.

In°ation, Financial Markets and Long-Run Real Activity [1]

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Abstract

Empirical evidence suggests that real activity, the volume of bank lending activity, and the volume of trading in equity markets are strongly positively correlated. At the same time, in°ation and ¯nancial market activity are strongly negatively correlated (in the long-run), as are in°ation and the real rate of return on equity. In°ation and real activity are also negatively correlated in the long-run, particularly for economies with relatively high rates of in°ation. We present a monetary growth model in which banks and secondary capital markets play a crucial allocative function. We show that - at least under certain con¯gurations of parameters - the predictions of the model are consistent with these and several other observations about in°ation, ¯nance and long-run real activity.

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Introduction

Extensive empirical investigation has established the following facts about the long-run relationships between in°ation, ¯nancial market conditions, and real activity.

(1) There is a negative long-run relationship between in°ation and real economic performance, at least at su±ciently high levels of in°ation.[4]

(2) There is a positive correlation between economic performance and (a) the volume of bank lending activity, (b) the quantity of bank liabilities issued, and (c) the volume of trading in equity markets. (King and Levine, 1993a,b; Levine and Zervos, 1998; Atje and Jovanovic, 1993).

(3) At low-to-moderate long-run rates of in°ation, there is a strong negative correlation between in°ation and (a) the volume of bank lending activity, (b) the quantity of bank liabilities issued, and (c) the volume of trading in equity markets. However, at higher rates of in°ation these partial correlations essentially disappear. (Boyd, Levine and Smith, 1996).

(4) For countries with low-to-moderate rates of in°ation, there is a pronounced negative correlation between in°ation and real equity returns. For countries where in°ation is su±ciently high, this correlation vanishes. (Nelson, 1976; Fama and Schwert, 1977; Gultekin, 1983; Boyd, Levine and Smith, 1996).

(5) As economies develop, equity markets generally become more important relative to banks. (Gurley and Shaw, 1960; Levine 1997).

Our purpose in this paper is to develop a theoretical model that is capable of confronting this

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[4] Barro (1995) asserts that there is evidence of a negative relationship at all rates of in°ation. Bruno and Easterly (1998) and Bullard and Keating (1995) ¯nd support for the notion that this negative relationship emerges only when rates of in°ation exceed some threshold. Levine and Renelt (1992) and Clark (1997) also question whether there is a uniformly negative relationship between in°ation and real activity independently of the prevailing rate of in°ation.

full set of observations. Such a model, clearly, must have several features. First there must be a role for intermediaries, as well as equity (secondary capital) markets, and the level of intermediary and equity market activity must be related to long-run real performance. Second, money must play a role in the economy, and changes in monetary policy (that affect the long-run rate of inflation) must be capable of affecting long-run real activity, as well as financial market conditions.

In order to produce an economy with these features, we utilize an overlapping generations model with production, similar to that of Diamond (1965). However, we modify Diamond's model in several important respects. First, we allow for the existence of multiple technologies for producing capital. Some of these technologies are simple in nature, meaning that (a) anyone can operate them, (b) they produce capital quickly, and (c) they are subject to no informational asymmetries. Others are more complex, meaning that (a) only a subset of individuals can operate them, (b) they involve a relatively lengthy gestation period,[5] and (c) informational asymmetries arise. More specifically, we assume that capital production using long-gestation technologies is subject to a standard costly state verification (CSV) problem.

The second central feature of the analysis is that agents employing long-gestation production technologies require some external finance in order to operate their projects. In particular, we assume that the long-gestation technology can be operated only on a relatively large scale; a scale that requires agents to invest more than just their own funds. In contrast, the short-gestation technology can be operated at any scale. As a result, no external finance is required to utilize it.[6]

[5] The importance of long gestation periods in capital production has been emphasized by authors from Bohm-Bawerk (1891) to Kydland and Prescott (1981). The specific model we adopt resembles Bencivenga, Smith and Starr (1995, 1996) in the nature of the gestation lags in capital production.

[6] Hicks (1969) identifies the central feature of the industrial revolution to be the employment of large scale technologies requiring the provision of external finance for their operation.

However, whenever long-gestation capital production technologies are in use, external finance is a necessity. Given the presence of the CSV problem, this external funding will naturally be provided through intermediaries (Diamond, 1984; Williamson, 1986). As a consequence, intermediary activity will play an important role in the allocation of investment, and in the determination of long-run real activity levels.

Third, given our assumptions on agents' life-cycles and the gestation of certain capital investments, agents utilizing long-lived capital production technologies will find it necessary to sell claims to the ownership of the capital they produce in secondary capital markets. Hence, whenever these technologies are in use, capital formation and equity market activity must be positively related, at least under certain technical conditions that we deduce.

These features of the economy create a role for agents to trade equity, and to hold intermediary liabilities. In addition we will obviously assume that the government issues a third asset that can be held by agents - fiat money. Here, as in Diamond (1965) or Tirole (1985), we simply treat money as an additional asset that can be held by any agent; it plays no special role in transactions. Thus, in equilibrium, the real return on money must be equated to the real return on competing assets. However, this feature of the model is not central to our analysis; it would be straightforward to create a role for money in transactions - and an additional role for banks - along the lines described by Champ, Smith and Williamson (1996) or Schreft and Smith (1997). We do not follow this route for two reasons. First, the simpler Diamond-Tirole formulation matches quite well the empirical relationship between inflation and real equity returns. And second, the Diamond-Tirole model generally gives rise to a Mundell-Tobin effect - that is to a positive relationship between inflation and long-run output levels - in the absence of the other features we have introduced. Thus our formulation makes it clear that these features are essential in allowing our model to confront all of the empirical observations described above.

Under the assumption that the supply of outside money grows at a constant, exogenously selected rate, we provide conditions under which there will be two or more steady state equilibria. These steady states are di®erentiated by their capital stocks and levels of real activity, as well as by the level of activity in their ¯nancial markets. In the low-capital-stock steady state agents will utilize either the commonly available or the long-gestation capital production technology. In the former case clearly no ¯nancial market activity is necessary. In the high-capital-stock steady state, on the other hand, production of capital will occur using the long-gestation technology; hence both intermediaries and equity markets will be active. For this steady state, we are able to state conditions under which higher steady state levels of real activity are associated with higher volumes of both bank lending and equity market activity. We also state conditions under which equity market activity rises relative to bank lending activity as the steady state capital stock rises.

With respect to in°ation, we show that higher rates of money creation (steady state in°ation) lead to lower levels of real activity, in the high-capital-stock steady state. They also reduce the real return on all assets, including equity, and higher money growth is (under conditions we state) detrimental both to bank lending activity and to the volume of trading in equity markets. Moreover, it is also possible to state conditions under which the negative relationship between in°ation and long-run real activity becomes more pronounced at higher rates of in°ation, as many have argued is true empirically.

Finally we spend some time analyzing the properties of equilibrium dynamics, in a neighborhood of each steady state. We are able to show that the low-capital-stock steady state is a saddle; hence there are always equilibrium paths that approach it. In addition, we illustrate by example that the high-capital-stock steady state may be either a source or a saddle. If it is a saddle, it too can potentially be approached from some combination of initial conditions. Our examples further demonstrate the possibility that the stability properties of the high-capital-stock steady state depend upon the rate of money growth.[7] As the rate of money creation rises, it can transpire that the high activity steady state is transformed from a saddle to a source. Hence it can no longer be approached, and if the economy converges to a steady state, it must converge to the low activity steady state. Consequently, for economies whose in°ation rate exceeds some critical level, the only \relevant" steady state may be the low activity one. Thus economies with high enough rates of in°ation can display discretely lower long-run levels of real and ¯nancial market activity than their low in°ation counterparts. Moreover, economies in low activity steady states (those with high rates of in°ation) will display a much di®erent correlation pattern between in°ation and ¯nancial market conditions than will economies in high activity steady states (those with lower rates of in°ation).

What accounts for the existence of multiple steady states, and for the other ¯ndings we have described? In this economy, a steady state equilibrium has the property that the return on loans and the return on equity must equal the prevailing real rate of return on money.[8] This is given, in a steady state, by the exogenously determined rate of money growth. In the presence of the CSV problem, the return on loans depends on two factors: the marginal product of capital, and the quantity of internal ¯nance provided.[9] Thus the required rate of return on loans can be obtained either by having a relatively low stock of capital, along with a relatively high marginal product of

[7] Boyd and Smith (1998) and Huybens and Smith (1998) obtain some analytical results on this point in much simpler, but related models.

[8] Again we emphasize that this feature of the economy is not essential to our results. What is necessary is that higher in°ation lowers long-run real returns. Negative associations between in°ation and real returns on a variety of savings instruments are well-documented empirically.

[9] The latter point was ¯rst made by Bernanke and Gertler (1989).

capital, a low level of income, and a low level of internal finance; or by having a relatively high stock of capital, a correspondingly high level of internal finance, but a low marginal product of capital. These two methods of delivering a particular rate of return typically yield two steady state equilibria. The high activity steady state will also have a high level of internal finance, and the higher the steady state capital stock, the higher will be the amount of internal finance provided.[10]

If the steady state real return is lowered by an increase in the steady state rate of inflation, in the high activity steady state the result is a decline in the quantity of internal finance provided. This produces the required decline in the real return on loans. It also exacerbates the severity of the CSV problem and leads to more extensive rationing of credit. As a result, the capital stock falls. And, under the conditions we describe, so does the volume of financial market activity. Thus this model can potentially account for all of the observations cited above.

The paper proceeds as follows. Sections 1 and 2 describe the environment and the nature of trade in the model. Section 3 analyzes steady state equilibria. Section 4 discusses steady state financial market activity and its relation to inflation. Section 5 briefly discusses local dynamics, while section 6 concludes.

1 The Model

We examine an economy consisting of an infinite sequence of two-period lived, overlapping generations. Each generation is identical in size and composition, and contains a continuum of agents with unit mass. Throughout, we let $t = 0, 1, \ldots$ index time.

At each date a single final good is produced using a constant returns to scale technology with

[10] Hamid and Singh (1992) document that, as an empirical matter, countries with high income levels also - on average - have high fractions of capital investments that are financed internally.

capital and labor as inputs. Let K_t denote the time t capital input, and L_t denote the time t labor input of a representative firm. Then its final output is $F(K_t, L_t)$. We will assume that F is a CES production function with elasticity of substitution greater than 1, that is $F(K, L) \equiv [\alpha K^{\rho} + \beta L^{\rho}]^{\frac{1}{\rho}}$, with $0 < \rho < 1$. Thus F is increasing in each argument and strictly concave. In addition, if $k \equiv K/L$ is the capital-labor ratio, and if $f(k) \equiv F(k, 1) = [\alpha k^{\rho} + \beta]^{\frac{1}{\rho}}$ is the intensive production function, then $f' > 0 > f''$ holds $\forall k$, and in addition $\lim_{k \to 0} f'(k) = \infty$.

