y^d-\tau)}\right)$ hold, (83) is satisfied if $R^d \geq 1$. Hence we can conclude that the steady state is a saddle, and local dynamics are monotone, if $R^d \geq 1$, (78), and (79) all hold.

(b) When $\text{Trace}\, J > 0$ and $\text{Det}\, J < 0$, we know that the steady state is a saddle with oscillatory dynamics when $\text{Trace}\, J > 1 + \text{Det}\, J$ and $1 + \text{Trace}\, J + \text{Det}\, J > 0$. We already know that $\text{Trace}\, J > 1 + \text{Det}\, J$ when $R^d \geq 1$ is satisfied. From (82) we know that $1 + \text{Trace}\, J > -\text{Det}\, J$ (the negative eigenvalue exceeds $-1$) holds iff

$$\begin{aligned}&1 + 2\,\text{Trace}\, J + \left[\left(1-\pi_d^d-\pi_f^d\right)\left(1-\pi_f^f-\pi_d^f\right)\left(\frac{y^d-\tau\;\, y^f}{(w^d-\tau)\,w^f}\right)\left(R^d\right)^3\right] \\ &> \frac{\left(1-\pi_d^d-\pi_f^d\right)\left(y^d-\tau\right)}{(w^d-\tau)}\left(R^d\right)^2+\left(1-\pi_f^f-\pi_d^f\right)\frac{y^f}{w^f}R^d+\left[\frac{w^f x+\left(w^d-\tau\right)}{\sigma^f w^f\,(w^d-\tau)}\right]\pi_d^f y^f R^d.\end{aligned}$$

Using the definition of the trace in (76), this condition is equivalent to

$$1 + \text{Trace}\, J + \left[\left(1-\pi_d^d-\pi_f^d\right)\left(1-\pi_f^f-\pi_d^f\right)\left(\frac{y^d-\tau\;\, y^f}{(w^d-\tau)\,w^f}\right)\left(R^d\right)^3\right]$$

$$> \left[\frac{w^f x + \left(w^d - \tau\right)}{\sigma^f w^f \left(w^d - \tau\right)}\right] \left\{ \pi_d^f y^f R^d - \frac{\left[\pi_d^f y^f x + \pi_d^d \left(y^d - \tau\right)\right]\left(\pi_f^f + \pi_d^f\right) y^f}{\left(\pi_d^d + \pi_f^d\right)\left(y^d - \tau\right) + \left(\pi_f^f + \pi_d^f\right) y^f x} R^d \right\};$$

or to

$$1 + \text{Trace } J + \left[\left(1 - \pi_d^d - \pi_f^d\right)\left(1 - \pi_f^f - \pi_d^f\right)\left(\frac{\left(y^d - \tau\right) y^f}{\left(w^d - \tau\right) w^f}\right) R^d\right]$$

$$> y^f R^d \left[\frac{w^f x + \left(w^d - \tau\right)}{\sigma^f w^f \left(w^d - \tau\right)}\right]\left[\frac{\left(\pi_d^f \pi_f^d - \pi_d^d \pi_f^f\right)\left(y^d - \tau\right)}{\left(\pi_d^d + \pi_f^d\right)\left(y^d - \tau\right) + \left(\pi_f^f + \pi_d^f\right) y^f x}\right]$$

$$= \frac{\left(y^f R^d\right)^2}{\sigma^f w^f \left(w^d - \tau\right)} \left\{ \frac{\left(\pi_d^f \pi_f^d - \pi_d^d \pi_f^f\right)\left(y^d - \tau\right)\left[\left(1 - \pi_f^f - \pi_d^f\right) y^f x + \left(1 - \pi_d^d - \pi_f^d\right)\left(y^d - \tau\right)\right]}{\left(\pi_d^d + \pi_f^d\right)\left(y^d - \tau\right) + \left(\pi_f^f + \pi_d^f\right) y^f x} \right\};$$

Notice that a weak sufficient condition for this to hold, and therefore for $1 + \text{Trace } J > -\text{Det } J$ to obtain, is

$$1 \geq \frac{y^f \left(y^d - \tau\right)}{w^f \left(w^d - \tau\right)} \frac{\left(R^d\right)^2}{\sigma^f} \left\{ \frac{\left(\pi_d^f \pi_f^d - \pi_d^d \pi_f^f\right)\left[\left(1 - \pi_f^f - \pi_d^f\right) y^f x + \left(1 - \pi_d^d - \pi_f^d\right)\left(y^d - \tau\right)\right]}{\left(\pi_d^d + \pi_f^d\right)\left(y^d - \tau\right) + \left(\pi_f^f + \pi_d^f\right) y^f x} \right\}. \quad (84)$$

