Providing Managerial Incentives: Do Benchmarks Matter?*

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Abstract

In this paper we revisit the question of whether relative performance-adjusted contracts provide portfolio managers with adequate incentives to gather information. We show that when portfolio managers are constrained in their ability to short-sell, relative performance-adjusted or *linear* 'benchmarked' contracts can provide incentives for gathering more precise information. We also provide conditions under which they emerge as optimal linear contracts contracts.

We show that when borrowing constraints are present, the risk-averse manager's optimal effort is, under certain conditions, an increasing function of her share on the portfolio's return. In addition, the optimal contract offered by the risk-neutral client to the risk-neutral, short-selling constrained manager is shown to be a benchmarked contract for any value of the constraint. In the limit, as short-selling constraints are relaxed, the optimal contract is *first-best*.

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1 Introduction

In this paper we revisit the question of whether relative performance-adjusted contracts provide portfolio managers with adequate incentives to gather information. We show that when portfolio managers are constrained in their ability to short-sell, relative performance-adjusted or *linear* 'benchmarked' contracts can provide incentives for gathering more precise information. We also provide conditions under which they emerge as optimal linear contracts contracts.

Relative performance-adjusted contracts are compensation schemes for which the manager's wage consists of a percentage on her portfolio's net return over a given benchmark index plus, possibly a performance-free rent. Benchmark-adjusted compensation is allowed in the US by Section 205 of the Investment Advisers Act. According to the regulation, the fee must be a fulcrum fee where the incentive (penalty) component rises or falls symmetrically with the performance of the fund. Additionally, performance must be measured against an appropriate independent index, rather than in absolute terms. Empirical evidence shows that, even if not in an explicit way, the compensation of portfolio managers often depends upon their performance relative to a benchmark. A survey by Del Guercio and Tkac (1998) reports that 59% of mutual fund investors compared fund performance to that of an index. This percentage is lower than that in the pension fund industry where virtually in all cases manager's portfolio performance is compared to an index. Sirri and Tufano (1997) find that mutual fund investors seem to base their fund purchase decision on prior performance information, but do so asymmetrically, massively investing in funds that performed well (relative to a benchmark) the prior period but failing to flee lower performing funds at the same rate.

Households invest money through managers possibly because managers are better informed. So, can households use benchmarked contracts to screen managers with better information? Bhattacharya and Pfleiderer (1985) have shown that linear contracts, like the benchmarked contracts defined above, can indeed be used as a mechanism to screen out 'better informed' managers. In this context, an interesting question posed by Stoughton (1993) is whether linear contracts induce managers to improve the 'quality' (meaning precision) of their information. Stoughton (1993) as well as Admati and Pfleiderer (1997) show that benchmarked contracts do not provide managers with incentives to collect better information.

In this paper we show that Soughton (1993) and Admati and Pfleiderer's (1997) results are critically dependent on the assumption that manager's capacity to short-sell is unconstrained. We show that with short-selling constraints benchmarked contracts can induce effort on the part of both risk-averse and risk-neutral managers to gather better information. In our model, the *non-incentive* result occurs as a special case when the short-selling constraints tend to infinity in the limit.

The intuition behind our result is as follows: In forming her optimal portfolio, a manager whose reward depends on a benchmarked compensation scheme takes into account the percentage she gets to retain from the portfolio's return. A change in this percentage would not change her expected wealth because she can appropriately change the composition of her portfolio. As a result, a change in her share of the portfolio's return cannot change any of her actions. However, this is only true if she can short-sell to any extent she wants to. However, when she is constrained in her ability to short-sell this is no longer true. Now, she may no longer be able to form the portfolio she desires. Constrained in such a manner, the manager can increase her expected wealth only by increasing the precision of her signal (i.e. by reducing the variance of her returns). In our model, the manager can increase signal precision by putting in more effort. The extent to which she will want to expend more effort in gathering more precise information will depend on her compensation scheme, her risk aversion and her marginal disutility of effort.

Provided the manager is short-selling constrained, would her client (the household) offer her a benchmarked contract? Further, would the client impose short-selling constraints on the manager? For analytical tractability we pose these questions in a setting where both the client and the manager are risk neutral.[1] In this framework, Heinkel and Stoughton's (1994) model shows that benchmarking is optimal (in the class of linear contracts) for a particular exogenously given short-selling constraint. We show that, for *any* exogenously given short-selling constraint, the optimal linear contract is benchmarked and the manager receives all the marginal returns to the portfolio.[2] As the short-selling constraint goes to infinity the optimal contract remains a benchmarked contract. In the limit, the contract achieves the *first-best*. Therefore, the optimal contract does not impose any short-selling constraint, however it is benchmarked. Further, the need to screen out managers vanishes. Note that this is different from claiming that benchmarked constraints fail to achieve screening.

The fact that short-selling constraints are not imposed in the risk-neutral case does not imply that such a result should necessarily hold in the risk-averse case. Note that in the risk-neutral case, even in the absence of short-selling constraints, benchmarked contracts provide the manager with incentives to gather more precise information. This, we show, cannot happen in the risk-averse case. Our conjecture is that, in the latter case, the benefits of relaxing the short-selling constraint have to be traded off against the loss of inducing the manager to collect better information. At this point we may have both short-selling constraints and benchmarked contracts in the optimal menu of contracts.

The rest of the paper is organized as follows. Section 2 will introduce the model. The optimal portfolio of the risk-averse manager will be derived. It will be shown that when borrowing constraints are present the manager's optimal effort is, under certain conditions, an increasing function of her share on the portfolio's return. Section 3 analyzes the optimal contract offered by the risk- neutral client to the risk-neutral,

[1] In addition, since in this setting risk sharing is redundant, if benchmarking arises at all it will be due to effort enhancing reasons.

