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On Domestic Financial Market Frictions, Unrestricted International Capital Flows, and Crises in Small Open Economies [1]

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November, 1998

Abstract

We produce an example of a small open economy for which small increases in the world interest rate may induce a sharp decline in output and a precipitous depreciation of the nominal and the real exchange rate (RER). Due to a costly state verification (CSV) problem in domestic credit markets, combined with unrestricted international capital flows, our economy generates two long-run equilibria, one with low GDP and a relatively depreciated RER, and one with high GDP and a relatively appreciated RER. The first is always a saddle, while the second may be a sink or a source, depending on the level of the world interest rate. More precisely, there exists a critical level of the world interest rate above which the high-GDP steady state turns from a sink to a source. Hence unexpected increases in the world interest rate to "supercritical" levels may induce a "crisis" in the economy. This is identified in the model with the economy switching from an equilibriumpath approaching the high output steady state, to the saddlepath approaching the low output steady state. We simulate such a "crisis" trajectory for our model economy. In Mexico's recent history, periods of growth associated with an appreciation of the real exchange rate (RER) have alternated with periods of sharp contraction characterized by a depreciation of the RER. Our economy may display such behavior as an equilibrium response to changes in the world interest rate.

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[1] We thank Tim Kehoe, Todd Keister, Manuel Santos, Karl Shell and especially Bruce Smith for very helpful discussions. Jaime Calleja Alderete, Eduardo Camero Godínez, and Juan Vargas Hernández, provided excellent research assistance. All remaining errors are ours.

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Introduction

\Even with the best economic management, small open economies remain vulnerable. They are like small rowboats on a wild and open sea. Although we may not be able to predict it, the chances of eventually being broadsided by a large wave are signi¯cant no matter how well the boat is steered"

Joseph Stiglitz, (1998, p. 1)

In the wake of the 1990s crises in Latin America and East Asia, a debate has emerged on the causes of the dramatic economic downturns experienced by these regions. Some scholars favor the view that a ¯nancial panic, i.e. a sudden shift in market expectations and con¯dence was key to the turmoil, irrespective of structural factors.[4] Others claim that, while exacerbated by self-ful¯lling behavior, the crises really re°ected a deterioration in macroeconomic fundamentals in general, and in the external account in particular.[5] In the latter view, real exchange rate appreciation and growing current account imbalances resulted in the accumulation of foreign debt which served to ¯nance \overinvestment". This was made worse by explicit or implicit bank - and country - bailout guarantees, which led heavily leveraged domestic intermediaries to lend to \excessively" risky projects. The result was \excessive," and hence unsustainable, growth (Corsetti et al., 1998, p.18).

In this paper, we present a model of a small open economy, in which a small increase in an external factor - the world interest rate - can provoke a precipitous decline in economic activity. At subcritical world interest rates, our economy may follow an equilibrium path of sustainable growth, accumulation of foreign debt and real exchange rate appreciation. However, a small increase in the

[4] See, for example, Radelet and Sachs (1998).

[5] This view is supported by, for example, Corsetti et al. (1998), and Dornbusch and Werner (1994).

world interest rate may eliminate that equilibrium path, provoke a sharp devaluation in the nominal and real exchange rate, and put the economy on an equilibrium path of declining economic activity.

The key ingredient of our model is the interaction between an informational friction in domestic credit markets and international capital °ows. We consider a small open economy - one in the spirit of Diamond's (1965) neoclassical growth model - that produces two goods, one tradable and one not. We assume that capital investment for production in the domestic tradable sector is subject to a costly state veri¯cation (CSV) problem, and requires external ¯nance. This external funding is naturally provided through intermediated loans (Williamson, 1986).[6] Capital production for the non-tradable sector, on the other hand, is subject to no informational asymmetries. We consider the case where money is held due to a reserve requirement against bank loans and we assume that domestic residents can borrow and lend freely in international ¯nancial markets at the world (risk free) rate of interest - which they take as given and do not a®ect.[7] Thus, young agents combine young period income, along with credit obtained either at home or abroad, in order to make investments.

We use this framework to analyze a number of issues. First, we study steady state equilibria. We describe technical conditions under which there will be exactly two steady state equilibria. One long-run equilibrium has a relatively low level of output and a relatively high level of the real exchange rate (RER), i.e. a high price of the tradable good relative to the price of the non-tradable good. The second steady state has a relatively high level of output and a relatively low level of the RER. We show that the former is necessarily a saddle, while the latter may either be stable (a sink)

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[6] The CSV problem we introduce is of the type considered by Townsend (1979), Gale and Hellwig (1985), Williamson (1986, 1987), Bernanke and Gertler (1989), and most speci¯cally Boyd and Smith (1994, 1997, 1998).

[7] Hence our economy resembles closely the one-sector economy presented in Huybens and Smith (1998).

or unstable (a source).

We then describe conditions under which the high-output-low-RER steady state is a sink and hence can be approached. This occurs when, ceteris paribus, the interest rate in world capital markets is relatively low. However, we also describe conditions under which there exists a \critical" world interest rate. World interest rates exceeding this level will transform the high-output-low-RER steady state from a sink to a source. Unexpected increases in the world interest rate may therefore eliminate entire sets of high activity, low RER equilibrium paths, thereby inducing a crisis in the economy.

We proceed to simulate such a crisis. We provide an example of an economy which faces a subcritical world interest rate and follows an equilibrium path towards the high-output-low-RER steady state. This equilibrium path is characterized by growing investment and output, accumulation of foreign debt, and real exchange rate appreciation, all in an entirely sustainable fashion. The word \excessive" does not apply to this trajectory. However, when the world interest rate unexpectedly and permanently rises to a level above its critical value (however small the actual increase), the economy faces a crisis. Indeed, the path approaching the high-output-low-RER steady state is eliminated and the economy reverts to the saddlepath approaching the low-output-high-RER steady state. On impact, this switch between equilibrium paths implies a sharp nominal and real depreciation, and a precipitous decline in output as well as lending provided by international investors.

Such a sequence of events is qualitatively quite consistent with the recent economic history of Mexico. From 1980 to 1994, the Mexican economy experienced several periods of growth associated with a continuously appreciating RER. These episodes of growth were thrice interrupted (in 1982, 1985, and 1994) by sharp declines in output, in conjunction with a precipitous devaluation of the nominal and real exchange rate (Figure 1). Our model shows how such trajectories can arise as the equilibrium response to changes in the world interest rate. And indeed, each of the Mexican crisis episodes coincided with a marked increase in the world interest rate (Figure 2).

In the world presented in this paper the simple combination of a domestic informational friction with international capital °ows in the presence of money, may generate a \crisis" path induced by - possibly small - changes in external factors. What exactly accounts for these ¯ndings? Clearly, our results depend crucially on the existence of two long-run equilibria, and on their stability properties. In this economy, a steady state equilibrium is determined by the requirement that domestic investors deliver an expected return to lenders equal to the prevailing world interest rate. For capital investment in the tradable sector, which is subject to a CSV problem, there are typically two ways in which this can be obtained. One is for the domestic economy to have a relatively low capital stock in the tradable sector, and a correspondingly high marginal product of capital. This is associated with a low level of income, and a low level of internal ¯nance provided by domestic borrowers. The high marginal product of capital is attractive to international investors, but the low level of internal project ¯nance exacerbates the CSV problem. Alternatively, domestic borrowers may deliver the world interest rate in an economy with a relatively high capital stock in the tradable sector and hence a high level of income. Such economy presents a relatively low marginal product of capital, which is unattractive to investors, but this is exactly compensated by the high level of internal ¯nance which mitigates the costs due to CSV. Hence, if there is any long-run equilibrium, there are typically two of them, one with a high level of capital in the tradable sector, and one with a low level of capital in the tradable sector. Moreover, since labor is free to move between the tradable and the non-tradable sector, workers will, in equilibrium, earn the same wage in both sectors. Hence the steady state with the low capital labor ratio in the tradable sector will also have a low capital-labor ratio in the non-tradable sector, and a relatively low level of GDP. Moreover, equilibrium requires that the return to investment in capital for the non-tradable sector equals the world interest rate as well. This implies a negative correlation between the RER and the capital-labor ratio in the non-tradable sector, and thus between the RER and GDP.

To summarize, our economy displays two long-run equilibria, one with a low level of GDP and a relatively depreciated RER, and one with a high level of GDP and a relatively appreciated RER. Moreover, the second steady state is stable only when the world interest rate is below a certain critical value. Hence paths approaching this steady state can only be sustained for relatively low levels of the world interest rate. It is worth emphasizing that our results depend critically on the presence of both the CSV problem and international capital °ows. Without the informational friction in domestic credit markets and/or without international capital °ows, our economy would have a unique long-run equilibrium, and no crisis of the kind we have discussed would ever occur. Moreover, while imperfections in domestic credit markets are crucial to our results, the incidence of a crisis in our economy is independent of the presence of overly risky investment due to moral hazard problems induced by deposit insurance schemes.

A number of papers have been concerned with presenting an explanation for the 1994 Mexican currency crisis and the ensuing recession. Dornbusch and Werner (1994) emphasize the role of an \overvalued" RER and appeal to diverging speeds of adjustment in capital versus goods markets to propose a nominal devaluation with the purpose of avoiding a crisis. Flood, Garber and Kramer, (1996) explain the crisis as a speculative attack resulting from the inconsistency of ¯xing the exchange rate while maintaining ¯scal imbalance. Calvo and Mendoza (1996), Cole and Kehoe (1996), and Sachs, Tornell and Velasco (1996), stress self-ful¯lling prophecies and herd behavior. Our model has much in common with the latter set of papers, in particular an emphasis on the ¯nancial sector and the importance of conditions in international capital markets. However, the above-mentioned models mainly address the behavior of the economy in the proximity of an attack on the currency. In contrast, we emphasize how the interaction between the unrestricted operation of international capital markets and frictions in the domestic financial markets can produce an environment in which both prolonged periods of growth and RER appreciation, and episodes of sharp decline in output can arise.

The remainder of the paper proceeds as follows. Section 1 describes the environment while section 2 lays out the nature of trade in factor and credit markets. Section 3 analyzes general equilibrium in a small open monetary economy in the presence of legal restrictions on financial intermediation. We analyze both steady state and dynamical equilibria. Section 4 concludes.

1 The Model

We consider a small open economy inhabited by an infinite sequence of two-period lived, overlapping generations, plus an initial old generation. Each generation is identical in size and composition, and contains a continuum of agents with unit mass. Within each generation, agents are divided into two types: "potential borrowers" and "lenders". A fraction $\delta \in (0,1)$ of the population is potential borrowers. Throughout, we let $t = 0, 1, \ldots$ index time.

At each date there are two final goods, A and B. We will assume good A to be tradable, while good B is nontradable. The relative price between the two goods at date t will be denoted by $p_t \equiv p_t^b / p_t^a$. The inverse of p can be interpreted as the real exchange rate (RER), which we define as the domestic relative price of tradable to nontradable goods.[8,9]

[8] This definition of the RER follows Edwards (1989).

