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We analyze a stochastic growth model with lags in the operation of new technologies. Stock values are impacted by news on technological innovations and some other external shocks affecting the economy. Episodes of technology adoption may generate long fluctuations in the aggregate value of stocks. We asses the quantitative importance of various macroeconomic variables in accounting for both the observed volatility of stock values and the less pronounced volatility of real macroeconomic aggregates. Our analysis singles out price markups and leverage as key determinants of asset price volatility, and confers a rather limited role to adjustment costs, taxes, and labor and financial frictions.

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1 Introduction

This paper is concerned with macroeconomic determinants of the volatility of asset prices. Stochastic growth models have been fairly successful in accounting for comovements of real economic aggregates but have failed to offer plausible explanations for the volatility of stock values based on the variability of economic fundamentals. Hence, the modern literature in the interface between macroeconomics and finance has consistently appealed to several forms of sophisticated preferences, stochastic discounting, time varying risk, adjustment costs, bubbles, and noise trader risk. We shall here take a more traditional route, and present a model with a standard intertemporal preference representation under a CRRA utility function and constant discounting, and a mild form of adjustment costs for capital investment. We perform several quantitative exercises to explore determinants of the observed volatility in stock market values along with the much lower volatility of real economic aggregates.

In the neoclassical growth model, changes in total factor productivity (TFP), the relative price of capital, taxes, and frictions in labor and capital markets hardly generate any volatility of stock values [see Rouwenhorst (1995) for some numerical exercises]. As a matter of fact, these forces do not affect significantly the volatility and persistence of dividends and earnings under observable variations in consumption. Viewed in another way, capital is the only asset in the economy, and investment must fluctuate enormously to get desirable levels of volatility in stock values. The model's performance can be improved with adjustment costs [Christiano and Fisher (2003)], but these costs may seem implausibly high.

There is a burgeoning literature on macroeconomics and finance, but there is no general consensus on the main macroeconomic determinants of asset price volatility – which remain a puzzle to economists. A good part of the literature has focused on the equity premium [e.g., see Campbell (1999), Cochrane and Hansen (1992) and Mehra and Prescott (2003)]. There is much less work intended to offer a joint explanation for the volatilities of the stock market and other macroeconomic variables [cf., Backus, Routledge and Zin (2006), Christiano and

Fisher (2003), and Rouwenhorst (1995)]. In a recent paper, Jinnai (2009) argues in favor of two-sector models and evaluates the influence of intangible capital, leverage, recursive preferences, and adjustment costs on asset volatility and the equity premium.

We consider a stochastic growth model with lags in the operation of new technologies. Technological innovations arrive exogenously to the economy. These innovations, however, cannot be readily put into use and undergo a process of adoption embedded in the production of new varieties of intermediate goods. Asset prices incorporate the option value of technological innovations that remain to be adopted. This propagation mechanism is somewhat present in the partial equilibrium setting of Abel and Eberly (2005), in the tree economy of Garleanu, Panageas and Yu (2009), and in the learning model of Pastor and Veronesi (2008). In all these papers the value of the firm may differ from the replacement value of the stock of capital. In contrast to these authors, we carry out a quantitative general equilibrium analysis in which the volatility of asset prices is explained along with other macroeconomic fluctuations. Hence, the challenge for our model is to generate observed levels of volatility in stock markets while preserving the less pronounced volatility of real macroeconomic aggregates.

Our model is a simplified variant of those in Romer (1990) and Comin and Gertler (2006), but our objectives are quite different. Romer (1990) is concerned with innovations and economic growth and Comin and Gertler (2006) with a quantitative analysis of real economic fluctuations.[1] We decompose the value of the stock market into the replacement value of capital, the value of technology goods, and the option value of adopting present and future innovations. Then, episodes of technology innovation may generate sudden fluctuations in the aggregate value of stocks.

At a later stage we report some quantitative experiments. In spite of restricting the analysis to a simple CRRA utility function and a constant discount factor, our model can account for a sizable part of the volatility of stock market values. The model is solved numerically by a high-order approximation method that picks non-linearities in the evolution of stock prices. We assign parameter values to fit some basic facts in economic growth and business

[1] In a later paper, Comin, Gertler and Santacreu (2009) borrow extensively from our theoretical work and build their main intuition and economic ideas from our fundamental equation for asset pricing derived in Proposition 3.1 below. It seems that most of the shocks that they consider have a small influence on the volatility of asset markets. Their results could be driven by stochastic discounting and a strong form of adjustment costs, rather than by the main forces underlying our quantitative experiments that yield much more volatility of dividends.

fluctuations. Then, we perform several numerical exercises to see how innovations to the economy may affect the dynamics of stock prices and other aggregate variables. As in Greenwood and Jovanovic (1999) not all technological innovations will increase stock market prices, since the arrival of new technologies will depreciate the value of existing ones. To affect positively the stock market, technologies must command higher price markups. These price markups may arise from lower elasticities of substitution attached to the new technologies or from perturbations to the global demand. Apart from price markups, we also consider the effects of leverage, adjustment costs, taxes, and labor and financial frictions. A notable feature of these numerical exercises is that adjustment costs for capital investment, taxes, and labor and financial frictions can only have a significative impact on the volatility of asset values at the expense of implausible fluctuations in some other variables.

Several recent works are concerned with asset price movements and volatilities. Most of these papers focus on level effects, and do not carry out a joint study of the variability and comovements of the financial and real sectors. Hence, it may be useful to integrate these approaches into a business cycle framework. Geanakoplos, Magill and Quinzii (2004) contend that changes in stock values may be driven by demographic trends, whilst Lustig and Nieuwerburgh (2006) cite credit access from home equity collateral that may affect attitudes toward risk. A large body of research [Greenwood and Jovanovic (1999), Laitner and Stolyarov (2003) and Peralta-Alva (2006)] elaborates on the effects of IT on the dynamics of capital values. McGrattan and Prescott (2005) attribute some trends in stock values to changes in the tax system. Bernanke and Gertler (1999), Christiano et al. (2008), and Kiyotaki and Moore (1997) stress some important qualitative effects of monetary interventions and financial restrictions such as collateral requirements.

The paper will proceed as follows. We start in Section 2 with some empirical evidence on stock market fluctuations. In Section 3 we lay out our model of technology adoption and derive some qualitative properties of the solution with emphasis on a fundamental asset pricing equation that decomposes the stock price into the stock of physical capital and the value of adopted and unadopted technologies for the production of intermediate products. Section 4 is devoted to the computation and calibration of the model, and Section 5 reports various numerical experiments. We conclude in Section 6 with a further evaluation of our main findings.

2 Some Empirical Evidence

Figure 1 plots the evolution of the S&P index and the corresponding price-earnings (PE) ratio. The S&P index has been filtered by taking out our best fit for a deterministic exponential trend. Figure 2 portrays a centered ten-year moving average of S&P yearly returns. All these series display similar long-term cyclical behavior; indeed, as discussed below stock prices are a main driving force of these fluctuations.[2] Note that peak values occur in 1880, 1900, 1925, 1955 and 1995. Therefore, the amplitude of these long-term cycles can be up to 40 years.

Jovanovic and Rousseau (2001) associate these long fluctuations in the stock market with three technological revolutions: Electricity, World War II, and IT. These authors document long lags in the operation and diffusion of new technologies. Nicholas (2008) claims that innovation was a main driver of the stock market run-up of the late 1920s. Figure 3 is intended to illustrate the financial impact of the recent IT revolution. Market capitalization relative to GNP is decomposed into the values of three different groups of companies: (*i*) The incumbents, (*ii*) Companies originating in 1980-1990, and (*iii*) Companies originating after 1990. As one can see, most added stock value belongs to new corporations. In our model these newcomers would be reflected in the value added of local technology adopters. In later work, Jovanovic and Rousseau (2009) address the further issue of when new technologies may be implemented by either incumbents or new companies.

But our story is not only a story of technology adoption. As a matter of fact, there seem to be other trends associated with these cycles coming from factors such as price markups, productivity, population, housing prices, and taxes. Figure 4 plots the price of farmland in England and the price of oil which move in tandem – property values leading changes in oil prices. Figure 5 confirms that the S&P is also a good leading indicator of oil prices. Given the cost structure of the oil industry, oil prices must be associated with the volatility and persistency of price markups which in turn may be generated by global forces in demand and supply [Dvir and Rogoff (2009)]. Price markups are substantial in the US economy [Hall

[2] Since the early papers of Kleidon (1986), Marsh and Merton (1987) and others, researchers have emphasized the persistence and lower variability of dividends and earnings, which are highly correlated. After adjusting for interest and growth rates, Barsky and DeLong (1993) obtain that extrapolations of dividends over twenty-year periods can mimic reasonably well the volatility of stocks.

(1988)]. These markups can vary over time led by macro trends and are prominent in the innovation and product cycles [Rotemberg and Woodford (1995) and Broda and Weinstein (2007)]. Some new technologies may display low elasticities of substitution and enjoy high markups. In our simple economy below these changes will be reflected in increases of the aggregate price markup. Hence, as in some recent neo-Keynesian literature in our model the price markup evolves exogenously, but will not just be a free parameter. We estimate this exogenous law of motion for the price markup by a simulated moments estimator. The estimated values are actually quite close to the empirical estimates obtained from the dynamic evolution of oil prices. Moreover, the volatility and persistence of the price markup in the model is compatible with the observed volatility and persistence of dividends and earnings, and the fluctuation of various macroeconomic aggregates such as investment and consumption.

Besides technological innovations and price markups, we also consider TFP shocks. There is here extensive evidence from the business cycles literature [e.g., see Cooley and Prescott (1995)]. As discussed in more detail below, for annual data the volatility of stock market values is roughly ten times greater than the volatility of output, and almost four times greater than the volatility of physical investment. The volatility of the stock of physical capital is of the same order of magnitude as the volatility of output. TFP shocks display similar levels of volatility than output and of physical capital, and would seem an important determinant of real macroeconomic fluctuations. But even when we consider leverage, price rigidities and additional market frictions that may enhance some propagation mechanisms, TFP shocks alone are unable to generate reasonable levels of volatility for stock market values.

3 The Model

The economy is populated by a continuum of identical households. At every time $t = 0, 1, \cdots$, each agent demands quantities of the aggregate consumption good, supplies labor inelastically, and trades in the equity and bond markets. The aggregate consumption good is produced by a single firm with a constant returns to scale technology. Three inputs are involved in the production of this final commodity: Capital accumulated by the firm, labor, and a composite intermediate good. Both the firm and the consumer act competitively in all markets, but the sector of intermediate goods is composed of a continuum of monopolistic competitors. The range of available intermediate goods can be expanded by a fixed set of local adopters upon the arrival of new technologies. As in Romer (1990), an increase in the varieties of intermediate goods allows for a more efficient use of resources and augments TFP. The remaining source of change in TFP is an exogenous shock to the production function of the final good. We focus on the macroeconomic determinants of the volatility of stock values in this economy. Proposition 3.1 below puts forward an asset pricing equation which will be the basis for our later analysis.

