What to Stabilize in the Open Economy?: Some Notes on a Problem of Keynes [1]

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Abstract

We consider the question of how to \best" maintain price level stability in the open economy and evaluate three possible policy choices: (a) a constant money growth rate rule; (b) a ¯xed exchange rate; and (c) a policy of explicit commitment to a price level target. In each case we assume that policy is conducted by injecting or withdrawing reserves from the \banking system". In evaluating the three regimes, we adopt Keynes' criteria for a desirable policy: the best policy should leave the least scope for indeterminacy and \excessive" economic volatility. In a steady state equilibrium the choice of regime is largely irrelevant: any steady state equilibrium under one regime can be duplicated by an appropriate choice of the \control" variable under any other regime. However, we show that the set of equilibria under the three regimes is dramatically di®erent. When all countries follow the policy of ¯xing a constant rate of money growth, there are no equilibria displaying endogenously arising volatility and there is no indeterminacy of equilibrium. Under a regime of ¯xed exchange rates, indeterminacies and endogenously arising °uctuations are impossible if the country with the low \reserve deposit ratio" is charged with maintaining the ¯xed rate. Finally, when one country targets the time path of its price level, under very weak conditions, there will be indeterminacy of equilibrium and endogenously arising volatility driven by expectations. These observations also have serious implications for how an open economy might respond to various shocks under the di®erent regimes.

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\Is it more important that the value of a national currency should be stable in terms of purchasing power, or stable in terms of the currency of foreign countries?"

J.M. Keynes (1932, p.187)

Throughout history, many countries have been confronted with the issue of how best to \stabilize" their economies after periods of high in°ation. In the past this question often arose when countries \left gold" during wartime episodes in order to resort to in°ationary ¯nance, and then faced a choice about whether or not to go back on gold, and if so, how to accomplish this and what rate of exchange to adopt. And, both historically and more recently, countries experiencing sustained high rates of in°ation have confronted choices about what policies to adopt that will \stabilize" their price levels. This issue has preoccupied many economists, including Keynes in the 1920s.

Of course countries interested in stabilizing their price levels have many options. One is to commit to a low and (approximately) constant rate of money creation. This course of action is in accord with standard quantity theory prescriptions which appear in Friedman (1956) or Lucas (1972). Alternatively, an economy could ¯x its rate of exchange against that of another, relatively stable currency. Some have argued that this is the surest and most successful route to the attainment of internal price level stability.[5] Keynes, in contrast, was dubious of this prescription. In considering Britain's optimal post - World War I monetary policy, Keynes (1932, p. 214) argued that, \in the event of the Federal Reserve Board failing to keep dollar prices steady, sterling prices should not ... plunge with them merely for the sake of maintaining a ¯xed parity of exchange." Finally, a third alternative would be for a country to adopt the policy of explicitly committing to a ¯xed target

[5] For instance Vegh (1995, p. 42) asserts that \the evidence clearly suggests that, in hyperin°ationary situations, price stability can be the immediate result of using the exchange rate as a nominal anchor."

time path for the price level. Many have advocated this policy for the U.S.,[6] and some countries - such as Germany - constitutionally mandate such a policy.

How should an economy decide which type of policy is the \best" one to adopt? Here our thinking is strongly in°uenced by Keynes, who wrote eloquently on this topic in the 1920s. While we strongly disagree with Keynes' conclusion as to the \best" course of action, Keynes (1932) wrote down several features of a desirable policy. First, both in°ation and de°ation were to be explicitly avoided because both e®ect \what is always harmful, a change in the existing standard of value ..." [Keynes (1932), p. 189] Second, \the fault of the post - war regime [in Britain], under which the price level mainly depends on internal in°uence (i.e. internal currency and credit policy) ... is that it ... may act violently for merely transitory causes." (p. 198) Thus a good policy should try to limit the scope for excessive °uctuations. The same desideratum was proposed by Friedman (1960), who also argued that some policies are to be avoided because they leave scope for indeterminacies. And, in a world of many open economies, indeterminacies and excessive °uctuations that arise due to the policy of one country can be transmitted to all others.

Stability of Prices Versus Stability of Exchange

Our section heading is borrowed from Keynes (1932, p.195), who took the view - as noted above - that an adherence to something like a money supply rule left undue scope for an economy to act \violently" for \transitory" reasons. Hence, Keynes focused his attention on whether it was best to ¯x the exchange rate, or to target the time path of the price level.

> If, therefore, the external price level lies outside our control, we must submit either to our own internal price level or to our rate of exchange being pulled about by external in°uences. If the external price level is unstable, we cannot keep our own price level, and our exchanges stable. And we are compelled to choose. (p. 195)

[6] See, for instance, Hoskins (1991), who proposes a target price level path with a zero in°ation rate.

Keynes also had a strong opinion as to the correct choice of policy.

> The right choice is not necessarily the same for all countries. It must partly depend on the relative importance of foreign trade in the life of the country. Nevertheless, there does seem to be in almost every case a presumption in favor of the stability of prices ... (p. 196)

Thus Keynes concluded that

> a sound constructive scheme must provide: a method for regulating the supply of currency and credit [our emphasis] with a view to maintaining ... the stability of the internal price level. (p.213)

But how was the attainment of this stability to be achieved? According to Skidelsky (1992, p. 153), \the central policy proposal of the `Tract [on Monetary Reform]' was that monetary policy should be used to stabilize the price level ... The central claim of the `Tract' is that by varying the amount of credit to the business sector, the banking system could even out °uctuations in business activity."

Then,

> having achieved the `normal' price level, the monetary authority will then maintain it, as required, by o®setting a rise or fall in the public's cash balances (or the velocity of circulation) by varying the supply of credit. If it wants to expand credit it injects cash into the banking system. The banks liquidate their `surplus' cash reserves by increasing their loans to the public. [Skidelsky (1992), p. 157]

To summarize, then, Keynes thought that the best policy was to target the price level path. To do so, the central bank would inject or withdraw reserves, as necessary. And Keynes believed that the crucial feature of such a policy was that the injection or withdrawal of reserves into the banking system would a®ect the volume of credit extension in a sympathetic manner.

The Scope of the Analysis

The issue of which policy is \best" for maintaining price level stability is as important today as it was when Keynes wrote. In this paper we adopt Keynes' criteria for a desirable policy: the best policy should leave the least scope for indeterminacy and \excessive" economic volatility. In our analysis we are interested in attaining the stability of the price level, and we ignore what happens at any time prior to the implementation of a \stabilization." Thus we ask only about the e®ects of the choice of policy regime after this regime has been implemented.[7]

We consider three possible choices of a policy to be followed : (a) a constant money growth rate rule; (b) a ¯xed exchange rate; and (c) a policy of explicit commitment to a price level target. In each case we assume that policy is conducted as Keynes suggested; by injecting or withdrawing reserves into or from the \banking system." And, in each case, we then consider the complete set of perfect foresight equilibria under the policy regime in place.

In a steady state equilibrium, as we show, the choice of regime is largely irrelevant. Any steady state equilibrium under one regime can be duplicated by an appropriate choice of the \control variable" under any other regime. However, this fact obscures another of equal importance. The set of equilibria under the three regimes is di®erent, and typically dramatically so.

When all countries follow the policy of ¯xing a constant rate of money growth there is a unique equilibrium (the steady state) in our model. There are no equilibria displaying endogenously arising volatility and there is no indeterminacy of equilibrium. Therefore the constant money growth rate rule behaves well, according to Keynes' criteria.

The set of equilibria under a regime of ¯xed exchange rates turns out to depend dramatically on which country is charged with maintaining the ¯xed rate of exchange. In a two country world, if the country with the high \reserve - deposit ratio" is responsible for maintaining the ¯xed exchange rate, then there can easily be a steady state equilibrium, plus a continuum of nonstationary equilibria

[7] See Paal (1997a, b) for an analysis of transitions between a period of high in°ation and the implementation of a stabilization program.

that display endogenously arising fluctuations in all variables (other than the exchange rate). Under some configurations of parameters, there are also equilibria where fluctuations need not dampen over time. However, if the country with the low "reserve deposit ratio" is charged with maintaining the fixed exchange rate, then there is a unique equilibrium - the steady state. Indeterminacies and endogenously arising fluctuations are impossible. Thus a fixed exchange rate regime, with the "right" country charged with maintaining the rate, performs very well by Keynes' criteria. This conclusion is, of course, in direct opposition to the one reached by Keynes.

Finally, when one country targets the time path of its price level - as Keynes proposed for Britain - under very weak conditions there will be a steady state equilibrium, and a continuum of nonstationary equilibria. All nonstationary equilibria display damped endogenous oscillation en route to a steady state. Thus Keynes' most preferred policy actually leaves the most scope for indeterminacy, and for endogenously arising volatility driven by expectations.

Clearly our focus on the potential for endogenously generated volatility to emerge under these alternative regimes is consistent with Keynes' concern about "animal spirits." It is also consistent with Friedman's (1960 p.23) views about the optimal conduct of monetary policy:

> The central problem is not to construct a highly sensitive instrument that can continuously offset instability introduced by other factors, but rather to prevent monetary arrangements from themselves becoming a primary source of instability.

However, we are aware that many readers will take the view that what matters is how an economy responds to various shocks under the different regimes.

Our analysis has strong implications for the answer to that question. In particular, it is well-posed when both countries fix their rate of money creation. In addition, this question has an unambiguous answer under a regime of fixed exchange rates, if and only if the country with the low reserve deposit ratio maintains the fixed rate. Under the other conditions we analyze - in which there is a multiplicity of equilibria - there is a continuum of possible reactions to any unanticipated shock, either permanent or temporary. In short, many things can happen in response to any economically relevant shock.

Of course this is presumably part and parcel of Keynes' and Friedman's concern with conducting policy in a way that allows for indeterminacies. But, in any event, indeterminacies and endogenous volatility that arise in a nonstochastic model, such as ours, will have important consequences for how exogenous shocks impact on an economy in a stochastic framework as well.

Our vehicle for examining all of these issues is a two-period lived, overlapping generations model with two countries. To focus on issues of price stability alone, we consider a pure exchange economy with (international and domestic) borrowing and lending, and we assume that those engaged in the activity of lending are subject to reserve requirements in each country.[8] We believe that this model brings credit conditions to the forefront, as they clearly were in Keynes' thinking about the problems we have posed.

The remainder of the paper proceeds as follows. Section 1 lays out the model economy we consider, while sections 2, 3, and 4 analyze the set of perfect foresight equilibria under regimes (a), (b) and (c), respectively. Section 5 considers the behavior of the balance of trade under each regime, and section 6 concludes.

[8] See Miller and Todd (1995) and Chin and Miller (1996) for some related modelling exercises, although their focus is much di®erent from ours.

1 Environment

1.1 Description

We consider a world in which there are two countries; countries are indexed by $i = 1, 2.$ Each country is populated by a sequence of two-period lived, overlapping generations, along with an initial old generation. Throughout, we let $t = 1, 2, \ldots$ index time.

At each date agents consume some of a single good, which can neither be produced nor stored. Since there is a single good, all international trade takes the form of borrowing/lending. Within each country, agents are divided into two "groups," which we term borrowers and lenders. We let N_i, $i = 1, 2$, denote the total young population of country i, which is constant over time. Within country i, we let α_i $(1 - \alpha_i)$ denote the fraction of the population that is borrowers (lenders) at each date. Note that both $N_1 \neq N_2$ and $\alpha_1 \neq \alpha_2$ may hold.

Let c^i_{1t} (c^i_{2t}) denote the first (second) period consumption of a representative agent who is born in country i at date t. Then all agents, both borrowers and lenders, have the common utility function $u\left(c^i_{1t}, c^i_{2t}\right).$ For simplicity of calculation, we assume that this utility function has the logarithmic form

$$u\left(c^i_{1t}, c^i_{2t}\right) = \ln c^i_{1t} + \beta \ln c^i_{2t}.$$

We further assume that lenders are endowed with $w > 0$ units of the single good when young, and that they have a zero endowment when old. Borrowers, in contrast, have a zero endowment of the good when young, and an endowment of $y > 0$ when old. The assumption that each group is endowed in only a single period substantially simplifies calculations.

In order to avoid the potential indeterminacy of exchange rates discussed by Kareken and Wallace (1981), we assume that the activity of lending is subject to a reserve requirement in each country, and that these reserve requirements bind at each date. Let L_t^i denote the amount lent in country i at date t by a representative saver (in real terms), and let $\bar{m}_t^i$ denote the individual holdings of country i real balances by the same agent. Then country i imposes a reserve requirement of the following form:

$$\bar{m}_t^i \geq \lambda_i L_t^i; \quad i = 1; 2: \tag{1}$$

Note that reserve requirements are imposed on lending activity, and that the reserve requirement that applies is the one prevailing in the country in which the loan is made.