Agents are assumed to care only about old age consumption and, in addition, all agents are risk neutral. Thus all young period income is saved.

There are potentially three assets in our economy, money and investments in the two different technologies for converting final goods into capital. The two capital production technologies are indexed by $j = 1, 2$. Technology $j = 1$ is a simple capital production technology: one unit of the final good invested at t returns $R_1 > 0$ units of capital at $t + 1$. Technology $j = 2$ is a more complicated capital production technology, which has the following properties. First, only a fraction $\delta \in (0, 1)$ of the population - which we will call potential borrowers - has access to this technology. The remaining fraction $(1 - \delta)$ of the population - which we will call lenders - does not have access to the complicated capital production technology. Second, the technology is indivisible: each potential borrower has one investment project which can only be operated at the scale q. Third, when this technology is utilized, two periods are needed to obtain mature capital. Fourth, the return on investments in technology 2 is random. More specifically, then, $q > 0$ units of the final good invested in technology 2 at t yield zq units of capital in progress (CIP) at $t + 1$, and $R_2 zq$ units of capital at $t + 2$. The random variable z is iid (across borrowers and periods), and is realized at $t + 1$. We let G denote the probability distribution of z, and assume that G has a differentiable density function g with support $[0, \bar{z}]$. Let $\hat{z}$ be the expected value of z. Finally, we assume that this technology is subject to a standard CSV problem of the type introduced by Townsend (1979): only the project owner can costlessly observe z, while any agent other than the project owner can observe z only by bearing a fixed cost of $\gamma > 0$ units of capital in progress (CIP).[11]

Capital produced by the simple investment technology is a perfect substitute for capital produced in the alternative fashion. Moreover, we assume that the capital stock depreciates completely after being used in production.

With respect to endowments, all young agents are endowed with one unit of labor, which is supplied inelastically, and agents are retired when old. Individuals other than the old of period zero have no endowment of capital or final goods, while the initial old agents have an aggregate capital endowment of $K_0 > 0$; and an aggregate endowment of capital in progress, $CIP_0 > 0$.

2 Trade

2.1 Factor Markets

We assume that capital and labor are traded in competitive markets at each date. Then, letting w_t denote the time t real wage rate and ρ_t the time t capital rental rate, the standard factor pricing relationships obtain:

$$\rho_t = f'(k_t) \tag{1}$$

$$w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t). \tag{2}$$

Clearly $w'(k) > 0$ holds.

[11] That is, in verifying the project return, γ units of CIP are used up. This assumption is responsible for the simple form assumed by the expected return to lenders under credit rationing [see equation (15) below].

2.2 Credit Markets

All young agents at t supply one unit of labor inelastically, earning the real wage rate w_t. However, we will assume that this young period income does not suffice to run a capital production project of type $j = 2$:

Assumption 1 $q > w(k_t)$ for all "relevant" values of k_t.

Thus, potential borrowers must obtain external financing to invest in technology 2. Let b_t denote the amount borrowed (in real terms) at t by the operator of a funded type 2 project ; clearly

$$b_t = q - w(k_t). \tag{3}$$

We can think of this borrowing as being intermediated (Williamson, 1986).

If potential borrowers attempt to obtain external funding they do so by announcing loan contract terms. These announced contract terms are either accepted or rejected by intermediaries: borrowers whose terms are accepted then receive funding and operate their projects. A loan contract consists of the following objects. First, there is a set of project return realizations A_t for which verification of the project return occurs at t. Verification of project returns does not occur if $z \in B_t \equiv [0, \bar{z}] - A_t$.[12] Second, if $z \in A_t$, then it is possible to make the contractual repayment contingent on the project return. Thus if $z \in A_t$ we denote the promised payment (per unit borrowed) by $R_t(z)$. On the other hand, if $z \in B_t$ then the loan payment cannot meaningfully depend on the project return, and the loan contract offers an uncontingent payment of x_t (per unit borrowed) for all $z \in B_t$. All payments specified by any contract are in real terms.

[12] We thus abstract from stochastic state verification. In a similar context, Boyd and Smith (1994) show that the welfare gains from stochastic monitoring are trivial when realistic parameter values are assumed.

Loan contracts offered by borrowers are either accepted or rejected by intermediaries who - without loss of generality - we can think of as making all loans. Thus intermediaries take deposits, make loans, and conduct the monitoring of project returns. We assume that any lender can establish an intermediary. In equilibrium intermediaries will be perfectly diversified, earn zero profits, and have a nonstochastic return on their portfolios.[13]

Since agents are two-period lived, a young borrower who initiates a capital investment of type $j = 2$ will seek to sell his "immature" capital in a secondary market. Let u_t denote the price of one unit of capital in progress (CIP) at time t.

Intermediaries accept deposits taking the gross real return that must be paid on them - r_t between t and $t + 1$ - as given, and they act as if they can obtain any desired quantity of deposits at that rate. It follows that intermediaries are willing to accept loan contract offers yielding an expected return no less than r_t. Thus loan contract offers must satisfy the expected return constraint

$$\int_{A_t} [R_t(z)b_t - u_{t+1}\gamma] g(z)dz + x_t b_t \int_{B_t} g(z)dz \geq r_t b_t. \tag{4}$$

In particular, expected repayments must at least cover the intermediary's cost of funds - $r_t b_t$ - plus the real expected monitoring cost

$$u_{t+1}\gamma \int_{A_t} g(z)dz.$$

The expected monitoring cost depends on u_{t+1}, of course, because γ units of capital in progress are expended when project returns are verified. Finally, project owners must have the appropriate incentives to correctly reveal when a monitoring state has occurred. This requires that

$$R_t(z) \leq x_t; \quad z \in A_t. \tag{5}$$

[13] As a result, intermediaries need not be monitored by their depositors. See Krasa and Villamil (1992) for a consideration of intermediaries that cannot perfectly diversify risk.

In addition, contractually specified repayments must be feasible for the borrower, so that

$$R_t(z) \leq \frac{u_{t+1}zq}{b_t}; \quad z \in A_t \tag{6}$$

$$x_t \leq \inf_{z \in B_t} \left[\frac{u_{t+1}zq}{b_t}\right]. \tag{7}$$

Equations (6) and (7) state that repayments never exceed the real value of the CIP yielded by an investment project, which in state z is $u_{t+1}zq$ at $t+1$.

Borrowers announce contract terms in order to maximize their own expected utility subject to the constraints (4)-(7). Therefore, announced loan contracts at date t will be selected to maximize

$$u_{t+1}\hat{z}q - b_t \int_{A_t} R_t(z)g(z)dz - x_t b_t \int_{B_t} g(z)dz$$

subject to these constraints.

At an optimum, borrowers offer a standard debt contract (modified for the presence of internal finance). In particular, the borrower either repays x_t (principal plus interest) or else defaults. In the latter case the lender verifies the project return, and retains the proceeds of the project net of monitoring costs. Formally,

Proposition 1 Suppose $q > b_t$. Then the optimal contractual loan terms satisfy

$$R_t(z) = \frac{u_{t+1}zq}{b_t}; \quad z \in A_t \tag{8}$$

$$A_t = [0, \frac{x_t b_t}{(u_{t+1}q)}) \tag{9}$$

$$r_t = \int_{A_t} \left[R_t(z) - \frac{u_{t+1}\gamma}{b_t}\right] g(z)dz + x_t \int_{B_t} g(z)dz. \tag{10}$$

The proof of Proposition 1 is standard,[14] and we omit it here.

[14] See Gale and Hellwig (1985) or Williamson (1986, 1987).

For future reference, substituting (8) and (9) into (10) yields

$$\int_{A_t}\left[R_t(z)-\frac{u_{t+1}\gamma}{b_t}\right]g(z)dz+x_t\int_{B_t}g(z)dz$$

$$=\quad x_t-\left(\frac{u_{t+1}\gamma}{b_t}\right)G\left(\frac{x_tb_t}{(u_{t+1}q)}\right)-\frac{u_{t+1}q}{b_t}\int_0^{\frac{x_tb_t}{(u_{t+1}q)}}G(z)dz\equiv\pi\left[x_t;\frac{b_t}{u_{t+1}}\right]=r_t. \tag{11}$$

The function π gives the expected return to a lender as a function of the gross loan rate, x_t, the amount of external finance required, b_t, and the future relative price of CIP, u_{t+1}.

It will be useful in what follows to put some additional structure on the function π. In particular, we will assume the following.

Assumption 2 $g(z)+(\frac{\gamma}{q})g'(z)\geq 0$; for all $z\in[0,\bar{z}]$.

Assumption 3 $\pi_1[0;(\frac{b_t}{u_{t+1}})]>0$.

Assumption 2 implies that $\pi_{11}<0$. Assumptions 2 and 3 imply that the function π has the configuration depicted in figure 1. Evidently, given b_t/u_{t+1}, there is a unique value of x_t which maximizes the expected return that can be offered. We denote this value by $\hat{x}(b_t/u_{t+1})$, where the function $\hat{x}$ is defined implicitly by

$$\pi_1\left[\hat{x}\left(\frac{b_t}{u_{t+1}}\right);\frac{b_t}{u_{t+1}}\right]\equiv 1-\left(\frac{\gamma}{q}\right)g\left[\hat{x}\left(\frac{b_t}{u_{t+1}}\right)\frac{b_t}{(u_{t+1}q)}\right]-G\left[\hat{x}\left(\frac{b_t}{u_{t+1}}\right)\frac{b_t}{(u_{t+1}q)}\right]\equiv 0. \tag{12}$$

Equation (12) and assumption 3 imply that

$$\hat{x}\left(\frac{b_t}{u_{t+1}}\right)\frac{b_t}{(u_{t+1}q)}\equiv\eta \tag{13}$$

where $\eta>0$ is a constant satisfying $1-(\frac{\gamma}{q})g(\eta)-G(\eta)\equiv 0$. When all potential borrowers offer the interest rate $\hat{x}(b_t/u_{t+1})$; project return verification occurs iff $z\in[0;\eta)$.