Condition (84), in turn, is satisfied if

$$\pi_d^d + \pi_f^d \geq 0.5 \quad (85)$$

$$\pi_f^f + \pi_d^f \geq 0.5 \quad (86)$$

$$1 \geq \frac{y^f \left(y^d - \tau\right)}{w^f \left(w^d - \tau\right)} \frac{\left(R^d\right)^2}{\sigma^f} \left(\pi_d^f \pi_f^d - \pi_d^d \pi_f^f\right) \quad (87)$$

all hold. Since $R^d < \frac{w^f}{\left(1 - \pi_f^f - \pi_d^f\right) y^f}$, (87) holds if

$$\sigma^f \left(1 - \pi_f^f - \pi_d^f\right)^2 \geq \frac{w^f \left(y^d - \tau\right)}{y^f \left(w^d - \tau\right)} \left(\pi_d^f \pi_f^d - \pi_d^d \pi_f^f\right) \quad (88)$$

Hence, if (80), (81), $\sigma^f R^d > 1$, (85), (86), and (88) all hold, then $1 + \text{Trace } J > -\text{Det } J > 0$ and the steady state is a saddle. Local dynamics are oscillatory.

(c) It remains to be shown that there is a unique equilibrium path approaching the steady state for both case (a) and case (b). Given $x_0$, the money market clearing condition, (42), determines

$m_0^f$ and hence $p_0^f$. The value of $p_0^d = \frac{p_0^f}{x_0}$ is then determined as well. Given $p_0^d$ and the holdings of foreign money (if any) by initial old agents in the domestic economy, the domestic market clearing condition, equation (13), determines $R_0^d$. Given $R_0^d$, the domestic market clearing condition for loans, equation (52), gives $i_0^d$. Using $x_0$ and $i_0^d$ in the market clearing condition for international loans, equation (53) gives $i_0^f$. We then obtain $R_0^f$ from the foreign loan market clearing condition, equation (21). Finally, using $x_0$ and $R_0^d$ in equations (54) and (57), we obtain the equilibrium sequence $\left\{x_t; R_t^d\right\}$. Clearly, $x_0$ is not determined by any equilibrium condition, hence one state variable is not predetermined. Since the dimension of the stable manifold is one when either (a) or (b) obtains, this implies that there is a unique equilibrium path in both cases. ∎

## Appendix D: Proof of Proposition 7

From equations (64) and (65), it is clear that dollarization implies a reduction in world steady state seigniorage revenue if and only if

$$\left(\pi_f^f + \pi_d^f\right) y^f + \left(\frac{\sigma^d - 1}{\sigma^d}\right)\left(\frac{\sigma^f}{\sigma^f - 1}\right)\left(\pi_d^d + \pi_f^d\right) y^d > \left(\pi_f^f + \pi_d^f\right) y^f + \frac{\left(\pi_d^d + \pi_f^d\right)\pi_d^f y^f}{\sigma^f \pi_d^f + \pi_d^d\left(\sigma^f - 1\right)}.$$

When $\sigma^f > 1$, this condition is equivalent to

$$\left(\frac{\sigma^d - 1}{\sigma^d}\right)\left(\frac{\sigma^f}{\sigma^f - 1}\right) y^d > \frac{\pi_d^f y^f}{\sigma^f \pi_d^f + \pi_d^d\left(\sigma^f - 1\right)},$$

or to

$$y^d \left(\frac{\sigma^d - 1}{\sigma^d}\right)\left(\pi_d^f + \pi_d^d\right)\left(\sigma^f\right)^2 - \left[y^d \left(\frac{\sigma^d - 1}{\sigma^d}\right)\pi_d^d + \pi_d^f y^f\right]\sigma^f + \pi_d^f y^f > 0. \qquad (89)$$