Finally, depending on the policy regime under which the government of country i operates, it must engage in a set of taxes and transfers designed to expand or contract the money supply as needed. In accordance with Keynes' notion that monetary policy works by affecting the supply of credit, we assume that all injections or withdrawals of money are accomplished via lump-sum transfers or taxes that are made to (or paid by) young lenders. This assumption guarantees that a monetary injection (withdrawal) has the effect of expanding (contracting) the availability of credit. We assume that governments make lump-sum transfers only to residents of their own country, and we let τ_t^i denote the lump-sum transfer received at t by young lenders residing in country i.

1.2 Behavior of Agents

Let R_t^i denote the gross real rate of interest on loans in country i between t and t + 1, and let p_t^i denote the time t price level in country i. The law of one price implies that the exchange rate between the currencies at t is simply the ratio of the price levels: $e_t = \frac{p_t^2}{p_t^1}$:

A young lender residing in country j at t chooses a quantity of loans to make in each country

(L_t^1, L_t^2), a quantity of each country's real balances to hold $(\bar{m}_t^1, \bar{m}_t^2)$, and a consumption vector (c_{1t}, c_{2t}) to solve the following problem:

$$\max \ \ln c_{1t}^j + \beta \ln c_{2t}^j$$

subject to (1);

$$c_{1t}^j + L_t^1 + L_t^2 + \bar{m}_t^1 + \bar{m}_t^2 \le w + \tau_t^j \tag{2}$$

$$c_{2t}^j \le R_t^1 L_t^1 + R_t^2 L_t^2 + \bar{m}_t^1 \left(\frac{p_t^1}{p_{t+1}^1} \right) + \bar{m}_t^2 \left(\frac{p_t^2}{p_{t+1}^2} \right). \tag{3}$$

If the reserve requirement binds in each country at each date (which means that $R_t^i > \frac{p_t^i}{p_{t+1}^i}$; $i = 1, 2, \forall t \ge 1$), then this problem can be transformed as follows. Let $s_t^i = L_t^i + \bar{m}_t^i = (1 + \lambda_i) L_t^i$ denote total savings invested in country i, and let $\theta_i \equiv \frac{\lambda_i}{(1+\lambda_i)}$. Here θ_i is the fraction of investments in country i held in the form of required reserves. Then a young lender residing in country j can be regarded as choosing a vector $(c_{1t}^j, c_{2t}^j, s_t^1, s_t^2)$ to maximize $\ln c_{1t}^j + \beta \ln c_{2t}^j$ subject to

$$c_{1t}^j + s_t^1 + s_t^2 \le w + \tau_t^j \tag{4}$$

$$c_{2t}^j \le \left[(1 - \theta_1) R_t^1 + \theta_1 \frac{p_t^1}{p_{t+1}^1} \right] s_t^1 + \left[(1 - \theta_2) R_t^2 + \theta_2 \frac{p_t^2}{p_{t+1}^2} \right] s_t^2. \tag{5}$$

Clearly, an absence of arbitrage opportunities requires that

$$(1 - \theta_1) R_t^1 + \theta_1 \frac{p_t^1}{p_{t+1}^1} = (1 - \theta_2) R_t^2 + \theta_2 \frac{p_t^2}{p_{t+1}^2}; \quad t \ge 1. \tag{6}$$

Equation (6) implies that the net return to savings - inclusive of reserve holdings - is the same in each country. When (6) holds, the solution to (4) sets

$$s_t^1 + s_t^2 = \frac{\beta}{1+\beta} \left(w + \tau_t^j \right). \tag{7}$$

For borrowers matters are substantially more simple, since borrowers are not subject to the analog of a reserve requirement. Indeed, a young borrower in country i at t simply chooses a loan quantity, l_t^i and a consumption vector (c_{1t}^i, c_{2t}^i), to maximize $\ln c_{1t}^i + \beta \ln c_{2t}^i$ subject to

$$c_{1t}^i \le l_t^i \tag{8}$$

$$c_{2t}^i \le y - R_t^i l_t^i. \tag{9}$$

The solution to this problem sets

$$l_t^i = \frac{y}{(1+\beta)\, R_t^i}; \quad i = 1, 2. \tag{10}$$

With respect to the initial old generation, each initial old agent residing in country i is endowed with the initial per capita money supply of that country, $M_0^i > 0$; which is exogenously given. Old agents use this currency to purchase consumption goods from the initial young generation.

1.3 Equilibrium

A perfect foresight equilibrium is a set of sequences $\{R_t^i\}$, $\{p_t^i\}$, and $\{m_t^i\}$, $i = 1, 2$, that satisfies four conditions. First, savers must perceive the same gross return from lending in either country, taking account of the reserve requirements that prevail in each location. Thus (6) must hold at each date. Second, sources and uses of funds must be equal. Sources of funds at each date are the savings of young lenders; uses are loans plus the accumulation of real balances. Thus

$$\begin{aligned} N_1(1-\alpha_1)\frac{\beta}{1+\beta}\left(w+\tau_t^1\right) + N_2(1-\alpha_2)\frac{\beta}{1+\beta}\left(w+\tau_t^2\right) &= \\ N_1\alpha_1\frac{y}{(1+\beta)\,R_t^1} + N_2\alpha_2\frac{y}{(1+\beta)\,R_t^2} + N_1 m_t^1 + N_2 m_t^2; \quad t \ge 1. \end{aligned} \tag{11}$$

The sequences $\{\tau_t^i\}$ are determined by the government budget constraints that obtain under each policy regime, as described below, and by the sequences of per capita real balances $\{m_t^i\}$. Under the assumption that the reserve requirement binds in each country, the per capita level of real balances, in equilibrium, must equal λ_i times the per capita quantity of loans in each country. Hence

$$m_t^i = \lambda_i \alpha_i \frac{y}{(1+\beta) R_t^i} \equiv \frac{\theta_i}{1-\theta_i} \alpha_i \frac{y}{(1+\beta) R_t^i}; \quad t \geq 1; \; i = 1;2 \tag{12}$$

must hold in equilibrium. Finally, the reserve requirement binds in each country iff

$$R_t^i > \frac{p_t^i}{p_{t+1}^i}; \quad t \geq 1; \; i = 1;2: \tag{13}$$

2 Equilibrium: Constant Money Growth Rates

In this section we describe the set of perfect foresight equilibria under the assumption that each country maintains a constant (for all time) rate of money growth. Thus, in this section, the government of each country stabilizes its rate of money creation.

To be more specific, in this section the per capita money supply of country i, M_t^i; evolves according to

$$M_{t+1}^i = \sigma_i M_t^i; \quad t \geq 0; \; i = 1;2: \tag{14}$$

Monetary injections are, of course, accomplished via lump-sum transfers to young lenders. Thus the government budget constraint of country i implies that

$$(1-\alpha_i)\tau_t^i = \frac{M_t^i - M_{t-1}^i}{p_t^i} = m_t^i \left(\frac{\sigma_i - 1}{\sigma_i}\right); \quad t \geq 1; \; i = 1;2: \tag{15}$$

In order to determine an equilibrium, we substitute (12) into (11) and (15), and use the results to obtain

$$\beta w\left[N_1(1-\alpha_1)+N_2(1-\alpha_2)\right] =$$
$$\frac{N_1\alpha_1 y}{R^1_t}\left[1+\frac{\theta_1}{(1-\theta_1)}\frac{(\sigma_1+\beta)}{(1+\beta)\sigma_1}\right]+\frac{N_2\alpha_2 y}{R^2_t}\left[1+\frac{\theta_2}{(1-\theta_2)}\frac{(\sigma_2+\beta)}{(1+\beta)\sigma_2}\right];\quad t\geq 1. \tag{16}$$

Equation (16) describes a relationship between R^1_t and R^2_t that must obtain for sources and uses of funds to be equated. In order to write (16) more compactly, define

$$\begin{aligned}
\phi &\equiv \frac{\beta w}{1+\beta}\left[N_1(1-\alpha_1)+N_2(1-\alpha_2)\right];\\
f(\theta_1,\sigma_1) &\equiv \frac{N_1\alpha_1 y}{1+\beta}\left[1+\frac{\theta_1}{(1-\theta_1)}\frac{(\sigma_1+\beta)}{(1+\beta)\sigma_1}\right];\\
g(\theta_2,\sigma_2) &\equiv \frac{N_2\alpha_2 y}{1+\beta}\left[1+\frac{\theta_2}{(1-\theta_2)}\frac{(\sigma_2+\beta)}{(1+\beta)\sigma_2}\right].
\end{aligned}$$

Then ϕ gives the total world availability of savings absent any transfers, while $\frac{f(\theta_1,\sigma_1)}{R^1_t}\left[\frac{g(\theta_2,\sigma_2)}{R^2_t}\right]$ gives the sum of loan demand and real balances, net of transfers, as a function of monetary policy (the reserve requirement and the rate of money growth) in country 1 (2). With these definitions, (16) reduces to

$$R^2_t = R^1_t\left[\frac{g(\theta_2,\sigma_2)}{\phi R^1_t - f(\theta_1,\sigma_1)}\right];\quad t\geq 1. \tag{17}$$

It is also the case that equation (6) - requiring the equality of returns to savings across countries - must obtain. To see its implications, we note that by definition

$$\frac{p^i_t}{p^i_{t+1}} \equiv \frac{m^i_{t+1}}{\sigma_i m^i_t};\quad t\geq 1;\ i=1,2; \tag{18}$$

must be satisfied at each date. Using (12) in (18) yields

$$\frac{p^i_t}{p^i_{t+1}} = \frac{R^i_t}{\sigma_i R^i_{t+1}};\quad t\geq 1;\ i=1,2. \tag{19}$$

Equation (19) asserts that the gross real rate of interest at t + 1 equals the gross nominal rate of interest at t, divided by the gross rate of money creation. Or, equivalently, (19) asserts that the gross nominal rate at t + 1 equals the gross nominal rate at t, multiplied by the ratio of the gross rate of inflation between t + 1 and t + 2 and the rate of money creation. In any event, substituting (19) into (6) produces

$$R_t^1\left[(1-\theta_1)+\frac{\theta_1}{\sigma_1 R_{t+1}^1}\right]=R_t^2\left[(1-\theta_2)+\frac{\theta_2}{\sigma_2 R_{t+1}^2}\right];\quad t\geq 1. \tag{20}$$

Upon substituting (17) into (20) and rearranging terms, we obtain the following difference equation governing the evolution of $\left\{R_t^1\right\}$

$$\frac{g(\theta_2;\sigma_2)}{\phi R_t^1-f(\theta_1;\sigma_1)}=\frac{\left[(1-\theta_1)+\frac{\theta_1}{\sigma_1}\left(R_{t+1}^1\right)^{-1}\right]}{(1-\theta_2)+\frac{\phi\theta_2}{\sigma_2 g(\theta_2;\sigma_2)}-\left[\frac{\theta_2 f(\theta_1;\sigma_1)}{\sigma_2 g(\theta_2;\sigma_2)}\right]\left(R_{t+1}^1\right)^{-1}} \tag{21}$$

Equation (21) gives R_{t+1}^1 as an increasing function of R_t^1. Having determined an equilibrium sequence $\left\{R_t^1\right\}$, (17) gives $\left\{R_t^2\right\}$, (19) then gives the equilibrium sequence of national inflation rates, while $\frac{\sigma_i M_0^i}{p_1^i}=\frac{\theta_i}{1-\theta_i}\alpha-\frac{y}{(1+\beta)R_1^i}$ gives the initial price level in each country.

We now proceed to characterize equilibrium sequences $\left\{R_t^1\right\}$ satisfying (21). We begin with steady state equilibria.

2.1 Steady State Equilibria

In a steady state, $\frac{p_t^i}{p_{t+1}^i}=\frac{1}{\sigma_i}$, $i=1,2$. Then equation (6) reduces to

$$(1-\theta_1)R^1+\frac{\theta_1}{\sigma_1}=(1-\theta_2)R^2+\frac{\theta_2}{\sigma_2}, \tag{22}$$

where we now omit time subscripts. Similarly, equation (17) is

$$R^2=R^1\left[\frac{g(\theta_2;\sigma_2)}{\phi R^1-f(\theta_1;\sigma_1)}\right]. \tag{23}$$

Figure 1 depicts equations (22) and (23) diagrammatically. Evidently, (22) defines an upward sloping locus in the figure, and it is easy to show that (23) defines a downward sloping locus. Hence (22) and (23) have a unique intersection, and there is a unique steady state equilibrium. Of course in order for the reserve requirement to bind in each country in the steady state, $R^i > \frac{1}{\sigma_i}$ must hold, $i = 1; 2$.