2.3 Credit Rationing

A well known feature of the environment just described - which was originally noted by Gale and Hellwig (1985) and Williamson (1986, 1987) - is that it permits the existence of unfulfilled demand for credit. In particular, if all borrowers desire to operate their projects at date t, the total (per capita) demand for funds is δq. The total per capita supply of saving is $w(k_t)$ at t. Therefore credit demand must exceed credit supply, and hence credit must be rationed, if the following assumption holds for all $t \geq 0$.

Assumption 4 $\delta q > w(k_t)$.

When credit rationing exists, however, it also must be the case that

$$x_t = \hat{x}\left(\frac{b_t}{u_{t+1}}\right). \tag{14}$$

Equation (14) asserts that all potential borrowers are offering the interest rate that maximizes a prospective lender's expected rate of return. As a result, rationed (unfunded) potential borrowers cannot obtain credit simply by offering an alternative set of loan contract terms, since doing so reduces the expected return perceived by (all) lenders. Thus if assumption 4 and equation (14) hold at date t, credit rationing is an equilibrium outcome. We focus here on economies where credit is rationed at all dates.[15]

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[15] When credit is rationed, the probability that any project will have to be monitored, ex post, is simply $G(\eta)$. Thus the monitoring probability - and, by implication, the probability of bankruptcy - is independent of any endogenous variables. The result is a substantial technical simplification, as is illustrated by the relatively simple expression in equation (15) describing the expected return received by a lender.

Of course credit rationing is clearly a widespread phenomenon in developing countries (McKinnon, 1973), and there is substantial evidence of significant rationing of credit even in the United States (Japelli, 1990). Therefore this does not seem to be an empirically unreasonable assumption.

2.3.1 Payoffs Under Credit Rationing

We now describe the expected payoffs received by lenders and (funded) borrowers when credit is rationed. For lenders, equations (11) and (14) imply that the expected return on bank deposits (and loans) at t satisfies

$$
\begin{aligned}
r_t &= \pi\left[\hat{x}\left(\frac{b_t}{u_{t+1}}\right); \frac{b_t}{u_{t+1}}\right] \equiv \frac{u_{t+1}q}{b_t}\left\{\hat{x}\left(\frac{b_t}{u_{t+1}}\right)\frac{b_t}{(u_{t+1}q)} - \left(\frac{\gamma}{q}\right) G\left[\hat{x}\left(\frac{b_t}{u_{t+1}}\right)\frac{b_t}{(u_{t+1}q)}\right] - \int_0^{\frac{\hat{x}b_t}{(u_{t+1}q)}} G(z)dz\right\} \\
&\equiv \frac{u_{t+1}q}{b_t}\left[\eta - \left(\frac{\gamma}{q}\right)G(\eta) - \int_0^{\eta} G(z)dz\right].
\end{aligned} \tag{15}
$$

In particular, the return on savings between t and $t+1$ is proportional to the ratio u_{t+1}/b_t when credit rationing obtains.

It is also possible to show that the expected utility of a funded borrower at t under credit rationing is given by

$$
u_{t+1}\hat{z}q - r_{t+1}b_t - u_{t+1}\gamma G\left[\hat{x}\left(\frac{b_t}{u_{t+1}}\right)\frac{b_t}{(u_{t+1}q)}\right] \equiv u_{t+1}q\left[\hat{z} - \left(\frac{\gamma}{q}\right)G(\eta)\right] - r_{t+1}b_t.
$$

Since any potential borrower could always forego investing in his project and deposit his income in a bank instead, all potential borrowers can guarantee themselves the utility level $r_{t+1}w_t$. Thus (potential) borrowers wish to operate their projects (under credit rationing) iff

$$
u_{t+1}q\left[\hat{z} - \left(\frac{\gamma}{q}\right)G(\eta)\right] - r_{t+1}b_t \geq r_{t+1}w_t. \tag{16}
$$

We now define

$$
\begin{aligned}
\phi &\equiv \hat{z} - \left(\frac{\gamma}{q}\right)G(\eta) \\
\psi &\equiv q\left[\eta - \left(\frac{\gamma}{q}\right)G(\eta) - \int_0^{\eta} G(z)dz\right].
\end{aligned} \tag{17}
$$

The parameter ϕ is the expected project yield per unit invested, net of CIP consumed by monitoring, under credit rationing. The parameter ψ determines the expected return on deposits under credit rationing, since

$$r_t = \psi \frac{u_{t+1}}{b_t} \equiv \psi \frac{u_{t+1}}{[q - w(k_t)]}. \tag{18}$$

We now observe that (16) is satisfied iff

$$\phi u_{t+1} \geq r_t. \tag{19}$$

Equations (18) and (19) imply that

$$\phi[q - w(k_t)] \geq \psi \tag{20}$$

must hold for all $t \geq 0$ in order for borrowers to wish to operate their projects. Given assumption 4, a sufficient condition for (20) to obtain is that

$$(1 - \delta)\phi q \geq \psi. \tag{21}$$

2.4 Money

The initial old at time zero are endowed with the initial per capita money supply $M_{-1} \geq 0$. Thereafter, the money supply grows at the constant (gross) rate $\sigma \geq 1$, which the government selects once and for all at $t = 0$. Therefore

$$M_{t+1} = \sigma M_t; \quad t \geq -1. \tag{22}$$

We let the government have an endogenous real expenditure level of g_t (per capita) at time t.[16] The government budget constraint implies that

$$g_t = \frac{M_t - M_{t-1}}{p_t}. \tag{23}$$

[16] The analysis would be unaltered if monetary injections ocurred via lump-sum transfers to old lenders.

Letting $m_t \equiv M_t/p_t$ denote the per capita stock of real balances, (22) and (23) imply that

$$g_t = \left(\frac{\sigma - 1}{\sigma}\right)m_t.$$

3 General Equilibrium Conditions and Steady States

In this section, we present equilibrium conditions and examine steady state equilibria in which the same capital production technology (j), is in use permanently. We then present conditions determining the equilibrium choice of investment technology. We begin with the case where the short-gestation capital production technology is in use.

3.1 Steady State Equilibria when the Production Technology $j=1$ is Utilized

When the economy produces only type $j = 1$ capital, an equilibrium in which capital investments coexist with money at all dates must satisfy the no-arbitrage condition

$$R_1 f'(k_{t+1}) = \frac{p_t}{p_{t+1}}; \quad t \geq 0. \tag{24}$$

By definition,

$$\frac{p_t}{p_{t+1}} \equiv \frac{m_{t+1}}{m_t}\frac{M_t}{M_{t+1}} \equiv \frac{m_{t+1}}{\sigma m_t}; \tag{25}$$

so that (24) can be rewritten to yield

$$R_1 f'(k_{t+1}) = \frac{m_{t+1}}{(\sigma m_t)}; \quad t \geq 0. \tag{26}$$

In addition, "sources" and "uses" of funds must be equal in equilibrium. If we let i_t denote the per capita quantity of resources invested in capital production at t, then an equality between sources and uses of funds requires that

$$w(k_t) = i_t + m_t; \quad t \geq 0;$$

since young agents save all of their wage income. Of course,

$$k_{t+1} = R_1 i_t; \quad t \geq 0;$$

and therefore

$$m_t = w(k_t) - \frac{k_{t+1}}{R_1}; \quad t \geq 0. \tag{27}$$

In steady state, equations (26) and (27) reduce to

$$R_1 f'(k) = \frac{1}{\sigma}; \tag{28}$$

$$m = w(k) - \frac{k}{R_1}. \tag{29}$$

Given our assumptions on the production technology f, it is clear that a unique monetary steady state k_s exists when the economy produces only type $j = 1$ capital.

3.2 Steady State Equilibria when the Production Technology $j=2$ is Utilized

When the economy produces only type $j = 2$ capital, an equilibrium in which money is valued, and in which loans to capital producers are made at all dates requires that the returns on these two assets must be equalized,

$$r_t = \psi \frac{u_{t+1}}{[q - w(k_t)]} = \frac{p_t}{p_{t+1}}; \quad t \geq 0. \tag{30}$$

Furthermore, in order for young lenders at time $t+1$ to buy CIP at the price u_{t+1}, it must be the case that, in equilibrium, these claims to capital ownership yield the same return as bank deposits or money between periods $t+1$ and $t+2$. Therefore

$$\frac{R_2 f'(k_{t+2})}{u_{t+1}} = r_{t+1} = \frac{p_{t+1}}{p_{t+2}}; \quad t \geq 0 \tag{31}$$

must be satisfied as well. Equations (30) and (31) then imply the following equilibrium condition

$$r_t r_{t+1} = \psi \frac{R_2 f'(k_{t+2})}{[q - w(k_t)]} = \frac{p_t}{p_{t+1}} \frac{p_{t+1}}{p_{t+2}}; \quad t \geq 0. \tag{32}$$

Using equation (25), (32) can be rewritten as

$$\psi \frac{R_2 f'(k_{t+2})}{[q - w(k_t)]} = \frac{1}{\sigma^2} \frac{m_{t+2}}{m_t}; \quad t \geq 0. \tag{33}$$

As before, it must also be the case that "sources" and "uses" of funds are equated. If μ_t denotes the fraction of potential borrowers who are funded at t, then the "uses" of funds in real terms at t is $\delta q \mu_t$, plus the real value of CIP purchased by young agents at time t, plus real balances; that is $\delta q \mu_t + \delta q \mu_{t-1} \phi u_t + m_t$. "Sources" of funds are simply per capita savings; that is $w(k_t)$. Therefore

$$w(k_t) = \delta q \mu_t + \delta q \mu_{t-1} \phi u_t + m_t \tag{34}$$

must hold at all dates. Since $k_{t+2} = R_2 \phi i_t = R_2 \phi \delta q \mu_t$, equation (34) can be rewritten as

$$w(k_t) = \frac{k_{t+2}}{R_2 \phi} + \frac{k_{t+1}}{R_2} u_t + m_t; \quad t \geq 0. \tag{35}$$

Moreover, equation (31) implies that $u_t = R_2\sigma f'(k_{t+1})(m_t/m_{t+1})$. Substituting this result in (35) yields

$$m_t = w(k_t) - k_{t+1}\sigma f'(k_{t+1})\left(\frac{m_t}{m_{t+1}}\right) - \frac{k_{t+2}}{R_2\psi}, \quad t \geq 0. \tag{36}$$

For an economy which produces only type $j = 2$ capital, equations (33) and (36) describe the evolution of any equilibrium sequences $\{k_t, m_t\}$ when credit is rationed. In a steady state, this dynamical system reduces to

$$\sqrt{R_2\psi\frac{f'(k)}{[q - w(k)]}} = \frac{1}{\sigma} = r; \tag{37}$$

$$m = w(k) - k\sigma f'(k) - \frac{k}{R_2\psi}. \tag{38}$$

We now define the function $H(k)$ by

$$H(k) \equiv \frac{f'(k)}{[q - w(k)]}. \tag{39}$$

Then, in a steady state equilibrium where only type $j = 2$ capital is produced, the per capita capital stock satisfies the following condition:

$$H(k) = \frac{1}{R_2\psi\sigma^2}. \tag{40}$$

It will clearly be necessary to establish some properties of the function H. These are stated in Lemma 1.