[2] Our results should not be interpreted in a quantitative sense. They are only suggestive.

short-selling constrained manager. The optimal contract will be explicitly derived. For any value of the constraint, the optimal contract will be shown to be a benchmarked contract. In the limit, the optimal contract is *first-best* and the need to screen managers vanishes.

2 The risk-averse manager

Suppose a given investor endowed with a unit of wealth who faces an investment opportunity set that consists just of two assets: a risky asset with known gross rate of return $\tilde{x}$ and a bond paying a risk-less gross rate of return r. The investment horizon is just one period. At the beginning of the period (t_0) the investor decides the optimal portfolio allocation so as to maximize her expected utility of wealth at t_1. Let θ denote the amount of money invested in the risky asset. Therefore, investor's (random) wealth at the end of the period will be

$$\tilde{W}(\theta) = (1-\theta)r + \theta\tilde{x}. \tag{1}$$

Given that the initial wealth of the investor is one unit, any θ strictly greater than one or strictly smaller than zero would represent a leveraged position at investor's optimal portfolio: either she short-sells the risk-less bond ($\theta > 1$) going long on the risky asset or vice versa ($\theta < 0$).

The optimal portfolio of the manager

The manager will be assumed to have valuable information for selecting stocks but no information about the pervasive or market-wide factors that affect stock price movements. In the literature of investment performance[3] this is known as *selectivity* versus *timing* ability. In our model there exists only one *timing* portfolio: the bond. This represents the (informational) benchmark for households. The return on this benchmark is therefore a known and fix constant that will be normalized to $r = 0$.[4] The manager is supposed to have 'private' *selectivity* information concerning

[3] A partial list of references includes Fama (1972), Jensen (1972), Merton (1981), Admati, Bhattacharya, Pfleiderer and Ross (1986) and Connor and Korajczyk (1995).

[4] In our model, the client is interested in 'active' management. This means she is only willing to pay for valuable 'private' information she cannot learn herself. By normalizing the return on the bond to zero the return on the manager's portfolio is (implicitly) a benchmarked return. In other words, if the manager behaves as a 'passive' investor (i.e. replicates the benchmark) her expected performance-adjusted reward will be zero.

We could alternatively assume, like in Zwiebel (1995), the benchmark is a risky asset and that only a zero-measure of managers invest in the non-benchmark asset. Then, assuming for instance that asset's returns are normally distributed, the common support of the real line for both returns plus the zero-measure assumption allows the market to ignore the non-benchmark asset in forming inferences. If there is a sufficiently large number of clients and managers the market will accurately infer the (ex-post) benchmark's return from the market average.

the return on the risky asset. According to the standard definition of *selectivity* information (see, e.g., Admati, Bhattacharya, Pfleiderer and Ross, *ibid*) this kind of information is *statistically independent* of the return on the timing portfolio. In our model, since the timing portfolio has been identified with a constant, the informational advantage of the manager is thus defined as *selectivity* ability.

As it is standard in this literature, let us model the informational advantage of the manager as a signal on the risky asset return shuch that

$$\tilde{y} = \tilde{x} + \tilde{\epsilon}, \tag{2}$$

with $\tilde{\epsilon}$ the noise term. To simplify the notation (and without loss of generality) let $\tilde{x}$ be now distributed as a standard normal variable. This is merely a normalization that will help to keep the model as parsimonious as possible. We assume $\tilde{\epsilon} \sim \mathcal{N}(0, \sigma^2)$, with $\sigma^2 < \infty$ such that higher σ^2 implies a less precise information signal. Finally, the return on the risky asset and the noise term are assumed to be uncorrelated.

According to (2), and provided the distributional assumptions on $\tilde{x}$ and $\tilde{\epsilon}$, $\tilde{y}$ is normally distributed:

$$\tilde{y} \sim \mathcal{N}\left(0, \frac{1}{H}\right), \tag{3}$$

with $H = (1 + \sigma^2)^{-1} \leq 1$ the signal precision.

After receiving the signal and before any decision is made the manager updates her believes about the conditional (on the signal realization) distribution of the risky asset. By Bayes rule[5]

$$\tilde{x}\,|\,y \sim \mathcal{N}\left(Hy, 1 - H\right). \tag{4}$$

The manager has to put some effort e prior to learning the value of the signal. Like in Stoughton (1993) it will be assumed that the variance of the signal's noise is a hyperbolic function of effort, such that

$$e = \frac{1}{\sigma^2}.$$

This implies that the (posterior) precision of the manager's private signal is an increasing and concave function of effort. More concretely,

$$H(e) = \frac{e}{1 + e}. \tag{5}$$

Given (5) equation (4) can be re-written as follows:

[5] We will follow the standard notation whereby a symbol with a tilde on top will represent the variable and the same symbol, without a tilde, its realization.

$$\tilde{x}\,|\,y\;\sim\mathcal{N}\left(\frac{e}{1+e}y,\frac{1}{1+e}\right).$$

Our analysis will be restricted to a very specific (though standard) type of the contracts: linear performed-adjusted contracts. According to this type of contracts the manager's fee consits of a percentage $0\leq\alpha\leq 1$ of the return on the portfolio she manages on behalf of her client, the household. Together with this performance-adjusted component the contract may include a (performace-free) fix wage F.

We set $\underline{\kappa}=0$ and $\overline{\kappa}=\kappa>0$. This means the manager is not allowed to short-sell the risky asset. He is allowed, however, to short-sell the risfreee asset and leverage her investment on the risky asset up to $\kappa-1$ (assuming $\kappa>1$). In case $\kappa<1$ the manager is then obliged to invest at least $1-\kappa$ in the riskfree asset. According to equations (3) and (4), $E_y[\tilde{x}\,|\,y]=0$. Thus, since we normalized $r=0$, this preserves the (ex-ante) symmetry with regard to the shot-selling constraints.[6] Therefore, confining the analysis to one short-selling constraint will simplify the algebra without qualitatively affecting the results.