3.1 The household

The representative household supplies one unit of labor inelastically, and has preferences over infinite streams of consumption. Preferences are represented by the expected discounted objective:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \right\} \tag{1}$$

where c_t denotes the quantity of consumption at t, with $0 < \beta < 1$ and $\sigma \geq 0$. This agent may participate in financial markets by trading shares of an aggregate stock a_t and units of a risk-free bond b_t. The aggregate stock yields a stochastic dividend d_t, and the bond sells at a predetermined gross interest rate R_t. For initial positive asset holdings $\hat{a}, \hat{b}$, the optimization problem of the representative agent is to choose a stochastic sequence of consumption, shares of the aggregate stock, and units of the risk-free bond $\{c_t, a_t, b_t\}_{t \geq 0}$ to attain the maximum utility in (1) subject to the sequence of budget constraints

$$c_t + q_t a_t + b_t = \omega_t + (q_t + d_t) a_{t-1} + R_t b_{t-1} \tag{2}$$

$$q_t a_t + b_t \geq 0, \ \ t = 0, 1, 2, \cdots \tag{3}$$

for a given sequence of stock prices q_t, rates of interest R_t, and unitary wages ω_t. Note that (3) is a simple borrowing limit which in this representative agent economy entails no loss of generality.

3.2 The production sector

The firm producing the final good accumulates capital and buys labor and intermediate goods. TFP of the firm is stochastic, and represented by a random variable θ_t. At every date t there is a mass A_t of intermediate goods that enter into the production of the final good. These intermediate goods are bundled together in a composite good M_t defined by a CES technology $M_t = [\int_0^{A_t} m_{s,t}^{\frac{1}{\vartheta_t}} ds]^{\vartheta_t}$ where $m_{s,t}$ denotes the amount of intermediate good s bought by the firm at time t and $\vartheta_t > 1$ follows an exogenous stochastic process to be specified below.

Given initial levels of capital and debt $\hat{k}, \hat{B} > 0$, the firm chooses stochastic sequences of investment, labor, debt, and intermediate goods $\left\{i_t,\ l_t,\ B_t,\ (m_{s,t})_{s\in[0,A_t]}\right\}_{t\geq 0}$ so as to maximize the present value of dividends:

$$E_0 \left\{ \sum_{t=0}^{\infty} \eta_t d_t^f \right\} \tag{4}$$

subject to

$$d_t^f \equiv Y_t - i_t - \omega_t l_t - \int_0^{A_t} p_{s,t} m_{s,t} ds + B_t - R_t B_{t-1} \tag{5}$$

$$Y_t \equiv \theta_t \left[\gamma \left(k_t^{\alpha} l_t^{1-\alpha}\right)^{\rho} + (1-\gamma) M_t^{\rho}\right]^{\frac{1}{\rho}},\ 0 < \gamma < 1,\ -\infty < \rho < 1 \tag{6}$$

$$k_{t+1} = (1-\delta)\, k_t + g(i_t/k_t) k_t,\ \text{and}\ B_t \leq \bar{B}_t. \tag{7}$$

In this simple framework, we could introduce a complete set of contingent claims without changing equilibrium allocations. Note that η_t is a state price converting income of period t to period 0, and $p_{s,t}$ denotes the price of intermediate good s at time t. The physical capital stock depreciates at a constant rate $\delta \geq 0$. Capital accumulation is also subject to adjustment costs which are represented by function g. This latter function is positive and concave with $g(\delta) = \delta$ and $g'(\delta) = 1$. The curvature of this function limits the volatility of capital investment.

Our definition of dividends in (5) includes financial leverage. This is convenient for several extensions of the model in our numerical experiments below on debt policies with additional financial restrictions in which the debt bound in (7) will be binding.[3] As discussed in Hall (2001), debt policies have been quite volatile: Pay-outs to debt holders have been fairly erratic in recent decades.

Besides stock prices, interest rates, and wages, the firm considers that TFP and price markups evolve exogenously. Stochastic variables θ_t and ϑ_t are governed by the following stationary first-order autoregressive processes

$$\ln(\theta_t) = \psi^\theta \ln(\theta_{t-1}) + \sigma_\theta \varepsilon_t^\theta \tag{8}$$

$$\ln(\vartheta_t) = \psi_0^\vartheta + \psi_1^\vartheta \ln(\vartheta_{t-1}) + \psi_2^\vartheta \varepsilon_t^\vartheta \tag{9}$$

where $\psi^\theta, \psi_1^\vartheta \in (0,1)$, ψ_2^ϑ, $\sigma_\theta > 0$, $\varepsilon_t^\theta \sim N(0,1)$, and $\ln(\varepsilon_t^\vartheta) \sim N(0, \sigma_\vartheta)$.

Monopolistic competition prevails in the market for intermediate goods. Each variety s is supplied by an independent producer. Without loss of generality, the production process adopts the following simple form: One unit of good s requires only one unit of the final good. Then, producer of variety s picks an optimal pricing strategy $p_{s,t}$ and quantity $m_{s,t}$ from inspection of the downward-sloping demand for the product by the firm producing the aggregate commodity – after assuming a fixed set of prices and quantities for all other varieties. More precisely, for each time period t producer of variety s maximizes the amount of profits:

$$\pi_{s,t} \equiv \max_{m_{s,t}} \{p_{s,t} m_{s,t} - m_{s,t}\} \tag{10}$$

where $p_{s,t}$ as a function of $m_{s,t}$ can be read off from the the inverse demand function

$$p_{s,t} = \left(\frac{m_{s,t}}{M_t}\right)^{\frac{1-\vartheta_t}{\vartheta_t}} p_t \tag{11}$$

with $p_t = \left(\int_0^{A_t} p_{s,t}^{\frac{1}{1-\vartheta_t}} ds\right)^{1-\vartheta_t}$.

[3] Actually, in the above complete markets model there is an arbitrary set of debt policies for the aggregate firm that can be generated as competitive equilibria without changing consumption and investment allocations. This is because of the equivalence of holding the bond and stock contemplated in the Modigliani-Miller theorem, and the representative household is the owner of the aggregate firm.

Production of intermediate goods may be discontinued because of exogenous factors. Let ϕ be the probability of survival of a technology at every date t. Let $V_{s,t}$ be the present value of operating technology s from the beginning of time t:

$$V_{s,t} = E_t \left\{ \sum_{r=t}^{\infty} \frac{\eta_r}{\eta_t} \phi^{r-t} \pi_{s,r} \right\}. \tag{12}$$

By the symmetry embedded in our model, $\pi_{s,t}$ and $V_{s,t}$ are the same for all s.

3.3 Technology adoption

Technological innovations arrive exogenously to the economy. The total stock of technological innovations Z_t evolves according to the law of motion

$$Z_t = \phi Z_{t-1} + \mu x_t \tag{13}$$

with normalizing constant $\mu > 0$ and

$$\ln x_t = \psi^x \ln x_{t-1} + \sigma_x \varepsilon_t^x \tag{14}$$

where $\psi_x \in (0,1), \sigma_x > 0,$ and $\varepsilon_t^x \sim N(0,1)$.

Technologies are put into use by local adopters. The adoption sector is composed of a continuum of agents $i \in [0,1]$ that behave competitively. Each adopted technology sells at price V_t to a producer of intermediate goods. Let A_t^i be the stock of already adopted technologies by agent i, and $\lambda(H_t^i)$ the probability of adopting a new technology after investing the amount of resources H_t^i. An adopter can undertake a diversified menu of projects, and hence we assume that her aggregate productivity is not subject to uncertainty. The stock A_{t+1}^i follows the law of motion

$$A_{t+1}^i = \lambda(H_t^i)\phi \left[Z_t - A_t^i \right] + \phi A_t^i. \tag{15}$$

The optimal amount of expenditure H_t^i is derived from the following Bellman equation in which the value function is the option value J_t^i of a new technology:

$$J_t^i = \max_{H_t^i} \left\{ -H_t^i + \phi E_t \left[\frac{\eta_{t+1}}{\eta_t} \left(\lambda(H_t^i) V_{t+1} + \left(1 - \lambda(H_t^i)\right) J_{t+1}^i \right) \right] \right\}. \tag{16}$$

As is well known, this equation can be computed recursively by the method of successive approximations. It follows that the optimal amount of expenditure H_t^i is the same for all i. We then let the aggregate stock of adopted technologies $A_{t+1} = \int A_{t+1}^i di$.

3.4 Equilibrium and Asset Prices

In our model the exogenous state variables are the stock of available technologies Z_t, the addition of new varieties x_t, the TFP index θ_t, and the price markup ϑ_t. The endogenous state variables are the capital stock k_t, and the stock of adopted technologies A_t. The remaining variables are determined as solutions of the model from the above optimization problems, the market clearing and feasibility conditions, and the laws of motion of the exogenous state variables.

As suggested above, we adopt the convention that the stock market value includes all the above three production sectors. That is, $q_t a_t$ comprises the value of the objective (4) for the firm producing the final good, plus the discounted net value of profits over the set of intermediate goods and technology adoption. Hence, the aggregate dividend $d_t = d_t^f + \pi_t A_t - H_t(Z_t - A_t)$. In what follows we assume that the aggregate net supply of the asset equals one (i.e. $a_t = 1$) so that q_t corresponds to the value of the stock market. Therefore, market clearing in the stock and bond markets requires $a_t = 1$, and $b_t = B_t$. For the aggregate commodity, market clearing holds if

$$Y_t - A_t m_t = c_t + i_t + H_t(Z_t - A_t) \tag{17}$$

where Y_t denotes gross production of the final good and m_t is the quantity of each intermediate good produced. Hence, $A_t m_t$ is the cost of the composite intermediate good. The left-hand side of equation (17) is then the value added in terms of the aggregate good generated in this economy – which can be broken down into consumption, investment in physical capital, and investment in adopting new technologies.

The first-order conditions for the representative household correspond to the usual no-arbitrage conditions for the aggregate stock and the risk-free bond:

$$1 = E_t \left\{ \frac{\eta_{t+1}}{\eta_t} \left(\frac{d_{t+1} + q_{t+1}}{q_t} \right) \right\} \tag{18}$$

$$1 = E_t \left\{ \frac{\eta_{t+1}}{\eta_t} R_t \right\}. \tag{19}$$

The firm producing the final good will always demand positive amounts of each factor. Hence, the first-order conditions for the maximization of the objective in (4) will always hold with equality, and the imposed bounds on the amount of leverage would be taken into consideration when these are binding. In the adoption sector, optimal positive expenditure in new varieties requires:

$$1 = \lambda'(H_t)\phi E_t \left\{ \frac{\eta_{t+1}}{\eta_t} \left(V_{t+1} - J_{t+1} \right) \right\}. \tag{20}$$

It follows that for a concave function $\lambda(H)$ the optimal expenditure H is positively correlated with the expected difference between the value of adopted and non-adopted varieties.
The next proposition is central to our study. It shows that the value of the stock market comprises the value of adopted technologies and the option value to adopt new technologies.