Lemma 1 The function H satisfies

(a) $\lim_{k\to 0} H(k) = \infty$

(b) $\lim_{k\to \hat{k}} H(k) = \infty$ where $\hat{k} \equiv w^{-1}(q)$

(c) $H'(k) \leq (\geq) 0$ iff $k \leq (\geq) f^{-1}(q)$, and

(d) $kH(k) > (=, <) 1$ iff $k > (=, <) f^{-1}(q)$.

The proof of Lemma 1 is presented in appendix A.

Lemma 1 implies that the function H has the configuration depicted in figure 2, and it is clear from this picture and equation (40) that there are potentially two steady states, k_{c1} and k_{c2}, when the economy produces only type $j = 2$ capital.

3.3 The Equilibrium Choice of Capital Production Technology

Whether type $j = 1$ or type $j = 2$ capital will be produced in equilibrium depends on the respective rates of return on these alternative capital production technologies. More precisely, we can state the following proposition.

Proposition 2 *In a steady state equilibrium, type $j = 2$ capital will be produced iff*

$$f'(k)[q - w(k)] < \frac{R_2\psi}{(R_1)^2}.$$

The proof of proposition 2 is presented in the appendix B.

Corollary 1 *Let $\underline{k}$ be defined by*

$$f'(\underline{k})[q - w(\underline{k})] = \frac{R_2\psi}{(R_1)^2}.$$

Then type $j = 1$ capital will be produced in steady state if $k < \underline{k}$; while type $j = 2$ capital will be produced in steady state if $k > \underline{k}$.

Proof. Clearly the expression $f'(k)[q - w(k)]$ is a decreasing function of k. In combination with proposition 2, this establishes the result.

We can now distinguish between two cases.

Case 1: $\underline{k} < f^{-1}(q)$: This is the situation depicted in figure 3. For case 1, the following proposition is immediate from an examination of figure 3.

Proposition 3 (a) Suppose that $\left(R_2\psi\sigma^2\right)^{-1} > H[f^{-1}(q)]$. Then there are exactly two steady state values of k, denoted by k_1 and k_2 in figure 3. If $R_1 f'(\underline{k}) \leq 1/\sigma$, then k_1 satisfies (28). If $R_1 f'(\underline{k}) > 1/\sigma$, then k_1 is given by the smallest solution to (40). In each case k_2 is the largest solution to (40).
(b) Suppose that $\left(R_2\psi\sigma^2\right)^{-1} < H[f^{-1}(q)]$. Then there is no monetary steady state with credit rationing.

Figure 4 depicts the consequences (for the steady state capital stock) of an increase in the rate of money creation, when case 1 obtains. As is apparent from the figure, an increase in the money growth rate (the steady state inflation rate), increases the steady state capital stock in the low-capital stock steady state, but decreases the steady state capital stock in the high-capital-stock steady state. The relationship between the rate of money creation and the steady state capital stock for case 1 is presented in figure 5.

Case 2: $\underline{k} > f^{-1}(q)$: This is the situation depicted in figure 6. For case 2, the following proposition can be deduced from that figure.

Proposition 4 (a) Suppose that $1/\sigma > R_1 f'(\underline{k})$. Then there are exactly two steady state values of k, denoted by k_1 and k_2 in figure 6. The value k_1 satisfies (28) while k_2 is the largest solution of (40).
(b) Suppose that $1/\sigma < R_1 f'(\underline{k})$. Then there is no monetary steady state with credit rationing.

The result of increasing the money growth rate (the steady state inflation rate) is depicted in figure 7. As in case 1, an increase in the rate of money creation increases the steady state capital stock in the low-capital stock steady state, but decreases the steady state capital stock in the high-capital-stock steady state. The relationship between the rate of money creation and the steady state capital stock for case 2 is presented in figure 8.

It remains to state conditions under which the steady state level of real balances is positive. For steady states with the type 1 capital production technology in use, our assumptions on $f(k)$ imply that real balances are necessarily positive - in the steady state - if $R_1 w(\underline{k}) > \underline{k}$ is satisfied. For steady state equilibria where the type 2 capital production technology is utilized, real balances will be positive necessarily if $w(k_2) - \sigma k_2 f'(k_2) > k_2/(\phi R_2)$ holds. Moreover, for steady states determined by (40) it is necessary to verify that (i) credit is rationed, and (ii) borrowers prefer to borrow rather than lend. The former condition will be satisfied if $k_2 < \phi R_2 \delta q$ holds, while the latter will be satisfied if $\phi[q - w(k_2)] \geq \psi$ obtains.

From this initial analysis it is clear that our economy is capable of reproducing several of the empirical facts laid out in the introduction. First, the high-capital-stock steady state displays a negative relationship between inflation and real activity [see point (1) of the introduction]. Moreover, in case 1 it is easy to verify that this relationship becomes more pronounced at high rates of inflation. Second, as is apparent from equation (37), the real return on equity holdings, r, is negatively related to inflation [see point (4) of the introduction]. Indeed, the real return on equity falls one-for-one with increases in the inflation rate. This is consistent with the large empirical literature that finds an essentially zero correlation between inflation and nominal equity returns (Nelson, 1976; Fama and Schwert, 1977; Gultekin, 1983; Boyd, Levine and Smith, 1996). And finally, a high level of real activity is associated with a high level of internal project finance, as is true empirically (Hamid and Singh, 1992).

What is the economic intuition behind these results? An increase in the money growth rate, ceteris paribus, reduces the steady state return on money. Hence, for money and other assets to be held simultaneously, the return on these assets has to decrease as well. For steady states in which all capital is produced using technology 1, and which therefore satisfy (28), this implies an increase in the steady state capital stock. For steady states in which all capital is produced using technology 2, and which thus satisfy (40), the decrease in the return on money likewise has to be accompanied by a decrease in the return on loans, as well as a decrease in the return to equity holdings. Given the presence of the CSV problem, the consequences of this observation depend on the nature of the steady state equilibrium that obtains. In the low-capital-stock steady state, a decrease in the rate of return on money implies an increase in the steady state capital stock. When the capital stock increases, the level of internal finance of investment projects rises as well, which by itself tends to mitigate the CSV problem and increase the return on loans. However, on the downward sloping portion of the function $H(k)$, the higher level of internal finance fails to compensate for the reduction in the marginal product of capital. Hence an increase in the per capita capital stock leads to the required fall in the rates of return on loans and equity holdings. In the high-capital-stock steady state the same two effects are at work. However, on the upward sloping portion of $H(k)$, the consequences of a change in the level of internal finance dominate the consequences of a change in the marginal product of capital. Therefore a higher steady state rate of inflation leads to a fall in the steady state capital stock. The implied reduction in the provision of internal finance more than offsets the effect of the increase in the marginal product of capital, and again the rate of return on loans and equity falls in the necessary way.

4 Financial Market Activity and Inflation

We now proceed to discuss the volume of financial market activity in steady states, and to examine how this is related both to the level of real activity, and to the rate of inflation. We are able to show that under certain technical conditions, equity market and bank lending activity are both positively related to the level of real activity in the high-capital-stock steady state of our economy.

This accords well with the empirical facts presented in point (2) of the introduction. Moreover, we show that as the level of real activity increases in that steady state, the importance of equity market activity relative to bank lending activity increases as well, which accounts for the empirical regularity mentioned in point (5) of the introduction. Of course, when case 2 obtains, no equity market activity or bank lending activity takes place in the low capital stock steady state.

In the next section we will also show that - at least in case 2 - the low-capital-stock steady state is always a saddle. In addition we will provide a set of examples with the following features. When the rate of money creation is su±ciently low, the high-capital-stock steady state is a saddle, with a two-dimensional stable manifold. As a result, the high activity steady state can be approached from some combination of initial conditions. Then, over some range of money growth rates, higher rates of money creation can - in the high activity steady state - lead to a reduction in the level of real activity, equity market activity and bank lending activity, and to a decline of the importance of equity markets relative to bank lending. However, once the steady state rate of in°ation reaches some critical level, the high-capital-stock steady state is transformed from a saddle to a source. Consequently the high activity steady state cannot be approached, and the low-capital-stock steady state is the only economically relevant steady state equilibrium of our economy. An increase in the money growth rate above some critical level is therefore accompanied by a sharp decrease in real activity, if a steady state is attained, while ¯nancial markets shut down altogether when case 2 obtains. Further increases in the rate of money creation then have no additional e®ects on the level of ¯nancial intermediation or equity market activity. These results accord well with the evidence presented in point (3) of the introduction. Once the rate of in°ation exceeds some threshold level, the association between further increases in in°ation and ¯nancial market activity disappears.

For the remainder of the analysis, we will focus on economies where case 2 obtains. Thus, we will henceforth adopt the following assumption.

Assumption 5 $\underline{k} > f^{-1}(q)$.

We now introduce two measures of steady state financial market activity, with the first representing the level of equity market activity, and with the second representing bank lending activity. Our measure of equity market activity, E, represents the real value of CIP sold in secondary capital markets relative to the size of the economy, that is

$$E(k) \equiv \frac{\phi\delta\mu q u}{f(k)}.$$

In effect, then, the function $E(k)$ represents the ratio of the total value of trading in secondary capital markets to GDP, for each possible value of the capital stock consistent with technology 2 being in use. Since in steady state, $\phi\delta\mu q = \frac{k}{R_2}$, while $u = f'(k)\sqrt{R_2/[\psi H(k)]}$, an alternative expression for this ratio is given by

$$E(k) = \frac{kf'(k)}{f(k)}\sqrt{\frac{1}{R_2\psi H(k)}}. \tag{41}$$

In addition, we introduce a measure of bank lending activity, B, which represents the real value of intermediated lending relative to the size of the economy for each value of the capital stock consistent with technology 2 being in use. B is given by

$$B(k) \equiv \frac{\delta\mu[q - w(k)]}{f(k)}.$$

Of course δ is the fraction of potential borrowers in the population, of whom a fraction μ actually receive credit, while each funded borrower receives a loan of $q - w(k)$. Hence per capita bank lending is given by $\delta\mu[q - w(k)]$, and $B(k)$ expresses the volume of bank lending to the private sector relative to GDP. An alternative expression for $B(k)$ is obtained by noting that $\delta\mu = k/(R_2\phi q)$, so that

$$B(k) = \frac{kf'(k)}{R_2\phi q f(k)}\frac{1}{H(k)}. \tag{42}$$

Finally, we introduce a measure of the relative importance of equity market versus banking activity,

$$EB(k) = \frac{E(k)}{B(k)} = \phi q \sqrt{\frac{R_2 H(k)}{\psi}}. \tag{43}$$

We can now state the following proposition.