[9] In Galor (1992), one sector produces capital while the other sector produces the single consumer good, which

Both goods are produced using a constant returns to scale technology with capital and labor as inputs. These factors of production are immobile internationally. We assume that the capital input for production in the tradable good sector is qualitatively different from the capital input for production in the non-tradable sector, so that capital inputs cannot be transferred between sectors. Let K_t^j denote the time t capital input of type j, let L_t^j denote the time t labor input, and let $k^j \equiv \frac{K^j}{L^j}$ be the capital-labor ratio of a representative firm in sector $j \in \{a, b\}$. Output of the tradable good is then given by

$$Y_t^a = F_a(K_t^a, L_t^a) = L_t^a f_a(k_t^a) \tag{1}$$

while that of the nontradable good is

$$Y_t^b = F_b(K_t^b, L_t^b) = L_t^b f_b(k_t^b), \tag{2}$$

where $f_j(k^j) \equiv F_j(k^j, 1)$ denotes the intensive production function in sector j. F_j satisfies the following conditions: it is increasing in each argument, homogeneous of degree 1, strictly concave, and $F_j\left(0, L^j\right) = F_j\left(K^j, 0\right) = 0$ holds, for all K^j, L^j. In addition, $f_j' > 0 > f_j''$ holds $\forall k^j$, and f_j satisfies the standard Inada conditions. Finally, we assume that capital in both sectors depreciates fully in production.

Labor market clearing requires that

$$L_t = L_t^a + L_t^b.$$

All young agents are endowed with one unit of labor, which is supplied inelastically, and agents are retired when old. Individuals other than the old of period zero have no endowment of capital

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does not allow for an interpretation of p in terms of the RER. Of course, another way in which our model differs from Galor, is the presence of money and financial market frictions.

or final goods, while the initial old agents have an aggregate type a capital endowment of $K_0^a > 0$, and an aggregate type b capital endowment of $K_0^b > 0$.

Agents of all types are assumed to care only about old age consumption, and, in addition, all agents are risk neutral. Furthermore, we will assume that lenders consume only good B, while potential borrowers consume only good A.[10]

Potential borrowers and lenders are also differentiated by the fact that each potential borrower has access to a stochastic linear technology for converting date t tradable goods into date t + 1 capital for the production of good A. Lenders have no access to this technology.

The type a capital investment technology has the following properties. First, it is indivisible: each potential borrower has one investment project which can only be operated at the scale q. In particular, $q > 0$ units of the tradable good invested in one project at t yield zq units of type a capital at t + 1, where z is an iid (across borrowers and periods) random variable, which is realized at t + 1. We let G denote the probability distribution of z, and assume that G has a differentiable density function g with support $[0, \bar{z}]$. Then $\hat{z} \equiv \int_0^{\bar{z}} z g(z)\,dz$ is the expected value of z.

Second, the amount of type a capital produced by any investment project can be observed costlessly only by the project owner. Any agent other than the project owner can observe the return on the project only by bearing a fixed cost of $\gamma > 0$ units of type a capital.[11]

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[10] The choice of these particular preferences is motivated by the following considerations. First, we want the optimal contract between borrowers and lenders to be a standard debt contract. In order to accomplish this we need risk neutrality. Moreover, while not crucial for our results, assuming preferences which imply constant expenditure shares on the two goods greatly simplifies the dynamic analysis.

[11] That is, in verifying the project return, γ units of type a capital are used up. The assumption that capital is consumed in the verification process follows Bernanke and Gertler (1989), and is reponsible for the simple form assumed by the expected return to lenders under credit rationing [see equation (8) below].

Finally, we impose two assumptions on the distribution of z:

Assumption 1 $q > \gamma g(0)$:

Assumption 2 $g(z) + (\frac{\gamma}{q})g'(z) \geq 0$; for all $z \in [0; \bar{z}]$:

These assumptions imply a simple structure for the lender's the profit function and thus greatly simplify our analysis.[12]

We assume that the investment technology for capital of type b is as in Diamond (1965). One unit of tradable good invested at time t delivers one unit of sector b capital at time $t+1$: All agents, regardless of their type, have access to the type b capital investment technology.

2 Trade

For future reference, let $\kappa_t \in (0; 1)$ be the fraction of the total labor force employed in the tradable good sector at t, so that $\kappa_t = \frac{L_t^a}{L_t}$: We can then rewrite equations (1) and (2) in their intensive form,

$$y_t^a = \frac{Y_t^a}{L_t} = \frac{L_t^a Y_t^a}{L_t L_t^a} = \kappa_t f_a(k_t^a) \tag{3}$$

and

$$y_t^b = \frac{Y_t^b}{L_t} = \frac{L_t^b Y_t^b}{L_t L_t^b} = (1 - \kappa_t) f_b(k_t^b): \tag{4}$$

2.1 Factor Markets

We assume that capital and labor are traded in competitive markets at each date. Let ρ_t^a denote the time t capital rental rate in sector a, ρ_t^b the time t capital rental rate in sector b, and w_t the time

[12] See section 3 below. For a more extensive discussion of assumptions 1 and 2, see Boyd and Smith (1997, 1998), Huybens and Smith (1998) or Antinolfi and Huybens (1998).

t real wage rate, all in units of the tradable good. Then the standard factor pricing relationships obtain:

$$\rho_t^a = f_a'(k_t^a) \tag{5}$$

$$\rho_t^b = p_t f_b'(k_t^b) \tag{6}$$

$$w_t = w_a(k_t^a) = f_a(k_t^a) - k_t^a f_a'(k_t^a) = w_b(p_t; k_t^b) = p_t \left[f_b(k_t^b) - k_t^b f_b'(k_t^b) \right]. \tag{7}$$

Clearly, given our assumptions on the production technology, $w_a'(k_t^a) > 0$, $\frac{\partial w_b(p_t; k_t^b)}{\partial k_t^b} > 0$; and $\frac{\partial w_b(p_t; k_t^b)}{\partial p_t} > 0$.

2.2 Credit Markets

At t, all young agents supply one unit of labor inelastically, earning the real wage rate w_t. For lenders this income is saved in the form of money, assets issued abroad, investments in type b capital, or loans to domestic producers of type a capital.

Potential borrowers have young period income w_t, and we will assume that the scale of the project is large relative to their income.

Assumption 3 $q > w_a(k_t^a)$ for all "relevant" values of k_t^a.

Hence, producers of type a capital must obtain external financing, $q - w_t > 0$, to operate the projects. We can think of all credit extension as being intermediated in the manner described by Williamson (1986). That is, intermediation arises because it allows the economy to avoid the duplication of verification costs which would arise if loans were held by individual lenders. Intermediaries behave competitively, taking as given the rates of return to currency, and risk-free assets issued abroad, and simply choosing whether to accept or reject the loan contract terms offered by potential entrepreneurs who seek external funding for their projects. In equilibrium, intermediaries will be perfectly diversified, and earn a non-stochastic return on their lending portfolio.

It is well known that in the CSV environment, the optimal loans contracts offered by potential borrowers (producers of type a capital) take the form of a standard debt contract.[13] In particular, a funded entrepreneur either repays a non state-contingent gross loan interest rate x_t or defaults. In the latter case, the intermediary monitors the project outcome, and retains all proceeds net of monitoring costs. All payments specified by these contracts are in terms of the tradable good. When assumptions 1 and 2 hold, the expected return to lending, $\pi(x_t)$, implied by such standard debt contracts, is a strictly concave function of the loan interest rate x_t which reaches its maximum at $\hat{x}_t$ such as depicted in Figure 3. For relatively low loan interest rates, expected returns rise as x_t increases, because the rise in gross repayments of principal plus interest, $x_t[q - w_t]$, outweighs the rise in costs due to an increased number of bankruptcies. However, beyond $\hat{x}_t$, a further increase in the loan interest rate produces a decrease in the expected return to lending. Indeed, for loan interest rates higher than $\hat{x}_t$, the fraction of projects going bankrupt becomes so high, that the increased costs associated with monitoring, and the fact that bankrupt firms cannot fully repay principal plus interest, dominate the increase in gross repayments which one would otherwise expect.

This environment is also characterized by the fact that unfulfilled demand for credit or "credit rationing" may arise.[14] When credit rationing obtains, all potential entrepreneurs offer the loan in-

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[13] For a formal derivation of this contract, as well as other results described in this section, see Boyd and Smith (1997, 1998), Huybens and Smith (1998) or Antinolfi and Huybens (1998). To facilitate the refereeing process, we have reproduced these derivations in a referee's appendix.

[14] The idea that the equilibrium loan interest rate does not necessarily clear the loan market in an economy with informational problems, was first noted by Stiglitz and Weiss (1981). Gale and Hellwig (1985) and Williamson (1986, 1987) formalized this notion for the CSV environment.

terest rate $\hat{R}_t$ that maximizes the expected return for a prospective lender.[15] Unfunded entrepreneurs cannot then alter loan contract terms in order to obtain credit since this would simply reduce the expected return to any potential lender. Thus credit rationing may be an equilibrium outcome. We henceforth focus on economies in which equilibrium credit rationing arises at all dates.[16] When all potential entrepreneurs offer the gross loan interest rate $\hat{R}_t$ that maximizes an intermediary's expected return, the critical project return at which a borrower's project income exactly covers loan principal plus interest is independent of the level of the capital stock, and we denote it by θ. Thus at all dates project verification occurs iff $z \in [0, \theta)$.

When credit is rationed, the expected return to lending at time t can be expressed as

$$r_t = \pi(\hat{R}_t) = \frac{\rho^a_{t+1}}{[q - w_t]} q \left[\theta - \left(\frac{\gamma}{q}\right) G(\theta) - \int_0^{\theta} G(z)dz \right]. \tag{8}$$

Alternatively, the expected income to the intermediary from lending $q - w_t$ units at time t, is

$$r_t [q - w_t] = \pi(\hat{R}_t)[q - w_t] = \rho^a_{t+1} q\theta - \rho^a_{t+1}\gamma G(\theta) - \rho^a_{t+1} q \int_0^{\theta} G(z)dz.$$

Here the first term represents gross expected repayments of principal plus interest $\rho^a_{t+1} q\theta = \hat{R}_t [q - w_t]$, while the last two terms represent the intermediary's costs due to bankruptcies. The second term denotes the expected monitoring costs, while the third term stands for expected losses due to bankrupt firms' inability to fully repay principal plus interest.

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[15] Stiglitz and Weiss (1981) refer to $\hat{R}$ as the "bank-optimal" loan interest rate.

[16] As discussed in Boyd and Smith (1997, 1998), the assumption that credit rationing obtains is maintained because it results in a substantial technical simplification. However, credit rationing is clearly a widespread phenomenon, given that there is substantial evidence of significant rationing of credit even in the United States (Japelli, 1990). Moreover, as we show in section 3, our economy may display domestic credit rationing while at the same time exhibiting large and volatile capital inflows.

Defining

$$\psi \equiv q\left[\eta - \left(\frac{\gamma}{q}\right)G(\eta) - \int_0^{\eta} G(z)dz\right];$$

equation (8) implies that the expected return received by a lender at t, when credit is rationed, is equal to $\psi\frac{\rho^a_{t+1}}{[q-w_t]}$: In other words, under credit rationing the expected return to a lender is proportional to the ratio $\frac{\rho^a_{t+1}}{[q-w_t]}$; hence it only depends on the small open economy's time t and time t + 1 capital to labor ratio in the tradable sector.

It is also straightforward to demonstrate that the expected utility of a funded borrower under credit rationing is given by

$$\rho^a_{t+1}q\left[\hat{z} - \left(\frac{\gamma}{q}\right)G(\eta)\right] - r_t[q - w_t]:$$

Defining

$$\phi \equiv \hat{z} - \left(\frac{\gamma}{q}\right)G(\eta) > 0;$$

the expected payoff of a funded entrepreneur can be written as $\phi\rho^a_{t+1} - r_t[q - w_t]$: Here the first term is expected project income, while the second term represents expected loan repayments. We observe that the parameter ϕ represents the expected amount of type a capital produced per unit invested, net of monitoring costs, when credit is rationed.

