TESTING FOR ZERO AUTOCORRELATION IN THE PRESENCE OF STATISTICAL DEPENDENCE*

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ABSTRACT

The problem addressed in this paper is to test the null hypothesis that a time series process is uncorrelated up to lag K in the presence of statistical dependence. We propose an extension of the Box-Pierce Q test that is asymptotically distributed as chi-square when the null is true for a very general class of dependent processes that includes non-martingale difference sequences. The test is based on a consistent estimator of the asymptotic covariance matrix of the sample autocorrelations under the null. The finite sample performance of this extension is investigated in a Monte Carlo study.

1. INTRODUCTION

Box-Pierce (1970) proposed using the Q_K statistic to test the null hypothesis that the first K autocorrelations of a covariance stationary time series are zero. The Q_K statistic is the sample size times the sum of the squares of the first K sample autocorrelations. Assuming the observations are independent and identically distributed, the asymptotic covariance matrix of the vector of sample autocorrelations is the inverse of the sample size times the identity matrix. Hence, under the null, Q_K is asymptotically distributed as chi-square with K degrees of freedom, *provided* that the observations are independent and identically distributed. The test can be extended to settings with statistical dependence by using the true asymptotic covariance matrix of the sample autocorrelations, or a consistent estimator, in place of the identity matrix.

This extension has been carried for time series generated by a martingale difference sequence (MDS). For certain MDS processes, the asymptotic covariance matrix of the sample autocorrelations is diagonal. This case has been repeatedly addressed in the literature; see, for example, Taylor (1984), Diebold (1986) and Lo and MacKinlay (1989). Guo and Phillips (1998) have extended the test for the case where the asymptotic covariance matrix of the sample autocorrelations may be nondiagonal. This more general MDS setting is of special importance for financial time series. For example, the asymptotic covariance matrix of the sample autocorrelations is nondiagonal for a GARCH (1,1) model with asymmetric errors.

In this paper, we present an extension of the Box-Pierce test to the case where the time series may be generated by a non-MDS process. Our test statistic is in the spirit of a

Lagrange Multiplier or score statistic because it is based on a consistent estimator of the asymptotic covariance matrix of the sample autocorrelations under the null. Uncorrelated time series can be generated by non-MDS models including certain bilinear models (Granger and Terasvirta (1993)) and all-pass models (Breidt, Davis and Trindade (1999)). An all-pass model is an ARMA model in which all the roots of the AR polynomial are reciprocals of the roots of the MA polynomial. Both bilinear models and all-pass models have been used in finance applications; for example, see Bera and Higgins (1997) and Terdik (1999) for bilinear examples and Breidt, Davis and Trindade (1999) for all-pass examples.

The performance of the standard Box-Pierce Q_K test and extensions of the Q_K test are compared in Monte Carlo experiments. The examples used in the experiments include MDS processes with diagonal and nondiagonal asymptotic covariance matrices of the sample autocorrelations and non-MDS processes. The powers of the various extensions of the Q_K test are also investigated.

This paper is organized as follows. Our extension of the Q_K test is developed in Section 2. The results of the Monte Carlo experiments on the probability of making a Type I error are reported in Section 3 for MDS and non-MDS examples. The powers of the tests are reported in Section 4. The concluding remarks are in Section 5.

2. EXTENSIONS OF BOX-PIERCE

Notation. Let $y_1, \ldots, y_n$, denote a real-valued covariance stationary time series with mean μ. Define the lag-j autocovariance by $\gamma(j) = E(y_t - \mu)(y_{t-j} - \mu)$ and the lag-j autocorrelation by $\rho(j) = \gamma(j)/\gamma(0)$. Define the sample mean by $\hat{\mu} = (1/n)\sum_{t=1}^{n} y_t$ and the lag-j autocovariance by $\hat{\gamma}(j) = \sum_{t=j+1}^{n} (y_t - \hat{\mu})(y_{t-j} - \hat{\mu}) / n$. The usual estimator of $\rho(j)$ is $r(j) = \hat{\gamma}(j)/\hat{\gamma}(0)$.