We will assume that all the investment is carried out by the manager. The client will offer the manager a contract (F,α,κ). Effort has a private disutility to the manager given by the function $V(a,e)$. We assume $V(a,0)=0$, $V_e(a,0)=0$, $V_e(a,e)>0$, $V_{ee}(a,e)\geq 0$ and $V_a(a,e)\geq 0$. The effort level choice of the manager is not observed by the client. Given contract (F,α,κ), the manager chooses her optimal effort (and thus precision) so as to maximize her *unconditional* expected utility. After receiving the information signal the manager will decide her (utility maximazing) optimal portfolio allocation.

Proposition 1 *Given the contract (F,α,κ), the unconditional net expected utility for the risk-averse manager with CARA coefficient 'a' is*[7]

$$E_{t_0}[U\left(\tilde{W}_A(e\,|\,F,\alpha,\kappa)\right)]=exp\{V(a,e)\}\,f(e\mid F,\alpha,\kappa),\tag{6}$$

with

$$\begin{aligned}&f(e\,|\,F,\alpha,\kappa)=\\&\quad-\,exp(-aF)\left\{\frac{1}{2}+\left(\frac{1}{1+e}\right)^{1/2}\frac{1}{2}\Phi\left(\frac{(a\kappa\alpha)^2}{e}\right)\right.\\&\quad\left.+\,exp\left(\frac{(a\kappa\alpha)^2}{2}\right)\frac{1}{2}\left[1-\Phi\left(\frac{(a\kappa\alpha)^2}{e}(1+e)\right)\right]\right\}\end{aligned}\tag{7}$$

the unconditional gross expected utility function.

[6] We could alernatively define $\kappa=\overline{\kappa}-\underline{\kappa}>0$.

[7] M stands for manager. Φ represents the distribution function of a Chi-squared variable with one degree of freedom.

Proof: See the Appendix.

Given the contract (F, α, κ), the manager's optimal effort choice $e^\star$ solves

$$e^\star \in \arg\max_{e} \; E_{t_0}[U\left(\tilde{W}_{A}(e\,|\,F, \alpha, \kappa)\right)]. \tag{8}$$

In order to prove that $e^\star$ is an increasing function of α we need two previous results. These results will be stated in Lemmma 1 and Lemma 2.

Let us define the operator $\mathcal{J}$ as[8]

$$\mathcal{J}(e\,|\,F, \alpha, \kappa) = V_e(a, e) f(e\,|\,F, \alpha, \kappa) + f_e(e\,|\,F, \alpha, \kappa).$$

Lemma 1 *Provided a (local) maximum exist, $e^\star$ maximizes the unconditional net expected utilitity of the risk-averse manager with CARA coefficient 'a' if and only if*

$$\begin{aligned} \mathcal{J}(e^\star\,|\,F, \alpha, \kappa) &= 0, && (9)\\ \frac{f_{ee}}{f_e}(e^\star\,|\,F, \alpha, \kappa) &< \frac{V_{ee}}{V_e}(a, e^\star) - V_e(a, e^\star). && (10) \end{aligned}$$

Proof: The *first-order* condition for $e^\star$ to be a local maximum of program (8) is:

$$\begin{aligned} E_e[U\left(\tilde{W}_{A}(e^\star\,|\,F, \alpha, \kappa)\right)] &= \exp\{V(a, e^\star)\} \\ &\quad [V_e(a, e^\star) f(e^\star\,|\,F, \alpha, \kappa) + f_e(e^\star\,|\,F, \alpha, \kappa)] = 0. \end{aligned} \tag{11}$$

The *second-order* condition is:

$$\begin{aligned} E_{e\,e}[U\left(\tilde{W}_{A}(e^\star\,|\,F, \alpha, \kappa)\right)] &= \exp\{V(a, e^\star)\} \\ &\quad [V_{e\,e}(a, e^\star) f(e^\star\,|\,F, \alpha, \kappa) + V_e(a, e^\star) f_e(e^\star\,|\,F, \alpha, \kappa) + f_{e\,e}(e^\star\,|\,F, \alpha, \kappa)] < 0. \end{aligned} \tag{12}$$

Provided a maximum exists, given the *first-order* condition (11) and the definition of the operator $\mathcal{J}$, equation (9) characterizes the 'candidates' for a local maximum of problem (8).

From the *first-order* condition (11),

$$V_e(a, e^\star) = -\frac{f_e}{f}(e^\star\,|\,F, \alpha, \kappa). \tag{13}$$

Substituing (13) in equation (12) we arrive at condition (10).

[8]Subscripts e, α and ee denote first partial derivative with respect to effort and α, respectively, and second partial derivative with respect to effort.

Q.E.D.

Equation (10) compares the 'concavity' of the manager's *unconditional* gross expected utility function with the 'convexity' of the manager's disutility function of effort. Proof of Proposition 1 at the Appendix shows that $f(e \mid F, \alpha, \kappa)$ is an increasing and concave function of effort. Therefore, under our assumptions on $V(a, e)$, equation (10) in Lemma 1 says that if the manager's *unconditional* gross expected utility function is *sufficiently* concave relative to the 'convexity' of the disutility function of effort then, provided a maximum exists, $\mathcal{J}(e^\star \mid F, \alpha, \kappa) = 0$ is a necessary and sufficent condition for $e^\star$ to be the manager's optimal effort choice.

The following Lemma presents a result that will be necessary in proving Proposition 2.