Proposition 3.1 *In equilibrium the market value of the aggregate stock*

$$q_t = p_t^I k_{t+1} - B_t + V_t^+ A_t + J_t^+ \left(Z_t - A_t \right) + \xi_t \tag{21}$$

where $V_t^+ \equiv V_t - \pi_t, J_t^+ \equiv J_t + H_t, \xi_t \equiv E_t \left\{ \sum_{r=t+1}^{\infty} \frac{\eta_r}{\eta_t} J_r \left(Z_r - \phi Z_{r-1} \right) \right\}$, *and* $p_t^I \equiv \frac{1}{g_t'}$.

Therefore, the value of the stock market incorporates five components: The replacement cost of installed capital, the amount of debt, the value of adopted technologies, the option value of inventions currently available but not yet implemented, and the present value of future inventions expected to happen. These latter components are further sources of volatility in the stock market over the stock of capital and the value of adopted technologies. We will analyze the dynamic evolution of these components – as well as their correlation with real macro aggregates – under perturbations of the exogenous state variables.

4 Computation and Calibration of the Model

The equilibrium is computed numerically using a high-order perturbation method [Schmitt-Grohé and Uribe (2004)] that takes into account the high volatility of stock market prices. To check for accuracy of the computed solution, we have combined this approximation method with a numerical dynamic programming algorithm [Santos (1999)] for the computation of Bellman's equation (16).

Our purpose is to match different statistics of medium-term fluctuations observed in the data. Following Comin and Gertler (2006) we define medium-term cycles as those within a frequency band of 2 to 50 years. We use annual data from 1948 to 2004. Each aggregate variable is transformed in per-capita terms over the population aged 15 to 64. Then the data are filtered over various frequency bands using the filter of Christiano and Fitzgerald (2003). Output and the Solow residual are from the Bureau of Labor Statistics (BLS) nonfarm business sector. Consumption is measured as the sum of non-durables and services, and investment refers to private non-residential investment. Both series are obtained from the Bureau of Economic Analysis (BEA). The stock price, one-year interest rate, dividends, and earnings are from Robert Shiller's web page: http://www.econ.yale.edu/ shiller/data.htm. For financial variables there is especially the problem of relating the data to the model. There are certainly problems with dividends and stock prices as our data only refers to publicly traded companies. Moreover, Hall (2001) points out that reported measures of dividends should be adjusted as corporations may barter equity for the services of their employees, owners and founders. This situation is even much more delicate for joint ventures and startups where owners may provide a considerable amount of intangible resources. Hence, we only consider data on stock values and dividends for publicly traded companies without further adjustments. Note that these stocks may actually reflect the value of incoming companies because of the possibility of mergers and acquisitions as well as further competition.

Our baseline calibration of parameter values is displayed in Table 1. There are several ingredients in this calibration exercise. First, various standard parameters are taken from the literature. These include parameters defining the utility function, the aggregate production function, and adjustment costs. Second, regarding the sector of technology adoption [equation (16)], where there could be a wide range of microeconomic empirical estimates, parameter values are selected to match some macroeconomic data. As a matter of fact, to avoid a very high sensitivity of optimal expenditure H, we postulate an expenditure function that becomes fairly parsimonious with R&D expenditures. Third, for the estimation of the law of motion for TFP, markups, and technology innovation, we use a simulation-based estimation procedure along the lines of Santos (2009). This exercise yields an optimal estimation of the covariance matrix for these three shocks. The estimation of the covariance matrix may be of independent interest as it suggests how unobservable shocks to TFP, markups and technological innovations may be correlated in the data. And fourth, within plausible empirical bounds of debt to equity ratios [e.g., see Hall (2001), Rouwenhorst (1995) and references therein] we perform various numerical experiments for arbitrary debt policies followed by the aggregate firm. Then, we settle down on a simple debt policy which produces reasonable results for the volatility of dividends and other aggregate variables.

We assume an inelastic labor supply. As is well known, standard RBC models do not generate enough volatility in worked hours [see Cooley and Prescott (1995), Hansen and Wright (1992), and Kydland (1995)]. We could improve the performance of the model in this dimension by incorporating labor indivisibilities, or variable effort. But it turns out that these labor market refinements do not change significantly asset pricing volatility. Parameter σ in the utility function is set to 4, which is within the range of empirical estimates in many empirical studies.

We choose values for the set of parameters $(\beta, \alpha, \rho, \gamma, \delta)$ in line with the above business cycle literature. Parameter β is fixed at 0.95, leading to an annual interest rate of 5.26%. We make $\alpha = 0.33$ based on evidence of the average share of labor costs over total costs. There is no direct evidence on the elasticity of substitution between the composite intermediate good and capital (i.e., $1/(1-\rho)$). Given the specialized nature of this good, we assume that its production process is skill-intensive. Krusell et. al (2000) provide some estimates for the elasticity of substitution between capital and skilled labor. Following this evidence we fix parameter ρ in the production function at -0.6. The share of materials in gross output is assumed to be 0.5; this is in accordance with estimates for the manufacturing sector [see Jaimovich and Floetotto (2008)]. In the corresponding model with no uncertainty, we get this income share as a steady state value for $\gamma = 0.7$. The laws of motion of parameters θ and ϑ are estimated below jointly with those of Z to match some crucial second-order moments. In the model with no uncertainty, we get that the steady-state value for the intermediate producers' gross markup ϑ is equal to 1.18. This value is rather low as compared to the range of available estimates [Rotemberg and Woodford (1995) provide an overview of microeconomic evidence]. But as the markup shock stems from a log-normal distribution, by Jensen's inequality the simulated mean for parameter ϑ is 1.35 which seems a more reasonable value.

We reproduce the average investment to capital ratio in the data by assuming an annual depreciation rate δ of 0.09 in (7). We specify the adjustment cost function as in Jermann (1998):

$$g(i/k) = \frac{\delta^{\frac{1}{\varsigma}}}{1-\frac{1}{\varsigma}}(\frac{i}{k})^{1-\frac{1}{\varsigma}} + \frac{\delta}{1-\varsigma} \tag{22}$$

where the positive parameter ς is the elasticity of the investment to capital ratio with respect to Tobin's q. We let $\varsigma = 8$, in line with empirical evidence [see Jermann (1998), Jinnai (2009), and references therein]. As discussed below, our results present low sensitivity to this parameter.

Following empirical estimates in Hall (2007), parameter ϕ regarding the survival rate of each intermediate product is set to 0.98. The probability of adoption is determined by an exponential function

$$\lambda(H_t) = \Lambda H_t^{\kappa} \tag{23}$$

with $\Lambda > 0$ and $\kappa \in (0,1)$. We assign parameter values in conjunction with the laws of motion for A and Z below to replicate the volatility and persistence of patents and the ratio of R&D expenditures over output. The steady-state value for probability $\lambda(H)$ is 0.166, which yields an average adoption time of six years. Parameter κ then determines the volatility of expenditures in technology adoption. We come close to this volatility for κ equal to 0.80. Under these parameter values, we then get that the mean value of the ratio of adoption expenditures over GDP is 2.35 percent. This figure is roughly the ratio of R&D expenditures over GDP in the data. In our simulated exercises the optimal law of motion for the ratio of adoption expenditures over output ($RH_t \equiv H_t(Z_t - A_t)/(Y_t - A_t m_t)$ is approximated by the expression

$$RH_t = R_0 + R_1(Z_t - A_t) \tag{24}$$

where R_0 and R_1 are constants with $R_1 > 0$. This approximate policy function has low computational cost, and tracks down the level and volatility of R&D data in a more parsimonious way.

We assume that the data counterpart for the number of adopted technologies (i.e., $A_{t+1} - \phi A_t$) is the number of patent applications.[4] Parameter values for the exogenous stochastic process (8)–(9) and (14) are selected from a loss function defined over weighted second-order moments of output, investment, and the stock market subject to replicating the variance, and first-order autocorrelation of the Solow residual and patent applications, together with the correlation between patents and output, over medium-term cycles in the data. (See Santos (2009) for a related calibration exercise.) We postulate the following law of motion for the shock process:

$$\begin{bmatrix} \varepsilon_t^{\theta} \\ \varepsilon_t^{x} \\ ln(\varepsilon_t^{\vartheta}) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \eta_{21} & \eta_{22} & 0 \\ \eta_{31} & \eta_{32} & \eta_{33} \end{bmatrix} \begin{bmatrix} u_t^{\theta} \\ u_t^{x} \\ u_t^{\vartheta} \end{bmatrix} \tag{25}$$

with $u_t^{\theta}, u_t^{x}, u_t^{\vartheta} \sim N(0,1)$. From this estimation we can see from Table 1 that the markup shock seems highly persistent with autoregressive parameter $\psi_1^{\vartheta} = 0.965$. Using data from Dvir and Rogoff (2009), we obtain similar persistence values from univariate estimations over various subperiods of the detrended price of oil. This high persistence is compatible with the behavior of stock values and dividends in the model. The oil data actually shows less variance for the innovation shock. Hence, the innovation shock may also reflect technological changes in the elasticity of substitution of intermediate goods.[5] This is actually confirmed in Table 1 for $Corr(\varepsilon^{x}, ln(\varepsilon^{\vartheta})) = 0.70$. Technological innovations are therefore associated with high price markups [Broda and Weinstein (2007)]. Moreover, $Corr(\varepsilon^{\theta}, ln(\varepsilon^{\vartheta})) = 0.90$, which suggests strong correlated effects between the Solow residual and price markups.

[4]The series considered is the number of patent applications obtained from U.S. Patent and Trademark Office for 1970–2004, and from Historical Statistics of the United States series W–99 for 1948 to 1970.

[5]Taking $\psi_0^{\vartheta} = 0$, our estimates for an univariate regression of oil prices are: $\psi_1^{\vartheta} = 0.93$ and $\sigma_{\vartheta} = 0.09$ for 1900–2008, with $\psi_1^{\vartheta} = 0.96$ and $\sigma_{\vartheta} = 0.10$ for the most recent subperiod 1950–2008, and $\psi_1^{\vartheta} = 0.93$ and $\sigma_{\vartheta} = 0.12$ for the shorter, recent subperiod 1970–2008. Note that in our baseline calibration of Table 1 we actually get $\psi_0^{\vartheta} = -0.145$ but for all our simulations we always obtain that ϑ_t is greater than 1.

In our baseline calibration the amount of debt B_t is specified as an exogenous process that incorporates a fixed component capturing long-term debt and a variable part representing short-term debt. The fixed component is a fraction τ^{debf} of the unleveraged value of the stock in the steady state q^{UL}. The variable part is specified as a constant fraction τ^{debv} of the factor payments to labor and the intermediate goods. More specifically,

$$B_t \equiv \tau^{debf} q^{UL} + \tau^{debv} \left[\gamma(1-\alpha) \left(k_t^{\alpha} l_t^{1-\alpha} \right)^{\rho} + (1-\gamma) M_t^{\rho} \right] \theta_t^{\rho} Y_t^{(1-\rho)} \tag{26}$$

We set $\tau^{debf} = 0.2$ and $\tau^{debv} = 0.5$. These values imply a mean value for the share of total debt over the unleveraged value of the stock equal to 29.3%. The short-term component is 42.69% of total debt, which is in line with different estimates [see Hall (2001), Rajan and Zingales (1995), and Rouwenhorst (1995)].