Proposition 1 (a) Equations (22) and (23) have a unique solution satisfying $R^i > \frac{1}{\sigma_i}$; $i = 1; 2$, if either of the following two conditions holds:

(i) $\sigma_1 f(\theta_1; \sigma_1) \geq \phi$ <u>and</u> $\sigma_2 g(\theta_2; \sigma_2) \geq \phi$:

(ii) $\frac{g(\theta_2;\sigma_2)(1-\theta_2)}{[A-\sigma_1 f(\theta_1;\sigma_1)]} > \frac{1}{\sigma_1} - \frac{\theta_2}{\sigma_2}$ <u>and</u> $\sigma_2 \geq \sigma_1$:

(b) The reserve requirement binds in country 1 ($R^1 > \frac{1}{\sigma_1}$) if the world economy is a classical case economy, in Gale's (1973) sense, if $\sigma_1 \geq 1$, and if $\frac{\sigma_1 - 1}{\sigma_1} \geq \theta_2 \left(\frac{\sigma_2 - 1}{\sigma_2}\right)$: The reserve requirement binds in country 2 if it binds in country 1 and $\sigma_2 \geq \sigma_1$:

The proof of proposition 1 appears in appendix A. The proposition asserts, among other things, that the reserve requirement binds in each country if the world economy is a classical case economy, according to Gale's (1973) definition, if rates of money creation are nonnegative, and if rates of money creation are not too dissimilar in the two countries.

2.1.1 Comparative Statics

In this section we briefly describe how the steady state equilibrium values of R^1 and R^2 depend on the monetary policy parameters σ_1, σ_2, θ_1, and θ_2 selected by the two countries. We begin with the rates of money creation σ_1 and σ_2.

It is straightforward to demonstrate that $\frac{\partial R^2}{\partial \sigma_1} < 0$ and $\frac{\partial R^1}{\partial \sigma_2} < 0$ both hold. Intuitively, increases in the rate of money creation in country 1 (2) reduce the steady state real return on reserves issued by that country. As a consequence, R^2 (R^1) tends to fall in order to maintain the "no-arbitrage" condition (22). At the same time, an increase in σ_1 (σ_2) leads to an expansion of credit as a result of the increased transfers made to young agents in country 1 (2). This also acts to exert downward pressure on the loan rate R^2 (R^1).

By the same reasoning, the expansion of credit associated with a higher value of σ_1 (σ_2) tends to put downward pressure on R^1 (R^2). However, this effect tends to be counteracted by the fact that a more rapid rate of money creation in country 1 (2) reduces the return on reserves in country 1 (2), and that that factor puts upward pressure on R^1 (R^2) in order to satisfy the "no-arbitrage" condition (22). Thus the partial derivatives $\frac{\partial R^i}{\partial \sigma_i}$ do not have an intuitively obvious sign. Nonetheless some results are available on the signs of these derivatives.

Proposition 2 (a) Suppose that $\frac{\theta_2}{\sigma_2} \geq \frac{\theta_1}{\sigma_1}$ and

$$\text{(i)} \quad \phi \geq f(\theta_1;\sigma_1)\left[\sigma_1 + \left(\frac{\rho}{1+\rho}\right)\right]$$

are satisfied. Then $\frac{\partial R^1}{\partial \sigma_1} > 0$.

(b) Condition (i) is satisfied if $\sigma_1 \geq 1$ and

$$\text{(ii)} \quad \phi \geq \sigma_1 f(0;\sigma_1)(1-\theta_1)^{-1}\left[1 + \left(\frac{\rho}{1+\rho}\right)(\sigma_1)^{-1}\right]$$

both hold.

(c) $\frac{\partial R^1}{\partial \sigma_1} < 0$ holds if $\frac{\theta_1}{\sigma_1} \geq \frac{\theta_2}{\sigma_2}$ and if

$$\text{(iii)} \quad f(0;\sigma_1) \geq \frac{(1+\beta)\phi}{\rho}.$$

(d) Suppose that $\frac{\theta_1}{\sigma_1} \geq \frac{\theta_2}{\sigma_2}$ and

$$\text{(iv)} \quad \phi \geq g(\theta_2;\sigma_2)\left[\sigma_2 + \left(\frac{\rho}{1+\rho}\right)\right]$$

are satisfied. Then $\frac{\partial R^2}{\partial \sigma_2} > 0$:

(e) Condition (iv) is satisfied if $\sigma_2 \geq 1$ and

$$\text{(v)}\quad \psi \geq \sigma_2 g(0;\sigma_2)(1-\theta_2)^{-1}\left[1+\left(\frac{\beta}{1+\beta}\right)(\sigma_2)^{-1}\right]$$

both hold.

(f) $\frac{\partial R^2}{\partial \sigma_2} < 0$ holds if $\frac{\theta_2}{\sigma_2} \geq \frac{\theta_1}{\sigma_1}$ and if

$$\text{(vi)}\quad g(0;\sigma_2) \geq \frac{(1+\beta)\psi}{\beta}:$$

The proof of proposition 2 is given in appendix B.

Parts (a) and (b) of the proposition indicate that an increase in country 1's rate of money creation must raise R^1 if $\sigma_1 \geq 1$, $\frac{\theta_2}{\sigma_2} \geq \frac{\theta_1}{\sigma_1}$, and condition (ii) holds. Condition (ii) requires that the demand for loans in country 1 [$f(0;\sigma_1)$] - at a zero net rate of interest - does not exceed a certain fraction of world savings (ψ). When this condition holds, country 2 absorbs enough credit so that an increase in σ_1 does not have a strong negative effect on R^2: As a result, R^1 must rise with an increase in σ_1 in order to maintain the international equality of rates of return. Part (c) of the proposition states the opposite conclusion: if $\frac{\theta_1}{\sigma_1} \geq \frac{\theta_2}{\sigma_2}$, and if country 1 loan demand is large relative to world savings, $\frac{\partial R^1}{\partial \sigma_1} < 0$ must obtain.

With respect to the reserve requirements θ_i, it is easy to verify that $\frac{\partial R^i}{\partial \theta_i} > 0$; $i = 1; 2$. Intuitively, an increase in the reserve requirement in country i forces investors in that country to hold more of their country i assets in the form of low-yielding currency. As compensation (or to maintain the international equality of rates of return), R^i must rise.

The signs of $\frac{\partial R^2}{\partial \theta_1}$ and $\frac{\partial R^1}{\partial \theta_2}$ are more complicated to deduce. We state some results in the following proposition.

Proposition 3 (a) $\frac{\partial R^1}{\partial \theta_2} \leq (>)\ 0$ holds iff

$$\text{(i)}\quad I^2\left(\frac{\sigma_2-1}{\sigma_2}\right)\left[\frac{g(0;\sigma_2)}{g(\theta_2;\sigma_2)}\right] \geq (<)\frac{1+\beta}{\beta};$$

where $I^2=\sigma_2R^2$ is the gross nominal rate of interest in country 2 (in steady state).

(b) A sufficient condition for $\frac{\partial R^1}{\partial \theta_2}>0$ is that

$$\text{(ii)}\quad I^2\left(\frac{\sigma_2-1}{\sigma_2}\right) \leq \frac{1+\beta}{\beta}:$$

(c) $\frac{\partial R^2}{\partial \theta_1} \leq (>)\ 0$ holds iff

$$\text{(iii)}\quad I^1\left(\frac{\sigma_1-1}{\sigma_1}\right)\left[\frac{f(0;\sigma_1)}{f(\theta_1;\sigma_1)}\right] \geq (<)\frac{1+\beta}{\beta};$$

where $I^1=\sigma_1R^1$ is the gross nominal rate of interest in country 1 (in steady state).

(d) A sufficient condition for $\frac{\partial R^2}{\partial \theta_1}>0$ is that

$$\text{(iv)}\quad I^1\left(\frac{\sigma_1-1}{\sigma_1}\right) \leq \frac{1+\beta}{\beta}:$$

Proposition 3 is proved in appendix C. Parts (a) and (b) of the proposition indicate that $\frac{\partial R^1}{\partial \theta_2}>0$ will obtain if σ_2 is not "too far" above one, and if I^2 is not too large. Likewise, parts (c) and (d) imply that $\frac{\partial R^2}{\partial \theta_1}>0$ will hold if σ_1 is not "too far" above one, and if I^1 is not too large.

2.2 Dynamics

The dynamical evolution of $\{R_t^1\}$ is described by equation (21). Figure 2 depicts the locus defined by (21): that locus is easily shown to be upward sloping, and to pass through the point $\left(\frac{f(\theta_1;\sigma_1)}{A}, \frac{f(\theta_1;\sigma_1)\theta_2}{A\theta_2+\sigma_2(1-\theta_2)g(\theta_2;\sigma_2)}\right)$: The latter point obviously lies below the 45° line in the figure. Finally, as $R_t^1 \uparrow \left[\frac{f(\theta_1;\sigma_1)}{A}+\frac{g(\theta_2;\sigma_2)(1-\theta_2)}{A(1-\theta_1)}+\frac{\theta_2}{\sigma_2(1-\theta_1)}\right]$, $R_{t+1}^1 \uparrow \infty$: Thus, at the unique steady state, the locus defined by (21) crosses the 45° line from below.

As is apparent from Figure 2, the steady state is unstable. In addition, it is straightforward to demonstrate that any candidate equilibrium paths with R_1^1 in excess of (below) its steady state value have $R_t^1 \to 1$ ($R_t^2 \to 1$): It is readily shown that along such paths either (a) the no-arbitrage condition is eventually violated, or (b) the reserve requirement eventually fails to bind in at least one of the countries. We therefore have the following proposition.

Proposition 4 When both countries fix their rate of money creation there is a unique equilibrium (the steady state) with the reserve requirement binding in each economy.

Thus, according to the criteria proposed by Keynes and Friedman, the policy where country 2 fixes its money growth rate has several desirable properties.

3 Equilibrium: Fixed Exchange Rates

In this section we describe the set of perfect foresight equilibria under a regime of fixed exchange rates. Thus Keynes' "stability of exchange" prevails. To fix ideas we assume that country 2 is responsible for maintaining the rate of exchange. Therefore the money supply of country 2 must be adjusted in each period as necessary to keep the exchange rate at its fixed level. Country 1, on the other hand, is free to determine the time path of its money supply. As in section 2, we assume that the money supply of country 1 grows at the exogenously given constant gross rate σ_1: In both countries changes in the money supply continue to be accomplished via lump-sum transfers to young lenders. This enables us to capture Keynes' notion that changes in the money supply and changes in the supply of credit are intimately linked.

3.1 Determination of Equilibrium

Let e be the fixed rate of exchange maintained by country 2. The law of one price implies that

$e = p_t^2 / p_t^1$. Hence since the exchange rate is held constant,

$$\frac{p_t^2}{p_{t+1}^2} = \frac{p_t^1}{p_{t+1}^1}; \quad t \geq 1 \tag{24}$$

must hold. Moreover (18) - and hence (19) - continue to hold in country 1. We therefore have

$$\frac{p_t^2}{p_{t+1}^2} = \frac{p_t^1}{p_{t+1}^1} = \frac{R_t^1}{\sigma_1 R_{t+1}^1}; \quad t \geq 1. \tag{25}$$

Substituting (25) into (6) yields the condition that the sequences $\left\{R_t^1\right\}$ and $\left\{R_t^2\right\}$ must satisfy in order to equate the returns to saving internationally:

$$(1 - \theta_2) R_t^2 = (1 - \theta_1) R_t^1 + (\theta_1 - \theta_2) \frac{R_t^1}{\sigma_1 R_{t+1}^1}; \quad t \geq 1. \tag{26}$$

It remains to describe the government budget constraints of the two economies. Since the conduct of policy in country 1 is unaltered, its government budget constraint continues to be given by (15). However, for country 2, the government budget constraint is now given by

$$(1 - \alpha_2)\tau_t^2 = \frac{M_t^2 - M_{t-1}^2}{p_t^2} = m_t^2 - m_{t-1}^2 \frac{p_{t-1}^2}{p_t^2} = m_t^2 - m_{t-1}^2 \frac{p_{t-1}^1}{p_t^1}; \quad t \geq 2; \tag{27}$$

where the last equality follows from (24).

Under the assumption that the reserve requirement binds in each country $\forall t \geq 1$, we may substitute (12) into (15) and (12) and (25) into (27). Employing the results in (11) yields the following condition under which sources and uses of funds are equated:

$$\begin{aligned}
&(1 + \beta)\phi R_t^1 - (1 + \beta) f(\theta_1; \sigma_1) = \\
&(1 + \beta) g(\theta_2; \sigma_1) \frac{R_t^1}{R_t^2} - N_2 \left(\frac{\theta_2}{1 - \theta_2}\right)\left(\frac{\beta}{1 + \beta}\right)\left(\frac{\alpha_2 y}{\sigma_1}\right) \frac{R_t^1}{R_t^2} + \\
&N_2 \left(\frac{\theta_2}{1 - \theta_2}\right)\left(\frac{\beta}{1 + \beta}\right)\left(\frac{\alpha_2 y}{\sigma_1}\right) \frac{R_{t-1}^1}{R_{t-1}^2}; \quad t \geq 2.
\end{aligned} \tag{28}$$

Lagging equation (26) one period and solving for R_t^1; we also have that

$$R_t^1 = \frac{\left(\frac{\theta_1 - \theta_2}{\sigma_1}\right)\left(\frac{R^1_{t-1}}{R^2_{t-1}}\right)}{1 - \theta_2 - (1-\theta_1)\frac{R^1_{t-1}}{R^2_{t-1}}}; \quad t \geq 2: \tag{29}$$

Substituting (29) into (28), defining $x_t = \frac{R_t^1}{R_t^2}$ to be the ratio of the gross loan rates in the two countries, and using the definition of $g(\theta_2; \sigma_1)$, we have that the equilibrium sequence $\{x_t\}$ evolves according to

$$\left[\frac{N_2\alpha_2 y}{1+\beta}\right]\left[1+\beta+\frac{\theta_2}{1-\theta_2}\right]x_t =$$
$$\frac{(1+\beta)\phi\left(\frac{\theta_1-\theta_2}{\sigma_1}\right)x_{t-1}}{1-\theta_2-(1-\theta_1)x_{t-1}} - \left(\frac{N_2\alpha_2 y}{1+\beta}\right)\left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\beta}{\sigma_1}\right)x_{t-1} - (1+\beta)f(\theta_1;\sigma_1); \quad t \geq 2: \tag{30}$$

Given a sequence $\{x_t\}$ that satisfies (30), the equilibrium sequence $\{R_t^1\}$ can be recovered from (29), and $\left\{\frac{p_t^1}{p_{t+1}^1}\right\}$ can then be deduced from (25). It is readily verified that x_1 is a free initial condition.[9]

In order to characterize the set of perfect foresight equilibria, we begin with a consideration of steady states. We then turn our attention to dynamical equilibria.