Lemma 2 *Let $\Phi(x) = \int_0^x \phi(s)\, ds$, $x \geq 0$ represent the distribution function of Chi-squared variable with one degree of freedom.*

$$\phi(s) = \frac{1}{\sqrt{2\pi}} exp\{-s/2\}\, s^{-\frac{1}{2}}$$

is the corresponding density function. Then, for any $0 \leq x < \infty$,

$$\phi(x) - \frac{1}{2}[1 - \Phi(x)] > 0. \tag{14}$$

Proof: Let us define $A(x)$ as follows:

$$A(x) = \frac{1}{2}[1 - \Phi(x)].$$

We will firt show that if $\phi(x) = A(x)$ for any $0 \leq x < \infty$ this x is unique.

When x tends to zero,

$$\begin{aligned} \lim_{x \to 0} \phi(x) &= \infty, \\ \lim_{x \to 0} A(x) &= \frac{1}{2}. \end{aligned} \tag{15}$$

Taking derivatives of both functions:

$$\begin{aligned} \phi'(x) = \quad & \tfrac{\partial}{\partial x}\phi(x) &= -\frac{1}{2}\phi(x)(1 + \frac{1}{x}), \\ A'(x) = \quad & \tfrac{\partial}{\partial x}A(x) &= -\frac{1}{2}\phi(x). \end{aligned}$$

Therefore,

$$A'(x) > \phi'(x) \quad \text{for all } 0 \le x < \infty. \tag{16}$$

Both functions are continous over the whole domain of x. Given (15) and (16), since $\lim_{x\to\infty} \phi(x) = \lim_{x\to 0} A(x) = 0$, we conclude that if $\phi(x)$ and $A(x)$ ever cross for any $0 \le x < \infty$ they just cross once.

Now we will show that they do not cross even once. We will prove it by contradicition.

Let us assume there exits $0 \le x < \infty$ such that $\phi(x) = A(x)$.

Given the definition of both functions, this implies

$$\frac{1}{\sqrt{2\pi}}\exp\{-x/2\}\, x^{-\frac{1}{2}} = \frac{1}{2}\int_0^x \frac{1}{\sqrt{2\pi}}\exp\{-s/2\}\, s^{-\frac{1}{2}}\, ds, \tag{17}$$

$$0 \le x < \infty. \tag{18}$$

Integrating by parts the second the second term in equation (17) we obtain

$$\int_x^\infty \exp\{-s/2\}\, s^{-\frac{3}{2}}\, ds = 0,$$

that can only be true when $x \to \infty$. But this contradicts (18).

Therefore, given (15), since both functions never cross for any $0 \le x < \infty$, Lemma 2 is proved.

Q.E.D.

Proposition 2 *Let $e^\star$ be the optimal effort choice of the risk-averse manager with CARA coefficient 'a' when she is offered the contract (F, α, κ) with $\kappa < \infty$. Then, $e^\star$ is increasing in α.*

Proof: See the Appendix.

The intuitition behind Proposition 2 is as follows: Off-equilibrium, if the manager wants to increase her expected payoff, she can either improve the accuracy of her information (which is costly in terms of disutility) or borrow more money at a fixed interest rate. It is clear that, as long as the manager is 'liability unbounded', she will always be able to rebalance her optimal portfolio (via short-selling) for any α offered by the client so as to keep her *unconditional* expected utility constant. In such a case both raw and relative performance-adjusted linear contracts will fail to motivate the manager in collecting better information. However, this is no longer the case when the manager's borrowing capacity is constrained. As a simplification assume that the manager can borrow only upto a certain limit at a fixed interest rate (or borrow over the limit at an interest rate of infinity). Now, when the manager has limited borrowing capacity, she might indeed want to gather more precise information. The extent to which she will want to expend more effort in this regard will depend on her compensation scheme, her risk aversion and her marginal disutility of effort.

The *incentive* result stated at Proposition 2 is driven by our assumption on the manager's short-selling constraint $\kappa < \infty$. Corollary 1 shows that the *non-incentive* results in Soughton (1993) and Admati and Pfleiderer (1997) are embeded in our model as a special case when $\kappa \to \infty$.

Corollary 1 *Let $e^\star$ be the optimal effort choice of the risk-averse manager with CARA coefficient 'a' when she is offered the contract (F, α, κ). Then, if $\kappa \to \infty$, $e^\star$ is independent of α.*

Proof: From equation (13) in Lemma 1, the *first-order* condition of the manager's optimal effort can be written as

$$V_e(a, e^\star) = -\frac{f_e}{f}(e^\star \,|\, F, \alpha, \kappa).$$

It is easy to prove that

$$\lim_{\kappa \to \infty} \frac{f_e}{f}(e^\star \,|\, F, \alpha, \kappa) = \frac{1}{2(1+e^\star)},$$

independent of α. Therefore, since $V_e(a, e^\star)$ is also independent of α it follows that, when $\kappa \to \infty$, the manager's optimal effort $e^\star$ is independent of α.

Q.E.D.

3 The optimal contract

Last section showed that, differently from Soughton (1993) and Admati and Pfleiderer (1997), linear contracts can induce information gathering effort on the part of the risk-averse manager when the short-selling capacity of the later is constrained. In our model, the *non-incentive* result occurs as a special case when the short-selling constraints tend to infinity in the limit.

This section addresses the following question: Provided the manager is short-selling constrained, would her client (the household) offer her a benchmarked contract? As in Heinkel and Stoughton (1994), we will set the optimal contract problem in a risk-neutral framework where both the manager and the household are risk-neutral. This is done for two reasons: First, for the sake of simplicity. Second, we are interested in whether benchmarked contracts would optimally arise for reasons other than risk-sharing.