Proposition 5 Let $k^\dagger$ be defined by $\frac{k^\dagger f'(k^\dagger)}{[q - w(k^\dagger)]} = \frac{(1+\rho)}{(1-\rho)}$; and let $k^\ddagger$ be defined by $\frac{k^\ddagger f'(k^\ddagger)}{[q - w(k^\ddagger)]} = \frac{1}{(1-\rho)}$. Then

(a) $f^{-1}(q) < k^\ddagger < k^\dagger$

(b) $E'(k_2) > 0$ iff $k_2 < k^\dagger$,

(c) $B'(k_2) > 0$ iff $k_2 < k^\ddagger$, and

(d) $EB'(k_2) > 0$.

The proof for proposition 5 appears in appendix C. Obviously, for economies which produce only type $j = 1$ capital, $E = B = 0$.

We now present some examples which illustrate the effect of an increase in the steady state rate of inflation on the steady state levels of output, equity market activity, bank lending activity and on the relative importance of equity markets versus intermediated lending. Our examples set $f(k) = \left(0.1025k^{0.5} + 1.5\right)^2$, $q = 3$, $g(z) = 1/\bar{z}$ with $\bar{z} = 33.33$, $\gamma = 75.49$, $R_1 = 10$; $R_2 = 1$, and in addition $\delta > 0.99$ holds. For these parameter values, $\phi = 10.5$ and $\psi = 3$. Moreover, $\underline{k} = 5.42$, while $f^{-1}(q) = 5.12$, so that case 2 obtains.

Example 2 For $\sigma = 1.25$, the low-capital-stock steady state is $k_1 = 4.89$ and the high-capital-stock steady state is $k_2 = 8.94$. For the high-capital-stock steady state, $E = 0.212$, $B = 0.02525$, and $EB = 8.4$.

Example 3 For $\sigma = 1.26$, the low-capital-stock steady state is $k_1 = 4.99$ and the high-capital-stock steady state is $k_2 = 8.55$. For the high-capital-stock steady state, $E = 0.209$, $B = 0.02517$, and $EB = 8.3$.

So, as the rate of money creation and the steady state rate of inflation increase from 25 to 26 percent, real activity in the low-capital-stock steady state increases, while output in the high-capital-stock steady state decreases. As real activity in the high-capital-stock steady state decreases, so does the level of equity market and bank lending activity. At the same time, the importance of equity market activity relative to bank lending activity decreases as well.

5 Local Dynamics

We now turn briefly to an analysis of the local stability properties of steady state equilibria. This analysis permits us to formalize some of the discussion in the previous section and - in particular - it allows us to illustrate how increases in the rate of inflation can transform the high activity steady state from a saddle to a source. We have already described the implications of this observation.

For equilibria with $k \leq \underline{k}$, only the short-gestation capital production technology is utilized. It is then immediate that the behavior of our economy is identical to the behavior of the Diamond (1965) model. As a result, given our focus on case 2 economies, the low-activity steady state is necessarily a saddle (Azariadis, 1993, Chapter 26).

When $k \geq \underline{k}$ holds, on the other hand, the long-gestation capital production technology is employed and matters are substantially more complicated. To see this, notice that equations (33), and (36) describe the equilibrium evolution of the sequences $\{k_t, m_t\}$ under credit rationing. The pair of equations (33) and (36) is obviously a system of two second order difference equations, which can be alternatively represented as follows. Let

$$k_{t+1} = y_t, \tag{44}$$

and

$$m_{t+1} = z_t. \tag{45}$$

Then (33) can be written as

$$y_{t+1} = R_2\phi\left[w(k_t) - y_t\frac{m_t}{z_t}\sigma f'(y_t) - m_t\right]; \tag{46}$$

while (36) becomes

$$z_{t+1} = R_2\psi\sigma^2 m_t\frac{f'\left\{R_2\phi\left[w(k_t) - y_t\frac{m_t}{z_t}\sigma f'(y_t) - m_t\right]\right\}}{[q - w(k_t)]}; \quad t \geq 0. \tag{47}$$

We now linearize the dynamical system consisting of equations (44) - (47) in a neighborhood of (any) steady state equilibrium (k, m, y, z). Then we have

$$(k_{t+1} - k, m_{t+1} - m, y_{t+1} - y, z_{t+1} - z)' = J(k_t - k, m_t - m, y_t - y, z_t - z)'$$

where J is the Jacobian matrix

$$J = \begin{bmatrix} \frac{\partial k_{t+1}}{\partial k_t} & \frac{\partial k_{t+1}}{\partial m_t} & \frac{\partial k_{t+1}}{\partial y_t} & \frac{\partial k_{t+1}}{\partial z_t} \\ \frac{\partial m_{t+1}}{\partial k_t} & \frac{\partial m_{t+1}}{\partial m_t} & \frac{\partial m_{t+1}}{\partial y_t} & \frac{\partial m_{t+1}}{\partial z_t} \\ \frac{\partial y_{t+1}}{\partial k_t} & \frac{\partial y_{t+1}}{\partial m_t} & \frac{\partial y_{t+1}}{\partial y_t} & \frac{\partial y_{t+1}}{\partial z_t} \\ \frac{\partial z_{t+1}}{\partial k_t} & \frac{\partial z_{t+1}}{\partial m_t} & \frac{\partial z_{t+1}}{\partial y_t} & \frac{\partial z_{t+1}}{\partial z_t} \end{bmatrix};$$

with all partial derivatives evaluated at the appropriate steady state. Expressions for these derivatives are given in appendix D.

The characteristic equation for J takes the form

$$\lambda^2\left\{\left[\frac{\partial y_{t+1}}{\partial y_t}-\lambda\right]\left[\frac{\partial z_{t+1}}{\partial z_t}-\lambda\right]-\frac{\partial z_{t+1}}{\partial y_t}\frac{\partial y_{t+1}}{\partial z_t}\right\}-\lambda\left\{\frac{\partial y_{t+1}}{\partial m_t}\frac{\partial z_{t+1}}{\partial y_t}-\frac{\partial z_{t+1}}{\partial m_t}\left[\frac{\partial y_{t+1}}{\partial y_t}-\lambda\right]\right\}+$$
$$\lambda\left\{\frac{\partial y_{t+1}}{\partial k_t}\left[\frac{\partial z_{t+1}}{\partial z_t}-\lambda\right]-\frac{\partial z_{t+1}}{\partial k_t}\frac{\partial y_{t+1}}{\partial z_t}\right\}+\frac{\partial y_{t+1}}{\partial k_t}\frac{\partial z_{t+1}}{\partial m_t}-\frac{\partial z_{t+1}}{\partial k_t}\frac{\partial y_{t+1}}{\partial m_t}=0. \tag{48}$$

We have not made any significant progress in providing a general characterization of the local stability properties of steady state equilibria when $k \geq \underline{k}$. However, we have produced a series of numerical examples that illustrate the effect of an increase in the money growth rate on the stability properties of the high-capital-stock steady state.

As in the previous section, our examples assume that $f(k) = \left(0.1025k^{0.5} + 1.5\right)^2$, $q = 3$, $g(z) = 1/\bar{z}$ with $\bar{z} = 33.33$, $\gamma = 75.49$, $R_1 = 10$, $R_2 = 1$, and that $\delta > 0.99$ holds. For these parameter values, $\phi = 10.5$ and $\psi = 3$. And, as before, $\underline{k} = 5.42$, while $f^{-1}(q) = 5.12$, so that case 2 obtains:

Example 4 For $\sigma = 1.25$, the parameters of the economy coincide with those for example 1, and the steady state equilibrium values of interest are described there. In addition, at the high-capital-stock steady state, $\mathrm{mod}(\lambda_1) = \mathrm{mod}(\lambda_2) = 1.41$; and $\mathrm{mod}(\lambda_3) = \mathrm{mod}(\lambda_4) = 0.99$, so the steady state is a saddle, with a two-dimensional stable manifold. Paths approaching the steady state oscillate as they do so.

Example 5 For $\sigma = 1.30$, the low-capital-stock steady state has $k_1 = 5.36$ and the high-capital-stock steady state has $k_2 = 6.34$. For the high-capital-stock steady state, $E = 0.19$, $B = 0.02363$, and $EB = 8.07$. Moreover, at the high-capital-stock steady state, $\lambda_1 = -1.81$; $\lambda_2 = -1.14$; and $\mathrm{mod}(\lambda_3) = \mathrm{mod}(\lambda_4) = 1.01$, so the steady state is a source.

Thus, for this set of examples, low rates of money growth result in a determinate steady state. In particular, there exists a unique dynamical equilibrium path that approaches the high activity steady state. However, as the money growth rate (and the steady state rate of in°ation) increase, the economy crosses a \threshold" and the high-capital-stock steady state becomes a source. The equilibrium behavior of the economy must change dramatically, and if the economy approaches any steady state, that must obviously be the low-capital-stock steady state. Then not only will real activity be low, but so will ¯nancial market activity. Moreover, further increases in in°ation - at least in a case 2 economy - can have no incremental e®ects on the volume of ¯nancial market activity. These predictions of the model are quite consistent with a number of the empirical ¯ndings noted in the introduction.

In addition, as example 3 illustrates, dynamical equilibrium paths approaching the high activity steady state can easily display endogenously arising volatility that dampens only very slowly. This is not possible here unless banks and secondary capital markets are active. Thus, as argued by Keynes (1936) and Friedman (1960) - and many others - the operation of the ¯nancial system can readily give rise to endogenous °uctuations along perfect foresight equilibrium paths.

6 Conclusions

As an empirical matter, there is a strong positive association between measures of both bank lending activity and the volume of trading in equity markets - on the one hand - and real activity on the other. In addition, in°ation and real activity are negatively correlated, particularly for economies with relatively high rates of in°ation. It is also true that in°ation and the development of the ¯nancial system are very negatively correlated, as are in°ation and real equity returns. Finally, there is some empirical evidence in favor of thresholds: once the rate of in°ation exceeds some critical level and stays there, there are strong observed reductions in the level of real activity (\in°ation crises," in Bruno and Easterly's terminology), and the empirical relationship between inflation and financial market activity flattens substantially.

