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Money, Credit and Default International Investors

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Money, Credit and Default

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Abstract
This paper develops a quantitative model of unsecured debt, default, and money demand for heterogenous agents economies. The paper generates a theory of money demand for the case in which money is a dominate asset that is not needed to carry-out transactions. In this environment holding money helps the agents to smooth their consumption during those periods in which they are excluded from credit markets following a default in their debts. In the model the welfare of the individuals is affected by the inflation rate: high inflation rates preclude individuals of using money as an asset that helps them smooth their consumption profile but low inflation rates tend to make softer the punishment for default making it difficult to sustain high levels of debt at equilibrium. This two opposite effects imply that in equilibrium the inflation rate that maximizes individuals welfare is positive but not too high.

Keywords: Default, Inflation, Money, Endogenous Borrowing Constraint.

JEL Classification: F34; F36; F42

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1 Introduction

We extend the Aiyagari (1994) model of heterogeneous agents that face idiosyncratic earnings shocks and incomplete credit markets by introducing bankruptcy, and unbacked money that cannot be liquidated to pay off unsecured debts. These extensions allow us to show: first, in equilibrium agents hold a part of their wealth in the form of money balances, second, the existence of a second asset which is exempt from liquidation increases the probability of bankruptcy, and third, as in Zame (1993) "voluntary" default improves the efficiency of equilibrium allocations.

The model in this paper is closely related to the literature on unsecured debt and consumer bankruptcy (see for example Chatterjee et al. (2005,2006), Livshits (2003, 2007), etc.). It departs from this literature by focusing on how the equilibrium of the model with bankruptcy is modified when an additional asset in which the individuals can store their wealth is introduced to the model. Also, in the model the emphasis is on "voluntary" bankruptcy vs. "involuntary" bankruptcy, and as a consequence, consumer bankruptcy acts in this framework as a way to complete credit markets, favoring consumption smoothing by individuals in the economy. The possibility of filling for bankruptcy reduces the level of precautionary savings that the agents require in order to smooth consumption when they face uncertain individual earnings.

The paper is also related to the literature on the role of incomplete markets in supporting a demand for unbacked money at equilibrium (see for example, Bewley (1977,1980,1983,1986)). The main departure of the model from this literature is that in here credit access is endogenous rather than exogenous, and therefore the demand for money that arises does not depend on ad-hoc exogenous credit constraints. Credit constraints are endogenous and because these constraints are a function of the individuals' earnings, they differ across individuals. Additionally the credit market is not open to all individuals. Only those individuals that are not in a default state have access to the market. As a consequence we observed an endogenous segmentation of the credit markets.

In our model economy filling for bankruptcy is an optimal choice that allows households to improve the efficiency of the non-contingent contracts by reducing the size of the precautionary savings they need to protect themselves against the possibility of a stream of negative earning shocks. However, bankruptcy entitles a cost: a defaulting individual is excluded from credit markets during a random number of periods. In this environment, money can be used as an alternative asset that helps to smooth consumption during the periods in which the agents are out of the credit markets. In such way we obtain a theory of the existence of money even when money is an asset that is not needed to carry-out transactions in the economy, and that does not generate a positive return.

Therefore, as in the models of Bewley (1986, undated) the demand of money at equilibrium is a function of the degree of market incompleteness of the credit markets. But, as we said before, the main difference of this article with those previous studies is that in the context of this model the degree of market incompleteness is to some extent endogenous. Individuals'decision of declaring bankruptcy and therefore to be excluded from the credit markets for a random number of periods is endogenous. Also, the probability that an individual will fill for bankruptcy in the next period will determined the credit limits that this particular individual faces.

In economies where the cost of going into default tends to infinite there are not tighter limits to borrowing than the natural limit of debt defined by Aiyagari (1994). In those economies, given the observed negative return in money balances (that corresponds to the inflation rate), the equilibrium money demand is zero. In such case the optimal level of inflation is undetermined.

In economies where it is possible to declare bankruptcy tighter borrowing limits arise and the welfare of the agents in this economy depends on the inflation rate: high levels of inflation preclude the agents from using money as a way to achieve a smoother path for consumption; however, low inflation rates make the cost of bankruptcy relatively low, and as a consequence, it is harder to support high levels of unsecured debt at equilibrium.

The paper proceeds as follows. In the next section, we describe a the model with unsecured debt and un-backed money. Section 3 presents the numerical findings. In the last section we conclude.