5 Numerical Experiments

This section contains several numerical experiments to assess model's predictions. Our primary interest is to quantify the importance of several macroeconomic factors on the volatility of stock market values. As a guide to understand the influence of external forces in the dynamics of the model, we first compute impulse-response functions for shocks in TFP, new technologies, and markups. In a second set of experiments we check the ability of the model to reproduce various second-order moments found in the data. We also discuss some other extensions of the model: Taxes, labor market frictions, financial constraints, and monetary policy.

5.1 Impulse-Response Functions

This first set of exercises focuses on the response of the different components of stock market values of Proposition 3.1 to a one-time perturbation of our forcing variables θ, x and ϑ. To gain further intuition we also report the response of the risk-free interest rate. From inspection of all these graphs, it will appear that the model has long propagation effects that may last for over 50 years. This is not actually a major concern in the sequel. As discussed below, some simulated variables present lower persistence than the empirical counterparts.

Figure 6 exhibits the percentage deviation from the steady state value for the stock price q, and its different components, as well as the risk-free interest rate R, after an increase in θ by one standard deviation. The effects are a bit stronger and more persistent than in the neoclassical growth model because of the extra propagation mechanism in the market of intermediate goods. An increase in TFP stimulates consumption and capital accumulation. Then, the interest rate goes down to accommodate convergence back to the steady state. Given the functional form for production function (6) positive changes in θ and k raise profits per variety π, which jointly with the lower interest rates results in increments in q, V, J, and ζ. Hence, there are further incentives to invest in technology adoption: H will go up, and the amount of adopted varieties A will increase over time. This last additional channel generates a more persistent response to the shock.

Figure 7 exhibits the response of the above variables to an increase of one standard deviation in x. Note that under our baseline calibration the stock market value q and its components go down because the arrival of new technologies will depress the amount of profits π per variety. Investment in technology adoption increases at the expense of consumption and capital investment. Output peaks later on because adoption of new technologies stirs up the productivity of capital and labor, and so both consumption and investment must rebound.

Figure 8 replicates the same experiment for an increase of one standard deviation of the markup ϑ. As expected an increase in ϑ boosts the stock market considerably. Capital investment and consumption go down. (Consumption goes down even in spite of the wealth effect.) Although there is more expenditure in technology adoption we also get that output does slow down.

To summarize, for our benchmark calibration of the model positive changes in θ and ϑ lead to extended increases in the stock price q, with more pronounced effects for changes in ϑ. Widening the range of available technologies Z may actually decrease the stock price q, as the arrival of new technologies depresses the price of existing ones.

5.2 Second-Order Moments

The foregoing exercises should be helpful to gain some intuition for the simulated moments that we now pass to present. We first focus on our benchmark calibration, and then comment on some other extensions which either have no effect on the volatility of stock values or display too much volatility in investment, dividends, and earnings. These simulations are obtained from equilibrium paths with 3000 observations, where the first 1000 observations have been dropped to avoid initial conditions effects.

To isolate the role of debt, in Table 2 we present volatilities when debt is equated to zero, so that equity is the only asset in the economy. As established in our calibration exercise the volatility and persistence of the Solow residual and patents are matched to those of the data. The volatility of the stock market is 14.27 as compared to 31.41 in the data. The volatilities of all real variables are in line with the business cycle literature. We nevertheless observe low volatilities in interest rates, price-dividends, and price-earnings ratios. As discussed below, these low volatilities indicate that stock values may be influenced by financial forces. The volatility of the interest rate is much higher than in related experiments with the neoclassical growth model. If adjustment costs are taken out of the model, then the volatility of investment goes from 8.77 percent to 9.61 percent, and the stock market volatility goes from 14.27 percent to 14.11 percent. Therefore, the introduction of adjustment costs leads to a mild drop in the volatility of capital investment. But it actually has a minor effect on the volatility of the stock market, since adjustment costs may crowd out expenditures in technology adoption.

Table 3 refers to the same volatilities for our baseline calibration of the model with debt. Here, for exogenous considerations the stockholders of the aggregate firm follow debt policy (26) and optimize the objective in (4) over the other decision variables.[6] We see that the volatility of the stock market is now 22.72 percent as compared to 31.41 percent in the data. Therefore, the model can provide for over two-thirds of the actual volatility in the data. The introduction of debt then raises the volatility from 14.27 percent to 22.72 percent, which is about a fifty-percent increase. Observe that this increase is attained at the expense of a higher volatility of dividends and earnings, but these figures seem still plausible. Indeed, in reality dividends is a policy variable: Companies may want to smooth out dividends (and even earnings) or engage in share repurchases. Hall (2001) concludes that pay-outs to debt holders have been quite erratic, and the pay-out yield (the ratio of total cash extracted

[6] As discussed above, in view of the Modigliani-Miller theorem this debt policy may be generated as an equilibrium outcome. Also, below we go over a more general setting in which this debt policy may actually be optimal for exogenously binding constraints under an active monetary policy and interest rate frictions.

by security owners to the market value of equity and debt) has been anything but steady. Moreover, our model includes all expenditures in technology adoption, which in practise may be publicly subsidized, and there are no lags or time-to-build for investment. These simplifying assumptions may affect the volatility of dividends.

From both Tables 2 and 3 we can see that the presence of debt brings the variances of price-dividends and price-earnings much closer to those in the data, and confirms that leverage should be an important ingredient of asset pricing. Leverage provides an additional channel affecting the volatility of dividends. Moreover, the volatility of stock values can now be linked to pay-outs to both debt and equity holders.

We also can see from these tables that the the model lags behind regarding the volatility of the stock market for the long-term cyclical component (i.e., in the frequency band of 8 to 50 years), rather than for the shorter term cyclical component (2 to 8 years). But for the risk-free interest rate the problem is actually inverted: The model generates much less volatility for the shorter term cyclical component. It seems that there are possibly some missing effects of active monetary policies in the short run, and there could be some missing long-term propagation factors coming from global uncertainty, international investment opportunities, and exchange rates.

Figure 9 decomposes the aggregate stock value as in our fundamental asset pricing equation (21). Note that the replacement value of physical capital ranges between 15 to 60 percent of the aggregate stock plus debt, the value of the intermediate goods sector ranges between 30 to 70 percent of the aggregate stock plus debt, whereas the option value of adopting technologies in the future lies between 10 to 30 percent of the aggregate stock plus debt. These figures seem quite plausible. It is common in the literature [e.g., Hall (2001)] to see that the weight of the replacement value of capital in the total stock value may get down to one fourth of its peak value in periods of high activity or technological innovations.

Table 4 reports on the persistence of the variables in the model and the data. As already pointed out, the model is set up to match the persistence of the Solow residual and patents in the data, and the persistence of the price markup is almost the same as the observed persistence of oil prices. As we can see, the model can reproduce fairly well all the autocorrelations observed in the data including the persistence of the stock market which is slightly lower in the model.

Table 5 displays contemporaneous correlations of output with other macro aggregates. Note that these empirical estimates are associated with high standard errors, and hence the confidence intervals are usually fairly wide. In most of the business cycle literature, output in the model is highly correlated with the Solow residual over the shorter term cyclical component, and output is also too highly correlated with the stock market. In our model, we still see a high correlation of output with the Solow residual, but output is more mildly correlated with the stock market. Output also correlates appropriately with dividends, earnings and the price-dividends ratio. Finally, Table 6 contains the contemporaneous correlations of the stock market with the other macro variables. The previous discussion on the correlation of the stock market with output does extend to the components of output. That is, both consumption and investment are not highly correlated with the stock market. The model replicates well the correlation of the stock market with patents and dividends, but it fails regarding the correlation of the stock market with the Solow residual, the risk-free interest rate, earnings, and the price-dividends and price-earnings ratios. These figures seem to suggest that while we have captured the correlation between innovations and stocks, we are probably missing some other determinants of interest rates and stock values. The problem seems quite complex because as discussed in Campbell (1999) the international evidence is not clear on the correlation of the stock market with interest rates.

In summary, for some debt policies the model can generate a sizable part of the volatility of the stock market, and can mimic various correlations and co-movements of macro aggregates in the data. The model fails to match the correlation of the stock market with the Solow residual, interest rates, earnings, and the price-dividends and price-earnings ratios. These failures may arise from lags in investment, active monetary and debt policies, exchange rates, and other financial developments in the global economy. We could test for leads and lags of the stock market over the cycle, and for several correlations on growth rates and returns rather than on levels. A systematic analysis of leads and lags across variables may have to be supported by a more elaborate calibration of the model that allows for a richer dynamic structure for the exogenous stochastic variables.

5.3 Extensions

Taxes : Taxes on corporate profits and dividends could greatly affect the stock market value as well as endogenous investment and dividends [cf. Hall (2001), McGrattan and Prescott (2005), and Poterba (2004)]. We have considered an exogenous process for taxes that is meant to fit the evolution of taxes on dividends in the US from data reported in McGrattan and Prescott (2003). This tax policy had a very small effect on the stock market. We should also remark that some activist fiscal policies on taxes and allowances for depreciation [Auerbach (2009)] have a damping effect in stock market values. After analyzing various arbitrary tax policies, we have concluded that taxes may strongly affect the volatility of asset values as they can change optimal dividend policies, but these desirable changes in the volatility of stock values are only obtained at the expense of excessive volatility in some real variables such as capital investment and consumption.

Labor Frictions : The model with variable labor does not improve significantly the volatility of the stock market [see Rouwenhorst (1995)]. The introduction of variable labor brings the same problems encountered in the business cycle literature [e.g., Hansen and Wright (1992) and Kydland (1995)]. Moreover, sticky wages, labor market rigidities and additional shocks to the labor markets seem to have a minor influence in the long-term volatility of the stock market. General and nested CES production functions for capital and labor and intermediate goods did not lead to much improvement of the current results. As a proxy for labor distortions, we have experimented with a persistent shock in the shares of labor and capital income. This distortion generated too much volatility in capital investment. Figure 10 plots the evolution of the income shares for labor, capital and intermediate goods for our benchmark calibration. We can see that the income share of labor hovers around 60 percent and it is also reasonably volatile. This share seems in accord with casual observations [cf. Krusell et. al. (2000)], although our labor income share does not include the fraction of proprietors income from the intermediate goods sector which should eventually be assigned to labor.