3.2 Steady State Equilibria

In a steady state, equation (28) reduces to

$$\frac{R^1}{R^2} \equiv x = \frac{\phi R^1 - f(\theta_1;\sigma_1)}{g(\theta_2;\sigma_1)}; \tag{31}$$

[9] Given $\{x_t\}_{t=1}^{\infty}$, (29) determines $\{R_t^1\}_{t=2}^{\infty}$, and therefore $\{R_t^2\}_{t=2}^{\infty}$. In addition, given x_1; equations (11) and (12) plus the t = 1 versions of the government budget constraints allow one to deduce R_1^1 (and hence, given x_1; R_2^1): Then, with R_1^1 determined, p_1^1 can be recovered from (12) and $m_1^1 = \frac{\sigma_1 M_0^1}{p_1^1}$: Since $p_1^2 = ep_1^1$ the initial price level in country 2 is also determined. Equation (12) and the definition $\frac{M_t^2}{p_t^2} \equiv m_t^2$ then allow us to recover the value of the time 1 money supply in country 2 required to maintain the fixed rate of exchange in that period. Notice that x_1 itself is not determined by any of these conditions, and hence is a free initial condition.

or, upon rearranging terms, to

$$\rho R^1 = g\left(\theta_2;\sigma_1\right)x + f\left(\theta_1;\sigma_1\right). \tag{32}$$

Similarly, equation (26) in steady state reduces to

$$R^1 = \frac{\frac{(\theta_1-\theta_2)}{\sigma_1}x}{1-\theta_2-(1-\theta_1)x}. \tag{33}$$

Equations (32) and (33) determine the steady state values R^1 and $R^2 = \frac{R^1}{x}$.

We now wish to depict the determination of equilibrium diagrammatically. Obviously (32) gives R^1 as a linear function of x. For equation (33) we have

$$\frac{\partial R^1}{\partial x}\Big|_{(33)} = \frac{(\theta_1-\theta_2)(1-\theta_2)}{\sigma_1\left[1-\theta_2-(1-\theta_1)x\right]^2}$$

and

$$\frac{\partial\left(R^1\right)^2}{\partial x^2}\Big|_{(33)} = \frac{2(1-\theta_1)}{\left[1-\theta_2-(1-\theta_1)x\right]}\frac{\partial R^1}{\partial x}.$$

Thus (33) defines an upward (downward) sloping, convex locus in Figure 3 if $\theta_1 > (<)\,\theta_2$. We now consider each case in turn.

Case 1: $\theta_1 < \theta_2$. This case is depicted in Figure 3.a. Here (33) describes a downward sloping locus, so obviously (32) and (33) have a unique solution. Equation (33) implies that this solution satisfies $R^1 > \frac{1}{\sigma_1}$ (the reserve requirement binds in country one) iff

$$\frac{\frac{(\theta_1-\theta_2)}{\sigma_1}x}{1-\theta_2-(1-\theta_1)x} > \frac{1}{\sigma_1} \tag{34}$$

holds. With $\theta_1 < \theta_2$, (34) is easily shown to reduce to $x < 1$. Moreover, since $x = \frac{R^1}{R^2}$, if $x < 1$ obtains, then $R^2 > R^1 > \frac{1}{\sigma_1}$ necessarily holds in the steady state. Hence the reserve requirement binds in both countries iff the solution to (32) and (33) satisfies $x < 1$. It is easy to verify that $x < 1$ will indeed obtain iff

$$\sigma_1 f\left(\theta_1;\sigma_1\right) + \sigma_1 g\left(\theta_2;\sigma_1\right) > \rho. \tag{35}$$

This condition is necessarily satisfied if $\sigma_1 \geq 1$; and if the world economy is a classical case economy in Gale's (1973) sense.

Case 2: $\theta_1 > \theta_2$. Under this configuration of parameters, which is depicted in Figure 3.b, equation (33) describes an increasing, convex locus. Hence once again, (32) and (33) necessarily have a unique intersection. An argument similar to the one just given establishes that - at this intersection - $R^1 > \frac{1}{\sigma_1}$ holds iff $x > 1$. The same condition is easily shown to imply $R^2 > \frac{1}{\sigma_1}$. Hence the reserve requirement binds in both countries (in steady state) iff $x > 1$: Again the solution to (32) and (33) can be shown to satisfy $x > 1$ iff (35) holds. Therefore, for the remainder of this section, we assume that parameter values obey (35).

3.2.1 Comparative Statics

As before, we investigate how the steady state equilibrium values R^1 and R^2 depend on the choices of policy parameters. The issues regarding the partial derivatives $\frac{\partial R^i}{\partial \theta_j}$ are essentially the same as in section 2 (except that with a fixed rate of exchange $\sigma_2 = \sigma_1$ holds in a steady state). Thus we focus here only on how R^1 and R^2 depend on the common rate of money creation, σ_1:

The nature of the relationship between σ_1 and the real rate of interest in each country does depend to some extent on which case obtains. However, it is possible to state the following result.

Proposition 5 (a) In case 1 ($\theta_2 \geq \theta_1$), $\frac{\partial R^1}{\partial \sigma_1} < 0$ holds. Moreover, $\frac{\partial R^2}{\partial \sigma_1} < 0$ holds if $\theta_2 - \theta_1$ is sufficiently small. (b) In case 2 ($\theta_1 > \theta_2$), $\frac{\partial R^2}{\partial \sigma_1} < 0$ necessarily holds. In addition, $\frac{\partial R^1}{\partial \sigma_1} < 0$ is satisfied if $\theta_1 - \theta_2$ is sufficiently small.

Proposition 5 is proved in appendix D. The proposition indicates that an increase in the (common) rate of money growth will lower the real rate of return in each country, if the levels of the reserve requirement across countries are not "too different." However, it is possible that if the reserve requirements differ too dramatically, an increase in σ_1 will raise the real rate of interest in one country, while lowering it in the other.

3.3 Dynamical Equilibria

As is true for an analysis of steady state equilibria, equilibrium dynamics vary considerably under fixed exchange rates, depending on whether $\theta_1 > (<)\, \theta_2$ holds. Again we consider each case in turn.

Case 1: $\theta_1 < \theta_2$. Differentiating equation (30), one obtains that

$$\left[\frac{N_2\alpha_2 y}{1+\beta}\right]\left[1+\beta+\frac{\theta_2}{1-\theta_2}\right]\frac{\partial x_t}{\partial x_{t-1}} = \frac{(1+\beta)\,\phi\,(\theta_1-\theta_2)\,(1-\theta_2)}{\sigma_1\left[1-\theta_2-(1-\theta_1)\,x_{t-1}\right]^2} - \left(\frac{N_2\alpha_2 y}{1+\beta}\right)\left(\frac{\theta_2}{1-\theta_2}\right)\frac{\beta}{\sigma_1}. \qquad (36)$$

Hence, when $\theta_1 < \theta_2$, (30) gives x_t as a decreasing function of x_{t-1}. The relevant equilibrium law of motion is depicted in Figure 4.a.

The unique steady state equilibrium may be either asymptotically stable, or unstable. If it is asymptotically stable, then there exist perfect foresight equilibrium paths that display damped oscillation as they approach the steady state.[10] Hence there is a continuum of equilibria, all but one of which (the steady state) display endogenously arising volatility. On the other hand, if the steady state is unstable, there may be a unique equilibrium (the steady state). The following proposition states conditions under which the steady state is asymptotically stable (unstable).

Proposition 6 (a) The steady state is asymptotically stable (unstable) if

$$\left[\frac{N_2\alpha_2 y}{1+\beta}\right]\left[1+\beta+\left(\frac{\theta_2}{1-\theta_2}\right)\left(1-\frac{\beta}{\sigma_1}\right)\right] > (<)\, \frac{(1+\beta)\,\phi\,(\theta_2-\theta_1)\,(1-\theta_2)}{\sigma_1\left[1-\theta_2-(1-\theta_1)\,x\right]^2}. \qquad (37)$$

(b) Since $x \in \left(\frac{1-\theta_2}{1-\theta_1}, 1\right)$ must hold in order for $R^1 > \frac{1}{\sigma_1}$ to be satisfied, a necessary condition for the steady state to be asymptotically stable is that

$$\left[\frac{N_2\alpha_2 y}{1+\beta}\right]\left[1+\beta+\left(\frac{\theta_2}{1-\theta_2}\right)\left(1-\frac{\beta}{\sigma_1}\right)\right] \geq \frac{(1+\beta)\,\phi\,(1-\theta_2)}{\sigma_1\,(\theta_2-\theta_1)}. \qquad (38)$$

[10] An example of such a path is depicted in Figure 4.a.

(c) Fix θ_1, θ_2, N_2, α_2, y, σ_1, β, and ϕ. Suppose that (38) and $\phi > \sigma_1 g(\theta_2, \sigma_1)$ hold. Then there exist values N_1, α_1 and w such that the steady state is asymptotically stable. Moreover, there necessarily exist values N_1, α_1 and w such that there exist equilibria displaying two period cycles ($x_t = x_e$, t even, $x_t = x_o$, t odd, $x_e \neq x_o$).

The proof of proposition 6 appears in appendix E.

Proposition 6 states that there are conditions under which a policy of maintaining a fixed exchange rate will lead to an indeterminacy of equilibrium, and to the possibility of endogenously generated volatility. Indeed, there are conditions under which there will be equilibria displaying volatility which does not dampen over time. In fact, it is straightforward to show that whenever the steady state equilibrium under fixed exchange rates is asymptotically stable, there necessarily exists a two-period cycle.

As this observation indicates, it is not difficult to produce an example of an economy that displays undamped oscillation. For instance, here is one such example.

Example 1 Let $N_1 = 4.96$, $\alpha_1 = 0.6$, $N_2 = 2$, $\alpha_2 = 0.5$, $\beta = 1$, $w = 13.08$ and $y = 3$. Moreover, let it be the case that $\sigma_1 = 1.5$, $\theta_1 = 0.1$, and $\theta_2 = 0.9$. For these parameter values, $\phi = 19.5$, $f(\theta_1, \sigma_1) = 4.87$ and $g(\theta_2, \sigma_1) = 12.75$. The steady state values of the two nominal interest rates for this economy are $I^1 = 1.0577$ and $I^2 = 1.5191$. However, there is also an equilibrium where $I_t^1 = 1.0574$ and $I_t^2 = 1.5165$, t odd, and $I_t^1 = 1.0579$, $I_t^2 = 1.5217$, t even. This equilibrium is attained by setting $x_1 = 0.6972$.

Economically speaking, the reason that there can be perfect foresight equilibria displaying endogenous volatility is as follows. If loan rates are expected to be relatively low in country 2 at date t then that country's loan demand will be relatively high. As a consequence, lenders must acquire additional reserves in country 2 in order to expand their lending there. Therefore, the demand for the currency of country 2 must rise. In order to maintain the fixed rate of exchange country 2 must accommodate this demand by increasing its money stock. In doing so - as argued by Keynes - they also expand the supply of credit, thereby validating the expectation of low real loan rates. But, if the real rate of interest is low in country 2 at t relative to that in country 1, the condition $\mu_2 > \mu_1$ implies that country 1's loan rate must be expected to fall between t and t + 1 in order to equate the expected return on loans between the two countries [see equation (26)]. As a result, country 1's loan rate will be low relative to that in country 2 at t + 1, resulting in a cycle of oscillating interest rates and price levels in each country. The oscillation that emerges may or may not dampen, and it is clearly the result of a self-fulfilling prophecy.

Summary. When $\mu_2 > \mu_1$ holds - or, in other words - when the country responsible for maintaining the fixed rate of exchange also has a relatively high reserve requirement, it is quite easy to generate an indeterminacy of equilibrium. Moreover, if there is more than one equilibrium path, all but one of the equilibrium paths will display oscillation in both interest rates and price levels. Finally, for some configuration of parameters, there will be equilibria displaying undamped oscillation.