We have attempted here to produce a theoretical framework that can - at least under some configurations of parameter values - account for these findings and the other observations noted in the introduction. Any model capable of doing so must contain - at a minimum - the following features. There must be a role for banks, secondary capital markets, and money, and at least some factors that increase the rate of inflation must also affect real activity, the financial system, and the real rate of return on equity. The model must also contain a mechanism explaining why matters change when the rate of inflation exceeds some critical value.

We have produced a model that has all of these features. To do so, we have started with a quite conventional neoclassical growth model (Diamond, 1965), and introduced into it two technologies for producing capital. One is very simple: it has a relatively short gestation period and anyone can operate it. The other is more complex. Only certain people can run it, it must be operated on a large scale, it involves a relatively lengthy gestation period for capital, and it has attached to it a CSV problem. The combination of the CSV problem and the long-gestation period of this technology implies that its use must be accompanied by banking and secondary capital market activity.

In this framework we have described conditions under which there are exactly two steady state equilibria (with credit rationing): one with a relatively low and one with a relatively high capital stock. In the high-capital stock steady state, both banks and equity markets are active. The same thing may or may not be true of the low-capital stock steady state. Moreover, we have shown that inflation and real activity must be negatively correlated in the high activity steady state. It can also easily happen that this negative relationship will become more pronounced at relatively high levels of inflation. In addition, we have stated conditions such that real activity and the volume of financial market activity are positively correlated in the high activity steady state. When this is the case, obviously inflation and financial market activity will be inversely related as well.

Finally, we have illustrated that the high-capital-stock steady state may be a saddle for low rates of money growth. However, once the rate of money creation (inflation) exceeds some critical level, the high activity steady state can be transformed from a saddle to a source. Thus, thresholds can easily exist: the behavior of the economy must differ dramatically depending on whether the steady state rate of inflation is above or below this threshold level. This implication of the model is again consistent with several pieces of empirical evidence described above.

In addition to explaining several established observations, our model yields some new testable implications of its own. For instance, in a case 1 economy, the model predicts that the correlations among inflation, financial market conditions, and real activity will differ strongly across three distinct situations: (i) $k < \underline{k}$, (ii) $k \in (\underline{k}, f^{-1}(q))$; and (iii) $k > f^{-1}(q)$: In a case 2 economy, these correlations will change as $k < \underline{k}$ or $k > \underline{k}$ hold. Of course these implications of the analysis might be difficult to test in practice, as the critical values $\underline{k}$ and $f^{-1}(q)$ might be hard to identify empirically.

Admittedly, in order to obtain all of these results we have had to make some strong assumptions. A particularly strong assumption has been placed on the production technology: we have assumed that capital and labor are highly substitutable in production ($\rho > 0$). It would be interesting to derive modifications of the analysis that would allow us to relax this sort of condition in the future.

APPENDIX

A. Proof of Lemma 1

Part (a) of Lemma 1 is immediate from $\lim_{k \to 0} f'(k) = \infty$ and assumption 1. Part (b) is also obvious. For (c), it is easy to verify that

$$H'(k) = -f''(k)\frac{[f(k) - q]}{[q - w(k)]^2};$$

establishing the result. Part (d) follows from

$$kH(k) \equiv \frac{kf'(k)}{[q - w(k)]} = \frac{kf'(k)}{[q - f(k) + kf'(k)]}.$$

B. Proof of Proposition 2

Type $j = 2$ capital will be produced in steady state iff the internal rate of return on investments in technology 2 exceeds that on investments in technology 1. From equations (28) and (37), this condition obtains iff

$$\sqrt{R_2 \left(\frac{f'(k)}{[q - w(k)]}\right.} > R_1 f'(k). \qquad (49)$$

Rearranging terms in equation (49) establishes the result.

C. Proof of Proposition 5

(a) Given our assumption that $0 < \rho < 1$, it follows that $1 < \frac{1}{(1-\rho)} < \frac{(1+\rho)}{(1-\rho)}$, which implies $1 < \frac{k^{\ddagger} f'(k^{\ddagger})}{[q - w(k^{\ddagger})]} < \frac{k^{\dagger} f'(k^{\dagger})}{[q - w(k^{\dagger})]}$. From Lemma 1 it is then obvious that $f^{-1}(q) < k^{\ddagger} < k^{\dagger}$, which establishes part (a).

(b) Differentiating equation (41) and rearranging terms yields $\frac{kE'(k)}{E(k)} = \frac{w(k)}{f(k)} - \frac{1}{2}kw'(k)\left\{\frac{q - w(k) + kf'(k)}{[kf'(k)[q - w(k)]]}\right\}$. Therefore, $E'(k) > 0$ holds iff $\frac{kf'(k)[q - w(k)]}{f(k)} > \frac{1}{2}\left[\frac{kw'(k)}{w(k)}\right][q - w(k) + kf'(k)]$. Since $f(k)$ has the CES form $f(k) = [\alpha k^{\rho} + \beta]^{\frac{1}{\rho}}$, with $0 < \rho < 1$, it is straightforward to show that $E'(k) > 0$ iff $\frac{kf'(k)}{[q - w(k)]} < \frac{(1+\rho)}{(1-\rho)} = \frac{k^{\dagger} f'(k^{\dagger})}{[q - w(k^{\dagger})]}$. Lemma 1 then implies that $k^{\dagger} > f^{-1}(q)$ is satisfied. Therefore, since $k_2 > f^{-1}(q)$ necessarily holds it follows that $E'(k_2) > 0$ obtains iff $k_2 < k^{\dagger}$.

(c) Differentiating equation (42) and rearranging terms, we obtain $\frac{kB'(k)}{B(k)} = \left[\frac{w(k)}{f(k)}\right] - \frac{kw'(k)}{[q-w(k)]}$. Therefore, $B'(k) > 0$ obtains iff $\frac{[q-w(k)]}{f(k)} > \frac{kw'(k)}{w(k)}$. Since $f(k) = [\alpha k^{\rho} + \beta]^{\frac{1}{\rho}}$, with $0 < \rho < 1$, it is easy to show that $B'(k) > 0$ holds iff $\frac{kf'(k)}{[q-w(k)]} < \frac{1}{(1-\rho)} = \frac{k^{\dagger\dagger}f'(k^{\dagger\dagger})}{[q-w(k^{\dagger\dagger})]}$. Lemma 1 then clearly implies that $k^{\dagger\dagger} > f^{-1}(q)$ holds. In addition, since $k_2 > f^{-1}(q)$; it follows that $B'(k_2) > 0$ is satisfied iff $k_2 < k^{\dagger\dagger}$.

(d) $H'(k_2) > 0$ necessarily holds and the result is then immediate from (43).

D. Elements of the Jacobian

The elements of the Jacobian matrix are given by the following expressions:

$$\frac{\partial k_{t+1}}{\partial k_t} = 0 \tag{50}$$
$$\frac{\partial k_{t+1}}{\partial m_t} = 0 \tag{51}$$
$$\frac{\partial k_{t+1}}{\partial y_t} = 1 \tag{52}$$
$$\frac{\partial k_{t+1}}{\partial z_t} = 0 \tag{53}$$
$$\frac{\partial m_{t+1}}{\partial k_t} = 0 \tag{54}$$
$$\frac{\partial m_{t+1}}{\partial m_t} = 0 \tag{55}$$
$$\frac{\partial m_{t+1}}{\partial y_t} = 0 \tag{56}$$
$$\frac{\partial m_{t+1}}{\partial z_t} = 1 \tag{57}$$
$$\frac{\partial y_{t+1}}{\partial k_t} = R_2\phi w'(k) \tag{58}$$
$$\frac{\partial y_{t+1}}{\partial m_t} = -R_2\phi\left[1 + \sigma\frac{kf'(k)}{m}\right] \tag{59}$$
$$\frac{\partial y_{t+1}}{\partial y_t} = R_2\phi\sigma\left[w'(k) - f'(k)\right] \tag{60}$$
$$\frac{\partial y_{t+1}}{\partial z_t} = R_2\phi\sigma\frac{kf'(k)}{m} \tag{61}$$
$$\frac{\partial z_{t+1}}{\partial k_t} = m\frac{w'(k)}{f'(k)}\left[\frac{1}{R_2\psi\sigma^2} + R_2\phi f''(k)\right] \tag{62}$$

$$
\begin{aligned}
\frac{\partial z_{t+1}}{\partial m_t} &= 1 + R_2 \cdot m \frac{w'(k)}{k f'(k)} \left(1 + \sigma \frac{k f'(k)}{m} \right) && (63) \\
\frac{\partial z_{t+1}}{\partial y_t} &= - R_2 \cdot \sigma m f''(k) \left(1 - \frac{w'(k)}{f'(k)} \right) && (64) \\
\frac{\partial z_{t+1}}{\partial z_t} &= - R_2 \cdot \sigma w'(k). && (65)
\end{aligned}
$$