2 The Model

We extend the Aiyagari (1994) model of heterogeneous agents that face idiosyncratic earnings shocks and incomplete credit markets by introducing "voluntary" default risk and unbacked money that can not be liquidated to pay off unsecured debts.

Technology

There is one good produced by a constant return production function, $F(K_t, L_t)$ where K_t is the aggregate capital stock that depreciates at rate δ and N_t is the aggregate labor in efficiency units. There is no aggregate uncertainty.

Households

The model economy is populated by a continuum of infinitely live households that maximize expected discounted utility

$$E \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where c_t is the household consumption and $\beta_t \in (0, 1)$ is the discount rate and $u(.)$ is continuously differentiable, strictly concave, and monotonically increasing. Each household is endowed with one unit of time, and each period they receive an idiosyncratic labor efficiency shock, $e_t \in [\underline{e}, \overline{e}] \subset R_{++}$, that follows a discrete Markov process, $\nu(e_t, e_{t+1})$.

Markets are incomplete and in order to smooth consumption agents can save or borrow using non-contingent one-period unsecured bonds, $b_t \in [\underline{b}, \overline{b}] \subset R$, or storing wealth in un-backed money balances, $m_t \in [\underline{m}, \overline{m}] \subset R_+$.

Households have the option to declare bankruptcy. If a household files for bankruptcy its debts are discharged and its level of unsecured debt is set to zero, $b_t = 0$. During a stochastic number of periods filers face two types of punishments: first, they will be excluded of the credit market, and second, they will experience a loss equal to a fraction θ of their labor earnings. The current period consumption of these households is given by the following budget constraint:

$$c^d(e, m, m') = (1 - \theta)ew + \frac{m}{1 + \pi} - m' + t,$$

where w is the real wage per efficiency unit, π is the inflation rate, and t ia a lump-sum transfer from the government to the households. Note that the unbacked money balances, m, can not be liquidated to pay off unsecured debts. Then, total resources that are exempt from liquidation in the case of bankruptcy are equal to $(1 - \theta)we_t +$

$\frac{m}{1+\pi}$. One key difference between $(1-\theta)we_t$ and $\frac{m}{1+\pi}$ is that the first type of resources at the disposition of bankrupted households is beyond the control of these households, while the second type of resources is under the control of these households. In other words, households choose how much of their wealth they hold in the form of monetary balances.

If a household has access to the credit market and the household does not fill for bankruptcy, then the current period consumption is given by the following budget constraint:

$$c^{nd}(e, b, b', m, m') = ew + b - qb' + \frac{m}{1+\pi} - m' + t.$$

where q is the price bond.

The optimization problem of a household is defined recursively using two distinct value functions. V^d is the value of being excluded from the credit market, while W is the value of not being excluded from the credit market. Consider a household with a state (e, m) which is excluded from the credit market. $V^d(e, m)$ is given by solving

$$V^d(e, m) = \max_{m'} \left\{ u(c^d(e, m, m')) + \beta E_{e'} \left[(1-\mu) W(e', 0, m') + \mu V^d(e', m') \right] \right\}$$

where $(1-\mu)$ is the probability of coming back to the credit market in the next period and start with an level of unsecured debt $b = 0$. Then, the value of not being excluded from the credit market is given by solving:

$$W(e, b, m) = \max_{d \in \{0,1\}} \left\{ \max_{b', m'} \left\{ u(c^{nd}(e, b, b', m, m')) + \beta E_{e'} W(b', m', e') \right\}, V^d(e, m) \right\},$$

and the household chooses to fill for bankruptcy, $d(e, b, m) = 1$, if

$$V^d(e, m) \geq \max_{b', m'} \left\{ u(c^{nd}(e, b, b', m, m')) + \beta E_{e'} W(b', m', e') \right\}.$$

Bankers

There is a continuum with measure one of identical, infinitely lived bankers. The individual banker is risk neutral. Bankers have complete information about households. They observe the total level of borrowing b_{t+1}, the current persistent labor efficiency shock e_t, and the borrower's money balances, m_t. This information allows the bankers to forecast the default probability of each individual, $d(e_t, m_t, b_{t+1})$, and price each bond $q(e_t, b_t, m_t, b_{t+1})$ according to their zero profit condition.