Since any potential borrower always has the option of foregoing his project, saving his wage income, and earning the return on deposits offered by banks or the world interest rate on directly held foreign assets, it is always necessary to verify that potential borrowers will prefer to borrow, rather than lend, in equilibrium. Letting $\tilde{r}_t$ denote the highest prevailing alternative return, borrowers prefers borrowing under credit rationing iff

$$\phi\rho^a_{t+1}q - r_t[q - w_t] \geq \tilde{r}_t w_t: \qquad (9)$$

In section 3 we will specify conditions under which (9) is satisfied. We henceforth analyze equilibria in which credit is rationed and potential entrepreneurs prefer borrowing over lending. Hence all potential entrepreneurs would prefer to run their investment projects, but only some of them will actually receive the external funding needed to do so.

2.3 International Asset Flows and Money

We assume international asset flows to be unrestricted, hence foreigners can make deposits in domestic banks, and domestic residents can accumulate foreign assets. Assets issued abroad are default-risk free and earn the world gross real interest rate, r^*. We will denote the domestic net per capita holdings of foreign assets by s_t, which is measured in units of the tradable good. Domestic residents take the world interest rate as given, and the assumption that the domestic country is small implies that activity in the domestic country does not influence that rate. The initial old own the initial stock of net real international assets, s_{-1}.

The old at time zero are also endowed with an initial per capita money stock of $M_{-1} > 0$. Thereafter, the government prints money at a constant rate $\sigma \geq 1$ which it selects once and for all. Thus

$$M_{t+1} = \sigma M_t; \quad t \geq -1. \tag{10}$$

Seignorage thus finances an endogenously determined stream of government expenditures. For simplicity, we assume that the government purchases only the tradable good A. In addition, let p_t^a and p_t^b be the domestic money prices of the tradable and the non-tradable good, respectively. Our small open economy cannot influence the price of the tradable good in terms of foreign currency, and domestic residents take this price, p_t^{a*}, as given. Then if we let e_t denote the price of foreign currency in terms of domestic currency, the law of one price implies that $p_t^a = e_t p_t^{a*}$. We will adopt the normalization $p_t^{a*} = 1 \ \forall t$, so that $p_t^a = e_t \ \forall t$. Denoting by $m_t = \frac{M_t}{p_t^a}$ the real per capita supply of domestic currency in terms of good A, and by g_t the real per capita government expenditures at time t, the government budget constraint implies that

$$g_t = \frac{M_t - M_{t-1}}{p_t^a} = \left(\frac{\sigma - 1}{\sigma}\right) m_t; \quad t \geq 1.$$

Moreover, the government imposes a reserve requirement on any loans to domestic producers of type a capital. More precisely, lenders (which can be thought of as intermediaries) are required to hold domestic real balances equal to at least λ times the value of their loan portfolio.

3 General Equilibrium

Let π_t denote the fraction of potential borrowers who do obtain credit at t. Since each funded borrower borrows $q - w_t$, total (per capita) loans equal $\delta\pi_t(q - w_t)$. Thus, in the aggregate

$$m_t \geq \lambda\delta\pi_t(q - w_t); \quad t \geq 0; \tag{11}$$

must hold. This legal restriction can be thought of as a conventional reserve requirement. If $\frac{p_t^a}{p_{t+1}^a} = \frac{m_{t+1}}{\sigma m_t} < r^*$ holds, then the reserve requirement in (11) is binding, and domestic intermediaries will hold exactly λ units of domestic real balances per unit lent. Clearly, for the reserve requirement to be binding in steady state, the world interest rate must exceed the inverse of the domestic rate of money creation. We henceforth consider only equilibria in which the reserve requirement binds, and thus maintain the following assumption:

Assumption 4 $r^* > \frac{1}{\sigma}$.

When the reserve requirement binds, the return on $1 + \lambda$ units of funds deposited at t, of which one unit is lent and λ units are held as reserves, is given by

$$\frac{1}{1+\lambda}\left\{\left(\frac{f_a'(k^a_{t+1})}{[q-w_a(k^a_t)]}\right)+\lambda\frac{p^a_t}{p^a_{t+1}}\right\};\quad t\geq 0. \tag{12}$$

Letting $\theta=\frac{\lambda}{1+\lambda}$; we can express the expected return to bank deposits by

$$(1-\theta)\left(\frac{f_a'(k^a_{t+1})}{[q-w_a(k^a_t)]}\right)+\theta\frac{p^a_t}{p^a_{t+1}};\quad t\geq 0;$$

where θ is the fraction of any intermediary's portfolio held in (required) reserves. For foreign assets, bank deposits and investments in sector b capital production to be held simultaneously in the domestic economy, the gross return on these alternative assets must be equalized at each date. Therefore,

$$r^*=(1-\theta)\left(\frac{f_a'(k^a_{t+1})}{[q-w_a(k^a_t)]}\right)+\theta\frac{p^a_t}{p^a_{t+1}}=\rho^b_{t+1}=p_{t+1}f_b'\left(k^b_{t+1}\right);\quad t\geq 0. \tag{13}$$

Moreover, it is the case that "sources" and "uses" of funds must be equal. The "uses" of funds in real per capita terms at t is investment in type a capital, plus net private domestic holdings of foreign assets, plus holdings of real balances, plus investments in type b capital, $\delta q\mu_t+s_t+m_t+(1-\kappa_{t+1})k^b_{t+1}$. "Sources" of funds are simply per capita savings, $w_a(k^a_t)$. Therefore,

$$\delta q\mu_t=w_a(k^a_t)-s_t-m_t-(1-\kappa_{t+1})k^b_{t+1};\quad t\geq 0. \tag{14}$$

Under our assumption that returns on investment projects are iid across borrowers, the fact that there is a large number of borrowers implies that there is no aggregate randomness in this economy. In particular, the time $t+1$ per capita type a capital stock is simply $\hat{z}\delta q\mu_t=\hat{z}[w_a(k^a_t)-s_t-m_t-(1-\kappa_{t+1})k^b_{t+1}]$, less type a capital expended on monitoring at $t+1$. The amount of type a capital consumed by monitoring is simply $\gamma\delta\mu_t G(\eta)=\frac{\gamma}{q}G(\eta)[w_a(k^a_t)-s_t-m_t-(1-\kappa_{t+1})k^b_{t+1}]$ under credit rationing. Thus the time $t+1$ amount of capital of type a in per capita terms is:

$$\begin{aligned}\kappa_{t+1}k^a_{t+1} &= \left(\hat{z}-\left(\frac{\gamma}{q}\right)G(\eta)\right)[w_a(k^a_t)-s_t-m_t-(1-\kappa_{t+1})k^b_{t+1}]\\ &= \phi[w_a(k^a_t)-s_t-m_t-(1-\kappa_{t+1})k^b_{t+1}];\quad t\geq 0.\end{aligned} \tag{15}$$

In addition, when the reserve requirement binds, (11), (14) and (15) imply that

$$m_t = \frac{\lambda}{q}\frac{\kappa_{t+1}k^a_{t+1}}{\phi}[q - w_a(k^a_t)]; \quad t \geq 0. \tag{16}$$

Since by definition

$$\frac{p^a_t}{p^a_{t+1}} \equiv \frac{m_{t+1}}{m_t}\frac{M_t}{M_{t+1}} = \frac{m_{t+1}}{\sigma m_t} = \frac{\kappa_{t+2}k^a_{t+2}[q - w_a(k^a_{t+1})]}{\sigma\kappa_{t+1}k^a_{t+1}[q - w_a(k^a_t)]}; \quad t \geq 0$$

holds, equation (13) implies that

$$r^* = (1-\theta)\psi\frac{f'_a(k^a_{t+1})}{[q - w_a(k^a_t)]} + \frac{\theta}{\sigma}\frac{\kappa_{t+2}k^a_{t+2}[q - w_a(k^a_{t+1})]}{\kappa_{t+1}k^a_{t+1}[q - w_a(k^a_t)]}; \quad t \geq 0 \tag{17}$$

must be satisfied at each date.

Finally, we have two market clearing conditions. Market clearing for good A requires that consumption of the traded good, plus investment in type a and type b capital production, plus government consumption, must equal domestic production of this good, plus net imports:

$$y^a_t + r^* s_{t-1} = c^a_t + \frac{\kappa_{t+1}k^a_{t+1}}{\phi} + (1-\kappa_{t+1})k^b_{t+1} + s_t + g_t; \quad t \geq 1.$$

Equivalently,

$$\begin{aligned} &\kappa_t f_a(k^a_t) + r^* s_{t-1} = \\ &\delta\left\{\mu_{t-1} f'_a(k^a_t)[\kappa_t k^a_t - \psi] + (1-\mu_{t-1})\, r^* w_{t-1}\right\} + \frac{\kappa_{t+1}k^a_{t+1}}{\phi} + (1-\kappa_{t+1})k^b_{t+1} + s_t + g_t; \quad t \geq 1. \end{aligned} \tag{18}$$

Market clearing in the non-traded good sector requires that production of good B equals consumption of the same good:

$$y^b_t = (1-\kappa_t)\, f_b\left(k^b_t\right) = c^b_t = \frac{(1-\delta)\, r^* w_{t-1}}{p_t}; \quad t \geq 1. \tag{19}$$

Of course, for producers of type a capital to be willing to borrow, we need to check that (9) is satisfied at all dates. Equivalently,

$$\rho^a_{t+1}[\phi q - \psi] \geq r^* w_t \tag{20}$$

has to hold at every date. Moreover, equilibria need to satisfy $\mu_t < 1 \; \forall t$, for credit to be rationed in all periods. Further, along any equilibrium path, it has to be the case that $\eta_t \in (0, 1) \; \forall t$. Finally, we must verify that the reserve requirement binds in each period.

Parenthetically, what accounts for the existence of credit rationing in a small open economy; one which presumably can absorb large quantities of funds without disrupting world capital markets? The answer lies in the presence of the CSV problem. At date t, the domestic economy has an inherited sector a capital stock k^a_t. Given this type a capital stock - and the implied wage rate w_t - along with the perfect foresight nominal exchange rate sequence $\{p^a_t\}$, domestic borrowers must offer lenders an expected return consistent with the world rate of interest. The highest expected return they can offer is $\left(\frac{f'_a(k^a_{t+1})}{[q - w(k^a_t)]}\right)$; at t this is obviously determined by k^a_{t+1}. In order to offer the necessary expected return to lenders, it is clear from (13) that $\theta\left(\frac{f'_a(k^a_{t+1})}{[q - w(k^a_t)]}\right) \geq r^* - \left(\frac{1-\theta}{\sigma}\right)\frac{p^a_t}{p^a_{t+1}}$ must hold. So, given the nominal exchange rate sequence, world capital markets fund the largest quantity of domestic type a capital investment consistent with domestic borrowers being able to offer the required market rate of return. Any further type a capital investment would lower the marginal product of type a capital enough so that domestic borrowers could no longer compete in international capital markets. This is the source of credit rationing in the domestic economy.