In a risk-neutral environment characterized by moral hazard, we show that when the manager is short-selling constrained the optimal linear contract is benchmarked and the manager receives all the marginal returns to the portfolio. Our results confirm Heinkel and Stoughton (1994) for all borrowing limits. Additionally, as the short-selling constraint goes to infinity the optimal contract remains a benchmarked contract. At the limit, the contract achives the *first-best* and the need to screen out managers vanishes. This finding is to be differentiated from the claim that benchmarked contracts cannot screen agents. Our result stresses the point that, at the limit, the need to screen agents vanishes as benchmarked contracts deliver the *first-best*.

In this section we will assume that signal precision and effort are related according to the following function:

$$H(e) = \begin{cases} \lambda e & \text{if } e < \frac{1}{\lambda} \\ 1 & \text{otherwise,} \end{cases} \tag{19}$$

with $\lambda > 0$ the effort efficency parameter. Equation (19) assumes that (in principle) perfect accuracy can be achieved for a finite level of effort. This is just a simplifying assumption that will be shown to be harmless for our results. Given the latest equation manager's effort can always be expressed as a function of signal's precision, namely

$$e(H) = \frac{1}{\lambda}\min\{1, H\}. \tag{20}$$

Equation (20) will be called the effort function.

In the analysis to follow all the results will be expressed in terms of precision instead of effort. This will simplify the notation.

A. The manager's optimal portfolio

The client will offer to the manager a contract (F, α, κ). Optimal effort (and thus precision) choice given contract (F, α, κ) maximizes the manager's *unconditional* expected utility. After receiving the information signal the manager will then decide her (utility maximazing) optimal portfolio allocation. In this section we will assume that effort has a private disutility to the manager and that is measured in cost terms as given by the effort function (20).[9] The effort level choice is not observed by the client.

Proposition 3 *Given contract (F, α, κ), the unconditional expected utility for the risk-neutral manager with effort efficiency parameter λ is*

[9] This means we assume $V(a, e) = e$.

$$E\left[U\left(\tilde{W}_M(H)\right)\right] = \begin{cases} F + \frac{\alpha\kappa}{\sqrt{2\pi}} H^{1/2} - H/\lambda & \textit{if } H < 1 \\ \\ F + \frac{\alpha\kappa}{\sqrt{2\pi}} - 1/\lambda & \textit{otherwise.} \end{cases} \tag{21}$$

Proof: See the Appendix.

We show now that the *unconditional* expected utilility of wealth for the risk-neutral manager, $E[\tilde{W}_M(H)]$, is a concave function in H, the signal precision.

Taking the first derivative of (21) with respect to H yields

$$\frac{\partial}{\partial H} E[\tilde{W}_M(H)] = \begin{cases} \frac{\alpha\kappa}{2\sqrt{2\pi}} H^{-1/2} - 1/\lambda & \text{if } H < 1 \\ \\ 0 & \text{otherwise.} \end{cases} \tag{22}$$

If $H \to 1$ then the expected utility of wealth converges to an upper bound given by $F + \frac{\alpha\kappa}{\sqrt{2\pi}} - 1/\lambda$. If H approaches zero then the marginal expected utility tends to infinity.

Deriving (22) with respect to H again we obtain

$$\frac{\partial^2}{\partial H^2} E[\tilde{W}_M(H)] = \begin{cases} -\frac{\alpha\kappa}{4\sqrt{2\pi}} H^{-3/2} < 0 & \text{if } H < 1 \\ \\ 0 & \text{otherwise.} \end{cases} \tag{23}$$

Therefore setting (22) equal to zero will yield manager's optimal precision as a function of α and κ,

$$H(\alpha, \kappa) = \min\left\{1, \left(\frac{\lambda\alpha\kappa}{2\sqrt{2\pi}}\right)^2\right\}. \tag{24}$$

Equation (24) will be called the precision function. Notice that the precision function is independent of F. Let us conclude the manager's problem by normalizing her reservation utility to zero.

B. The client's problem

At time t_0 the client will offer a contract (F, α, κ) to the manager. The client does not observe the signal realization. Neither she observes λ, the manager's effort efficiency parameter. However, she can deduce manager's *conditional* portfolio (44) as a function of λ. Moreover, she knows the signal distribution (3). Therefore she can calculate her *conditional* (on the manager's precision) expected utility as well as the manager's precision as a function of λ.

The manager will carry on all the investment. Given the conditional distribution of the return on the risky asset (4), the client's *conditional* wealth expectation and variance will be[10]

[10] C stands for client.

$$E[\tilde{W}_C(H)] = (1-\alpha)\int_{\Re}\theta(y)\, E[\tilde{x}\,|\,y]\, g(y\,|\,H)\, dy - F, \tag{25}$$

$$Var[\tilde{W}_C(H)] = (1-\alpha)^2\int_{\Re}\theta^2(y)\, Var[\tilde{x}\,|\,y]\, g(y\,|\,H)\, dy, \tag{26}$$

with $g(y\,|\,H)$ the density function of the signal distribution (3).
Solving (25) we obtain the expected utility of wealth for he risk-neutral client

$$E\left[U\left(\tilde{W}_C(H)\right)\right] = \begin{cases} (1-\alpha)\frac{\kappa}{\sqrt{2\pi}}H^{1/2} - F & \text{if } H < 1 \\ \\ (1-\alpha)\frac{\kappa}{\sqrt{2\pi}} - F & \text{otherwise.} \end{cases} \tag{27}$$

The optimal contract will solve

$$\begin{aligned} \max_{F,\alpha,\kappa} \quad & E_{t_0}\left\{U\left[\tilde{W}_C\left(F, H(\alpha,k)\right)\right]\right\} \\ s.t.\quad & H(\alpha,\kappa) = \min\left\{1, \left(\tfrac{\lambda\alpha\kappa}{2\sqrt{2\pi}}\right)^2\right\}, \\ & E_{t_0}\left\{U\left[\tilde{W}_M\left(F, H(\alpha,\kappa)\right)\right]\right\} \geq 0, \\ & 1 \geq \alpha \geq 0. \end{aligned} \tag{28}$$

Two question arises from problem (28): Provided the manager is short-selling constrained, would her client (the household) offer her a benchmarked contract? Further, would the client impose short-selling constraints on the manager?