Government

There is a government, which is benevolent in the sense that its objective is to maximize the welfare of the consumers by choosing the level of inflation, π_t subject to the government budget constraint

$$m_{t+1} - \frac{m_t}{1+\pi_t} = T_t,$$

where T_t is the aggregate lump-sum transfer.

2.1 Characterization of the demand for unbacked money

In our model economy, if a household defaults, this household demands unbacked money to smooth consumption choosing m' to solve:

$$-\frac{\partial u(c^d(e,b,m))}{\partial c} + \beta\left[(1-\mu)E\frac{\partial V^d(e',m')}{\partial m} + \mu E\frac{\partial W(e',0,m')}{\partial m}\right] = 0.$$

Given that these type of households are not allowed to access the credit market $\frac{\partial V^d(e,m)}{\partial m}$ is, as in an Bewley economy, strictly positive. If a household does not default, this household demands unbacked money, m', and the unsecured bond, b', in a way that the following conditions are satisfied:

$$-\frac{\partial u(c^{nd}(e,b,m))}{\partial c} + \beta E\frac{\partial W(e',b',m')}{\partial m} = 0,$$

$$-q(e,b,m;b')\frac{\partial u(c^{nd}(e,b,m))}{\partial c} + \beta E\frac{\partial W(e',b',m')}{\partial b} = 0.$$

where

$$E\frac{\partial W(e',b',m')}{\partial m} = \frac{1}{1+\pi} - \sum_{e'}(1-d(e',b',m';b'))\frac{\partial q(e',b',m';b'')}{\partial m}b''(e',b',m')\nu(e,e') \quad (1)$$

and

$$E\frac{\partial W(e',b',m')}{\partial b} = \sum_{e'}(1-d(e',b',m';b'))\left[1-\frac{\partial q(e',b',m';b'')}{\partial b}b''(e',b',m')\right]\nu(e,e'). \quad (2)$$

The fist part of equation (1) says that increasing money holdings always give $\frac{1}{1+\pi}$ units of good tomorrow. The second part says that, conditional on not defaulting, we pay a lower bond prize tomorrow, given that $q(e,b,m;b')$ is not increasing in money (see figure 1). Equation (2) says that the opportunity cost of increasing money balances is only paid if the household chooses not to default in the next period. Then,

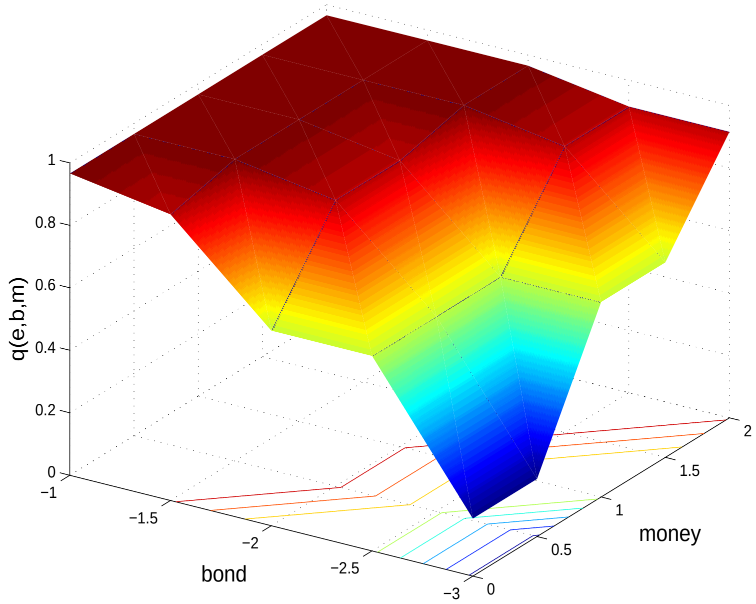

Figure 1: Price of bonds is increasing in money holdings.

if the household finds optimal to default with probability $d(e', b', m'; b') \in (0, 1)$, the following arbitrage condition holds:

$$\frac{1}{1+\pi} = \sum_{e'} (1 - d(e', b', m'; b')) \left\{ \frac{\left[1 - \left(\frac{\partial q(e', b', m'; b'')}{\partial b}\right) b''\right]}{q(e, b, m; b')} + \frac{\partial q(e', b', m'; b'')}{\partial m} b'' \right\} \nu(e, e') \quad (3)$$