3.1 The First Period

In the initial period, the goods market clearing conditions, and the government budget constraint take a different form than at any other date. At $t = 0$, the government budget constraint is

$$g_0 = \frac{(\sigma - 1)M_{-1}}{p^a_0};$$

with M_{-1} given. Moreover, the initial old, a fraction δ of which are good A lovers and a fraction $(1 - \delta)$ of which are good B consumers, own the initial stock of money, M_{-1}, the initial stocks of capital K_0^a and K_0^b; and the initial stock of net international assets s_{-1}. Hence, their per capita income at $t = 0$ is

$$K_0^a f_a'\left(\frac{K_0^a}{\eta_0}\right) + p_0 K_0^b f_b'\left(\frac{K_0^b}{1-\eta_0}\right) + r^* s_{-1} + \frac{M_{-1}}{p_0^a}.$$

Consequently, market clearing for good A and B at $t = 0$ become

$$\eta_0 f_a\left(\frac{K_0^a}{\eta_0}\right) + r^* s_{-1} = \delta\left[K_0^a f_a'\left(\frac{K_0^a}{\eta_0}\right) + p_0 K_0^b f_b'\left(\frac{K_0^b}{1-\eta_0}\right) + r^* s_{-1} + \frac{M_{-1}}{p_0^a}\right] + \frac{K_1^a}{\psi} + K_1^b + s_0 + g_0 \tag{21}$$

$$(1-\eta_0) f_b\left(\frac{K_0^b}{1-\eta_0}\right) = \frac{(1-\delta)}{p_0}\left[K_0^a f_a'\left(\frac{K_0^a}{\eta_0}\right) + p_0 K_0^b f_b'\left(\frac{K_0^b}{1-\eta_0}\right) + r^* s_{-1} + \frac{M_{-1}}{p_0^a}\right]. \tag{22}$$

Hence, given an initial domestic price level for good A, p_0^a, (or equivalently, an initial value for the nominal exchange rate), equations (22) and (7) jointly determine the $t = 0$ level of the RER, p_0, and of the fraction of the labor force employed in the domestic tradable sector, η_0. The $t = 0$ value for real balances is determined by $m_0 = \frac{\sigma M_{-1}}{p_0^a}$, which allows us to derive the value for $\eta_1 k_1^a$ (K_1^a) from (16). Equations (21) and (15) then jointly determine the $t = 0$ level of net holdings of foreign assets and $(1-\eta_1) k_1^b = K_1^b$.

The entire equilibrium sequence of future capital stocks in the tradable and non-tradeable sectors, employment in both sectors, real and nominal exchange rates, price levels, real money balances and holdings of net foreign assets can thus be determined given the initial price level, p_0^a. However, the value of p_0^a itself is not determined by any of these conditions, and hence is a free initial condition. The properties of dynamic equilibrium paths are analyzed below. We first turn our attention to stationary equilibria.

3.2 Steady State Equilibria

In a steady state equilibrium, all real endogenous variables are constant at all dates. Then, for there to exist no arbitrage opportunities,

$$r^* = (1 - \theta)\psi \frac{\rho^a(k^a)}{[q - w_a(k^a)]} + \frac{\theta}{\sigma} \tag{23}$$

must hold and this pins downs the long run capital labor ratio in the tradable sector. We now define the function $H(k^a)$ by

$$H(k^a) \equiv \frac{\rho^a(k^a)}{[q - w_a(k^a)]}. \tag{24}$$

Then

$$H(k^a) = \frac{1}{\psi(1 - \theta)} \left[r^* - \frac{\theta}{\sigma} \right] \tag{25}$$

is an alternative representation of (23). Since the function H plays a crucial role in determining steady state equilibria, we now collect some of its properties in Lemma 1.

Lemma 1 The function H satisfies

(a) $\lim_{k^a \to 0} H(k^a) = \infty$

(b) $\lim_{k^a \to \hat{k}^a} H(k^a) = \infty$ where $\hat{k}^a \equiv w_a^{-1}(q)$

(c) $H'(k^a) \leq 0; k^a \leq \left[f_a^{-1}(q)\right]$, and

(d) $H'(k^a) \geq 0; k^a \geq \left[f_a^{-1}(q)\right]$.

Proof. Part (a) of Lemma 1 is, of course, immediate from equation (24). Part (b) is also obvious. For (c) and (d), straightforward differentiation yields that

$$H'(k^a) = -f_a''(k_a)\frac{[f_a(k_a) - q]}{[q - w_a(k^a)]^2}$$

establishing the result. ■

Lemma 1 implies that the function H has the configuration depicted in Figure 4. The following proposition is now immediate.

Proposition 1 (a) Suppose that $\frac{1}{A(1-\theta)}[r^* - \frac{\theta}{\sigma}] > H[f_a^{-1}(q)]$. Then there are exactly two values of k^a, denoted by k_1^a and k_2^a ($k_1^a < k_2^a$) in Figure 3, that satisfy (25). (b) Suppose that $\frac{1}{A(1-\theta)}\left[r^* - \frac{\theta}{\sigma}\right] < H[f_a^{-1}(q)]$. Then there is no steady state equilibrium with credit rationing in the presence of international asset flows and a reserve requirement.

Hence, when the world interest rate is sufficiently large in the sense of Proposition 1.a, there will be exactly two steady state levels of k^a; denoted by k_1^a and k_2^a in Figure 4. What is the economics underlying this phenomenon? The answer to this question is related to the informational asymmetry in credit markets (the CSV problem) and to how this informational friction is affected by the ability of borrowers to provide internal financing for their projects. For foreign assets and bank deposits to be held simultaneously, it is necessary that domestic intermediaries provide depositors with a gross real return on bank deposits equal to the world gross interest rate, r^*. As (12) indicates, the gross return on bank deposits increases as the rental rate on type a capital increases, and it also increases as the level of internal finance increases. In the low k^a steady state, the rental rate of type a capital is relatively high. However, the level of internal finance - which depends on potential borrowers' young period income - is relatively low. Since it is well known that the ability of borrowers to contribute internal finance to a project acts to mitigate the CSV problem [Bernanke and Gertler (1989)], in this steady state the costs imposed by the informational asymmetry are relatively high. In the high k^a steady state, the situation is exactly the opposite. The rental rate of type a capital is relatively low, but potential borrowers can provide a relatively large amount of internal finance, which mitigates the CSV problem.

In order for both steady states to be legitimate equilibria, we have to check several conditions. First, the reserve requirement has to be binding in both steady states, which is guaranteed by

Assumption 4. Second credit rationing has to obtain for both k_1^a and k_2^a . This will be the case if

$$\delta q > w_a(k_j^a) - s_j - m_j - (1-\kappa_j)k_j^b; \; j = 1; 2: \tag{26}$$

Third, producers of type a capital must be willing to borrow, i.e. equation (20) must be satisfied in both steady states. A sufficient condition for this to hold is

$$\rho_a(k_2^a)[\phi - \psi] \geq r^* w_a(k_2^a): \tag{27}$$

Finally, we need to check that $\kappa \in (0; 1)$:

Once the steady state level for the capital-labor ratio in the tradable sector, k^a; is determined, the long run values for the capital-labor ratio in the non-tradable sector, the fraction of the labor force employed in the tradable sector, the RER, real money balances, and net holdings of foreign assets follow from:

$$k^b \left[\frac{f_b(k^b)}{k^b f_b'(k^b)} - 1 \right] = \frac{w_a(k^a)}{r^*}; \tag{28}$$

$$\kappa = 1 - (1-\delta) r^* \left[1 - \frac{k^b f_b'(k^b)}{f_b(k^b)} \right]; \tag{29}$$

$$p = \frac{r^*}{f_b'(k^b)}; \tag{30}$$

$$m = \frac{\lambda}{q} \frac{\kappa k^a}{\phi} [q - w_a(k^a)]; \tag{31}$$

$$s = w_a(k^a) - m - \frac{\kappa k^a}{\phi} - (1-\kappa)k^b: \tag{32}$$

Moreover, the long-run level of per capita GDP is

$$GDP = \kappa f_a(k^a) + p(1-\kappa) f_b(k^b) = \kappa f_a(k^a) + (1-\delta)(r^*)^2 \left[\frac{f_b(k^b)}{f_b'(k^b)} - 1 \right]: \tag{33}$$

How do the values of these variables compare in both long run equilibria? In order to make progress on this question, we henceforth turn to the analysis of a double Cobb-Douglas economy, assuming,

$$f_a(k^a) = Ak^{a\alpha} \qquad 0 < \alpha < 1;$$
$$f_b\left(k^b\right) = Bk^{b\beta} \qquad 0 < \beta < 1.$$

For such an economy, equations (28) - (30) and (33) reduce to:

$$k^b = \frac{\beta}{r^*}\frac{(1-\alpha)}{(1-\beta)}k^{a\alpha}; \tag{34}$$

$$\eta = 1-(1-\delta)(1-\beta)r^*; \tag{35}$$

$$p = \left(\frac{r^*}{\beta}\right)^{\beta}\left(\frac{1-\alpha}{1-\beta}k^{a\alpha}\right)^{1-\beta}; \tag{36}$$

$$GDP = k^{a\alpha}\left[\eta A+(1-\eta)B\frac{1-\alpha}{1-\beta}\right] = k^{a\alpha}\left\{A+(1-\delta)r^*\left[B(1-\alpha)-A(1-\beta)\right]\right\}. \tag{37}$$

Clearly, equation (35) implies that credit rationing steady state equilibria will only exists for $r^* < \frac{1}{(1-\delta)(1-\beta)}$, which we will henceforth maintain. We are now ready to state proposition 2, which follows immediately from equations (34) - (37).

Proposition 2 When production is governed by Cobb-Douglas technology in both sectors, and there exist two steady state equilibria; with $k_1^a < k_2^a$, then

(a) $\frac{dk^b}{dk^a} > 0$; and hence $k_1^b < k_2^b$;

(b) $\frac{d\eta}{dk^a} = 0$; and hence $\eta_1 = \eta_2$;

(c) $\frac{dp}{dk^a} > 0$, and hence $p_1 < p_2$;

(d) $\frac{dGDP}{dk^a} > 0$, and hence $GDP_1 < GDP_2$;

Proposition 2 states that the steady state with the relatively low capital-labor ratio in the tradable sector, also has a relatively low capital-labor ratio in the non-tradable sector. The fraction of the labor force employed in the tradable sector is the same in both steady states, while the RER is always higher in the low capital-labor ratio steady state than in the high capital-labor ratio steady state. Finally, the steady state with the low capital-labor ratios has a relatively low level of GDP. These results are due to the fact that labor is free to move between the tradable and the non-tradable sector, and to the requirement that the return to investment in capital for the non-tradable sector equals the return to on other assets. To summarize, our economy displays two long-run equilibria, one with a high level of GDP and a relatively appreciated RER, and a second with a low level of GDP and a relatively depreciated RER.[17]

Hence, in the long run, our economy exhibits a negative correlation between GDP and the RER. This, incidentally, accords well with the empirical evidence on national income and relative prices. As Summers and Heston (1991) report, there is a negative correlation between national income and $\frac{ep^*}{p}$, a commonly used measure of the RER. Moreover, they also provide evidence on the relationship between national income and the relative price of tradables and nontradables, which is exactly how we have defined the RER in this paper. Again, there is a negative correlation between a country's GDP and $\frac{p^a}{p^b}$: nontradables are relatively more expensive in high-income countries.

We conclude this section with an example of our economy which illustrates the results presented above.