Proposition 4 shows that when the manager's borrowing capacity is *exogenously* constrained the optimal contract is benchmarked regardless of the value of κ, the short-selling bound. Corollary 2 answers the second question: It the limit, the contract achieves the *first-best* and the need to screen out manager vanishes.

Proposition 4 *For any $\hat{\kappa}$ such that the signal precision is not perfect, the optimal contract offered by the risk-neutral client to the risk-neutral manager with effort efficiency parameter $\lambda > 0$ will be $(F(\hat{\kappa}), 1, \hat{\kappa})$ with*

$$F(\hat{\kappa}) = -\left(\frac{\hat{\kappa}}{2\sqrt{2\pi}}\right)^2\lambda. \tag{29}$$

Proof: See the Appendix.

Corollary 2 *When $\hat{\kappa} \to \infty$ the optimal contract is benchmarked and achieves the first-best.*

Proof: From proof of Proposition 4 at the Appendix we know that, given $\hat{\kappa} > 0$, there exists a contract $(F(\hat{\kappa}), \alpha^\star, \hat{\kappa})$ such that the optimal precision choice of the manager is an increasing function of $\hat{\kappa}$:

$$H(\alpha^\star, \hat{\kappa}) = \left(\frac{\lambda\hat{\kappa}}{2\sqrt{2\pi}}\right)^2.$$

Thus, when $\hat{\kappa} \to \infty$[11] then $H(\alpha^\star, \hat{\kappa}) \to 1$. In other words: By increasing $\hat{\kappa}$ (i.e. relaxing the manager's short-selling constraint) the manager's precision increases.

Given equation (29), $\lim_{\hat{\kappa}\to\infty} F(\hat{\kappa}) = -\infty$. Since optimal $\alpha^\star = 1$ for all $\hat{\kappa}$ then

$$lim_{\hat{\kappa}\to\infty}(F(\hat{\kappa}), \alpha^\star, \hat{\kappa}) = (-\infty, 1, \infty). \tag{30}$$

Equation (30) shows that, in the limit, the optimal contract is benchmarked . Plugging equation (30) into equations (21) and (27) we see that when $\hat{\kappa} \to \infty$ (and thus $H(\alpha^\star, \hat{\kappa}) \to 1$) both the client's and the manager's *unconditional* expected utility of wealth are unbounded. The *first-best* is achieved in the limit.

Q.E.D.

Appendix

This Appendix provides proofs of Propositions 1, 2, 3 and 4.

Proof of Proposition 1

Under the contract (F, α, κ) offered by the client, the manager's random wealth conditioned on the signal y will be:

$$\tilde{W}_M(y) = F + \alpha\theta\tilde{x}\,|\,y - V(a, e). \tag{31}$$

After receiving the information signal y the manager solves for her optimal portfolio θ as follows:

$$\max_{\theta(y)} \quad E_{t_0}[U\left(\tilde{W}_M(y)\right)]$$
$$\text{s.t. } 0 \leq \theta \leq \kappa.$$

According to (31) manager's final wealth is nomally distributed. Thus, since the agent is assumed to have a CARA utility function, manager's optimal portfolio decision problem becomes:

[11] Actually, given the effort function (20), this is going to be true for any $\hat{\kappa} \geq \frac{2\sqrt{2\pi}}{\lambda}$.

$$\max_{\theta} \qquad F + \alpha\theta\frac{e}{1+e}y - \frac{1}{2}a\alpha^2\theta^2\frac{1}{1+e}$$

$$\text{s.t. } 0 \leq \theta \leq \kappa.$$

Let us use $\mathcal{L}(\theta, \lambda_0, \lambda_\kappa)$ to denote the Lagrange functional of the last problem, with $\lambda_0,\ \lambda_\kappa \geq 0$ the Lagrange multipliers.

Since the latest program is concave, by the Kuhn-Tucker theorem, the optimal portfolio $\theta^\star$ satisfies (provided a maximum exists):

$$\frac{\partial}{\partial\theta}\mathcal{L}(\theta^\star, \lambda_0^\star, \lambda_\kappa^\star) = 0$$
$$0 \leq \theta^\star \leq \kappa,$$

together with the binding conditions

$$\begin{aligned} \theta^\star\lambda_0^\star &= 0 \\ (\theta^\star - \kappa)\lambda_\kappa^\star &= 0. \end{aligned}$$

This system of equations yields two *corner* solutions and an *interior* solution:

1. Lower bound *corner* solution: $\begin{cases} \theta^\star &= 0 \\ \lambda_0^\star &= -\frac{e}{1+e}\alpha y \\ \lambda_\kappa^\star &= 0 \end{cases}$

2. *Interior* solution ($\alpha > 0$): $\begin{cases} \theta^\star &= \frac{1}{\alpha}\frac{e}{a}y \\ \lambda_0^\star &= 0 \\ \lambda_\kappa^\star &= 0 \end{cases}$

3. Upper bound *corner* solution: $\begin{cases} \theta^\star &= \kappa \\ \lambda_0^\star &= 0 \\ \lambda_\kappa^\star &= \frac{e}{1+e}\alpha\left(y - \frac{a\kappa\alpha}{e}\right). \end{cases}$