Case 2: $\mu_1 > \mu_2$. An examination of equation (30) will quickly indicate that there exists a value $\bar{x} \in \left[0; \frac{1-\mu_2}{1-\mu_1}\right]$ such that if $x_{t-1} = \bar{x}$, then $x_t = 0$: Moreover, $x_t \geq 0$ holds iff $x_{t-1} \in \left[\bar{x}; \frac{1-\mu_2}{1-\mu_1}\right]$: For $x_{t-1} \in \left[\bar{x}; \frac{1-\mu_2}{1-\mu_1}\right]$, it is straightforward to verify that equation (30) defines x_t as an increasing, convex function of x_{t-1}. Finally, since

$$\lim_{x_{t-1} \to \frac{1-\mu_2}{1-\mu_1}} x_t = \infty;$$

equation (30) defines a locus in Figure 4.b with the configuration depicted there.

Evidently, the unique steady state must be unstable. Moreover, there can be no dynamical equilibrium paths with x_1 below its steady state level, since all such paths involve interest rates falling in a way which eventually leads the reserve requirement to be non-binding. In addition, there can be no dynamical equilibria with x_1 in excess of its steady state level, since if there was such a path, there would exist a date T such that $x_t > \frac{1-\mu_2}{1-\mu_1}$ would hold for $t \geq T$. But this is not consistent with positive interest rates, as will be clear from equation (29). Thus, when $\mu_1 > \mu_2$, there is a unique equilibrium - the steady state - under a regime of fixed exchange rates.

A comparison of cases 1 and 2 implies an immediate result: under a regime of fixed exchange rates - maintained by country 2 - there is the potential for indeterminacy and endogenous volatility when $\mu_2 > \mu_1$ holds. However, there is necessarily a unique equilibrium when $\mu_1 > \mu_2$ obtains. Thus there are strong reasons for the country with the lower level of reserve requirements to assume the responsibility for the maintenance of the fixed rate of exchange.

What is the economics underlying this result? When $\mu_2 > \mu_1$ holds, it is possible to construct equilibria in which R_t^2 is expected to be relatively low (high) at t because this leads to a high (low) demand for the reserves of country 2. As a result, in order to maintain the fixed exchange rate, country 2 must accommodate this by increasing (reducing) the supply of reserves. Following Keynes, this is done in a way that increases (reduces) the supply of credit, thereby validating the expectation of low (high) real loan rates in country 2.

When $\mu_2 > \mu_1$ holds, equation (26) implies that a temporarily low (high) rate of interest in country 2 relative to the steady state must be accompanied by falling (rising) rates of interest in country 1. Thus the relative position of the two countries tends to be reversed. However, when $\mu_1 > \mu_2$ holds, equation (26) implies that low (high) values of R_t^2 must be accompanied by rising (falling) values of R_t^1. Thus any current differences tend to be magnified over time, in a way that our analysis indicates is unsustainable. Consequently, when $\mu_1 > \mu_2$ holds, it is impossible to construct equilibria in which interest rates vary as a result of a self-fulfilling prophecy.

4 Equilibrium: Price Level Stabilization

We now provide a partial characterization of the set of perfect foresight equilibria under the assumption that country 2 adheres to a particular target time path for its price level. To be more specific, we assume that country 2 has a target value for its initial price level, p_1^2, and that thereafter, it commits to maintain a constant rate of inflation, so that $\frac{p_t^2}{p_{t+1}^2} = \rho$, $\forall\, t \geq 1$. Thus country 2 stabilizes its inflation rate, and to do so it must adjust the money supply each period as required to maintain the target inflation rate. As before, country 1 simply allows its money supply to grow at the constant gross rate σ_1. Obviously this formulation allows for the possibility that $\rho = 1$, so that country 2 commits to keeping its domestic price level stable. This was the policy advocated by Keynes.

4.1 Determination of Equilibrium

When country 2 follows the policy of setting $\frac{p_t^2}{p_{t+1}^2}$ equal to some target level ρ for all $t \geq 1$, equation (6) becomes

$$(1-\theta_2)R_t^2 + \theta_2\rho = (1-\theta_1)R_t^1 + \theta_1\frac{p_t^1}{p_{t+1}^1} = (1-\theta_1)R_t^1 + \theta_1\left(\frac{R_t^1}{\sigma_1 R_{t+1}^1}\right); \quad t \geq 1; \tag{39}$$

where the second equality follows from (19). In addition, the government budget constraint of country 2 now becomes

$$(1-\alpha_2)\tau_t^2 = m_t^2 - m_{t-1}^2\frac{p_{t-1}^2}{p_t^2} = m_t^2 - \rho m_{t-1}^2; \quad t \geq 2; \tag{40}$$

where, as before, the government accomplishes any required changes in the money supply by making lump-sum transfers to young lenders. Since country 1 follows the same policy as in Section 2, its government budget constraint continues to be given by equation (15). Substituting (15) and (40) into (11), we obtain our second equilibrium condition

$$\begin{aligned}&\left(\frac{\beta}{1+\beta}\right) w\left[N_1(1-\alpha_1)+N_2(1-\alpha_2)\right] = \\ &N_1\alpha_1\frac{y}{(1+\beta)R_t^1}+N_2\alpha_2\frac{y}{(1+\beta)R_t^2}+ \\ &N_1 m_t^1\left[1-\left(\frac{\beta}{1+\beta}\right)\left(\frac{\sigma_1-1}{\sigma_1}\right)\right]+\frac{N_2 m_t^2}{(1+\beta)}+N_2 m_{t-1}^2\rho\left(\frac{\beta}{1+\beta}\right);\quad t\ge 2.\end{aligned}\tag{41}$$

Finally, when the reserve requirement binds, the equilibrium values of real balances in each country continue to be given by equation (12). Upon substituting (12) into (41), and using our previous notation, we obtain

$$\begin{aligned}(1+\beta)\varphi &= (1+\beta)\frac{f(\theta_1;\sigma_1)}{R_t^1}+(1+\beta)\frac{g(\theta_2;\rho^{-1})}{R_t^2}- \\ &N_2\left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\beta}{1+\beta}\right)\rho\left(\frac{\alpha_2 y}{R_t^2}\right)+ \\ &N_2\left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\beta}{1+\beta}\right)\rho\left(\frac{\alpha_2 y}{R_{t-1}^2}\right);\quad t\ge 2.\end{aligned}\tag{42}$$

Equations (39) and (42) describe the evolution of the equilibrium sequences $\left\{R_t^1\right\}$ and $\left\{R_t^2\right\}$.

4.2 Steady State Equilibria

In a steady state, clearly (39) reduces to

$$(1-\theta_1)R^1+\frac{\theta_1}{\sigma_1}=(1-\theta_2)R^2+\theta_2\rho;\tag{43}$$

while (42) becomes

$$R^2=R^1\left[\frac{g(\theta_2;\rho^{-1})}{\varphi R^1-f(\theta_1;\sigma_1)}\right].\tag{44}$$

Evidently, (43) and (44) are identical to (22) and (23), except that σ_2 is replaced by $\frac{1}{\rho}$. In country 2 - under a regime of price level targeting - the steady state rate of money creation must equal the target inflation rate. As a consequence, all of the results stated in section 2 about the existence and the uniqueness of a steady state equilibrium with the reserve requirement binding in each country apply here as well, when σ_2 is replaced with ρ^{-1}.

4.3 Dynamical Equilibria

In order to analyze dynamical equilibria, it will be useful to transform equations (39) and (42) as follows. Define the variables q_t and z_t by $q_t = \frac{1}{R_t^1}$, and $z_t = \frac{1}{R_t^2}$. Then equation (39) can be rewritten in the form

$$q_t = \left[\frac{\sigma_1(1-\theta_2)}{\theta_1}\right]\left(\frac{q_{t-1}}{z_{t-1}}\right) + \frac{\sigma_1\theta_2\rho}{\theta_1} q_{t-1} - \frac{(1-\theta_1)\sigma_1}{\theta_1}; \quad t \geq 2; \tag{45}$$

while (42) becomes

$$\begin{aligned} &\left[(1+\beta)\, g(\theta_2; \rho^{-1}) - N_2\left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\beta}{1+\beta}\right)\rho\alpha_2 y\right] z_t = \\ &\left[(1+\beta)\phi - (1+\beta)\, f(\theta_1; \sigma_1) q_t - N_2\left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\beta}{1+\beta}\right)\rho\alpha_2 y\right] z_{t-1}; \quad t \geq 2. \end{aligned} \tag{46}$$

We henceforth work with the dynamical system consisting of equations (45) and (46).

For the remainder of this section we assume that a unique steady state (q, z) exists, and that it has the feature that the reserve requirement binds in each country. We then proceed to characterize local dynamics in a neighborhood of this steady state. In particular, we approximate the dynamical system (45) and (46) by

$$(q_t - q,\ z_t - z)' = J(q_{t-1} - q,\ z_{t-1} - z)'; \tag{47}$$

where J is the Jacobian matrix

$$J = \begin{bmatrix} \frac{\partial q_t}{\partial q_{t-1}} & \frac{\partial q_t}{\partial z_{t-1}} \\ \frac{\partial z_t}{\partial q_{t-1}} & \frac{\partial z_t}{\partial z_{t-1}} \end{bmatrix}$$

with all the partial derivatives evaluated at the steady state. We then analyze the dynamical system embodied in equation (47).

Let λ_1 (λ_2) denote the smallest (largest) eigenvalue of J. We are now prepared to state the following result.

Proposition 7 Suppose that there exists a steady state with the reserve requirement binding in each country. Suppose further that $\theta_1 \leq 0.5$ and that

$$1+\beta \geq \left(\frac{\theta_2}{1-\theta_2}\right)(\beta\rho - 1); \tag{48}$$

are both satisfied. Then $-1 < \lambda_1 < 0 < 1 < \lambda_2$ holds. In particular, the steady state is a saddle. Dynamical equilibrium paths approaching it display damped oscillation as they do so.

The proof of proposition 7 appears in appendix F.

Proposition 7 has two strong implications. First, it is easy to show that one has a free choice of initial conditions here. Thus when the steady state is a saddle - which happens under weak conditions - there is a one-dimensional indeterminacy of equilibrium if country 2 targets its price-level path. Second - again when the steady state is a saddle - all possible equilibria (except the steady state itself) exhibit endogenously arising volatility. In short, when country 2 follows the policy of targeting (or stabilizing) its price level path, this creates substantial scope both for the indeterminacy of equilibrium, and for "excessive" economic fluctuations. These will be observed in both countries.

Intuitively, the reason that economic fluctuations can arise in this context is as follows. Suppose that R_t^1 and R_t^2 are both expected to be "low" (relative to the steady state). Then loan demand at t will be correspondingly high, as - in consequence - will be the demand for reserves in country 2. In order to maintain its target path for the price level, country 2 will therefore have to expand its supply of reserves. In doing so, it contributes to the savings of young lenders, and hence to the supply of credit (à la Keynes). The result is the creation of downward pressure on interest rates (as expected). In addition, the no-arbitrage condition (39), along with the relationship between transfers and interest rates, then requires that - under weak conditions - R^1_{t+1} (and R^2_{t+1}) be "high" relative to the steady state. This expectation is again validated by transfers, and so on.

It bears emphasis that the conditions of proposition 7 are quite weak. The requirement that $\theta_1 \le 0.5$ implies only that the reserve ratio (requirement) in country 1 not exceed 50 percent. Similarly (48) will hold whenever $\rho \le (\beta\theta_2)^{-1} + \left(\frac{1-\theta_2}{\theta_2}\right)$ is satisfied. If $\theta_2 \le 0.5$ also holds, then, (48) will be satisfied whenever $\rho \le 1 + 2\beta^{-1}$. Thus (48) will typically obtain so long as country 2 does not attempt to create quite a substantial deflation. It seems, therefore, that we should generally expect a policy of price level targeting (stabilization) to generate both indeterminacies and endogenous volatility in both countries.

5 The Current Account

The kind of policy adopted by country 2 - and the sort of equilibrium that potentially obtains under it - has strong implications for the current account surplus or deficit of both countries. We now briefly explore how the current account of country 2 depends on its policy choices and on the equilibrium time path of interest rates.

The current account balance of country 2 at t is, of course, simply its income (endowment) minus its consumption in that period. Defining ψ_t to be the current account balance of country 2 at t, it follows that ψ_t is given by

$$\begin{aligned}\psi_t = {} & N_2(1-\alpha_2)w\left(\frac{\beta}{1+\beta}\right)\left(1-R^2_{t-1}\right) + \left(\frac{N_2\alpha_2 y}{1+\beta}\right)\left(\frac{R^2_t-1}{R^2_t}\right) \\ & - \left[\frac{N_2(1-\alpha_2)y}{1+\beta}\right]\tau^2_t - \left[\frac{N_2(1-\alpha_2)\beta}{1+\beta}\right]R^2_{t-1}\tau^2_{t-1} \end{aligned} \quad (49)$$

We now proceed to describe ψ_t under each policy regime.