Denote the vector of sample autocovariances as $\hat{\gamma} = (\hat{\gamma}(0), \hat{\gamma}(1), ..., \hat{\gamma}(K))'$, and the vector of population autocovariances as $\gamma = (\gamma(0), \gamma(1), \ldots, \gamma(K))'$. The vector of sample autocorrelations is $r = (r_1, \ldots, r_K)'$ and the vector of population autocorrelations is $\rho = (\rho_1, ..., \rho_K)'$. The vector $w_t = (w_{1t}, \ldots, w_{Kt})'$ has as its k-th component $w_{kt} = (y_t - \mu)(y_{t-k} - \mu)$ for k =1,…K, and $\hat{w}_t = (\hat{w}_{1t}, ..., \hat{w}_{Kt})'$ has as its k-th component $\hat{w}_{kt} = (y_t - \hat{\mu})(y_{t-k} - \hat{\mu})$ for k =1,…K.

We assume covariance stationarity and characterize dependence with the concept of near epoch dependence (NED) on a mixing set. Notice that we do not require strict stationarity, but we assume covariance stationarity in order for the autocovariances and the autocorrelations to be properly defined. Even the assumption of covariance stationarity could be dropped. In this case, the null hypothesis would be that the lag-j autocorrelations are zero for all t and j= 1, …, K. We note that our results can be extended to some nonstationary time series (in fact, the theoretical references that we employ are not restricted to the stationary framework), but, for simplicity, we only treat the stationary case.

ASSUMPTION 1. Let y_t be a covariance stationary process that satisfies $E|y_t|^s < \infty$ for some s> 4 and all t and is L_2-NED of size –1/2 on a process V_t where V_t is an α-mixing sequence of size –s/(s-4).

LEMMA 1. *Under ASSUMPTION 1 the vector of sample autocovariances* $\hat{\gamma}$ *satisfies the following Central Limit Theorem (CLT):*

$$\sqrt{\mathrm{n}}(\hat{\gamma}-\gamma) \Rightarrow N(0, 2\pi\mathrm{C})$$

where C is the spectral density matrix at zero frequency of the vector w_t *and has as its ij-th element*

$$c_{ij} = \sum_{d=-\infty}^{d=\infty} \{E(y_t-\mu)(y_{t-i}-\mu)(y_{t+d}-\mu)(y_{t+d-j}-\mu) - E(y_t-\mu)(y_{t-i}-\mu)E(y_{t+d}-\mu)(y_{t+d-j}-\mu)\};\ i, j = 0,1,\ldots,K.$$

Proof. First notice that

$$\sqrt{\mathrm{n}}\hat{\gamma}(\mathrm{k}) = \frac{1}{\sqrt{\mathrm{n}}}\sum_{t=k+1}^{n}\hat{\mathrm{w}}_{\mathrm{kt}} = \frac{1}{\sqrt{\mathrm{n}}}\sum_{t=k+1}^{n}\mathrm{w}_{\mathrm{kt}} + O_{\mathrm{P}}(\mathrm{n}^{-1/2})$$

since the remaining terms have the same order of magnitude as $\sqrt{\mathrm{n}}(\hat{\mu}-\mu)^2$ and $(\hat{\mu}-\mu) = O_{\mathrm{P}}(\mathrm{n}^{-1/2})$. Since each of the w_{kt} processes (normalized by the square root of the variance of $\sum_{t=k+1}^{n}\mathrm{w}_{\mathrm{kt}}$) satisfies Assumption 1 in De Jong and Davidson (1999), the CLT is a particular case of the functional CLT (Theorem 3.2) in De Jong and Davidson (1999).

LEMMA 2. *The vector* $\sqrt{nr}$ *is asymptotically normally distributed with asymptotic covariance matrix T where the ij-th element of T is given by*

$$\tau_{ij} = \gamma(0)^{-2}[c_{ij} - \rho(i)c_{0j} - \rho(j)c_{0i} + \rho(i)\rho(j)c_{00}]. \qquad (1)$$

Proof. The CLT is obtained by a straightforward application of the delta method.

Assuming T is known, $\mathrm{H_K}$: $\rho = 0$ can be tested using a test statistic of the form $\mathrm{nr'T^{-1}r}$, which asymptotically follows a chi-square with K degrees of freedom when $\mathrm{H_K}$ is true. A feasible test can be obtained either by replacing T by a known matrix or by estimating T. The Box-Pierce $\mathrm{Q_K}$ statistic replaces T with the identity matrix.