The optimal portfolio $\theta^\star$ can be as written as a function of the information signal y as follows (assuming $\alpha > 0$):

$$\theta(y) = \begin{cases} 0 & \text{if } y < 0 \\ \frac{1}{\alpha}\frac{e}{a}y & \text{if } 0 \leq y \leq \frac{a\kappa\alpha}{e} \\ \kappa & \text{if } y > \frac{a\kappa\alpha}{e}. \end{cases} \tag{32}$$

After substituing (32) into the expected utility function of the manager we obtain her *conditional* (net) expected utility of wealth

$$E_{t_0}[U\left(\tilde{W}_A(y)\right)] = \begin{cases} -\exp\{-aF + V(a,e)\} & \text{if } y < 0 \\ -\exp\{-a\left(F + \frac{e^2}{2a(1+e)}y\right) + V(a,e)\} & \text{if } 0 \le y \le \frac{a\kappa\alpha}{e} \\ -\exp\{-a\left(F + \frac{e}{1+e}\alpha\kappa\left(y - \frac{a\kappa\alpha}{2e}\right)\right) + V(a,e)\} & \text{if } y > \frac{a\kappa\alpha}{e}. \end{cases} \tag{33}$$

Then, integrating (33) over y we obtain manager's *unconditional* expected utility of wealth:

$$\begin{aligned} &E_{t_0}[U\left(\tilde{W}_A(e\,|\,F,\alpha,\kappa)\right)] = \\ &\quad -\exp\{-aF + V(a,e)\}\left\{\frac{1}{2} + \left(\frac{e}{1+e}\right)^{1/2}\int_0^{\frac{a\kappa\alpha}{e}} \frac{1}{\sqrt{2\pi}}\exp\{-\frac{e}{2}y\}\,dy \right. \\ &\quad \left. + \exp\{\frac{(a\kappa\alpha)^2}{2}\}\left(\frac{e}{1+e}\right)^{1/2}\int_{\frac{a\kappa\alpha}{e}}^{\infty} \frac{1}{\sqrt{2\pi}}\exp\{-\frac{e}{2(1+e)}(y + a\kappa\alpha)^2\}\,dy\right\}. \end{aligned}$$

After a little of algebra, we arrive at equation (6), the manager's *unconditional* net expected utility.

We will prove now that the gross *unconditional* expected utility of wealth $f(e\,|\,F,\alpha,\kappa)$ is an increasing and concave function of effort.

Taking the first derivative of (7) yields

$$f_e(e\,|\,F,\alpha,\kappa) = \frac{\partial}{\partial e}f(e\,|\,F,\alpha,\kappa) = \frac{1}{4}\exp\{-aF\}\left(\frac{1}{1+e}\right)^{3/2}\Phi\left(\frac{(a\kappa\alpha)^2}{e}\right) > 0. \tag{34}$$

Finally, deriving again with respect to e:

$$\begin{aligned} f_{ee}(e\,|\,F,\alpha,\kappa) = \frac{\partial^2}{\partial e^2}f(e\,|\,F,\alpha,\kappa) = \\ -\frac{1}{4}\exp\{-aF\}\left[\frac{3}{2}\left(\frac{1}{1+e}\right)^{5/2}\Phi\left(\frac{(a\kappa\alpha)^2}{e}\right)\right. \\ \left. + \frac{a\kappa\alpha}{\sqrt{2\pi}}\left(\frac{1}{e(1+e)}\right)^{3/2}\exp\{-\frac{(a\kappa\alpha)^2}{2e}\}\right] < 0. \end{aligned}$$

Q.E.D.

Proof of Proposition 2

Since $e^\star$ is optimal, by Lemma 1

$$\mathcal{J}(e^\star \,|\, F, \alpha, \kappa) = 0.$$

By the implicit function theorem

$$\frac{\partial}{\partial \alpha} e^\star = -\frac{\mathcal{J}_\alpha}{\mathcal{J}_e}(e^\star \,|\, F, \alpha, \kappa). \tag{35}$$

From (10) in Lemma 1, if $e^\star$ is the manager's optimal effort when she is offered the contract (F, α, κ) then

$$\mathcal{J}_e(e^\star \,|\, F, \alpha, \kappa) < 0. \tag{36}$$

Therefore, given (35), Proposition 2 will be proved if we show that $\mathcal{J}_\alpha(e^\star \,|\, F, \alpha, \kappa) > 0$.

Let us first study the sign of $f_\alpha(e^\star \,|\, F, \alpha, \kappa)$.

$$f_\alpha(e^\star \,|\, F, \alpha, \kappa) =$$
$$a(a\kappa)^2 \exp\{aF + \frac{(a\kappa\alpha)^2}{2}\} \left\{ \phi\left(\frac{(a\kappa\alpha)^2}{e^\star}(1 + e^\star) \right) - \frac{1}{2}\left[1 - \Phi\left(\frac{(a\kappa\alpha)^2}{e^\star}(1 + e^\star) \right) \right] \right\}.$$

From Lemma 2 we know that, for any contract (F, α, κ),

$$\phi\left(\frac{(a\kappa\alpha)^2}{e^\star}(1 + e^\star) \right) - \frac{1}{2}\left[1 - \Phi\left(\frac{(a\kappa\alpha)^2}{e^\star}(1 + e^\star) \right) \right] > 0$$

and, therefore, the manager's *unconditional* gross marginal utility of wealth is positive.

Given the definition of the operator $\mathcal{J}$,

$$\mathcal{J}_\alpha(e^\star \,|\, F, \alpha, \kappa) = V_e(a, e^\star) f_\alpha(e^\star \,|\, F, \alpha, \kappa) + f_{e\,\alpha}(e^\star \,|\, F, \alpha, \kappa). \tag{37}$$

From (34) is easy to see that $f_{e\,\alpha}(e^\star \,|\, F, \alpha, \kappa) > 0$. $V_e(a, e^\star) \geq 0$ by assumption. Finally, we just proved that $f_\alpha(e^\star \,|\, F, \alpha, \kappa) > 0$. Hence, according to (37),

$$\mathcal{J}_\alpha(e^\star \,|\, F, \alpha, \kappa) > 0.$$

Equations (35) and (36) complete the proof.