Example 1 Let $f_a(k^a) = k^{a^{0.5}}$; $f_b(k^b) = k^{b^{0.27}}$; $q = 2$; $g(z) = \frac{1}{\bar{z}}$ with $\bar{z} = 6.814$; $\gamma = 6.66$; and $\delta = 0.75$ hold. For these parameter values $Á = 5.11$ and $Ã = 5.33$. In addition we set $r^* = 1.0839$,

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[17] For the more general case of CES production, the fraction of the labor force employed in the tradable sector does depend on the levels of the capital-labor ratio's in both sectors. More precisely, the fraction of the labor force employed in the tradable sector is lower in the steady state with the relatively low capital-labor ratios iff the marginal rate of substitution between capital and labor in the non-tradable sector is bigger than one. However, all the other results of Proposition 2, as well as the results stated in the next section, are preserved for CES production, as long as the marginal rate of substitution between capital and labor is not "too far" below one for both sectors.

$\lambda = 2.2$ and $\sigma = 1.03$. Then the long-run values for the low-output steady state are, $GDP_1 = 1.864$, $k_1^a = 3.947$, $k_1^b = 0.339$, $\kappa = 0.8021$, $RER_1 = 0.547$, $s_1 = -0.379$. For the high-output steady state, the corresponding values are, $GDP_2 = 1.888$, $k_2^a = 4.052$, $k_2^b = 0.343$, $\kappa = 0.8021$, $RER_2 = 0.543$, $s_2 = -0.393$. It is easy to check for this example that in both steady states borrowers are willing to borrow, and an unfulfilled demand for credit exists.

3.3 Comparative statics

How do conditions in the international capital markets affect long-run equilibria in this small open economy? It follows immediately from equation (35) that an increase in the world interest rate, r^*, reduces the long-run level of the fraction of the labor force employed in the tradable sector. For the other variables, the effect of an increase in the world interest rate depends on which steady state obtains, as is illustrated in Figure 5. In the low k^a steady state, an increase in the world interest rate from r^* to $r^{*\prime}$ reduces k^a, while the opposite effect obtains in the high k^a steady state. It then follows immediately from equations (34), (36), and (37) that in the low k^a steady state, an increase in the world interest rate is met by a decrease in the capital labor ratio for the non-tradable sector, k^b, as well. At the same time, GDP declines for this steady state when $\frac{A}{1-\alpha} > \frac{B}{1-\beta}$, while the effect of an increase in the world interest rate on the RER is ambiguous. In the high k^a steady state, the same increase in the world interest rate raises the type a capital-labor ratio but has an ambiguous effect on k^b. The RER decreases in this case, while GDP increases when $\frac{B}{1-\beta} > \frac{A}{1-\alpha}$. We collect these results in proposition 3.

Proposition 3 The effect of an increase in the world interest rate on long-run equilibria is as follows:

(a) $\frac{\partial \kappa}{\partial r^*} < 0$;

(b) for the low-output, high RER steady state $\frac{\partial k_1^a}{\partial r^*} < 0$; $\frac{\partial k_1^b}{\partial r^*} < 0$; and $\frac{\partial GDP_1}{\partial r^*} < 0$ if $\frac{A}{1-\alpha} > \frac{B}{1-\beta}$;

(c) for the high-output, low RER steady state $\frac{\partial k_2^a}{\partial r^*} > 0$; $\frac{RER_2}{\partial r^*} < 0$; and $\frac{\partial GDP}{\partial r^*} > 0$ if $\frac{B}{1-\beta} > \frac{A}{1-\alpha}$.

Given that our economy displays two steady states, we clearly need to assess whether these long-run equilibria can both be attained, and how equilibrium paths are influenced by conditions in international credit markets. For that purpose, we now turn to an investigation of the dynamical equilibria in this economy.

3.4 Dynamics

In this section will show that the low-output-high RER steady state is always a saddle, while the high-output-low-RER steady state may be stable (a sink) or unstable (a source). We establish that the high-output-low-RER steady state is a sink when the world interest rate is "low enough". An economy in which those conditions are met, can either converge to the low-output-high-RER steady state by following the saddle path, or it may approach the high-output-low-RER steady state along any of a continuum of trajectories. Since this economy has a free initial condition, there is therefore potentially an indeterminacy of equilibria. Moreover, when the world interest rate is too high, we show that the high-output-low-RER steady state must be a source. It is this fact that can result in increases in the world interest rate leading to a "crisis" in the domestic economy, and we provide an example of a trajectory of economy that faces such a "crisis".

For the dynamic analysis we maintain the assumption of a double Cobb-Douglas economy, and we assume the tradable sector to be more capital intensive than the non-tradable sector, that is $\alpha > \beta$. Equation (17), which summarizes dynamic equilibria in our economy, can then be rewritten as follows:

$$r^* \frac{\left[q - w(k_t^a)\right]}{f_a'\left(k_{t+1}^a\right)} - \frac{\theta}{\sigma} \frac{\left[q - w\left(k_{t+1}^a\right)\right]}{f_a'\left(k_{t+2}^a\right)} \frac{\left[w\left(k_{t+2}^a\right) - r^*(1-\delta)(1-\beta) w\left(k_{t+1}^a\right)\right]}{\left[w\left(k_{t+1}^a\right) - r^*(1-\delta)(1-\beta) w(k_t^a)\right]} - (1-\theta)\psi = 0. \quad (38)$$

Letting

$$k_{t+1}^a = y_t, \tag{39}$$

equation (38) becomes

$$r^* \frac{[q - w(k_t^a)]}{f_a'(y_t)} - \frac{\theta}{\sigma} \frac{[q - w(y_t)]}{f_a'(y_{t+1})} \frac{[w(y_{t+1}) - r^*(1-\delta)(1-\beta)w(y_t)]}{[w(y_t) - r^*(1-\delta)(1-\beta)w(k_t^a)]} - (1-\theta)\psi = 0. \tag{40}$$

We henceforth work with the first order dynamical system consisting of equations (39) and (40). Linearizing this system in a neighborhood of a either steady state (k^a, y) yields

$$(k_{t+1}^a - k^a, y_{t+1} - y)' = J(k_t^a - k, y_t - y)'$$

where J is the Jacobian Matrix

$$J = \begin{bmatrix} \frac{\partial k_{t+1}^a}{\partial k_t^a} & \frac{\partial k_{t+1}^a}{\partial y_t} \\ \frac{\partial y_{t+1}}{\partial k_t^a} & \frac{\partial y_{t+1}}{\partial y_t} \end{bmatrix},$$

with all partial derivatives evaluated at the appropriate steady state. Those derivatives satisfy:

$$\frac{\partial k_{t+1}^a}{\partial k_t^a} = 0 \tag{41}$$

$$\frac{\partial k_{t+1}^a}{\partial y_t} = 1 \tag{42}$$

$$\frac{\partial y_{t+1}}{\partial k_t^a} = \frac{r^* w'(k^a) f_a'(k^a) \left\{ (1-\delta)(1-\beta)\frac{\theta}{\sigma}[q - w(k^a)] + w(k^a)[1 - r^*(1-\delta)(1-\beta)] \right\}}{\{ f_a''(k^a) w(k^a)[1 - r^*(1-\delta)(1-\beta)] - f_a'(k^a) w'(k^a) \} \frac{\theta}{\sigma}[q - w(k^a)]} \tag{43}$$

$$\begin{aligned} \frac{\partial y_{t+1}}{\partial y_t} = {} & 1 - \frac{r^* w'(k^a) f_a'(k^a)(1-\delta)(1-\beta)}{f_a''(k^a) w(k^a)[1 - r^*(1-\delta)(1-\beta)] - f_a'(k^a) w'(k^a)} - \\ & \frac{w(k^a) f_a'(k_a)[1 - r^*(1-\delta)(1-\beta)] \left[w'(k^a)\frac{\theta}{\sigma} - f_a''(k^a)(1-\theta)\psi \right]}{\{ f_a''(k^a) w(k^a)[1 - r^*(1-\delta)(1-\beta)] - f_a'(k^a) w'(k^a) \} \frac{\theta}{\sigma}[q - w(k^a)]} \end{aligned} \tag{44}$$

Let $T(k^a)$ and $D(k^a)$ denote the trace and determinant of J, respectively, as a function of k^a. Clearly, $T(k^a) = \frac{\partial y_{t+1}}{\partial y_t}$ and $D(k^a) = -\frac{\partial y_{t+1}}{\partial k_t^a}$. Moreover, under the assumption that $1 - r^*(1-\delta)(1-\beta) > 0$ (which is equivalent to $\kappa < 1$), we have that $T(k^a) > 0$ and $D(k^a) > 0$. When $T(k^a) > 0$ and $D(k^a) > 0$, it is well known that the relevant steady state is a saddle if

$T > 1 + D$; when $T < 1 + D$; the relevant steady state can be either a source or a sink.[18] It is a sink if $D < 1$ and a source if $D > 1$: When $T < 1 + D$ holds, paths approaching the steady state display locally monotone dynamics if $T^2 - 4D > 0$; when $T^2 - 4D < 0$; paths approaching the steady state display damped oscillation.

It is easy to show that $T(k^a)$ can be rewritten in terms of $D(k^a)$ as follows:

$$T(k^a) = 1 + D(k^a) + \frac{w(k^a)\left(r^* - \frac{\theta}{\sigma}\right)[1 - r^*(1-\delta)(1-\beta)]\{f_a'(k^a)w'(k^a) + f_a''(k^a)[q - w(k^a)]\}}{\{f_a''(k^a)w(k^a)[1 - r^*(1-\delta)(1-\beta)] - f_a'(k^a)w'(k^a)\}\frac{\theta}{\sigma}[q - w(k^a)]}: \tag{45}$$

In appendix A we prove the following result.

Lemma 2 $T(k^a) > (<)\, 1 + D(k^a) \iff f_a(k^a) < (>)\, q$:

Lemma 2 implies that the low-output-high-RER steady state is always a saddle. The high-output-low-RER steady state, on the other hand, is never a saddle; it is either a sink or a source.

We now wish to investigate conditions under which the high-output-low-RER steady state is a sink. When it is, it is possible for a small open economy to approach this steady state (from some initial conditions).

To begin, we let $k_2^a(r^*; \sigma; \theta)$ denote the capital-labor ratio in the high-output-low-RER steady state, as a function of parameters, and define the function $d(r^*; \sigma; \theta)$:

$$d(r^*; \sigma; \theta) \equiv D[k_2^a(r^*; \sigma; \theta)] \tag{46}$$

In the appendix B we show the following.

Lemma 3 (a) $d_1(r^*; \sigma; \theta) > 0$ holds if $1 - 2r^*(1-\delta)(1-\beta) > 0$; and

(b) $d_2(r^*; \sigma; \theta) > 0$:

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[18] See, for instance, Azariadis (1993, Chapter 6.4)

Thus, for relatively low levels of the world interest rate, the relevant determinant is increasing in the world interest rate, and it is always increasing in the rate of money creation. In other words, low rates of low interest rates and low levels of domestic money creation are conducive to the high-output-low-RER steady state being a sink.

Under what conditions is the high-output-low-RER steady state a sink (source)? The following proposition is proved in appendix C.

Proposition 4 The high-output-low-RER steady state is a sink (source) iff

$$\frac{\alpha\sigma r^*}{\theta} < (>) \frac{q - w(k_2^a)}{w(k_2^a)}.$$

Thus, since the right-hand side is decreasing in r^* and σ, the high-output-low-RER steady state is a sink if σr^* is sufficiently small. However, if either the world interest rate or the money growth rate is too high, the high-output-low-RER steady state must be a source.