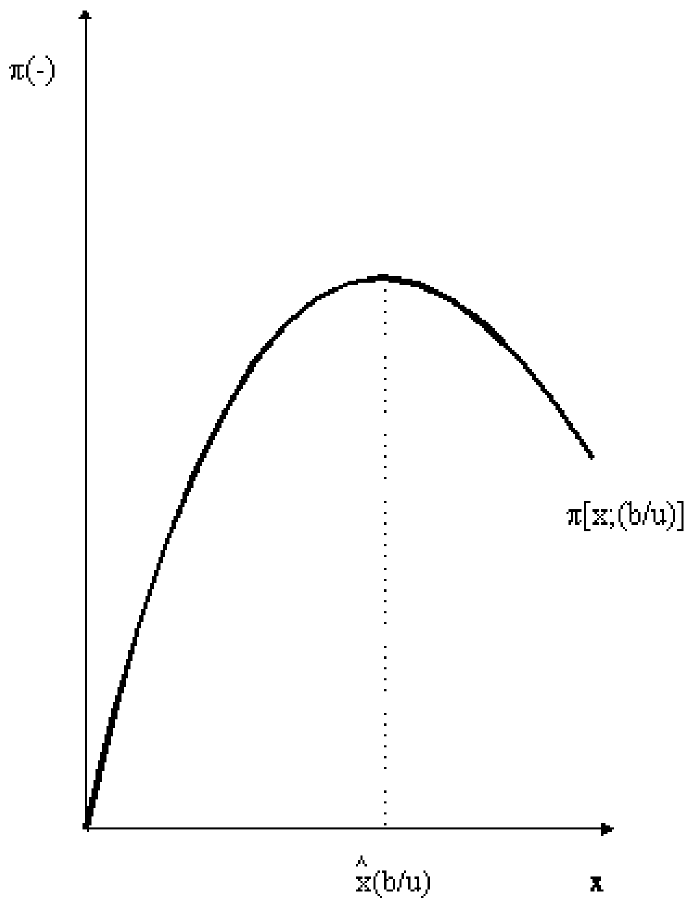

Figure 1

The Expected Return Function

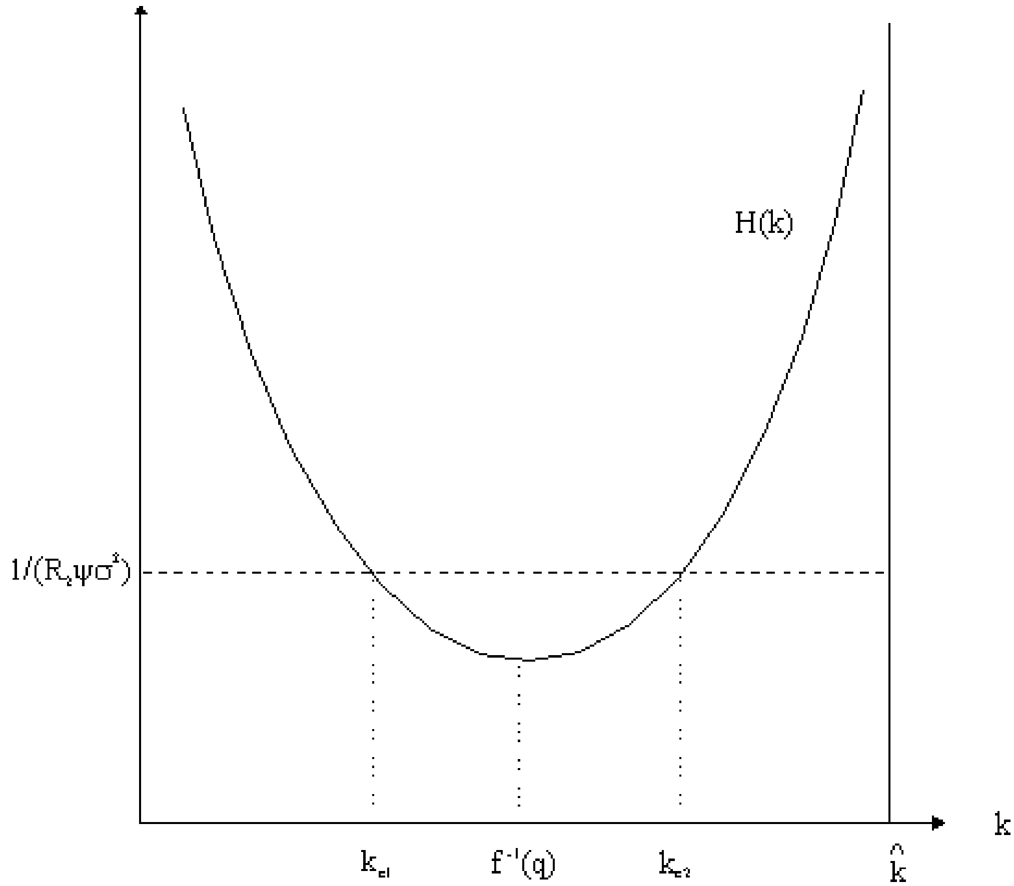

Figure 2

The Steady State Capital Stock with Long Gestation Investments

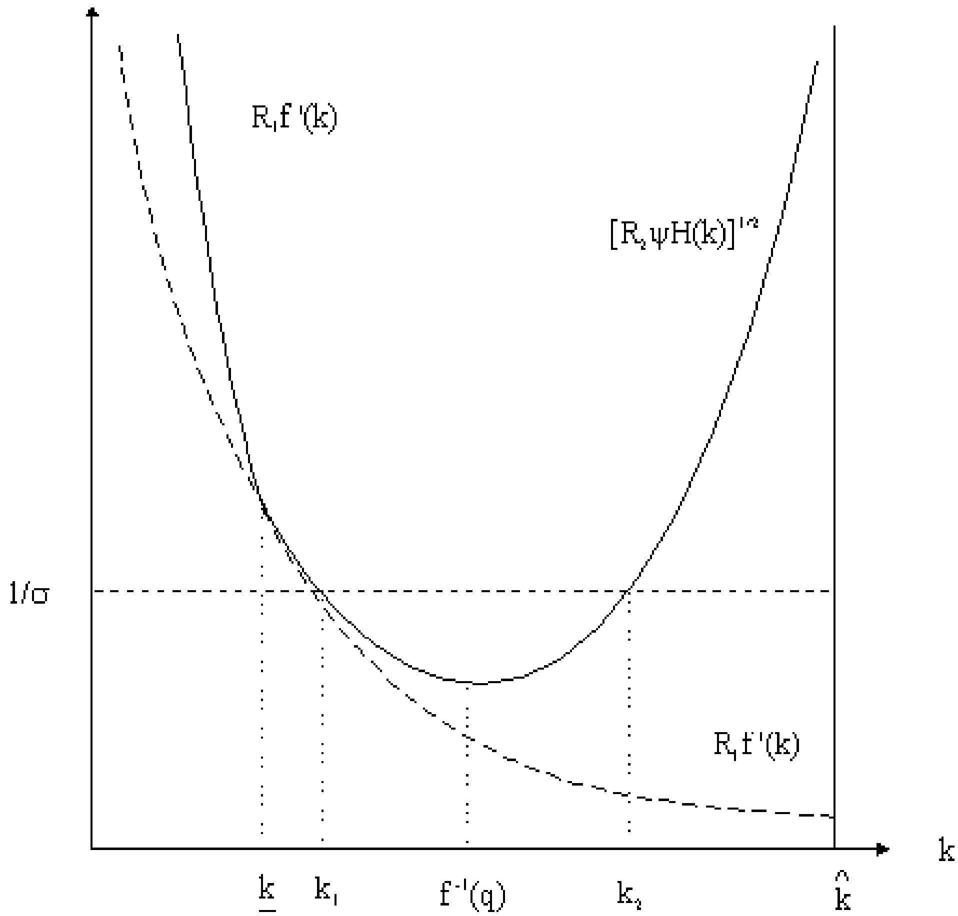

Figure 3

Steady States
Case 1: $\underline{k}<f'^{-1}(q)$

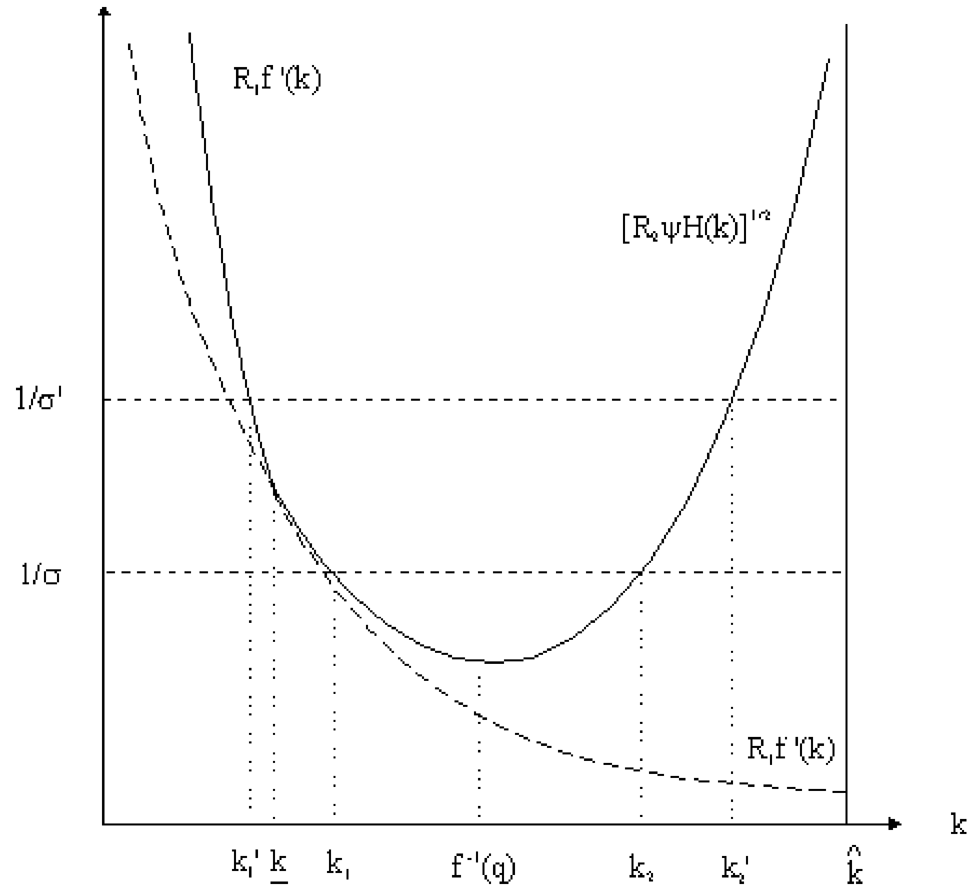

Figure **4**

The Effect of an Increase in
the Money Growth Rate:
Case 1

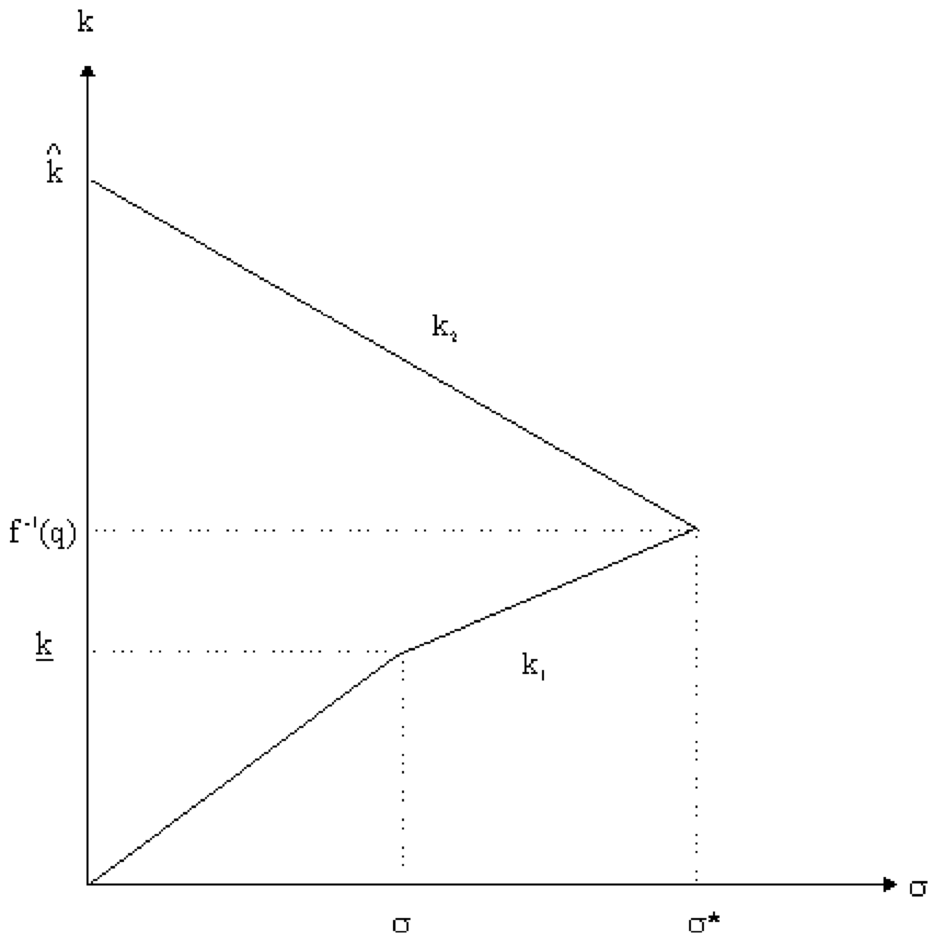