5.1 A Constant Money Growth Rate

When country 2 follows a policy of fixing its rate of money creation, τ_t^2 is given by equation (15). Substituting (15) for $i = 2$ into (49) yields (upon some rearrangement)

$$\begin{aligned} \psi_t \;=\; & N_2(1-\alpha_2)w\left(\frac{\beta}{1+\beta}\right)\left(1-R^2_{t-1}\right) + \left(\frac{N_2\alpha_2 y}{1+\beta}\right)\left[1-\left(\frac{\beta}{1+\beta}\right)\left(\frac{\sigma_2-1}{\sigma_2}\right)\left(\frac{\theta_2}{1-\theta_2}\right)\right] \\ & - \left[\frac{N_2\alpha_2 y}{(1+\beta)R^2_t}\right]\left[1+\left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\sigma_2-1}{\sigma_2}\right)(1+\beta)^{-1}\right] \end{aligned} \tag{50}$$

Since R^2_t is constant (at its steady state value), (50) reduces to

$$\begin{aligned} \psi \;=\; & \left(\frac{N_2\alpha_2 y}{1+\beta}\right)\left[1-\left(\frac{\beta}{1+\beta}\right)\left(\frac{\sigma_2-1}{\sigma_2}\right)\left(\frac{\theta_2}{1-\theta_2}\right)\right] + N_2(1-\alpha_2)w\left(\frac{\beta}{1+\beta}\right)\left(1-R^2\right) \\ & - \left[\frac{N_2\alpha_2 y}{(1+\beta)R^2}\right]\left[1+\left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\sigma_2-1}{\sigma_2}\right)(1+\beta)^{-1}\right]. \end{aligned} \tag{51}$$

Then, as in Kareken and Wallace (1978), country 2 can have either a permanent current account deficit or surplus, depending on the equilibrium value of R^2 and parameters.

It is straightforward to show that any policy actions undertaken in country 1, and which have the effect of raising R^2, reduce country 2's current account balance iff

$$\beta(1-\alpha_2)w > \alpha_2 y\left[1+\left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\sigma_2-1}{\sigma_2}\right)(1+\beta)^{-1}\right]\left(R^2\right)^{-2} \tag{52}$$

is satisfied. This condition, in turn, necessarily holds if $R^2 \geq 1$, and if country 1 is a net borrower, in equilibrium.

5.2 A Fixed Exchange Rate

Under a regime of fixed exchange rates, τ_t^2 is given by (27). Substituting (27) into (49) and rearranging terms, we obtain that

$$\begin{aligned} \psi_t \;=\; & N_2(1-\alpha_2)w\left(\frac{\beta}{1+\beta}\right) + \left(\frac{N_2\alpha_2 y}{1+\beta}\right) \\ & - \left[\frac{N_2\alpha_2 y}{(1+\beta)^2}\right]\left(\frac{\theta_2}{1-\theta_2}\right)\left[\left(R^2_t\right)^{-1} - \left(\frac{R^1_{t-1}}{\sigma_1 R^1_t R^2_{t-1}}\right)\right] \end{aligned}$$

$$
-N_2(1-\alpha_2)R^2_{t-1}\left(\frac{\beta}{1+\beta}\right)\Bigg\{ w + (1-\alpha_2)^{-1}\left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\alpha_2 y}{1+\beta}\right)\left[\left(R^2_{t-1}\right)^{-1} - \left(\frac{R^1_{t-2}}{\sigma_1 R^1_{t-1}R^2_{t-2}}\right)\right]\Bigg\}
$$
$$
-\left[\frac{N_2\alpha_2 y}{(1+\beta)R^2_t}\right]. \qquad (53)
$$

As is apparent from this expression, the time path of the current account can display quite complicated behavior in any equilibrium that exhibits oscillation.

5.3 A Target Price Level Path

If country 2 follows a policy of targeting its price level path, then τ^2_t is given by (40). Equations (40) and (49) imply that country 2's current account satisfies

$$
\begin{aligned}
\psi_t \;=\;& N_2(1-\alpha_2)w\left(\frac{\beta}{1+\beta}\right) + \left(\frac{N_2\alpha_2 y}{1+\beta}\right)\left(\frac{R^2_t-1}{R^2_t}\right) \\
& - \left[\frac{N_2\alpha_2 y}{(1+\beta)^2}\right]\left(\frac{\theta_2}{1-\theta_2}\right)\left[\left(R^2_t\right)^{-1} - \rho\left(R^2_{t-1}\right)^{-1}\right] \\
& - N_2(1-\alpha_2)\left(\frac{\beta}{1+\beta}\right) w R^2_{t-1} \\
& - N_2\alpha_2 y\left[\frac{\beta}{(1+\beta)^2}\right]\left(\frac{\theta_2}{1-\theta_2}\right)\left[1-\rho\left(\frac{R^2_{t-1}}{R^2_{t-2}}\right)\right]. \qquad (54)
\end{aligned}
$$

Again it is evident that, in any equilibrium displaying oscillation, the current account of each country can exhibit quite complicated behavior.

6 Conclusions

What kinds of policies afford the best prospects for "stability?". Authors like Friedman (1956) and Lucas (1972) have proposed constant money growth rates. And indeed, in the model we have described, constant rates of money growth allow no scope either for endogenously arising volatility, or for the indeterminacy of equilibrium. Many have also argued for fixed exchange rates. In the framework we have considered, a regime of fixed exchange rates allows no scope for indeterminacy or excess volatility if the country with low reserve requirements is responsible for maintaining the fixed rate of exchange. If this is not the way that responsibility for maintaining the rate of exchange is allocated, however, a regime of fixed exchange rates can leave considerable scope for indeterminacy and endogenous volatility.

Finally, a policy of price level targeting, which was advocated by Keynes and has recently been resurrected as a proposal, will typically allow wide latitude for both a severe multiplicity of equilibria, and for volatility that arises due to self-fulfilling prophecies. Thus, in our view, according to Keynes' own criteria, the policy he proposed actually performs the worst.

There are, of course, several possible extensions of the analysis that might alter this conclusion. One is the consideration of alternative methods for attaining a particular target path of the country 2 price level or rate of exchange. For instance, if country 2 introduces a currency board in our model, it is possible to demonstrate that the set of equilibria under a fixed exchange rate regime is qualitatively very similar to that which emerges with a constant rate of money creation (in both countries). However, this method of maintaining a fixed rate of exchange is not particularly faithful to Keynes' notion that monetary policy works - in part - by affecting the availability of credit. Nevertheless, further examination of different mechanisms for producing price level or exchange rate stability that are more faithful to Keynes' notion might yield some different results. A second extension that could be undertaken is a consideration of production economies. A study of production economies would permit us to analyze how real activity, as well as inflation rates and rates of interest, can behave under different choices of a policy regime.

In addition, we have focused on policy choices by "country 2;" always assuming that "country 1" maintains a constant rate of monetary expansion. How might the analysis be modified if country 1 is following a regime of price level targeting? This issue is easily amenable to analysis.

Finally, the model of "banking" we have utilized is clearly not very deep. One could reconsider the questions we have posed in a model where the banking system plays a far more significant allocative function. This would be an interesting topic for future investigation.

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APPENDIX

A. Proof of Proposition 1.

(a) That (22) and (23) have a unique solution is immediate from Figure 1. Moreover, as is clear from the figure, $R^1 > \frac{f(\theta_1,\sigma_1)}{A}$ and $R^2 > \frac{g(\theta_2,\sigma_2)}{A}$ must hold. Thus (i) guarantees that $R^i > \frac{1}{\sigma_i}$, $i = 1, 2$.

To establish that (ii) implies $R^1 > \frac{1}{\sigma_1}$, solve (22) for R^2 and substitute the result into (23) to obtain

$$(1-\theta_1)R^1 + \frac{\theta_1}{\sigma_1} - \frac{\theta_2}{\sigma_2} = \frac{R^1 g(\theta_2,\sigma_2)(1-\theta_2)}{\rho R^1 - f(\theta_1,\sigma_1)}. \tag{55}$$

Solving (55) yields the equilibrium value of R^1. Moreover, the left (right) - hand side of (55) is increasing (decreasing) in R^1. Thus $R^1 > \frac{1}{\sigma_1}$ holds if

$$\frac{g(\theta_2,\sigma_2)(1-\theta_2)}{\rho - \sigma_1 f(\theta_1,\sigma_1)} > \frac{1}{\sigma_1} - \frac{\theta_2}{\sigma_2} \tag{56}$$

is satisfied. But this is condition (ii). To show that $R^2 > \frac{1}{\sigma_2}$, note that $R^2 > \frac{1}{\sigma_2}$ holds iff $(1-\theta_1)R^1 + \frac{\theta_1}{\sigma_1} > \frac{1}{\sigma_2}$. But this is implied by $R^1 > \frac{1}{\sigma_1}$ and $\sigma_2 \geq \sigma_1$.

(b) Equation (55) implies that $R^1 > \frac{1}{\sigma_1}$ holds iff

$$\frac{g(\theta_2,\sigma_2)(1-\theta_2)}{\rho - \sigma_1 f(\theta_1,\sigma_1)} > \frac{(1-\theta_1)}{\sigma_1} + \frac{\theta_1}{\sigma_1} - \frac{\theta_2}{\sigma_2} = \frac{1}{\sigma_1} - \frac{\theta_2}{\sigma_2}. \tag{57}$$

Now the world is a classical case economy, in Gale's (1973) sense, iff

$$\frac{g(0,\sigma_2)(1-\theta_2)}{\rho - f(0,\sigma_1)} > 1 - \theta_2. \tag{58}$$

Moreover, it is easy to show that $\sigma_1 \geq 1$ implies that

$$\frac{g(\theta_2,\sigma_2)(1-\theta_2)}{\rho - \sigma_1 f(\theta_1,\sigma_1)} > \frac{g(0,\sigma_2)(1-\theta_2)}{\rho - f(0,\sigma_1)} \tag{59}$$

and $(1-\theta_2) \geq \frac{1}{\sigma_1} - \frac{\theta_2}{\sigma_2}$ holds iff $\frac{\sigma_1 - 1}{\sigma_1} \geq \theta_2 \left(\frac{\sigma_2 - 1}{\sigma_2}\right)$. Thus (58) and the latter condition imply the satisfaction of (57), and therefore imply that $R^1 > \frac{1}{\sigma_1}$. To show that $R^2 > \frac{1}{\sigma_2}$ holds, note that $R^2 > \frac{1}{\sigma_2}$ holds iff $(1-\theta_1)R^1 + \frac{\theta_1}{\sigma_1} > \frac{1}{\sigma_2}$. But this condition is implied by $R^1 > \frac{1}{\sigma_1}$ and $\sigma_2 \geq \sigma_1$.

B. Proof of Proposition 2.

(a) Solving equation (23) and substituting the result into (22) yields

$$R^1 \left\{ \left[\frac{g(\theta_2, \sigma_2)}{\rho R^1 - f(\theta_1, \sigma_1)} \right] - \left(\frac{1-\theta_1}{1-\theta_2} \right) \right\} = \frac{\theta_1}{\sigma_1(1-\theta_2)} - \frac{\theta_2}{\sigma_2(1-\theta_2)}. \tag{60}$$

Differentiating (60) with respect to σ_1 yields

$$\frac{\partial R^1}{\partial \sigma_1} \left(\left(\frac{R^2}{R^1} \right) \left[\frac{f(\theta_1, \sigma_1)}{\rho R^1 - f(\theta_1, \sigma_1)} \right] + \left(\frac{1-\theta_1}{1-\theta_2} \right) \right) = \frac{\theta_1}{(\sigma_1)^2 (1-\theta_2)} + \left[\frac{R^2 f_2(\theta_1, \sigma_1)}{\rho R^1 - f(\theta_1, \sigma_1)} \right]. \tag{61}$$

It is also the case that

$$\begin{aligned} f_2(\theta_1, \sigma_1) &= -\left(\frac{\theta_1}{1-\theta_1} \right) \left(\frac{\beta}{1+\beta} \right) \left(\frac{1}{\sigma_1} \right)^2 \left[\frac{N_1 \alpha_1 y}{1+\beta} \right] \\ &= -\left(\frac{\theta_1}{1-\theta_1} \right) \left(\frac{\beta}{1+\beta} \right) \left(\frac{1}{\sigma_1} \right)^2 f(0, \sigma_1) < 0 \end{aligned} \tag{62}$$

holds. Defining $x \equiv \frac{R^1}{R^2}$, it is then easy to show that $\frac{\partial R^1}{\partial \sigma_1}$ is opposite in sign to the term

$$R^1 \left(\frac{1}{x} \right)^2 \left[\frac{N_1 \alpha_1 y}{(1+\beta)\, g(\theta_2, \sigma_2)} \right] \left(\frac{\beta}{1+\beta} \right) - \left(\frac{1-\theta_1}{1-\theta_2} \right).$$