It remains to consider the scope for oscillatory dynamics. We now state the following proposition

Proposition 5 There exists a critical value of the world interest rate, r_c^*, such that

(a) $d(r_c^*, \sigma, \theta) = 1$.

(b) If $(1 - \alpha)\sigma r_c^* > \theta$, then as $r^* \uparrow r_c^*$, $T(k_2^a)^2 < 4D(k_2^a)$ holds.

Thus, paths approaching the high-output-low-RER steady state necessarily display damped oscillation for sufficiently high - but "sub-critical"- world interest rates.

Proposition 4 is proved in appendix D. The proposition asserts that there exists a critical value of the world interest rate, r_c^*, above which the high-output-low-RER steady state is a source. Hence, for world interest rates above that critical level, the entire class of equilibrium paths approaching the high-output steady state is eliminated. Moreover, if a "crisis" can occur due to "excessively high" world real interest rates, oscillatory dynamics will be observed for nearly \critical rates" of the world interest rate. Obviously, analogous results hold for increases in ¾:

We now present a sequence of examples for which the high-output-low-RER steady state, depending on the level of the world interest rate, is either a sink or a source with local dynamic behavior varying between monotone dynamics and endogenous oscillation. At the same time, this series of examples illustrates the comparative statics of long-run equilibria with respect to the world interest rate. As before, our examples assume that $f_a(k^a) = k^{a^{0:5}}$; $f_b(k^b) = k^{b^{0:27}}$; $q = 2:0$; $g(z) = \frac{1}{z}$ with $\bar{z} = 6:814$; ° $= 6:66$; and ± $= 0:75$ hold. For these parameter values, Á $= 5:11$ and Ã $= 5:33$. In addition we set ¾ $= 1:03$ and ¸ $= 2:2$. We then let $r^{¤}$ vary from 1.0839 to 1.0890, which gives the following results.

Example 2 For $r^{¤} = 1:0839$; · $= 0:8022$; $GDP_1 = 1:864$; $k_1^a = 3:947$; $k_1^b = 0:339$; $RER_1 = 0:547$; $s_1 =$ ¡$0:379$; while $GDP_2 = 1:887$; $k_2^a = 4:053$; $k_2^b = 0:3435$; $RER_2 = 0:543$; $s_2 =$ ¡$0:393$: Furthermore, $D(k_2^a) = 0:842 < 1$; and $T(k_2^a) = 1:838 < 1 + D(k_2^a)$ hold. Thus the high-output-low-RER steady state is a sink. Moreover, $T(k_2^a) > 2\sqrt{D(k_2^a)}$; therefore paths approaching this steady state display locally monotone dynamics.

Example 3 For $r^{¤} = 1:0865$; · $= 0:8017$; $GDP_1 = 1:738$; $k_1^a = 3:393$; $k_1^b = 0:313$; $RER_1 = 0:580$; $s_1 =$ ¡$0:305$; while $GDP_2 = 2:023$; $k_2^a = 4:657$; $k_2^b = 0:367$; $RER_2 = 0:516$; $s_2 =$ ¡$0:465$: Furthermore, $D(k_2^a) = 0:959 < 1$; and $T(k_2^a) = 1:911 < 1 + D(k_2^a)$ hold. Thus the high-output-low-RER steady state is a sink. Moreover, $T(k_2^a) < 2\sqrt{D(k_2^a)}$; therefore paths approaching this steady state display damped oscillation.

Example 4 For $r^{¤} = 1:089$; · $= 0:8012$; $GDP_1 = 1:683$; $k_1^a = 3:167$; $k_1^b = 0:302$; $RER_1 = 0:594$; $s_1 =$ ¡$0:273$; while $GDP_2 = 2:081$; $k_2^a = 4:930$; $k_2^b = 0:377$; $RER_2 = 0:505$; $s_2 =$ ¡$0:494$:

Furthermore, $D(k_2^a) = 1:015 > 1;$ and $T(k_2^a) = 1:946 < 1 + D(k_2^a)$ hold. Thus the high-output-low-RER steady state is a source.

For this set of examples, low levels of the world real interest rate result in an indeterminate steady state, but locally monotone dynamics. As the world interest rate increases, eventually endogenous volatility emerges. For even higher values of the world real interest rate, the high-output-low-RER steady state becomes a source and can not be approached.

This observation has the following implication. Consider an economy confronting a sub-critical world interest rate, and following a path approaching the high-output-low-RER steady state. An unanticipated increase in the world real interest rate that causes it to exceed its critical level, would eliminate all equilibrium paths approaching the high-output-low-RER steady state. Hence, the economy may be condemned to revert to an equilibrium path approaching the low GDP; high RER steady state. We will show by way of an example that our economy can indeed display such kind of behavior.

Consider the situation presented in Table 1 and Figure 6. The last four columns of this table depict an economy which faces $r^* = 1:0839$ at all dates. The economy converges monotonically to the high-output-low-RER steady state. Now consider the economy presented in the ¯rst four columns of Table 1. For the ¯rst four periods, this economy is exactly like the one presented in the last four columns. It faces $r^* = 1:0839;$ and follows an equilibrium trajectory approaching the high output steady state. However, at time $T = 5;$ this economy faces an unexpected and permanent increase in the world interest rate to a level $r^* = 1:089:$ As demonstrated in example 4, the high-GDP-low RER steady state is no longer stable. Hence the equilibrium trajectory that the economy was following for $t \in [0; 4]$ does no longer exist, and the economy switches to the saddlepath approaching the (new) low output steady state.[19] On impact, this switch provokes a 12 percent depreciation of the nominal exchange rate, and a 30 percent decrease in the net holdings of domestic assets by foreigners. Moreover, by time t = 6, there is a 6 percent decline in GDP, and a 3 percent depreciation of the real exchange rate.

Thus, our model is qualitatively consistent with the kind of episode recently experienced by the Mexican economy (Figure 1). From 1987-1994, the Mexican economy grew at real per capita rates ranging from 3 to 10 percent. Over the same period the RER appreciated consistently. Between 1993 and 1994, the real return on US treasury bills increased more than 2 percent. In late 1994, Mexico abandoned the ¯xed exchange rate it had adhered to, which was followed by a nominal depreciation of more than 100. In 1995 the Mexican economy entered a crisis in which real GDP per capita declined 7 percent, and the RER depreciated. Our model shows how such an episode can arise as an equilibrium response to changes in the world interest rate. It bears emphasis that without the CSV problem and/or without the presence of international capital °ow, our economy would exhibit a unique long-run equilibrium, and hence no crisis of the type presented here would ever occur.

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[19] We generate a numerical estimate of the saddle path using the following procedure presented in Parker and Chua (1989), and Kostelich, Yorke, and You (1996). We consider the linearized version of the equilibrium system around the low-output-high-RER steady state, and compute the saddle path for this linear approximation. This saddle path has two properties: it reaches the low-output-high-RER steady state, and is tangent to the saddle path of the original system at that point. We then consider initial conditions on a small segment of the saddle path for the linear approximation, and close to the saddle point. Finally, we apply the inverse map of the system to iterate this segment backward.

Table 1: A simulated \crisispath"

t	GDP_t	RER_t	e_t	s_t		GDP_t	RER_t	e_t	s_t
0	1.8544	0.5506	1	-0.3815		1.8544	0.5506	1	-0.3815
1	1.8587	0.5497	1.029	-0.3826		1.8587	0.5497	1.029	-0.3826
2	1.8622	0.5489	1.058	-0.3835		1.8622	0.5489	1.058	-0.3835
3	1.8675	0.5482	1.088	-0.3843		1.8651	0.5483	1.088	-0.3843
4	1.8685	0.5477	1.121	-0.3850		1.8675	0.5478	1.121	-0.3850
5=T	1.7966	0.5494	1.258	-0.2771		1.8695	0.5473	1.153	-0.3855
6	1.7507	0.5618	1.316	-0.2724		1.8711	0.5470	1.187	-0.3860
7	1.7200	0.5728	1.369	-0.2713		1.8725	0.5467	1.222	-0.3865
8	1.7033	0.5804	1.419	-0.2714		1.8737	0.5464	1.259	-0.3868
9	1.6879	0.5855	1.468	-0.2719		1.8747	0.5462	1.296	-0.3871
10	1.6803	0.5887	1.515	-0.2723		1.8757	0.5460	1.334	-0.3874
11	1.6755	0.5907	1.563	-0.2727		1.8763	0.5459	1.374	-0.3877
12	1.6727	0.5919	1.611	-0.2729		1.8769	0.5457	1.415	-0.3879
13	1.6709	0.5927	1.661	-0.2730		1.8775	0.5456	1.457	-0.3881
14	1.6698	0.5932	1.711	-0.2732		1.8780	0.5455	1.500	-0.3882
15	1.6692	0.5934	1.763	-0.2732		1.8784	0.5454	1.545	-0.3884
16	1.6688	0.5936	1.816	-0.2733		1.8788	0.5453	1.591	-0.3885
17	1.6685	0.5937	1.871	-0.2733		1.8791	0.5453	1.639	-0.3887
18	1.6684	0.5938	1.927	-0.2733		1.8794	0.5452	1.688	-0.3888
250	1.6682	0.5939		-0.2733		1.8877	0.5334		-0.3928

A number of papers have been concerned with presenting an explanation for the 1994 Mexican currency crisis and the ensuing recession. Dornbusch and Werner (1994) emphasize the role of an "overvalued" RER and appeal to diverging speeds of adjustment in capital versus goods markets to justify a nominal devaluation with the purpose of avoiding a crisis. Flood, Garber and Kramer, (1996) explain the crisis as a speculative attack resulting from the inconsistency of fixing the exchange rate while maintaining fiscal imbalance. Calvo and Mendoza (1996), Cole and Kehoe (1996), and Sachs, Tornell and Velasco (1996), stress self-fulfilling prophecies and herd behavior. Our model has much in common with the latter set of papers, in particular an emphasis on the financial sector and the importance of conditions in international capital markets. However, the above-mentioned models mainly address the behavior of the economy in the proximity of the attack on the currency. In contrast, we emphasize how the interaction between the unrestricted operation of international capital markets and frictions in the domestic financial markets can produce an environment in which both prolonged periods of growth and RER appreciation and episodes of sharp decline in output can arise.

A natural question raised by these observations is, can a small open economy insulate itself from a "crisis" induced by an increase in the world real interest rate? One possibility, as we now show, is to reduce the domestic rate of money creation. We present a sequence of examples for which the high-output-low-RER steady state, depending on the level of the money growth rate, is either a sink or a source, with local dynamic behavior varying between monotone dynamics and endogenous oscillation. Again, our examples assume that $f_a(k^a) = k^{a^{0.5}}$; $f_b(k^b) = k^{b^{0.27}}$; $q = 2$; $g(z) = \frac{1}{z}$ with $\bar{z} = 6.814$; $\gamma = 6.66$; and $\delta = 0.96$ hold. In addition, we set $r^* = 1.09$ and $\lambda = 2.2$. We will then let σ vary from 1.03 to 1.0222, which gives the following results.