Figure 5

The Relationship Between the Money Growth Rate
and the Steady State Capital Stock:
Case 1

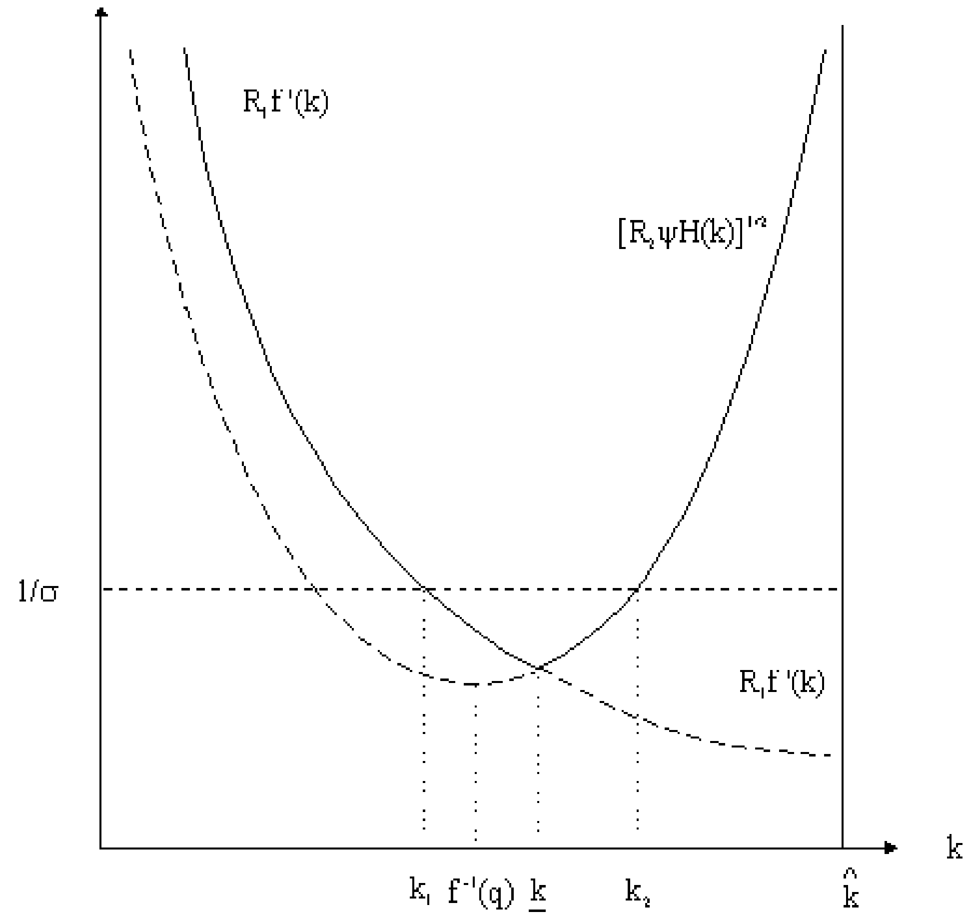

Figure 6

Steady States
Case 2: $\underline{k} > f^{-1}(q)$

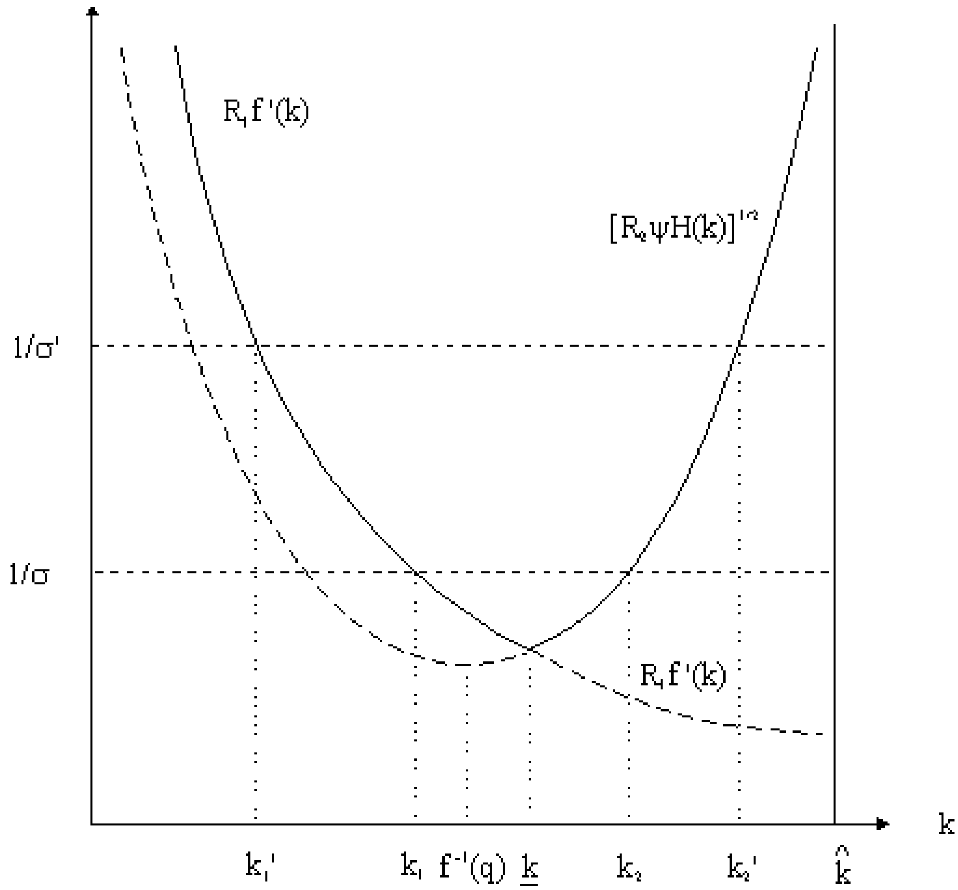

Figure 7

The Effect of an Increase in
the Money Growth Rate:
Case 2

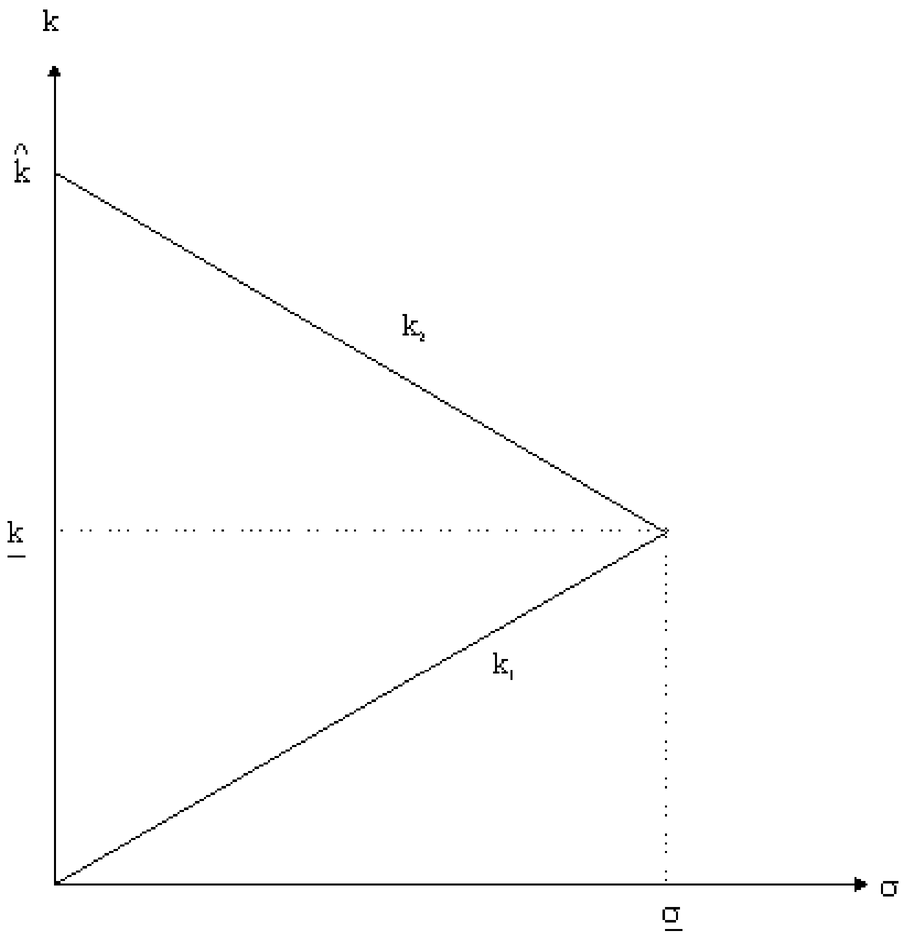

Figure 8

The Relationship Between the Money Growth Rate
and the Steady State Capital Stock:
Case 2