Monetary Policy and Leverage : Monetary policy may affect interest rates and hence the behavior of the stock market. Campbell (1999) reviews the international evidence and argues that there is no a clear correlation between interest rates and the stock market. Also, the finance literature finds that borrowing constraints and other market frictions have minor effects on the volatility of asset prices [e.g., Heaton and Lucas(1996)]. Here we have run two experiments and in both cases we get negative results. The first set of experiments captures the effects of monetary policy by introducing a government bond and public consumption and an exogenous law of motion for the risk-free interest rate. We match the volatility of the exogenous interest rate to that of actual interest rates for the cyclical component at frequencies of 2 to 8 years so that the interest rate is more volatile than in our original calibrated model. We even take account of the equity premium by introducing an interest rate subsidy paid by lump-sum taxation, and as in (7) we bound the amount of short-selling of the bond. Then, a debt policy of the form (26) could be optimal for the aggregate firm. Briefly, the effects of this interest-rate policy on the stock market and other macro aggregates are rather minor for reasonable bounds on the volatility and persistence of dividends. In a second set of experiments we introduce borrowing constraints that may limit the ability to invest in a given period. Rather than fixing those constraints to some constant limits as most of the literature [e.g., Heaton and Lucas (1996)], we assume that the role of monetary policy is to set up the level of leverage over time [cf. Geanakoplos 2009]. We consider different stochastic policies that exogenously affect the evolution of leverage, and all the results point in the direction that the stock market will remain essentially unaffected; for if not, dividends would fluctuate wildly.

6 Concluding Remarks

This paper explores macroeconomic determinants of asset price volatility in a general equilibrium model with lags in technology adoption. Technologies are embedded in the production of new varieties of intermediate goods. Stocks are impacted by the arrival of new technologies and other shocks affecting the economy. Our analysis builds on an asset pricing equation that decomposes the value of the stock market into the replacement value of capital, the value of existing technologies, and the option value of adopting new technologies.

This general equilibrium setting imposes a lot of discipline in our numerical experiments: Desired levels of volatility of asset prices usually come with pronounced changes in macroeconomic fluctuations. Since the value of the stock market can be computed as the expected discounted sum of future dividends, the lack of volatility of stock prices is already familiar from the early empirical studies of LeRoy and Porter (1981) and Shiller (1981). But it becomes much harder to deal with in model simulation as one cannot postulate exogenous specifications for the dividend process. To get an idea of related computations, in our simulations of the neoclassical growth model a volatility (i.e. standard deviation) of investment of the order of 6.5 percent translates into a volatility of the capital stock of the order of 2.3 percent. As the volatility of the stock market is about 31.41 percent it is then not surprising that many candidate variables will have a limited role in accounting for observed fluctuations of stock market values as they would require implausible fluctuations in other sectors.

Thus, we find that taxes, monetary policy interventions, financial and labor market frictions, and various CES formulations of the aggregate production function for labor and capital have minor effects on the long-term volatility of asset market values. Technological innovations can have significant effects if they come along with high markups and TFP changes. Leverage – short-term and long-term debt – appears to be an important source of stock market volatility, but it cannot be arbitrarily increased in the model as it leads to excessive volatility of dividends. Overall, we get that in our model the volatility of the stock market is of the order of 22.72 percent as opposed to 31.41 percent in the data. Hence, the model can deliver about 70 percent of the observed volatility. Paradoxically, the model gets close to the volatility of the cyclical component over the window of frequencies between 2 and 8 years, but does not match well the volatility of interest rates. Therefore, it appears that we may be missing activist monetary policies and some structural forces or long-term propagation mechanisms affecting the economy and the global financial markets.

Our model has other advantages over the traditional business cycle literature in which dividends and earnings are not sufficiently volatile[7] and the risk-free rate has almost zero variability. Our model displays higher volatility of dividends and earnings than in the data,[8] and a better volatility of the risk-free rate. Moreover, there are many other implications for financial markets that come from our fundamental asset pricing equation (21) since the stock

[7] In a closely related exercise that makes full use of our analytical work, Comin, Gertler and Santacreu (2009) claim to get rather low volatility of dividends, and hence we are doubtful that they could account for the volatility of stock values under standard parameterizations of preferences, technologies, and capital adjustment costs.

[8] As already discussed, companies may want to smooth out dividends; moreover, some further adjustments are needed in reported dividends figures to bring the model closer to the data [Hall (2001)].

market anticipates news and the evolution of technological innovations and other exogenous variables. In our model stock values are leading indicators which are only mildly correlated with output and other real variables [Campbell (1999)].

Let us briefly mention two extensions of this work. First, our calibration exercise could be improved to account for cross correlations over time. This calibration refinement may actually need to allow for a richer dynamic structure of the exogenous shocks. Hence, the state space may need to be expanded. This expansion of the state space could be effective to obtain higher long-term volatility. Second, the macro finance literature has struggled with several puzzles and empirical regularities, and the present model may be helpful to address some further issues guided by our fundamental asset pricing equation (21).

7 Appendix

Proof of Proposition 3.1: Recall that $d_t = d_t^f + d_t^i$, where d_t^f and $d_t^i \equiv \pi_t A_t - H_t(Z_t - A_t)$ are dividends generated by the final and intermediate goods sectors, respectively. Then, first order condition (18) for the household's problem jointly with the equilibrium condition $a_t = 1$ and the transversality condition $\lim_{T\to\infty} E_t \left\{ \frac{\eta_T}{\eta_t} q_T a_T \right\} = 0$ imply

$$q_t = E_t \left\{ \sum_{r=t+1}^{\infty} \frac{\eta_r}{\eta_t} d_r \right\}. \tag{27}$$

Similarly, using the Euler equation for capital accumulation and the transversality condition for the aggregate firm

$$\lim_{T\to\infty} E_t \left\{ \frac{\eta_T}{\eta_t} (p_t^I k_{T+1} - B_T) \right\} = 0, \tag{28}$$

we obtain

$$p_t^I k_{t+1} - B_t = E_t \left\{ \sum_{r=t+1}^{\infty} \frac{\eta_r}{\eta_t} d_r^f \right\}. \tag{29}$$

Finally, we must show

$$V_t^+ A_t + J_t^+ (Z_t - A_t) + \xi_t = E_t \left\{ \sum_{r=t+1}^{\infty} \frac{\eta_r}{\eta_t} d_r^i \right\}. \tag{30}$$

To this end, note that

$$
\begin{aligned}
&V_t^+ A_t + J_t^+ (Z_t - A_t) + \xi_t \\
&= E_t\left\{\frac{\eta_{t+1}}{\eta_t}\phi V_{t+1}\right\} A_t + E_t\left\{\frac{\eta_{t+1}}{\eta_t}\phi[\lambda_t(H_t)V_{t+1} + (1-\lambda_t(H_t))J_{t+1}]\right\}(Z_t - A_t) + \\
&+E_t\left\{\frac{\eta_{t+1}}{\eta_t}[J_{t+1}(Z_{t+1} - \phi Z_t) + \xi_{t+1}]\right\} \\
&= E_t\left\{\frac{\eta_{t+1}}{\eta_t}\ [V_{t+1}A_{t+1} + J_{t+1}(Z_{t+1} - A_{t+1}) + \xi_{t+1}]\right\},
\end{aligned}
$$

where the last equality comes after rearranging terms and letting $A_{t+1} = \phi\lambda_t(H_t)[Z_t - A_t] + \phi A_t$, and $Z_{t+1} - A_{t+1} = Z_{t+1} - \phi\lambda_t(H_t)[Z_t - A_t] - \phi A_t$. Hence,

$$
\begin{aligned}
&V_t^+ A_t + J_t^+ (Z_t - A_t) + \xi_t \\
&= E_t\left\{\frac{\eta_{t+1}}{\eta_t}\left[d_{t+1}^i + V_{t+1}^+ A_{t+1} + J_{t+1}^+(Z_{t+1} - A_{t+1}) + \xi_{t+1}\right]\right\}.
\end{aligned}
$$

Then, iterating forward this equation and the absence of bubbles in equilibrium [see Santos and Woodford (1997)] imply that (30) is satisfied.

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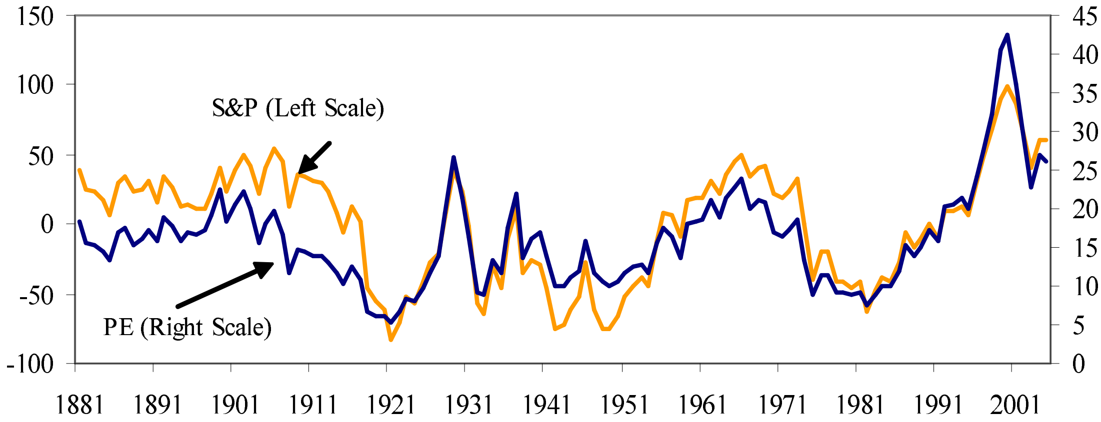
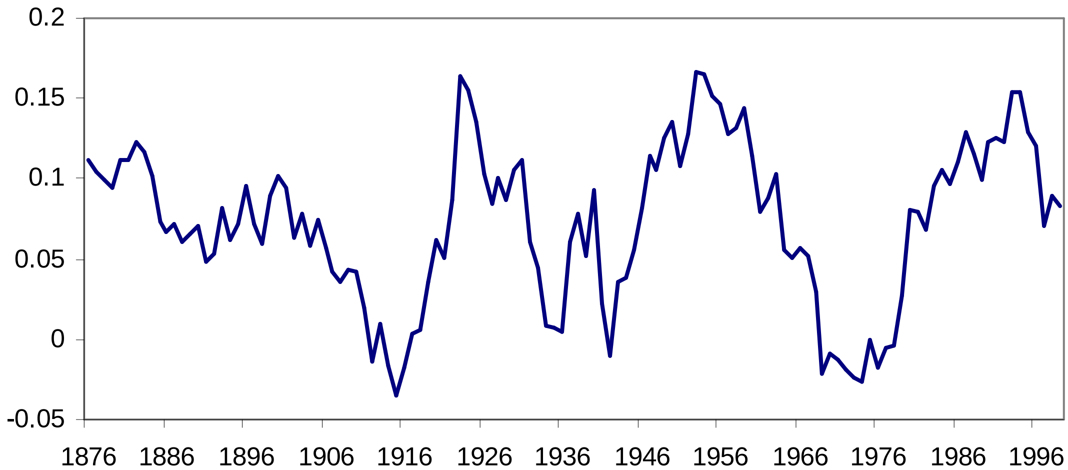
Figure 1: Evolution of S&P and PE