2.2 Stationary Equilibrium

An steady-sate competitive equilibrium is a set of strictly positive prices, w^*, $\hat{q}$, a non negative loan price vector $q(e, b, m; b')$ and inflation rate π^*, strictly positive quantities of aggregate labor,N^*, and capital, K^*, value functions $W(e, b, m)$, $V^d(e, m)$, policy functions $m' = g_m(e, b, m)$, $b' = g_b(e, b, m)$, $m' = g_m^d(e, m)$, $d(e, b, m)$ and a measure $\varphi(e, b, m)$ such that the following conditions hold:

1. Consumer's optimization. Given the inflation rate, π^*, and the price $q(e, b, m; b')$, the functions $W(e, b, m)$, $V^d(e, m)$solves consumers's problem. $m' = g_m(e, b, m)$, $b' = g_b(e, b, m)$, $m' = g_m^d(e, m)$ and $d(e, b, m)$ are optimal policy functions.

2. Competitive bankers make zero profits:

$$\hat{q} = \sum_{e'} [1 - d(e', b', m'; g_b(e', b', m'))] q(e, b, m; b').$$

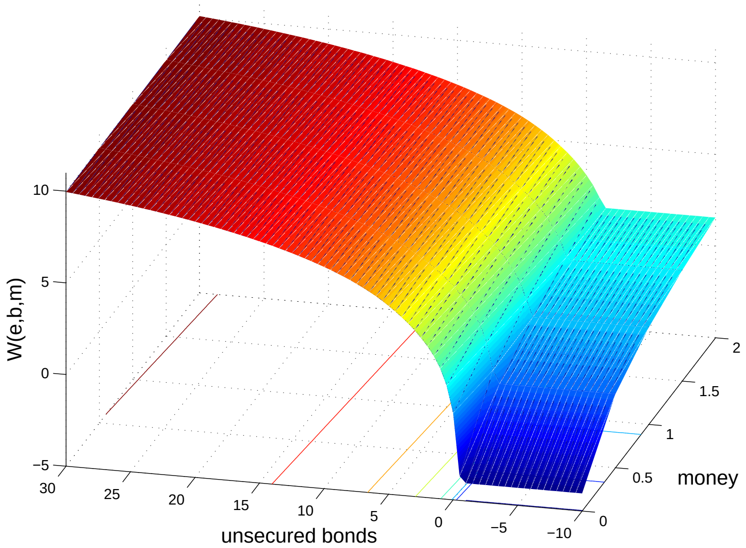

Figure 2: Value function is increasing in money holdings.

3. K^* and N^* solves the firm problem

4. Labor market clearing

$$N^* = \sum_S e\varphi(e,b,m)$$

where $S = [\underline{e}, \overline{e}] \times [\underline{b}, \overline{b}] \times [\underline{m}, \overline{m}]$

5. Capital market clearing

$$K^* = \sum_S \left\{(1-d(e,b,m))[q(e,b,m)g_b(e,b,m) + g_m(e,b,m)] + d(e,b,m)g_m^d(e,m)\right\}\varphi(e,b,m)$$

6. The stationary distribution $\varphi(e,b,m)$ is induced by exogenous $\nu(e,e')$ and $g_m(e,b,m)$, $g_b(e,b,m)$, $g_m^d(e,m)$ and $d(e,b,m)$ verifies

$$\varphi(b,m,l) = \sum_S \sum_{S'} 1_{\left\{(g_b(e,b,m),(1-d(e,b,m))*g_m(e,b,m)+d(e,b,m)g_m^d(e,b,m))=(b',m')\right\}}\nu(e,e')\varphi(e,b,m)$$

7. The inflation rate π verifies

$$\frac{T}{\pi} = \sum_S \left[(1-d(e,b,m;b'))g_m(e,b,m) + d(e,b,m;b')g_m^d(e,m)\right]\varphi(e,b,m)$$

Table 1: **Parameters**

Parameter Values	Baseline
Value Risk Aversion	2.75
Capital share	0.025
Productivity parameter	0.25
Autocorrelation of individuals labor income	0.7294
Volatility of individuals labor income s	0.0684
Discount Factor	0.955
Probability of reentering credit markets	0.5
Parameter of punishment	0.75

3 Findings

Our benchmark economy is an economy like the one in Aiyagari (1994): an economy populated by heterogenous agents that face idiosyncratic uncertainty in their labor earnings and that have access to a credit market where one-period non-contingent bonds are traded. The only credit limit that applies to the individuals in this economy is the "natural debt limit".