Thus, $\frac{\partial R^1}{\partial \sigma_1} \geq 0$ holds iff

$$\left(\frac{1-\theta_1}{1-\theta_2} \right) x^2 \geq \left(\frac{\beta}{1+\beta} \right) \left[\frac{N_1 \alpha_1 y R^1}{(1+\beta)\, g(\theta_2, \sigma_2)} \right] \equiv R^1 \left(\frac{\beta}{1+\beta} \right) \left[\frac{f(0, \sigma_1)}{g(\theta_2, \sigma_2)} \right]. \tag{63}$$

Now equation (22) can be rewritten in the form

$$1 = \left(\frac{1-\theta_1}{1-\theta_2} \right) x + \left[\left(\frac{\theta_1}{\sigma_1} \right) - \left(\frac{\theta_2}{\sigma_2} \right) \right] \frac{x}{(1-\theta_2) R^1}.$$

Since $\frac{\theta_2}{\sigma_2} \geq \frac{\theta_1}{\sigma_1}$ holds, it follows that $\left(\frac{1-\theta_1}{1-\theta_2} \right) x \geq 1$. Consequently, $\frac{\partial R^1}{\partial \sigma_1} \geq 0$ holds if $\frac{\theta_2}{\sigma_2} \geq \frac{\theta_1}{\sigma_1}$ and if

$$1 \geq R^2 \left(\frac{\beta}{1+\beta} \right) \left[\frac{N_1 \alpha_1 y}{(1+\beta)\, g(\theta_2, \sigma_2)} \right]. \tag{64}$$

It is also the case that (23) and $R^1 > \frac{1}{\sigma_1}$ imply that

$$R^2 = \frac{g(\theta_2;\sigma_2)\,R^1}{\phi R^1 - f(\theta_1;\sigma_1)} < \frac{g(\theta_2;\sigma_2)}{\phi - \sigma_1 f(\theta_1;\sigma_1)}. \tag{65}$$

But (64) and (65) imply that $\frac{\partial R^1}{\partial \sigma_1} > 0$ is satisfied if $\frac{\theta_2}{\sigma_2} \geq \frac{\theta_1}{\sigma_1}$ and if

$$1 \geq \left(\frac{\beta}{1+\beta}\right)\left[\frac{N_1\alpha_1 y}{(1+\beta)\left[\phi - \sigma_1 f(\theta_1;\sigma_1)\right]}\right]. \tag{66}$$

Moreover, $f(\theta_1;\sigma_1) \geq \frac{N_1\alpha_1 y}{(1+\beta)}$ holds: consequently $\frac{\theta_2}{\sigma_2} \geq \frac{\theta_1}{\sigma_1}$ and

$$1 \geq \left(\frac{\beta}{1+\beta}\right)\left[\frac{f(\theta_1;\sigma_1)}{\phi - \sigma_1 f(\theta_1;\sigma_1)}\right] \tag{67}$$

are sufficient for $\frac{\partial R^1}{\partial \sigma_1} > 0$. Rearranging terms in (67) yields condition (i) in the text.

(b) If $\sigma_1 \geq 1$ holds, then it follows from the definition of $f(\theta_1;\sigma_1)$ that

$$f(\theta_1;\sigma_1) \leq \frac{N_1\alpha_1 y}{(1+\beta)(1-\theta_1)} \equiv \frac{f(0;\sigma_1)}{(1-\theta_1)}. \tag{68}$$

Clearly, then, conditions (ii) and (68), along with $\sigma_1 \geq 1$, imply condition (i).

(c) Similar reasoning to that in part (a) indicates that $\frac{\partial R^1}{\partial \sigma_1} \leq 0$ holds iff

$$\left(\frac{\beta}{1+\beta}\right)\left[\frac{N_1\alpha_1 y R^1}{(1+\beta)\,g(\theta_2;\sigma_2)}\right] \geq \left(\frac{1-\theta_1}{1-\theta_2}\right)(x)^2$$

and that

$$1 = \left(\frac{1-\theta_1}{1-\theta_2}\right)x + \left[\left(\frac{\theta_1}{\sigma_1}\right) - \left(\frac{\theta_2}{\sigma_2}\right)\right]\frac{x}{(1-\theta_2)\,R^1}.$$

Therefore, since $\frac{\theta_2}{\sigma_2} \leq \frac{\theta_1}{\sigma_1}$, a sufficient condition for $\frac{\partial R^1}{\partial \sigma_1} \leq 0$ to hold is that

$$\left(\frac{\beta}{1+\beta}\right)\left[\frac{N_1\alpha_1 y R^2}{(1+\beta)\,g(\theta_2;\sigma_2)}\right] \geq 1. \tag{69}$$

Moreover, equation (23) implies that $R^2 > \frac{g(\theta_2;\sigma_2)}{\phi}$ holds. It follows that $\frac{\partial R^1}{\partial \sigma_1} < 0$ is satisfied if $\frac{\theta_2}{\sigma_2} \leq \frac{\theta_1}{\sigma_1}$, and if

$$\left(\frac{\beta}{1+\beta}\right)\left[\frac{N_1\alpha_1 y}{(1+\beta)\,\phi}\right] \equiv \left(\frac{\beta}{1+\beta}\right)\left[\frac{f(0;\sigma_1)}{\phi}\right] \geq 1. \tag{70}$$

Rearranging terms in (70) yields condition (iii).

The proofs of parts (d), (e) and (f) are similar to the proofs of parts (a), (b), and (c), respectively.

C. Proof of Proposition 3.

(a) Differentiating equation (60) with respect to θ_2 yields

$$\frac{\partial R^1}{\partial \theta_2}\left\{\left(\frac{R^2}{R^1}\right)\cdot\left(\frac{f(\theta_1;\sigma_1)}{\phi R^1 - f(\theta_1;\sigma_1)}\right)+\left(\frac{1-\theta_1}{1-\theta_2}\right)\right\} = \left[\frac{R^1 g_1(\theta_2;\sigma_2)}{\phi R^1 - f(\theta_1;\sigma_1)}\right] - \frac{1}{\sigma_2(1-\theta_2)}\left(I^2 - 1\right). \quad (71)$$

where

$$g_1(\theta_2;\sigma_2) = \left(\frac{N_2\alpha_2 y}{1+\beta}\right)\cdot\left(\frac{\sigma_2+\beta}{\sigma_2(1+\beta)}\right)\frac{1}{(1-\theta_2)^2}.$$

Then it is straightforward but tedious to show that $\frac{\partial R^1}{\partial \theta_2} \leq (>)\, 0$ holds iff

$$\left[\frac{(\sigma_2-1)R^1}{\phi R^1 - f(\theta_1;\sigma_1)}\right]\left(\frac{N_2\alpha_2 y}{1+\beta}\right) \geq (<) \frac{(1+\beta)}{\beta}. \quad (72)$$

The condition (72) is easily shown to be equivalent to condition (i) in the text.

(b) Part (b) follows immediately from condition (i) and the observation that $g(0;\sigma_2) \leq g(\theta_2;\sigma_2)$, $\forall \theta_2$.

The proofs of parts (c) and (d) are similar to the proofs of parts (a) and (b), respectively.

D. Proof of Proposition 5.

In a steady state equilibrium, R^1 satisfies the condition

$$R^1\left\{\frac{g(\theta_2;\sigma_1)}{\phi R^1 - f(\theta_1;\sigma_1)}\right\} = \left(\frac{1-\theta_1}{1-\theta_2}\right)R^1 + \frac{\theta_1-\theta_2}{\sigma_1(1-\theta_2)}. \quad (73)$$

Differentiating equation (73) with respect to σ_1 yields

$$\frac{\partial R^1}{\partial \sigma_1}\left\{\left(\frac{R^2}{R^1}\right)\cdot\left(\frac{f(\theta_1;\sigma_1)}{\phi R^1 - f(\theta_1;\sigma_1)}\right)+\left(\frac{1-\theta_1}{1-\theta_2}\right)\right\} =$$
$$\left[\frac{R^1 g_2(\theta_2;\sigma_1)}{\phi R^1 - f(\theta_1;\sigma_1)}\right] + \left\{\frac{R^1 g(\theta_2;\sigma_1)\, f_2(\theta_1;\sigma_1)}{[\phi R^1 - f(\theta_1;\sigma_1)]^2}\right\} + \frac{\theta_1-\theta_2}{(\sigma_1)^2(1-\theta_2)}. \quad (74)$$

Since $g_2(\theta_2;\sigma_1) < 0$ and $f_2(\theta_1;\sigma_1) < 0$ both hold, $\frac{\partial R^1}{\partial \sigma_1} < 0$ must be satisfied in case 1 ($\theta_2 \geq \theta_1$). And clearly, $\frac{\partial R^1}{\partial \sigma_1} < 0$ will hold in case 2 as well, if $|\theta_1 - \theta_2|$ is sufficiently small.

Differentiating equation (31), we also obtain the relation

$$\frac{\sigma_1}{R^2}\frac{\partial R^2}{\partial \sigma_1} = -\left(\frac{\sigma_1}{R^1}\frac{\partial R^1}{\partial \sigma_1}\right)\left[\frac{f(\theta_1;\sigma_1)}{\phi R^1 - f(\theta_1;\sigma_1)}\right] + \frac{\sigma_1 g_2(\theta_2;\sigma_1)}{g(\theta_2;\sigma_1)} + \frac{\sigma_1 f_2(\theta_1;\sigma_1)}{\phi R^1 - f(\theta_1;\sigma_1)}. \tag{75}$$

Thus $\frac{\partial R^2}{\partial \sigma_1} < 0$ holds iff

$$-\frac{\sigma_1 g_2(\theta_2;\sigma_1)}{g(\theta_2;\sigma_1)} - \frac{\sigma_1 f_2(\theta_1;\sigma_1)}{\phi R^1 - f(\theta_1;\sigma_1)} > -\left(\frac{\sigma_1}{R^1}\frac{\partial R^1}{\partial \sigma_1}\right)\left[\frac{f(\theta_1;\sigma_1)}{\phi R^1 - f(\theta_1;\sigma_1)}\right] \tag{76}$$

obtains. Using equation (74), equation (76) is easily shown to be equivalent to

$$-\frac{\sigma_1 g_2(\theta_2;\sigma_1)}{g(\theta_2;\sigma_1)} - \frac{\sigma_1 f_2(\theta_1;\sigma_1)}{\phi R^1 - f(\theta_1;\sigma_1)} > \left[\frac{\theta_2 - \theta_1}{(1-\theta_1) I^1}\right]\left[\frac{f(\theta_1;\sigma_1)}{\phi R^1 - f(\theta_1;\sigma_1)}\right];$$

where $I^1 = \sigma_1 R^2$. Obviously this condition must be satisfied in case 1 ($\theta_1 \geq \theta_2$), and it holds in case 2 as well if $\theta_2 - \theta_1$ is sufficiently small.

E. Proof of Proposition 6.

(a) Equation (36), along with the condition that $\theta_2 > \theta_1$, implies that the steady state is asymptotically stable (unstable) if

$$\frac{(1+\beta)\phi(\theta_1 - \theta_2)(1-\theta_2)}{\sigma_1[1-\theta_2-(1-\theta_1)x]^2} - \left(\frac{N_2\alpha_2 y}{1+\beta}\right)\left(\frac{\theta_2}{1-\theta_2}\right) - \frac{\beta}{\sigma_1} > (<) \left(\frac{N_2\alpha_2 y}{1+\beta}\right)\left(1+\beta+\frac{\theta_2}{1-\theta_2}\right). \tag{77}$$

Rearranging terms in (77) yields (37).

(b) Since $x > \frac{1-\theta_2}{1-\theta_1}$ holds, the right-hand side of (37) is decreasing in x. Hence, in order for the steady state to be asymptotically stable for any value of $x < 1$, it must be the case that (38) holds.

(c) Fix θ_1; θ_2 (with $\theta_2 > \theta_1$), N_2, α_2, y, σ_1, β, and ϕ. The steady state value of x is the unique solution to

$$g(\theta_2;\sigma_1)x + f(\theta_1;\sigma_1) = \frac{\phi(\theta_2-\theta_1)x}{\sigma_1[(1-\theta_1)x-(1-\theta_2)]}. \tag{78}$$

Since $\phi > \sigma_1 g(\theta_2;\sigma_1)$ holds, we can choose $f(\theta_1;\sigma_1)$ sufficiently close to $\frac{\phi}{\sigma_1} - g(\theta_2;\sigma_1)$ so that x is as close to one as desired. Thus, for $f(\theta_1;\sigma_1)$ sufficiently close to $\frac{\phi}{\sigma_1} - g(\theta_2;\sigma_1)$, (38) guarantees that the steady state is asymptotically stable.

Similarly, by allowing $f(\theta_1; \sigma_1)$ to become large, we can make x as close to $\frac{1-\theta_2}{1-\theta_1}$ as desired. Hence, from (37), the steady state will be unstable for $f(\theta_1; \sigma_1)$ large.

Now by varying N_1, α_1, and w appropriately, we can set $f(\theta_1; \sigma_1)$ anywhere in the interval $\left(\frac{\psi}{\sigma_1} - g(\theta_2; \sigma_1); \infty\right)$ while holding ψ fixed. Thus there are values N_1, α_1, and w such that the steady state is asymptotically stable, and similarly, there are values such that it is unstable.