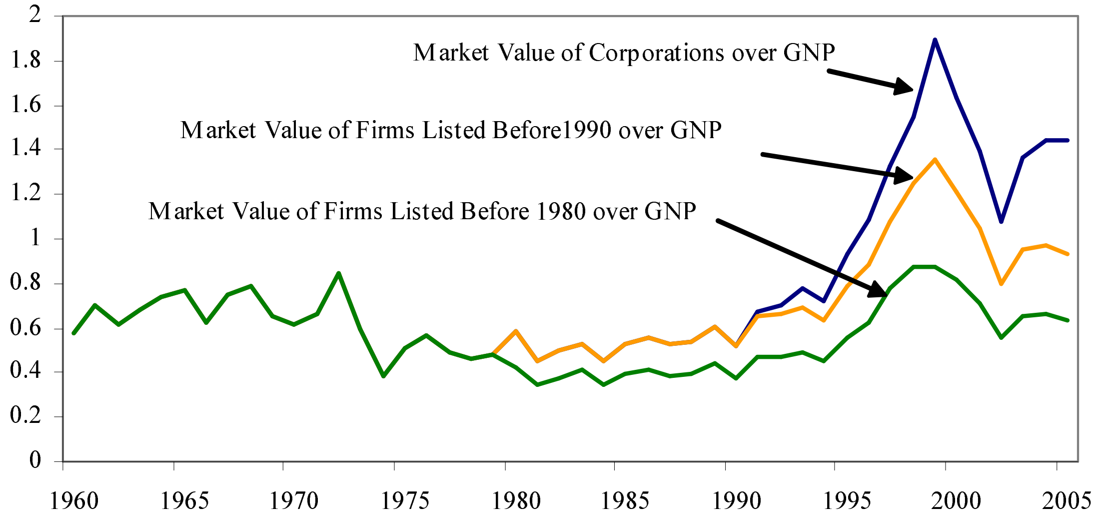
Source: Robert Shiller's web site: http://www.econ.yale.edu/ shiller/data.htm.
Note: Detrended S&P price index and Price-Earnings ratio. Annual data. Price-Earnings ratio is computed as current price over the average of earning for previous ten years.

Figure 2: Real Rate of Return of S&P



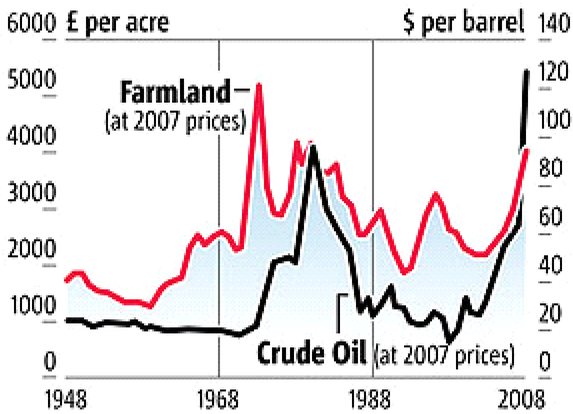
Source: Robert Shiller's web site.
Note: Centered ten-year moving average of the continuously compounded log return of S&P index. Annual data.

Figure 3: Market Value of Different Vintages



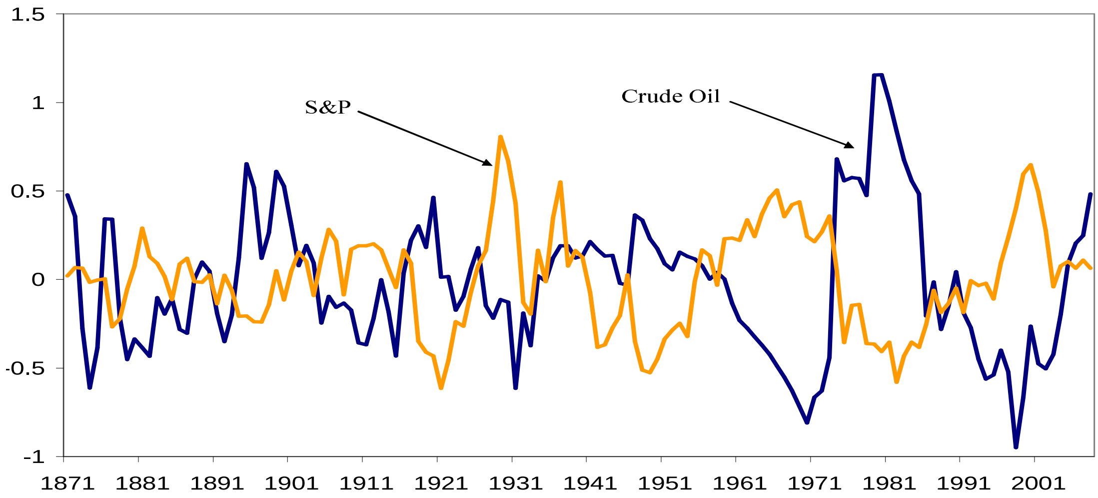
Sources: CRSP and NIPA.
Note: Market value of corporations over GNP for different vintages. Annual data from 1960 to 2005.

Figure 4: Farmland and Crude Oil Prices



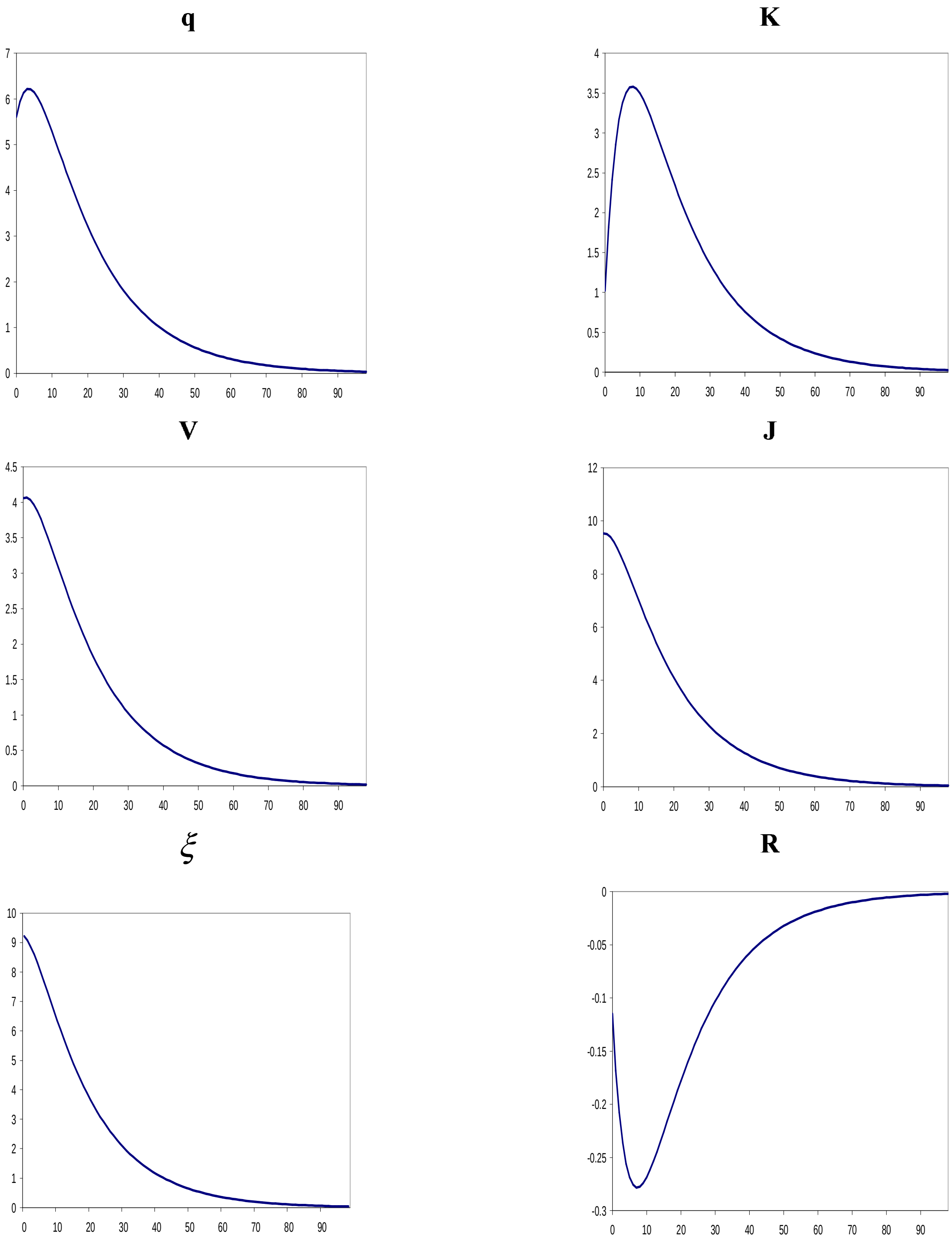
Source: http://www.thisismoney.co.uk

Figure 5: Evolution of S&P and Crude Oil Price



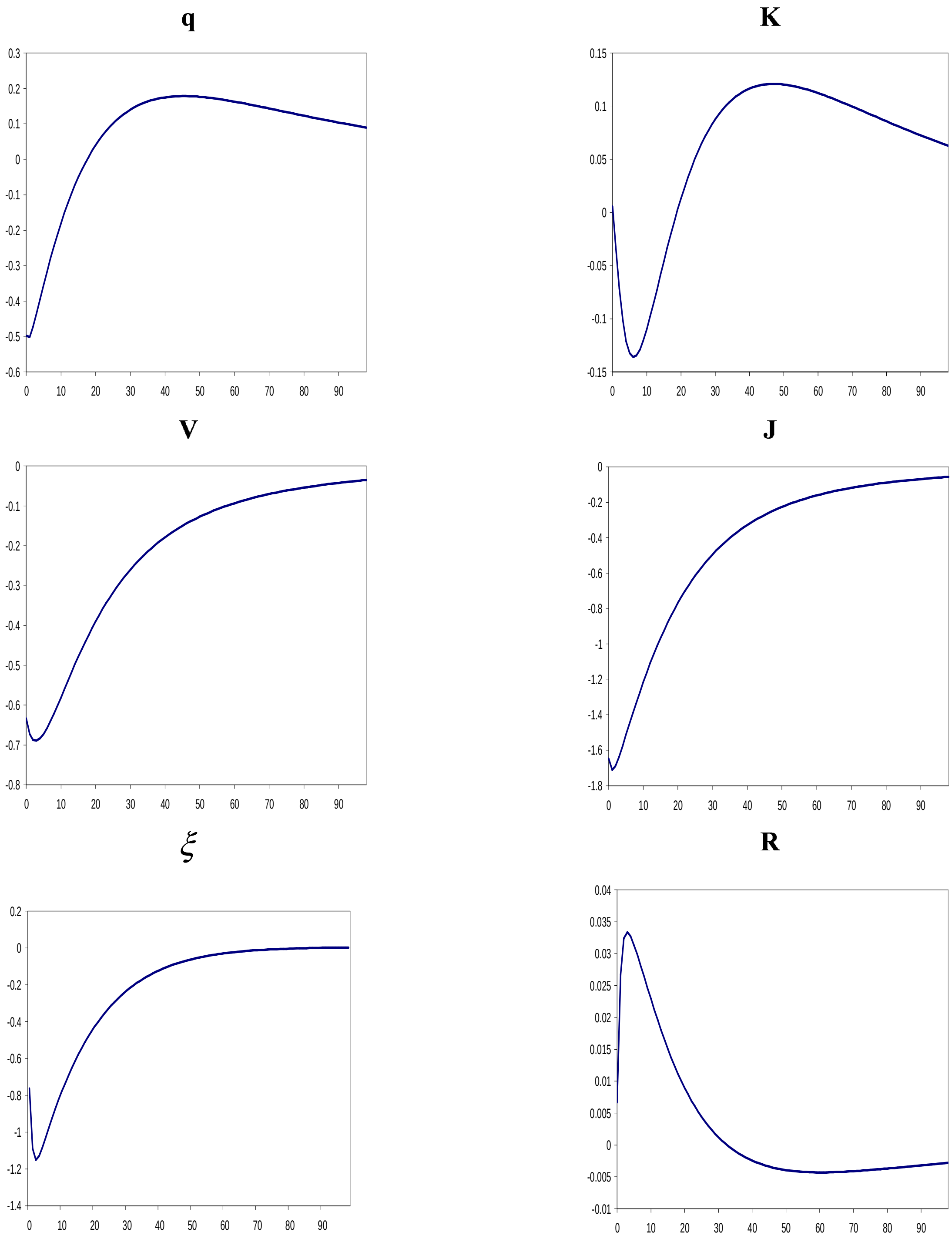
Sources: Robert Shiller's web site, BP Statistical Review of World Energy 2008 and US Department of Energy.
Note: Real S&P price index and U.S. Real Crude Oil Price (Dollars per Barrel). Annual data. The logarithm of each series has been filtered for frequencies of 2-50 years.