Q.E.D.

Proof of Proposition 3

Manager's optimal *conditional* portfolio solves the following problem:

$$\begin{aligned} \max_{\theta} \quad & E_{t_0}\left[U\left(\tilde{W}_M(\theta\,|\,y)\right)\right] \\ & s.t.\ 0 \le \theta \le \kappa, \end{aligned} \tag{38}$$

with

$$\tilde{W}_M(y) = F + \alpha\,\theta\,\tilde{x}\,|\,y - e \tag{39}$$

the manager's *conditional* wealth net of effort cost.

Given equation (20) effort disutility e can be expressed as a function of the signal precision H. Thus, the manager's *conditional* net wealth can be re-written as

$$\tilde{W}_M(y) = F + \alpha\,\theta\,\tilde{x}\,|\,y - \frac{1}{\lambda}\min\{1, H\}. \tag{40}$$

The expectation and variance of (40) are

$$\begin{aligned} E[\tilde{W}_M(y)] &= F + \left(\alpha\theta y - \frac{1}{\lambda}\right)\min\,\{1, H\}, & (41) \\ Var[\tilde{W}_M(y)] &= \alpha^2\theta^2 \max\,\{0, 1 - H\}. & (42) \end{aligned}$$

Since the manager is assumed to be risk-neutral, problem (38) becomes:

$$\begin{aligned} \max_{\theta} \quad & F + \left(\alpha\theta y - \frac{1}{\lambda}\right)\min\,\{1, H\} \\ & s.t.\ 0 \le \theta \le \kappa. \end{aligned} \tag{43}$$

From (43) we derive manager's optimal *conditional* portfolio

$$\theta(y) = \begin{cases} 0 & \text{if } y \le 0 \\ \kappa & \text{otherwise.} \end{cases} \tag{44}$$

Equation (44) shows that the optimal *coditional* portfolio of the risk-neutral manager is always *binding*, whatever the signal is.

Substituing (44) into (41) and integrating over y we arrive at equation (21).

Q.E.D.

Proof of Proposition 4

Let us fix $\kappa = \hat{\kappa} > 0$ in problem (28). Given the precision function (24) the optimal contract $(F^\star, \alpha^\star, \hat{\kappa})$ is such that

$$(F^\star, \alpha^\star, \hat{\kappa}) \in \arg\max_{F,a} \quad (1-\alpha)\frac{\alpha\hat{\kappa}^2}{4\pi}\lambda - F \tag{45}$$

$$\begin{aligned} s.t. \quad & F + \left(\tfrac{\alpha\hat{\kappa}}{2\sqrt{2\pi}}\right)^2 \geq 0 \\ & 1 \geq \alpha \geq 0. \end{aligned} \tag{46}$$

Since the function (45) and both constraints in (46) are concave in (F, α) the latest is a concave program and the optimal $(F^\star, \alpha^\star)$, provided a maximum exists, can be fully characterized by the Kuhn-Tucker theorem. Let us first define the Lagrangian operator as

$$\mathcal{L}(F, \alpha \,|\, \hat{\kappa}) = (1-\alpha)\alpha\frac{\hat{\kappa}^2}{4\pi}\lambda - F + \gamma_1\left[F + \left(\frac{\alpha\hat{\kappa}}{2\sqrt{2\pi}}\right)^2\lambda\right] + \gamma_2\alpha + \gamma_3(1-\alpha), \tag{47}$$

with $\gamma \equiv (\gamma_1, \gamma_2, \gamma_3) \geq 0$ the corresponding vector of multipliers.
If a maximum exists, optimal contract $(F^\star, \alpha^\star, \gamma^\star)$ satisfies

$$\begin{aligned} \frac{\partial\mathcal{L}}{\partial F}(F^\star, \alpha^\star \,|\, \hat{\kappa}) &= 0, \\ \frac{\partial\mathcal{L}}{\partial \alpha}(F^\star, \alpha^\star \,|\, \hat{\kappa}) &= 0, \\ F^\star + \left(\frac{\alpha^\star\hat{\kappa}}{2\sqrt{2\pi}}\right)^2\lambda &\geq 0, \\ 1 \geq \alpha^\star &\geq 0; \end{aligned}$$

together with the binding conditions

$$\begin{aligned} \gamma_1^\star\alpha^\star &= 0, \\ \gamma_2^\star(1-\alpha^\star) &= 0, \\ \gamma_3^\star\left[F^\star + \left(\frac{\alpha^\star\hat{\kappa}}{2\sqrt{2\pi}}\right)^2\lambda\right] &= 0. \end{aligned}$$

Solving this system of equations yields

$$
\begin{aligned}
F^\star &= F(\hat{\kappa}), \\
\alpha^\star &= 1, \\
\gamma_1^\star &= 1, \\
\gamma_2^\star = \gamma_3^\star &= 0.
\end{aligned}
$$

Therefore, $\alpha^\star$ and the optimal Lagrange multipliers are independent of $\hat{\kappa}$. Moreover, since $\alpha^\star > 0$ the optimal contract will be benchmarked for any value of $\hat{\kappa}$.

Given $\alpha^\star$ and $\hat{\kappa}$ equation (24) implies,

$$H(\alpha^\star, \hat{\kappa}) = \left(\frac{\lambda \hat{\kappa}}{2\sqrt{2\pi}} \right)^2 < 1. \tag{48}$$

Q.E.D.

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