With $\theta_1; \theta_2$, N_2, α_2, y, σ_1, β, and ψ held constant, then as N_1, α_1, and w are varied, (always in such a way as to hold ψ fixed), it is apparent from (36) that $\frac{\partial x_t}{\partial x_{t-1}}$ (evaluated at the steady state) varies only insofar as x varies. With the same parameters fixed, it is clear from (78) that x varies smoothly with $f(\theta_1; \sigma_1)$. Moreover, since

$$\lim_{f(\theta_1;\sigma_1) \to \frac{\psi}{\sigma_1} - g(\theta_2;\sigma_1)} \frac{\partial x_t}{\partial x_{t-1}} > -1$$

when (38) holds, and since

$$\lim_{f(\theta_1;\sigma_1) \to \infty} \frac{\partial x_t}{\partial x_{t-1}} < -1;$$

it follows that there is a finite value of $f(\theta_1; \sigma_1)$ such that $\frac{\partial x_t}{\partial x_{t-1}} = -1$. At this value of $f(\theta_1; \sigma_1)$ a flip bifurcation occurs, (Azariadis, 1993, pp. 95-97), and equilibria displaying two period cycles necessarily emerge in a neighborhood of this value.

Thus the claim is established if we can vary $f(\theta_1; \sigma_1)$ as desired by varying N_1, α_1, and w, while holding ψ constant. But $\psi > \sigma_1 g(\theta_2; \sigma_1)$ and the definitions of ψ and $f(\theta_1; \sigma_1)$ imply that this is indeed possible.

F. Proof of Proposition 7.

It is straightforward to show that the elements of J are given by

$$\frac{\partial q_t}{\partial q_{t-1}} = \frac{\sigma_1 \theta_2 \rho}{\theta_1} + \frac{\sigma_1 (1 - \theta_2)}{\theta_1 z} \tag{79}$$

$$\frac{\partial q_t}{\partial z_{t-1}} = -\frac{\sigma_1 (1 - \theta_2) q}{\theta_1 z^2} \tag{80}$$

$$\frac{\partial z_t}{\partial q_{t-1}} = -\frac{(1+\beta) f(\theta_1;\sigma_1)\left(\frac{\sigma_1\theta_2\rho}{\theta_1}\right)}{\left\{(1+\beta) g(\theta_2;\rho^{-1}) - N_2\left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\beta}{1+\beta}\right)\rho\alpha_2 y\right\}} - \frac{(1+\beta) f(\theta_1;\sigma_1)\left[\frac{\sigma_1(1-\theta_2)}{\theta_1 z}\right]}{\left\{(1+\beta) g(\theta_2;\rho^{-1}) - N_2\left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\beta}{1+\beta}\right)\rho\alpha_2 y\right\}} \tag{81}$$

$$\frac{\partial z_t}{\partial z_{t-1}} = \frac{\left\{(1+\beta) f(\theta_1;\sigma_1)\left[\frac{\sigma_1(1-\theta_2)q}{\theta_1 z^2}\right] - N_2\left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\beta}{1+\beta}\right)\rho\alpha_2 y\right\}}{\left\{(1+\beta) g(\theta_2;\rho^{-1}) - N_2\left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\beta}{1+\beta}\right)\rho\alpha_2 y\right\}} \tag{82}$$

Let D and T denote the determinant and trace of J respectively. Then it is easy to show that

$$D = -\left\{1 + \left(\frac{1-\theta_1}{\theta_1}\right) I^1\right\}\left\{\frac{N_2\left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\beta}{1+\beta}\right)\rho\alpha_2 y}{\left[(1+\beta) g(\theta_2;\rho^{-1}) - N_2\left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\beta}{1+\beta}\right)\rho\alpha_2 y\right]}\right\} < 0 \tag{83}$$

$$T = 2 + \left(\frac{1-\theta_1}{\theta_1}\right) I^1 + \frac{\left\{(1+\beta) f(\theta_1;\sigma_1)\left[\frac{\sigma_1(1-\theta_2)q}{\theta_1 z^2}\right] - (1+\beta) g\left(\theta_2;\rho^{-1}\right)\right\}}{\left\{(1+\beta) g(\theta_2;\rho^{-1}) - N_2\left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\beta}{1+\beta}\right)\rho\alpha_2 y\right\}}; \tag{84}$$

where $I^1 \equiv \sigma_1 R^1$ is the gross nominal rate of interest on loans in country 1, in the steady state. The assumption that the reserve requirement binds in the steady state implies that $I^1 > 1$.

Since $T^2 > 4D$ clearly holds, it follows that λ_1 and λ_2 are real numbers satisfying $\lambda_1 < 0 < \lambda_2$. Moreover, (84) implies that $T \geq \left(\frac{1-\theta_1}{\theta_1}\right) I^1$ iff

$$2 \geq \frac{\left\{(1+\beta) g\left(\theta_2;\rho^{-1}\right) - (1+\beta) f(\theta_1;\sigma_1)\left[\frac{\sigma_1(1-\theta_2)q}{\theta_1 z^2}\right]\right\}}{\left\{(1+\beta) g(\theta_2;\rho^{-1}) - N_2\left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\beta}{1+\beta}\right)\rho\alpha_2 y\right\}} \tag{85}$$

is satisfied. One consequence of this fact is that, when $\theta_1 \leq 0.5$ holds, the satisfaction of (85) implies that $T \geq \left(\frac{1-\theta_1}{\theta_1}\right) I^1 \geq I^1 > 1$. In particular the satisfaction of (85) implies that $\lambda_2 > 1$ holds. It follows that the steady state is either a source or a saddle; it is a saddle ($\lambda_1 > -1$) if $1 + T > -D$ [see Azariadis (1993), chapter 6.4].

Now $1 + T > -D$ is equivalent to the condition

$$1 + \left(\frac{1-\theta_1}{\theta_1}\right) I^1 + 2 + \frac{\left\{(1+\beta) f(\theta_1;\sigma_1)\left[\frac{\sigma_1(1-\theta_2)q}{\theta_1 z^2}\right] - (1+\beta) g\left(\theta_2;\rho^{-1}\right)\right\}}{\left\{(1+\beta) g\left(\theta_2;\rho^{-1}\right) - N_2\left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\beta}{1+\beta}\right)\rho\alpha_2 y\right\}}$$
$$> \left\{1 + \left(\frac{1-\theta_1}{\theta_1}\right) I^1\right\}\left\{\frac{N_2\left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\beta}{1+\beta}\right)\rho\alpha_2 y}{\left[(1+\beta) g(\theta_2;\rho^{-1}) - N_2\left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\beta}{1+\beta}\right)\rho\alpha_2 y\right]}\right\}. \tag{86}$$

Clearly a sufficient condition for (86) to hold, in turn, is that

$$(1+\beta)\, g\left(\theta_2; \rho^{-1}\right) \geq 2N_2 \left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\beta}{1+\beta}\right)\rho\alpha_2 y. \tag{87}$$

Thus, satisfaction of (87) implies that $1+T > -D$ $(\lambda_1 > -1)$. It is also apparent that (87) implies the satisfaction of (85). Consequently, $\theta_1 \leq 0.5$ and (87) imply that $\lambda_2 > 1$. Hence when (87) holds, the steady state is a saddle. To complete the proof, note that by definition (87) is equivalent to

$$1+\left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{1+\beta\rho}{1+\beta}\right) \geq 2\left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\beta\rho}{1+\beta}\right); \tag{88}$$

or to

$$1 \geq \left(\frac{\theta_2}{1-\theta_2}\right)\left(\frac{\beta\rho-1}{1+\beta}\right). \tag{89}$$

Figure 1: Steady State Equilibria under a Money Growth Rate Rule

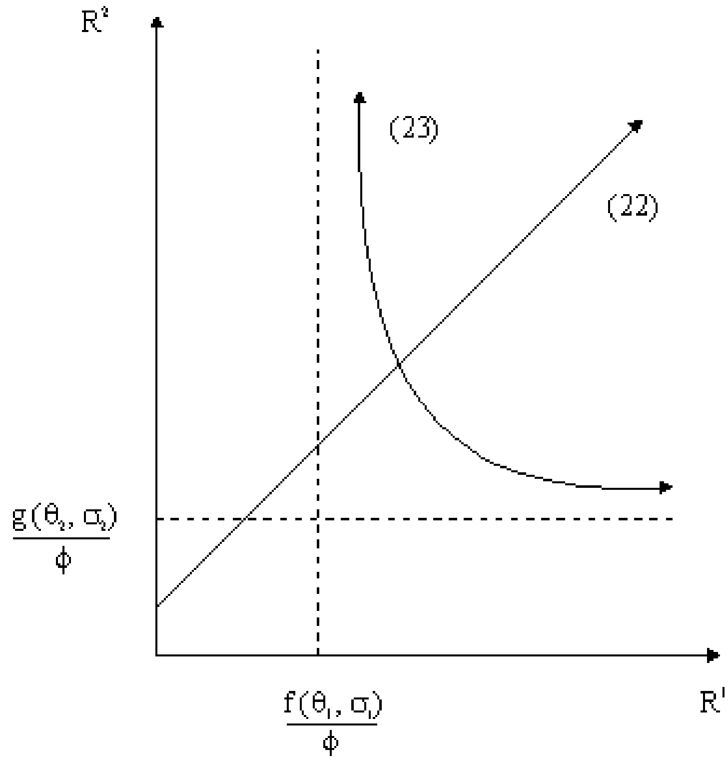


Figure 2: Dynamics under a Money Growth Rate Rule

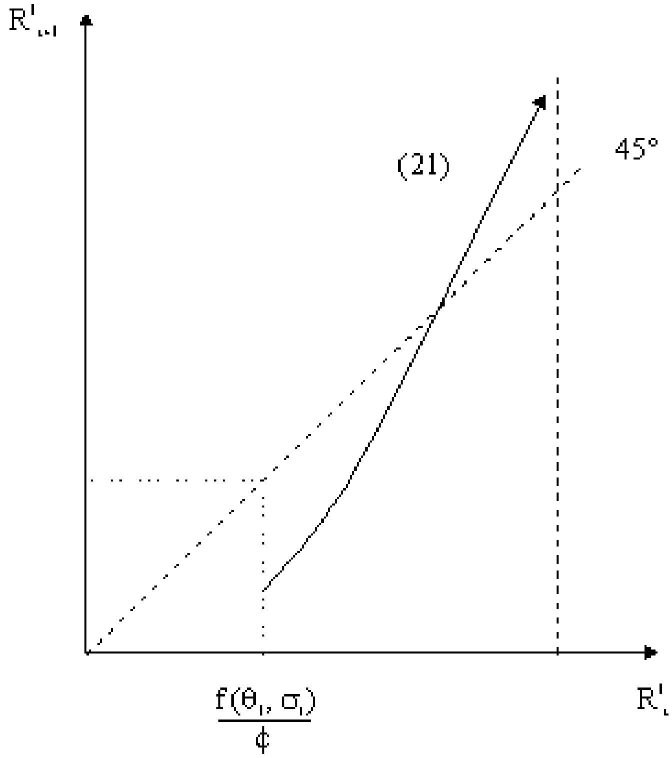
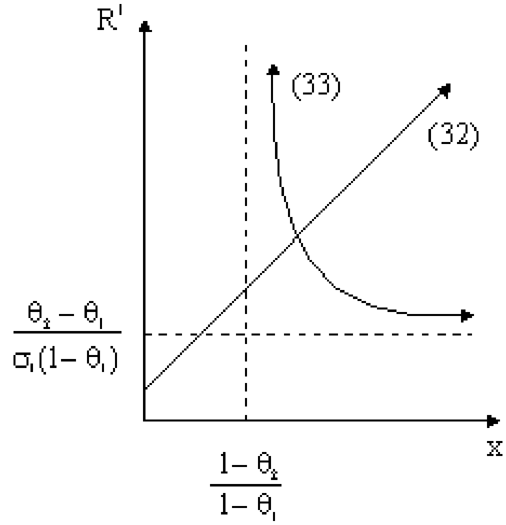


Figure 3: Steady State Equilibria under Fixed Exchange Rates

(a) $\theta_2 > \theta_1$



(b) $\theta_1 > \theta_2$

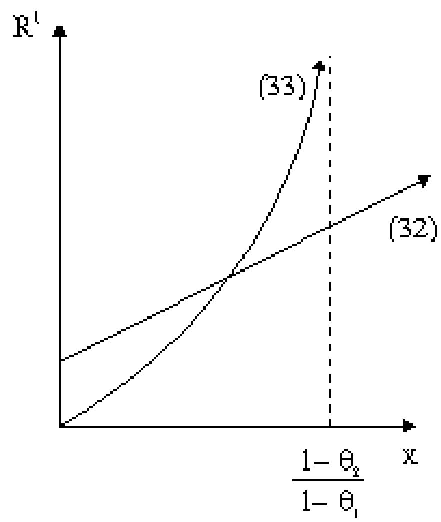


Figure 4: Equilibrium Law of Motion for x_t
under Fixed Exchange Rates