Figure 6: Impulse–Response Functions to a Shock in θ



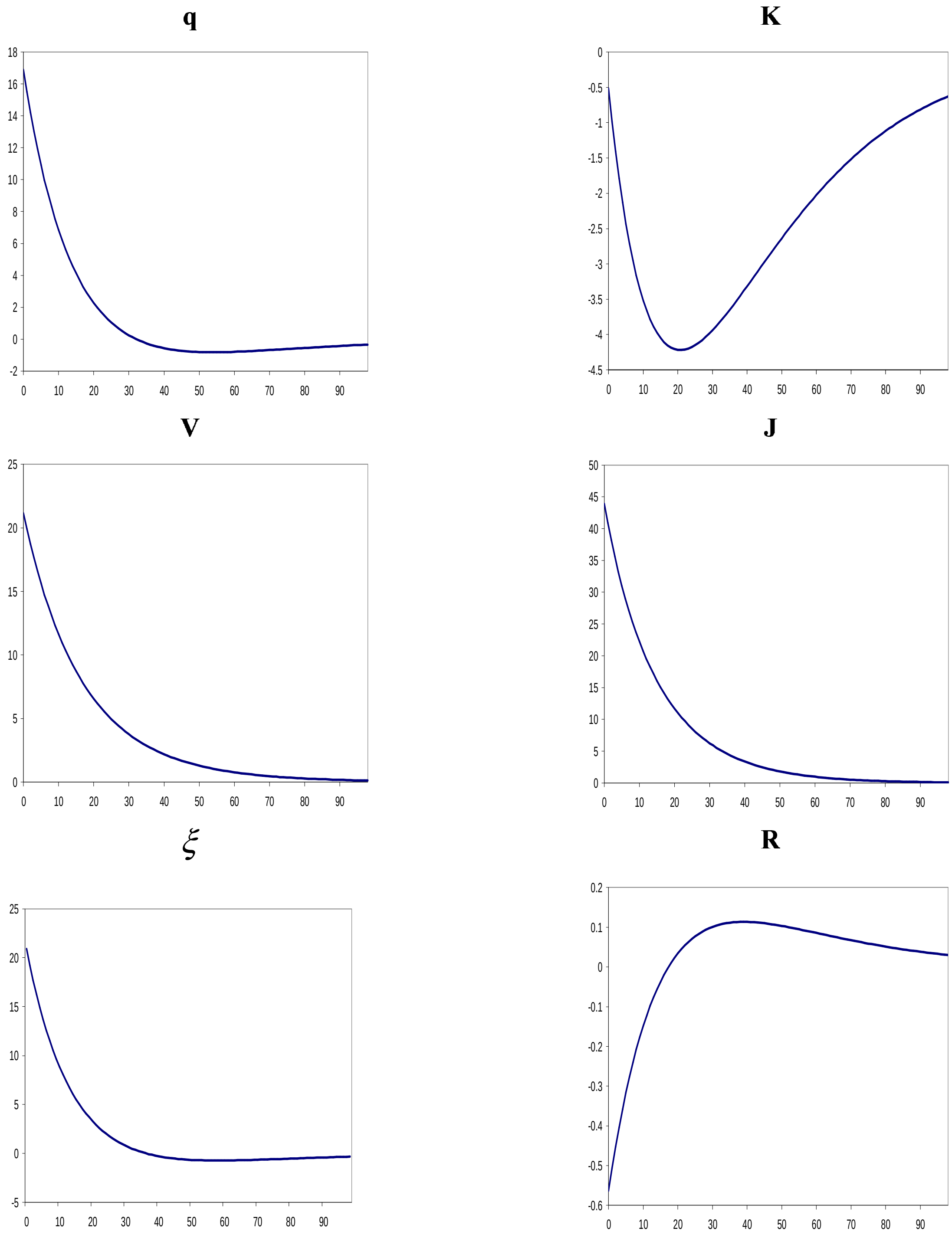
Note: Response of each variable to a positive, one–standard–deviation shock in θ. The y-axis measures percentage deviation from the steady-state value and the x-axis measures time in years. Variable q represents the value of the stock market, K denotes capital, V is the value of an installed technology, J is the value of a not–adopted technology, ξ is the present value of technologies available in the future, and R is the one-year risk-free interest rate.

Figure 7: Impulse–Response Functions to a Shock in x



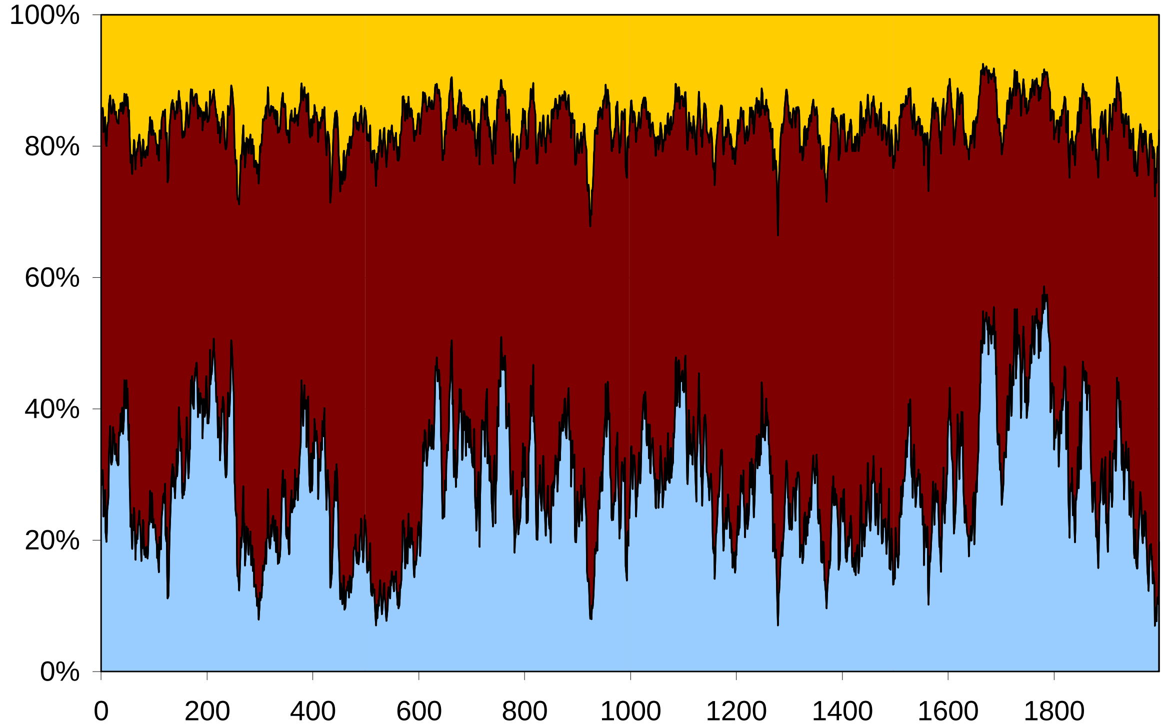
Note: Response of each variable to a positive, one–standard–deviation shock in x.

Figure 8: Impulse–Response Functions to a Shock in ϑ



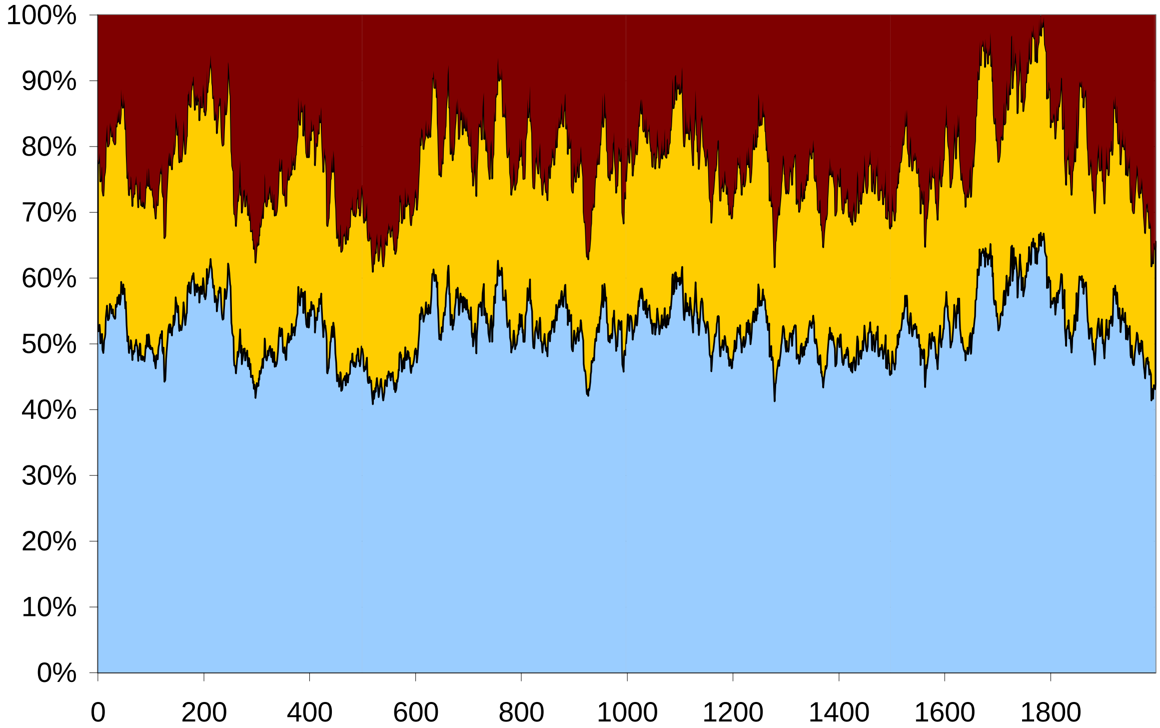
Note: Response of each variable to a positive, one–standard–deviation shock in ϑ.