We compared the previous economy to two different economies: First, a economy like the benchmark economy but where individuals are allowed to fill for bankruptcy, and second, an economy where individuals are allowed to fill for bankruptcy and where they can use money balances to store their wealth.

The numerical simulation of these economies assumes that one period in the model is the equivalent to a time period of a quarter. A summary of the parameters' values used in the simulation is presented in Table 1.

Table 2 summarizes our numerical findings. We summarize the results of these experiments as follows:

1. First, allowing a household to fill for bankruptcy decreases the level of precautionary savings,

2. Second, allowing a household to maintain a part of their wealth in the form of money balances increases the equilibrium probability of declaring bankruptcy by a factor of 10.

Table 2: **Numerical experiment**

Variables	Baseline	Bankruptcy No Money	Bankruptcy Money
Credit Limits/GDP	1147.00%	79.11%	79.01%
Interest Rate	4.444%	4.508%	4.512%
Probability of Default	0.000	0.024%	0.299%
Mean Risk Premium	0.000	0.054%	0.441%
Money Demand/GDP	0.000	0.000	3.587e-4
Savings/GDP	10.81%	10.70%	10.69%

It is worth discussing how unbacked money affects the decision to go into bankruptcy. In our model economy going into bankruptcy is always an optimal choice, because there is not "involuntary" default. Contrary to Chatterjee, et al. (2006), the current budget constraint correspondence

$$BC(e,b,m) = \left\{ c \in R_+, b' \in B, m' \in M : c \leq we + b - q(e,b,m,b')b' + \frac{m}{1+\pi} - m' \right\}$$

is never empty, because the maximum level of debt is required to be grater than the *natural debt limit*

$$\min(B) \geq -\phi = -we\hat{q}$$

where $\hat{q}$ is the risk-free price bond. Households choose to default if the present benefits of going into default are higher than the future cost of being excluded from the credit market. The future cost of filling for bankruptcy is the inability to smooth consumption during a stochastic number of periods. Allowing households to accumulated unbacked money balances increases the present benefits of defaulting and reduces the future costs of filling for bankruptcy.

Allowing households to use money balances to store wealth increases the present benefits of going into default by giving bankrupted households some control over the fraction of their resources that are exempt of liquidation. Additionally, holding money balances reduces the future cost of bankruptcy by allowing bankrupted households that have lost access to credit markets to have a level of consumption that differs from their labor earnings and that can be made less volatile than this income. Then allowing households to accumulated unbacked money balances must increase the equilibrium probability of default.

Moreover, as the probability of default increases the endogenous credit limits become tighter for the economy as a whole. In other words, the maximum level of debt that can be supported at equilibrium is lower the higher is the equilibrium probability of default. In the standard Aiyagari model tighter credit limits induce higher precautionary savings. However in our model economy tighter endogenous credit limits also generated more bankruptcy in equilibria. In a world of uncertainty and incomplete markets the possibility of default allows households to improve the efficiency of the non-contingent contracts by reducing the size of the precautionary savings they need in order to protect themselves against a stream of negative earnings shocks. Moreover, tighter credit limits expand the equilibrium amount of unbacked money that is hold by the households generating conditions to support money with a finite positive price level.[1]

4 Conclusions

We show that bankruptcy helps to complete the credit market by reducing the size of precautionary savings in the economy. Moreover, including money in models of unsecured debt helps to explain a much higher probability of default at equilibrium.

Our findings have several implications. First, we show that it is not always reasonable to think that in models with heterogeneous agents a closed economy with more enforcement problems in its credit market should have a higher savings rate, and higher investment levels than countries in which credit markets work better. We find that countries with more enforcement problems where agents are able to go into bankruptcy and choose to do so "voluntarily" should save and invest less than countries with less enforcement problems.

Second, different levels of inflation modify the incentives that individuals have to go into bankruptcy. The inflation rate has to opposite effects on the welfare of the individuals in the economy: In one hand, with high inflation rates holding money balances is very costly, and this asset does not help too much the individuals to smooth their consumption. In the other hand, high inflation rates make the punishment for default harsher, and allows the economy to support larger levels of debt at equilibrium. Analyzing numerically these opposite forces will allow us to determine an optimal level of inflation.

[1] In the limit, as the credit limits tends to zero the economy converge to a Bewley economy.

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