Note that (89) holds at $\sigma^f = 1$ and at $\sigma^f = \infty$. Moreover, the left-hand side of (89) is minimized at

$$\sigma^f = \frac{y^d \left(\frac{\sigma^d - 1}{\sigma^d}\right)\pi_d^d + \pi_d^f y^f}{2 y^d \left(\frac{\sigma^d - 1}{\sigma^d}\right)\left(\pi_d^d + \pi_d^f\right)}.$$

Hence if

$$\pi_d^f y^f \leq y^d \left(\frac{\sigma^d - 1}{\sigma^d}\right)\left(\pi_d^d + \pi_d^f\right); \qquad (90)$$

then (89) holds for all $\sigma^f \geq 1$. On the other hand, if (90) fails, then (89) holds for all $\sigma^f \geq 1$ if and only if

$$\pi^f_d y^d \geq \frac{\left[y^d\left(\frac{\sigma^d-1}{\sigma^d}\right)\pi^d_d + \pi^f_d y^f\right]^2}{4y^d\left(\frac{\sigma^d-1}{\sigma^d}\right)\left(\pi^d_d + \pi^f_d\right)}. \tag{91}$$

This establishes parts (a) and (b) of the proposition.

If (90) and (91) fail to obtain, then we have the situation depicted in Figure 4. Clearly, the left-hand side of (89) is positive in this case if $\left(\sigma^f - 1\right)$ is sufficiently small, or if $\sigma^f$ is sufficiently large. ■

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Figure 1:
Timing of Events Within a Period