Example 5 For ¾ = 1:03; · = 0:8012; GDP_1 = 1:683; k_1^a = 3:167; k_1^b = 0:302; RER_1 = 0:594; s_1 = ¡0:273; while GDP_2 = 2:081; k_2^a = 4:930; k_2^b = 0:377; RER_2 = 0:505; s_2 = ¡0:494: Furthermore, $D(k_2^a)$ = 1:016 > 1; and $T(k_2^a)$ = 1:946 < 1 + $D(k_2^a)$ hold. Thus the high-output-low-RER steady state is a source.

Example 6 For ¾ = 1:026; · = 0:8013; GDP_1 = 1:740; k_1^a = 3:404; k_1^b = 0:3133; RER_1 = 0:578; s_1 = ¡0:306; while GDP_2 = 2:020; k_2^a = 4:644; k_2^b = 0:366; RER_2 = 0:516; s_2 = ¡0:462: Furthermore, $D(k_2^a)$ = 0:955 < 1; and $T(k_2^a)$ = 1:908 < 1 + $D(k_2^a)$ hold. Thus the high-output-low-RER steady state is a sink. Moreover, $T(k_2^a) < 2\sqrt{D(k_2^a)}$; therefore paths approaching this steady state display damped oscillation.

Example 7 For ¾ = 1:0222; · = 0:9682; GDP_1 = 1:861; k_1^a = 3:939; k_1^b = 0:337; RER_1 = 0:548; s_1 = ¡0:377; while GDP_2 = 1:889; k_2^a = 4:062; k_2^b = 0:342; RER_2 = 0:542; s_2 = ¡0:392: Furthermore, $D(k_2^a)$ = 0:955 < 1; and $T(k_2^a)$ = 1:908 < 1 + $D(k_2^a)$ hold. Thus the high-output-low-RER steady state is a sink. Moreover, $T(k_2^a) < 2\sqrt{D(k_2^a)}$; therefore paths approaching this steady state display locally monotone dynamics.

For this set of examples, low levels of money growth rates result in an indeterminate steady state, but locally monotone dynamics. As the money growth rate and the steady state in°ation rate increase, eventually endogenous volatility emerges. For even higher rates of money growth, the high-output-low-RER steady state becomes a source and can no longer be approached.

4 Conclusions

We have examined a simple model of a small open economy where domestic residents can borrow and lend abroad, where a CSV problem is a source of frictions in domestic credit markets, and where domestic lending is subject to a reserve requirement.

We have described conditions under which there are exactly two steady state equilibria. One has a high level of output, a low RER, a relatively large quantity of internal ¯nance, and a correspondingly minor CSV problem. The second has a low level of output, a high RER, a low level of internal ¯nance, and a correspondingly severe CSV problem. Moreover, the low-output-high-RER steady state is necessarily a saddle, while the high-output-low-RER steady state is either a sink or a source.

We have analyzed the e®ects of conditions in international credit markets on steady state output and the steady state RER as well as on the stability properties of the steady states. The low-output-high-RER steady state is always a saddle, while the high-output-low-RER steady state may be stable or unstable. When the world interest rate is relatively low, the high-output-low-RER steady state is stable, and can thus be approached. However, there exists a critical level of the world interest rate above which the high-output steady state is transformed into a source. A - possibly small - unexpected increase in the world interest rate can thus eliminate an entire class of high-output-low-RER equilibrium trajectories, thereby inducing a \crisis". We have simulated a crisis path for our model economy, and found it to be qualitatively consistent with episodes like the one recently experienced by the Mexican economy. We have also shown that a \crisis" induced by an unanticipated increase in the world interest rate can potentially be avoided by reducing the money growth rate.

The Mexican crisis also suggests an interesting extension for our paper. In our current model, reducing the money growth rate in the face of a super-critical increase in the world interest rate may help to avoid a crisis. However, it has been argued that Mexican banks were in such bad shape before the crisis that tightening monetary policy in response to the sharp increase in world interest rates could have caused the banking system to collapse. In our model banks face no aggregate risk and thus the fragility of the banking system is not an issue when evaluating the e®ectiveness of monetary policy. Our model could be extended to incorporate ¯nancial vulnerability in the analysis.

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Appendix

A. Proof of Lemma 2

It follows from (56) that $T(k^a) > (<) 1 + D(k^a)$ if and only if:

$$T(k^a) = 1 + D(k^a) + \frac{w(k^a)\left[1 - r^*(1-\delta)(1-\beta)\right]\left[r^* - \frac{\theta}{\sigma}\{w'(k^a)f_a'(k^a) + f_a''(k^a)[q - w(k^a)]\}\right]}{\{f_a''(k^a)w(k^a)\left[1 - r^*(1-\delta)(1-\beta)\right] - f_a'(k^a)w'(k^a)\}\frac{\theta}{\sigma}[q - w(k^a)]}$$

$$> (<) 1 + D(k^a).$$

This condition is equivalent to

$$\frac{w(k^a)\left[1 - r^*(1-\delta)(1-\beta)\right]\left[r^* - \frac{\theta}{\sigma}\{w'(k^a)f_a'(k^a) + f_a''(k^a)[q - w(k^a)]\}\right]}{\{f_a''(k^a)w(k^a)\left[1 - r^*(1-\delta)(1-\beta)\right] - f_a'(k^a)w'(k^a)\}\frac{\theta}{\sigma}[q - w(k^a)]} > (<) 0.$$

Under the assumption that $1 - r^*(1-\delta)(1-\beta) > 0$ (which is equivalent to $\eta < 1$), this is true when

$$w'(k^a)f_a'(k^a) + f_a''(k^a)[q - w(k^a)] < (>) 0. \tag{A.1}$$

Noting that $w'(k^a) = -k^a f''(k^a)$, $w(k^a) = f_a(k^a) - k^a f'(k^a)$, and rearranging terms, equation (A.1) is equivalent to

$$q > (<) f_a(k^a).$$

B. Proof of Lemma 3

For the Cobb-Douglas economy,

$$d(r^*; \sigma; \theta) = \frac{\alpha r^*(1-\delta)(1-\beta)}{(1-\alpha)\left[1 - r^*(1-\delta)(1-\beta)\right] + \alpha} +$$

$$\frac{r^*(1-\alpha)\left[1 - r^*(1-\delta)(1-\beta)\right]}{\{(1-\alpha)\left[1 - r^*(1-\delta)(1-\beta)\right] + \alpha\}\frac{\theta}{\sigma}} k^a H(k^a)$$

It is easy to show that the derivative with respect to r^* of the first term in the sum is positive. Furthermore, the derivative of the same term with respect to σ is equal to zero. Now, observing that $\frac{dk_2^a}{dr^*} > 0$ and $H'(k^a) > 0$, straightforward differentiation of the second term in $d(r^*;\sigma;\theta)$ with respect to σ shows that $d_2(r^*;\sigma;\theta) > 0$.

Differentiating with respect to r^* the second term, it is easy to see that a sufficient condition for the derivative to be positive given that $\frac{dk_2^a}{dr^*} > 0$ and $H'(k^a) > 0$ is $1 - 2r^*(1-\delta)(1-\beta) > 0$. ■

C. Proof of Proposition 2

$D(k_2^a) < (>) 1$ iff

$$r^*(1-\delta)(1-\beta)w'(k_2^a)f_a'(k_2^a)\frac{\theta}{\sigma}[q - w(k_2^a)] + r^* w'(k_2^a)w(k_2^a)f_a'(k_2^a)[1 - r^*(1-\delta)(1-\beta)] < (>)$$

$$\left[w(k_2^a)f_a''(k_2^a)[1 - r^*(1-\delta)(1-\beta)] - w'(k_2^a)f_a'(k_2^a)\right]\frac{\theta}{\sigma}[q - w(k_2^a)].$$

Rearranging terms shows that this condition is equivalent to

$$\frac{\theta}{\alpha\sigma r^*} > (<) \frac{f_a'(k_2^a)w'(k_2^a)w(k_2^a)}{[q - w(k_2^a)][f_a'(k_2^a)w'(k_2^a) - f_a''(k_2^a)w(k_2^a)]},$$

which in turn can be simplified to

$$\frac{\theta}{\alpha\sigma r^*} > (<) \frac{w(k_2^a)}{[q - w(k_2^a)]}$$

by noticing that $f_a'(k_2^a)w'(k_2^a) = f_a'(k_2^a)w'(k_2^a) - f_a''(k_2^a)w(k_2^a)$. ■

D. Proof of Proposition 3

(a) Consider an equilibrium of the economy such that $d(r^*, \sigma, \theta) < 1$. It is easy to show that $\lim_{r^* \to \infty} d(r^*, \sigma, \theta) = \infty$, when k^a is an increasing function of r^*. From monotonicity and continuity of d it follows that there exists a value r_c^* such that $d(r_c^*, \sigma, \theta) = 1$.

(b) When $r^* = r_c^*$, $D(k_2^a) = 1$ and

$$T(k_2^a) = 2 + \frac{w(k_2^a)\left[1 - r_c^*(1-\delta)(1-\beta)\right]\left\{w'(k_2^a)f_a'(k_2^a) + f_a''(k_2^a)\left[q - w(k_2^a)\right]\right\}\left(r_c^* - \frac{\theta}{\sigma}\right)}{\left\{f_a''(k_2^a)w(k_2^a)\left[1 - r_c^*(1-\delta)(1-\beta)\right] - f_a'(k_2^a)w'(k_2^a)\right\}\frac{\theta}{\sigma}\left[q - w(k_2^a)\right]}.$$

Thus, when $r^* = r_c^*$, $T(k_2^a)^2 < 4D(k_2^a) = 4$ holds iff

$$\frac{w(k_2^a)\left[1 - r_c^*(1-\delta)(1-\beta)\right]\left\{w'(k_2^a)f_a'(k_2^a) + f_a''(k_2^a)\left[q - w(k_2^a)\right]\right\}\left(r_c^* - \frac{\theta}{\sigma}\right)}{\left\{f_a''(k_2^a)w(k_2^a)\left[1 - r_c^*(1-\delta)(1-\beta)\right] - f_a'(k_2^a)w'(k_2^a)\right\}\frac{\theta}{\sigma}\left[q - w(k_2^a)\right]} < 0.$$

This relationship holds iff

$$w'(k_2^a)f_a'(k_2^a) + f_a''(k_2^a)\left[q - w(k_2^a)\right] < 0.$$

Noticing that when $D(k_2^a) = 1$, $\left[q - w(k_2^a)\right] = w(k_2^a)\frac{\alpha\sigma r_c^*}{\theta}$, the above inequality is satisfied when $(1-\alpha)\sigma r_c^* > \theta$. Under this condition, $T(k_2^a) < 2$ at r_c^*, and the result follows from continuity.