Figure 9: Components of the Stock Market Value



Note: The bottom, blue area is the relative value of capital, the middle, red area is the relative value of adopted technologies, and the top, orange/yellow area is the relative value of unadopted technologies.

Figure 10: Factor Income Shares



Note: The bottom, blue area is the labor income share, the middle, orange/yellow area is the capital income share, and the top, red area is the share of intermediate sector's profits.

Table 1: Parameter Values

β	0.95
σ	4
α	0.33
γ	0.70
δ	0.09
ρ	-0.60
ς	8
κ	0.8
λ	0.166
ϕ	0.98
R_0	-0.0038
R_1	0.222
ψ^{θ}	0.82
ψ_0^{ϑ}	-0.145
ψ_1^{ϑ}	0.965
ψ_2^{ϑ}	0.1508
ψ^{x}	0.315
σ_{θ}	0.014
σ_{ϑ}	0.207
σ_{x}	0.23
$Corr(\varepsilon^{\theta}, ln(\varepsilon^{\vartheta}))$	0.90
$Corr(\varepsilon^{\theta}, \varepsilon^{x})$	0.49
$Corr(\varepsilon^{x}, ln(\varepsilon^{\vartheta}))$	0.70
τ^{Debf}	0.20
τ^{Debv}	0.50

Table 2: Standard Deviation

	2-50		2-8		8-50	
	Data	Model	Data	Model	Data	Model
Output	3.46	2.94	1.98	1.14	2.81	2.71
	(2.65; 4.27)		(1.53; 2.42)		(2.19; 3.42)	
Consumption	1.92	2.17	0.68	0.72	1.79	2.05
	(1.38; 2.47)		(0.55; 0.81)		(1.29; 2.28)	
Investment	8.86	8.77	4.35	3.73	7.66	7.93
	(7.09; 10.65)		(3.59 ; 5.10)		(5.69; 9.63)	
Solow Residual	2.43	2.43	1.14	1.19	2.12	2.17
	(1.95; 2.90)		(0.85; 1.43)		(1.69; 2.55)	
Patents	8.03	8.03	3.05	3.31	7.37	7.34
	(6.33; 9.73)		(2.26; 3.85)		(5.65; 9.09)	
Stock Market	31.41	14.27	9.28	5.47	29.95	13.18
	(25.22; 37.58)		(7.32; 11.25)		(24.08; 35.83)	
Dividends	10.27	9.25	3.37	3.32	9.59	8.63
	(8.21; 12.33)		(2.01; 4.73)		(7.47; 11.71)	
Earnings	17.92	9.97	11.68	4.66	12.53	8.81
	(13.45; 22.39)		(8.00; 15.36)		(10.20; 14.87)	
Interest Rate	2.48	0.80	1.68	0.30	1.80	0.74
	(1.77; 3.18)		(1.15; 2.22)		(1.16; 2.44)	
Price-Dividends	32.73	6.38	14.20	2.62	28.95	5.82
	(25.92; 39.54)		(11.39; 17.00)		(21.25; 36.66)	
Price-Earnings	24.65	6.13	9.05	1.95	22.89	5.81
	(19.87; 29.44)		(7.12; 10.97)		(18.48; 27.30)	

Note: Both data and model's simulations have been filtered for various frequency bands.

Table 3: Standard Deviation

	2-50		2-8		8-50	
	Data	Model	Data	Model	Data	Model
Output	3.46	2.94	1.98	1.14	2.81	2.71
	(2.65; 4.27)		(1.53; 2.42)		(2.19; 3.42)	
Consumption	1.92	2.17	0.68	0.72	1.79	2.05
	(1.38; 2.47)		(0.55; 0.81)		(1.29; 2.28)	
Investment	8.86	8.77	4.35	3.73	7.66	7.93
	(7.09; 10.65)		(3.59 ; 5.10)		(5.69; 9.63)	
Solow Residual	2.43	2.43	1.14	1.19	2.12	2.17
	(1.95; 2.90)		(0.85; 1.43)		(1.69; 2.55)	
Patents	8.03	8.03	3.05	3.31	7.37	7.34
	(6.33; 9.73)		(2.26; 3.85)		(5.65; 9.09)	
Stock Market	31.41	22.72	9.28	8.72	29.95	20.98
	(25.22; 37.58)		(7.32; 11.25)		(24.08; 35.83)	
Dividends	10.27	22.18	3.37	8.24	9.59	20.59
	(8.21; 12.33)		(2.01; 4.73)		(7.47; 11.71)	
Earnings	17.92	28.09	11.68	13.92	12.53	24.40
	(13.45; 22.39)		(8.00; 15.36)		(10.20; 14.87)	
Interest Rate	2.48	0.80	1.68	0.30	1.80	0.74
	(1.77; 3.18)		(1.15; 2.22)		(1.16; 2.44)	
Price-Dividends	32.73	12.70	14.20	7.32	28.95	10.38
	(25.92; 39.54)		(11.39; 17.00)		(21.25; 36.66)	
Price-Earnings	24.65	18.02	9.05	11.59	22.89	13.80
	(19.87; 29.44)		(7.12; 10.97)		(18.48; 27.30)	

Note: Both data and model's simulations have been filtered for various frequency bands.

Table 4: Autocorrelation

	2-50		2-8		8-50	
	Data	Model	Data	Model	Data	Model
Output	0.64	0.81	0.11	0.04	0.89	0.95
	(0.47; 0.80)		(-0.14; 0.38)		(0.77; 1)	
Consumption	0.84	0.86	0.22	0.06	0.93	0.95
	(0.68; 0.99)		(-0.03; 0.48)		(0.80; 1)	
Investment	0.72	0.78	0.14	0.04	0.91	0.94
	(0.55; 0.88)		(-0.11; 0.40)		(0.77;1)	
Solow Residual	0.73	0.73	0.10	0.03	0.92	0.93
	(0.60; 0.87)		(-0.16; 0.36)		(0.83; 1)	
Patents	0.81	0.81	0.08	0.23	0.93	0.93
	(0.70; 0.97)		(-0.17; 0.34)		(0.88; 1)	
Stock Market	0.87	0.81	-0.02	0.05	0.96	0.95
	(0.75; 0.98)		(-0.28; 0.24)		(0.87; 1)	
Dividends	0.81	0.83	0.21	0.04	0.94	0.96
	(0.66; 1)		(-0.02; 0.44)		(0.84; 1)	
Earnings	0.51	0.69	0.07	-0.09	0.86	0.95
	(0.31; 0.70)		(-0.19; 0.34)		(0.75; 0.96)	
Interest Rate	0.47	0.82	-0.03	0.01	0.90	0.96
	(0.19; 0.76)		(-0.28; 0.24)		(0.75; 1)	
Price-Dividends	0.76	0.57	-0.07	-0.17	0.95	0.94
	(0.62; 0.90)		(-0.26; 0.11)		(0.85; 1)	
Price-Earnings	0.82	0.46	-0.09	-0.22	0.95	0.95
	(0.67; 0.96)		(-0.20; 0.01)		(0.86; 1)	

Note: Both data and model's simulations have been filtered for various frequency bands.

Table 5: Correlation with Output

	2-50		2-8		8-50	
	Data	Model	Data	Model	Data	Model
Consumption	0.74	0.95	0.83	0.96	0.74	0.95
	(0.49; 0.98)		(0.70; 0.98)		(0.42; 1)	
Investment	0.58	0.86	0.83	0.93	0.50	0.85
	(0.22; 0.94)		(0.68; 0.97)		(0.04; 0.95)	
Solow Residual	0.62	0.94	0.83	0.99	0.55	0.93
	(0.33; 0.89)		(0.69; 0.98)		(0.18; 0.89)	
Patents	0.31	0.33	-0.22	0.29	0.49	0.34
	(0.03; 0.58)		(-0.48; 0.02)		(0.23; 0.76)	
Stock Market	0.47	0.66	-0.04	0.75	0.61	0.64
	(0.17; 0.73)		(-0.30; 0.21)		(0.30; 0.93)	
Dividends	0.57	0.45	0.20	0.59	0.66	0.43
	(0.33; 0.91)		(-0.05; 0.45)		(0.41; 0.93)	
Earnings	0.40	0.58	0.45	0.50	0.48	0.60
	(0.22; 0.61)		(0.22; 0.68)		(0.20; 0.72)	
Interest Rate	-0.26	-0.63	-0.50	-0.64	-0.11	-0.62
	(-0.63; 0.11)		(-0.73; -0.28)		(-0.56; 0.34)	
Price-Dividends	0.22	0.38	-0.40	0.23	0.43	0.44
	(-0.05; 0.50)		(-0.58; -0.22)		(0.09; 0.77)	
Price-Earnings	0.37	-0.07	-0.11	-0.03	0.52	-0.09
	(0.05; 0.70)		(-0.30; 0.06)		(0.17; 0.87)	

Note: Both data and model's simulations have been filtered for various frequency bands.

Table 6: Correlation with the Stock Market

	2-50		2-8		8-50	
	Data	Model	Data	Model	Data	Model
Output	0.47 (0.21; 0.72)	0.66	-0.04 (-0.29; 0.21)	0.75	0.61 (0.26; 0.94)	0.64
Consumption	0.61 (0.25; 0.95)	0.82	0.10 (-0.13; 0.34)	0.88	0.68 (0.29; 1)	0.81
Investment	0.14 (-0.30; 0.58)	0.25	-0.21 (-0.46; 0.04)	0.53	0.21 (-0.27 0.67)	0.19
Solow Residual	0.09 (-0.35; 0.50)	0.53	0.30 (0.11; 0.87)	0.73	0.05 (-0.46; 0.55)	0.48
Patents	0.53 (0.27; 0.78)	0.57	-0.07 (-0.33; 0.18)	0.25	0.63 (0.34; 0.89)	0.64
Dividends	0.75 (0.53; 0.97)	0.84	0.25 (0; 0.50)	0.62	0.81 (0.58; 1)	0.87
Earnings	0.19 (-0.08; 0.48)	0.76	0.09 (-0.16; 0.35)	0.55	0.28 (-0.16; 0.70)	0.82
Interest Rate	0.18 (-0.24; 0.61)	-0.98	-0.06 (-0.26; 0.15)	-0.97	0.28 (-0.25; 0.80)	-0.98
Price-Dividends	0.85 (0.69; 1)	0.32	0.90 (0.39; 1)	0.48	0.91 (0.72; 1)	0.28
Price-Earnings	0.95 (0.86; 1)	0.06	0.59 (0.53; 0.65)	0.08	0.96 (0.87; 1)	0.06

Note: Both data and model's simulations have been filtered for various frequency bands.