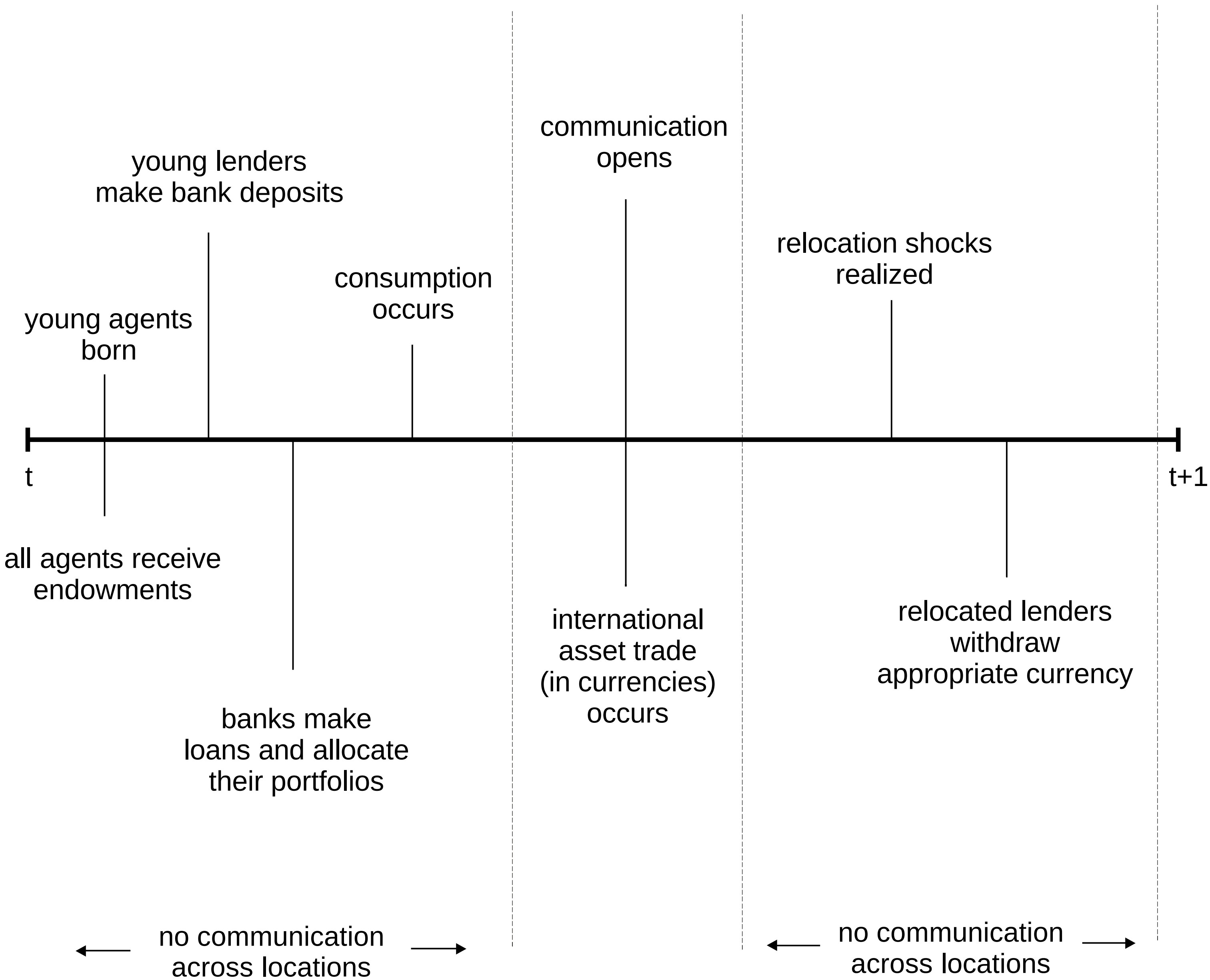


Figure 2:
Determination of a Steady State Equilibrium with International Capital Flows and Two Currencies

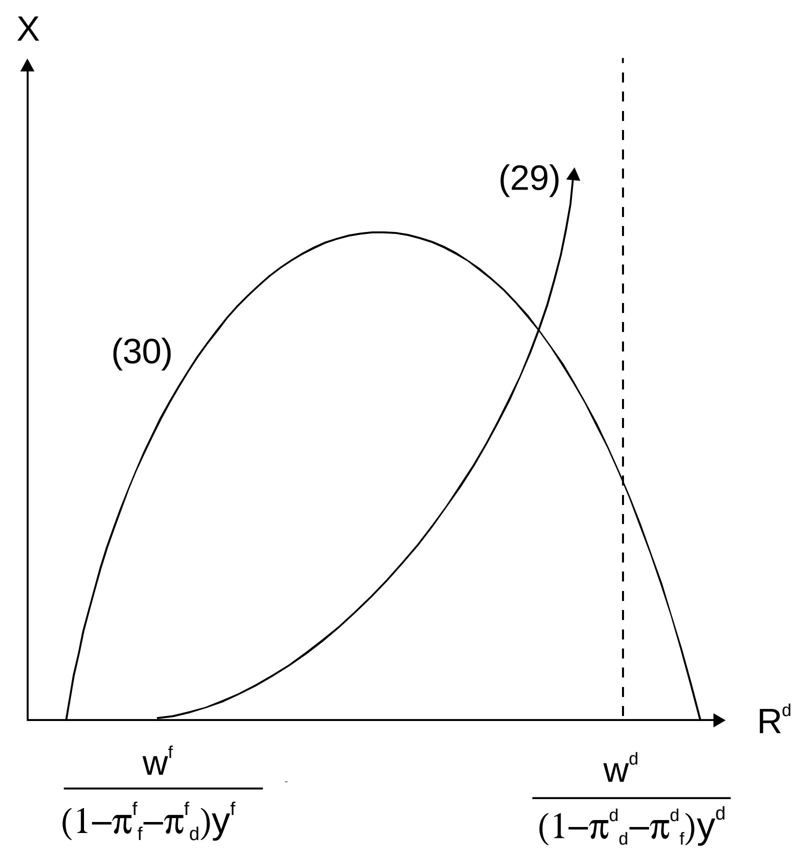


Figure 3:
Determination of a Steady State Equilibrium with
International Capital Flows and One Currency