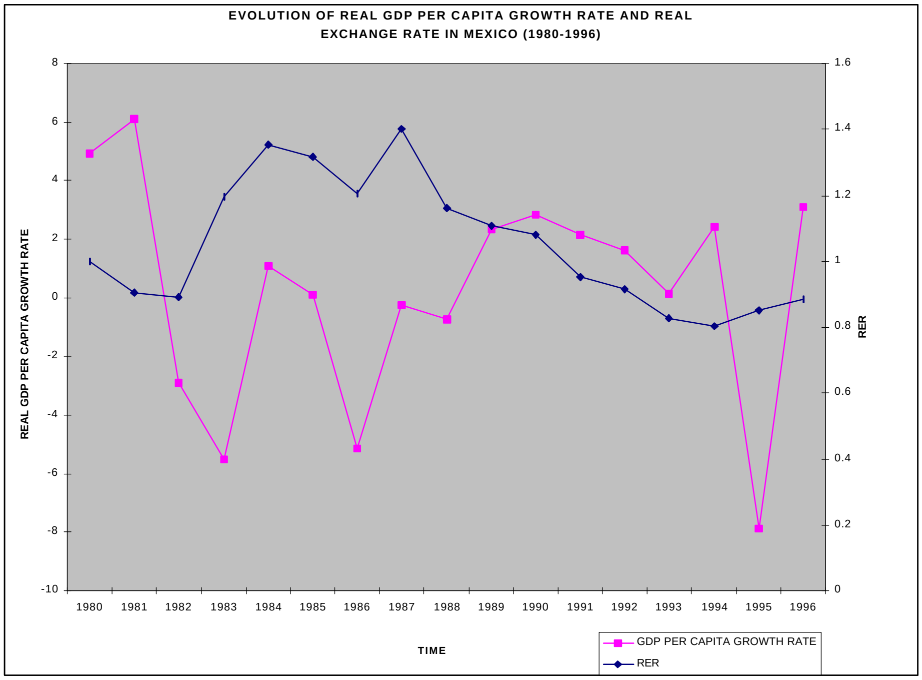

Figure 1: Evolution of Real GDP Per Capita Growth Rate and Real Exchange Rate in Mexico (1980-1996)

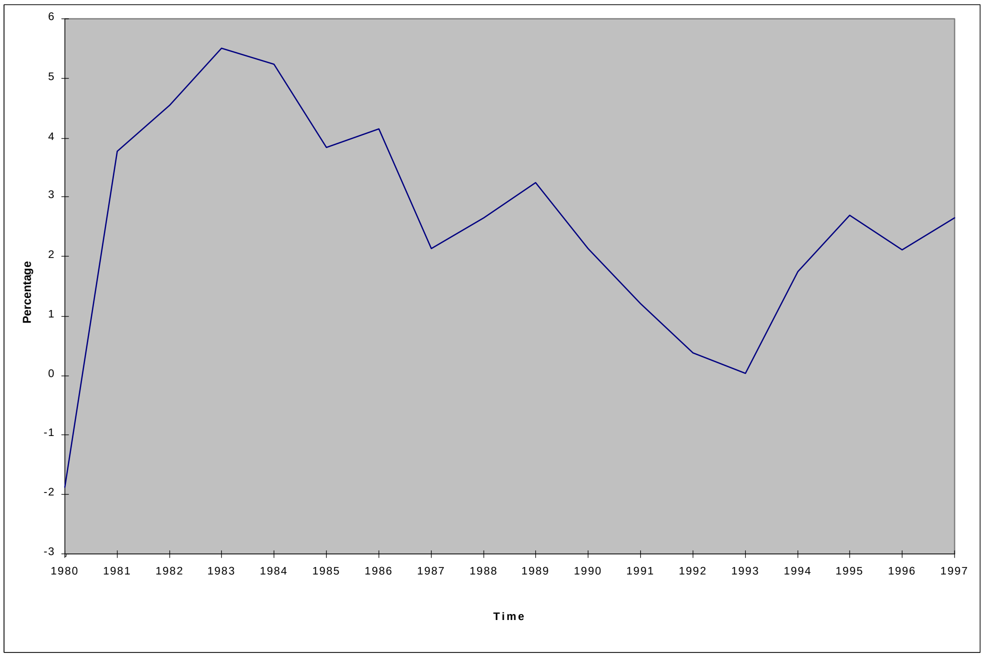

Figure 2: World Real Interest Rate

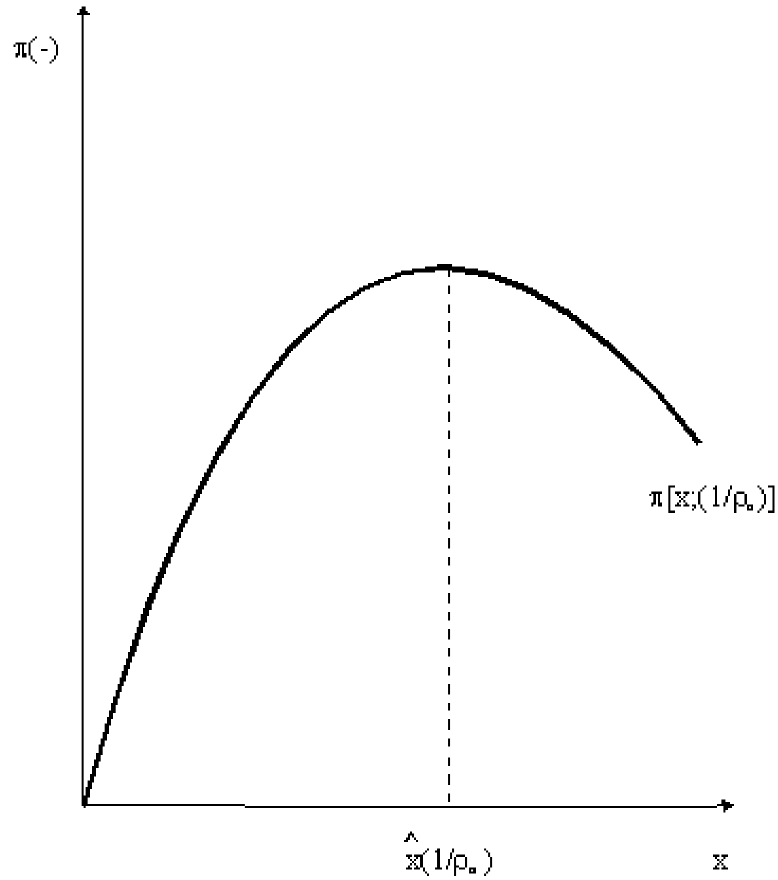

Figure 3.

The Expected Return Function

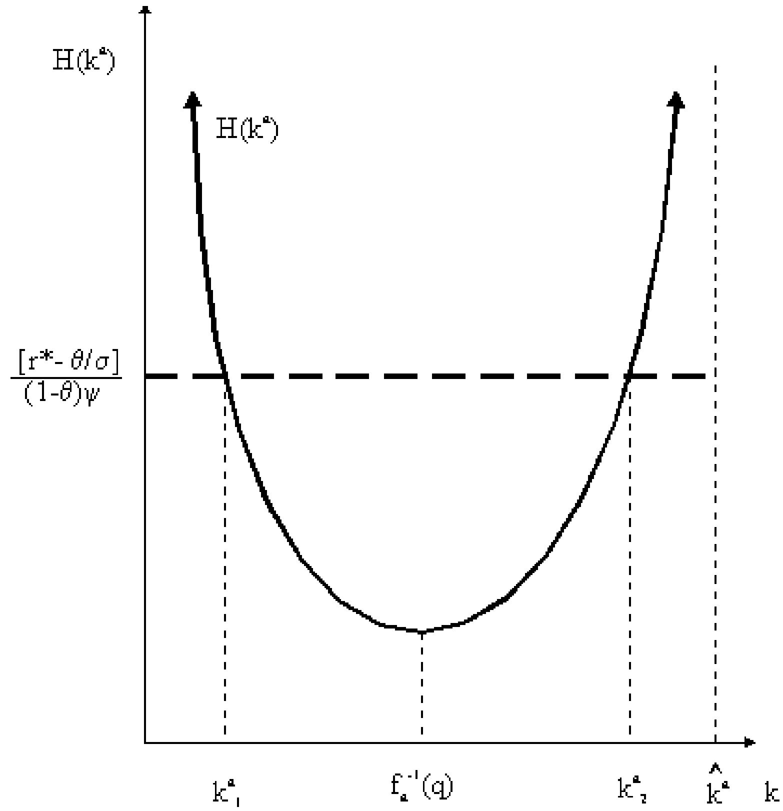

Figure 4.

The Open, Monetary Economy in the Presence of a Reserve Requirement

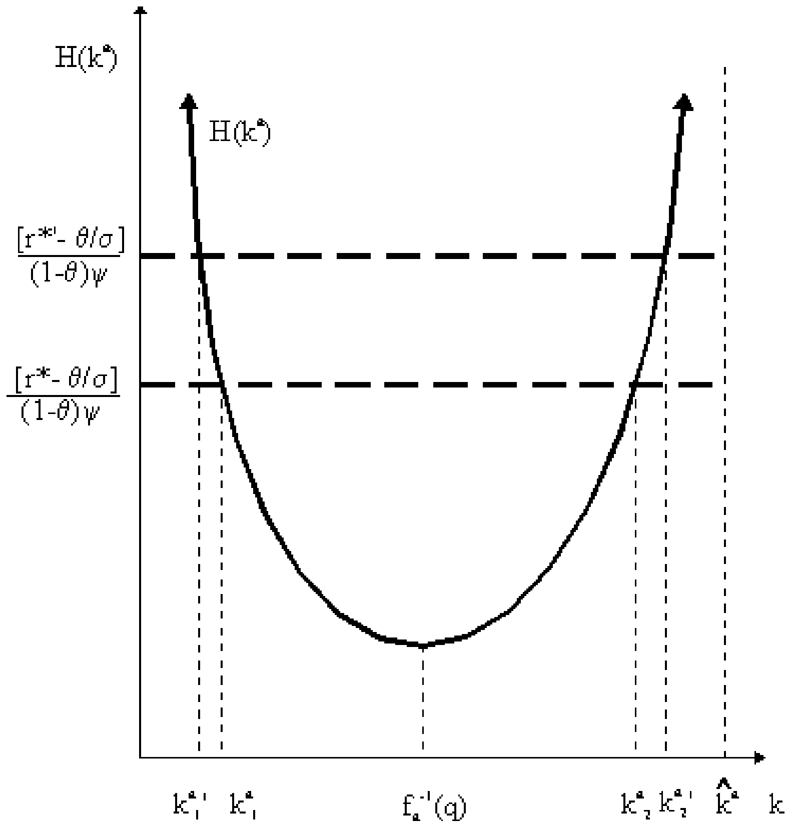

Figure 5.

The Effect of an Increase in the World Interest Rate

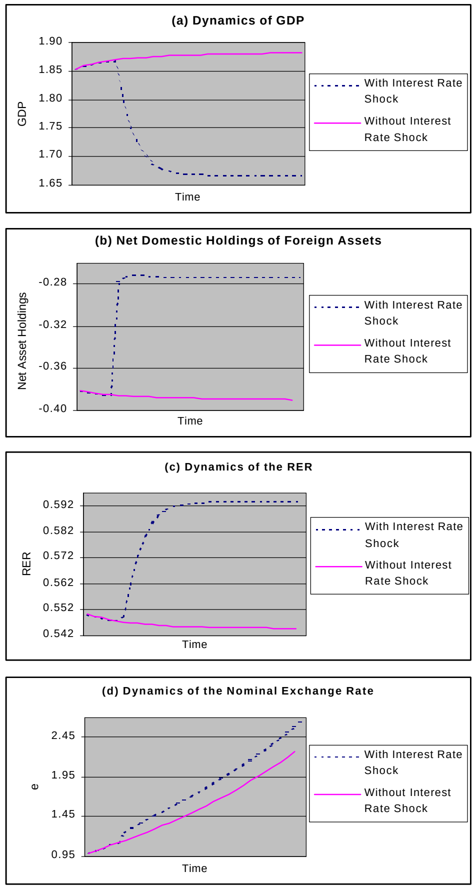

Figure 6: Trajectories of Convergence for Economies

with and without Interest Rate Shock