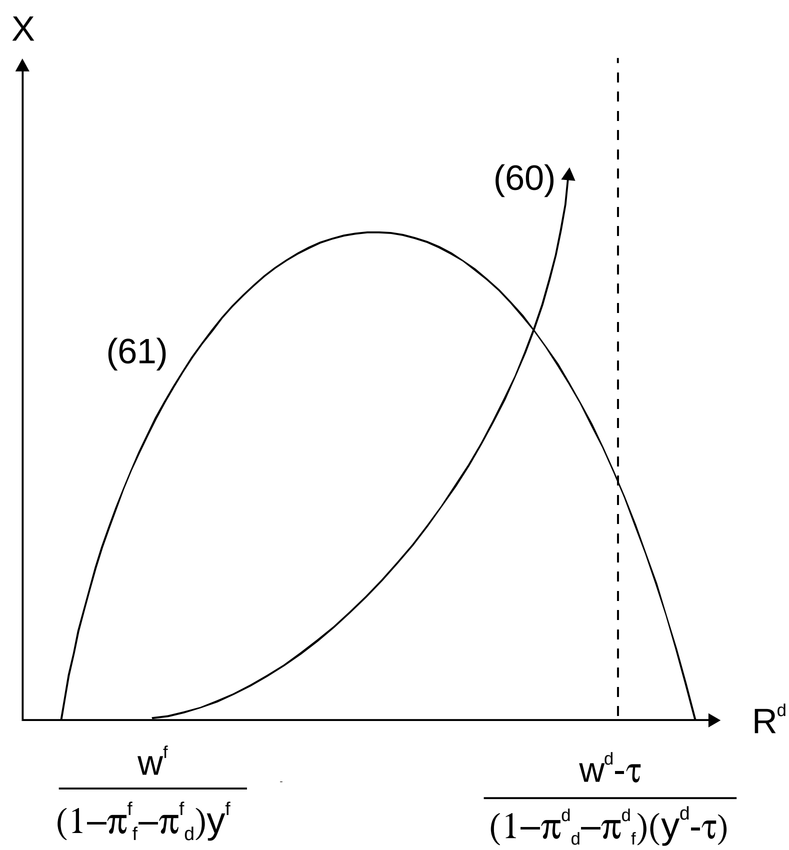
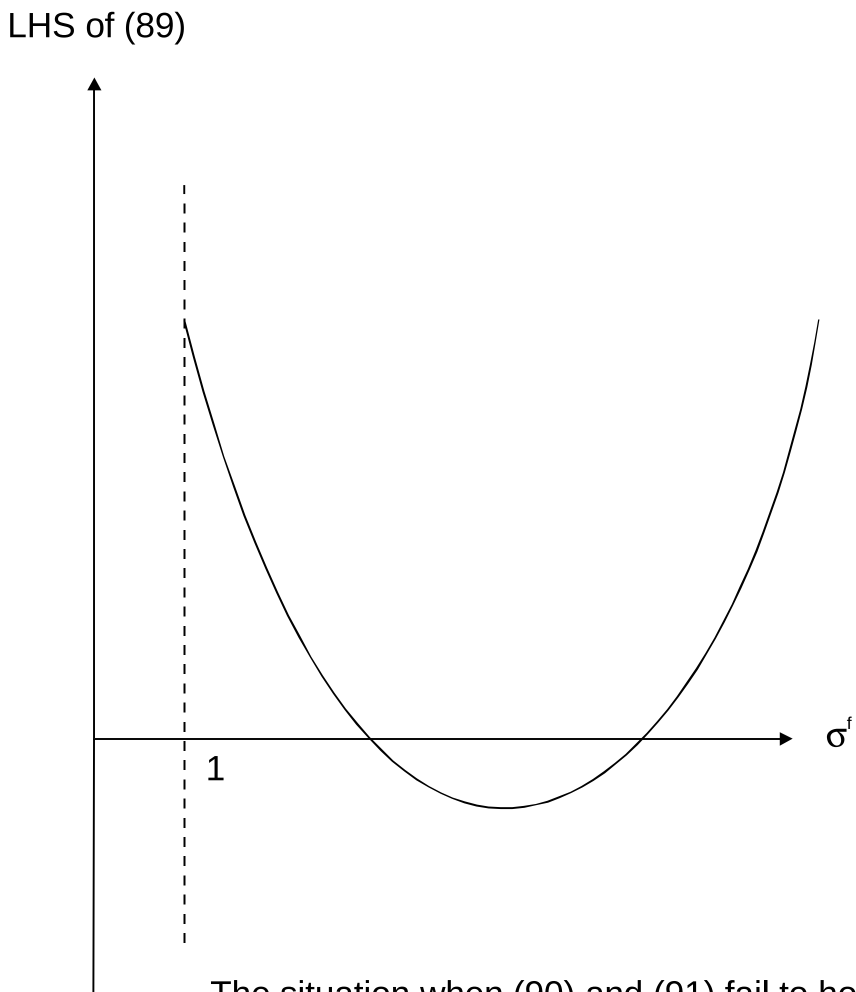


Figure 4:
The Fiscal Consequences of Dollarization




The situation when (90) and (91) fail to hold