Alternative Tests. The spirit of the LM, or score test, is to exploit the restrictions imposed by the null. Under H_K, the matrix T simplifies to $\tilde{T} = \{\gamma(0)^{-2}\tilde{C}\}$ where $\tilde{C}$ has as its ij-th element

$$\tilde{c}_{ij} = \sum_{d=-\infty}^{d=\infty} E(y_t - \mu)(y_{t-i} - \mu)(y_{t+d} - \mu)(y_{t+d-j} - \mu);\ i,j = 1,...,K. \tag{2}$$

The extension of the Q_K test we propose is based on the statistic $\tilde{Q}_K = nr'\hat{\tilde{T}}r$ where $\hat{\tilde{T}}$ is a consistent estimator of $\tilde{T}$ under H_K. As will be shown below, $\tilde{Q}_K$ is asymptotically chi-squared distributed with K degrees of freedom when H_K is true.

An important case in economics and finance is where the time series is a martingale difference sequence (MDS). For a MDS process, the only possible non-zero elements of $\tilde{C}$ are terms of the form $E(y_t - \mu)^2(y_{t-i} - \mu)(y_{t-j} - \mu)$. In (2) these occur at d = 0. Guo and Phillips (1998, Theorem 5) have developed a test of H_K, the GP_K test, for the MDS case. In our context, the GP_K test is a special case of the $\tilde{Q}_K$ test where $\tilde{c}_{ij}$ is replaced by the sample analog of $E(y_t - \mu)^2(y_{t-i} - \mu)(y_{t-j} - \mu)$.

The $\tilde{Q}_K$ test can be specialized further by assuming that $\tilde{c}_{ij} = 0$ except when d = 0 and i = j, in which case $\tilde{T}$ is diagonal and is denoted by T*. In this diagonal case,

$$c^*_{jj} = E(y_t - \mu)^2 (y_{t-j} - \mu)^2,\ j = 1,...,K, \tag{3}$$

and the diagonal elements of T* are

$$\tau_{jj}^* = c^*_{jj} / \gamma(0)^2 .$$

Following Lobato, Nankervis and Savin (1999), this modified Box-Pierce statistic is denoted by Q_K^*. It is constructed by replacing the diagonal matrix T* by a consistent estimator $\hat{T}^*$:

$$Q_K^* = n r'[\hat{T}^*]^{-1} r = n\sum_{j=1}^{K}[r(j)]^2 / \hat{\tau}_{jj}^* \qquad (4)$$

where $\hat{\tau}_{jj}^*$ is a consistent estimator of τ_{jj}^*:

$$\hat{\tau}_{jj}^* = [\, n^{-1}\sum_{t=j+1}^{n}(y_t - \hat{\mu})^2 (y_{t-j} - \hat{\mu})^2 / \hat{\gamma}(0)^2\,].$$

For the hypothesis that a single autocorrelation coefficient is zero, the Q_1^* test is equivalent to the GP_1 test.

General $\tilde{Q}_k$ Test. A consistent estimator of $\tilde{T}$ is required to implement the general $\tilde{Q}_K$ test. A consistent estimator can be obtained using $\hat{\gamma}(0)$ to estimate $\gamma(0)$ and a non-parametric estimator of the matrix $\tilde{C}$. Since the matrix $\tilde{C}$ is the spectral density matrix at zero frequency of the K dimensional vector process w_t under H_K, a nonparametric consistent time domain estimator of $\tilde{C}$ is given by

$$\hat{\tilde{C}} = \sum_j k\left(\frac{j}{\ell}\right) g(j) = \frac{1}{n}\sum_j \sum_t k\left(\frac{j}{\ell}\right)\hat{w}_t \hat{w}'_{t-j} \qquad (5)$$

where $g(j) = (1/n)\sum_t \hat{w}_t \hat{w}'_{t-j}$ with $\hat{w}_t$ defined as above, $\ell > 0$ is the bandwidth parameter, and $k(\bullet)$ is the kernel or lag window. We assume that the kernel and the bandwidth satisfy the following assumptions.

ASSUMPTION 2. The kernel $k(\bullet)$ belongs to K where K is the class of functions

$$K = \{k(\bullet) : \mathbb{R} \to [-1,1]\}$$

which is symmetric around zero, continuous at zero at all but a finite number of points and satisfies

$$k(0) = 1, \int_{-\infty}^{\infty} |k(x)|\, dx < \infty, \int_{-\infty}^{\infty} |\psi(\xi)|\, d\xi < \infty$$

where $\psi(\xi) = (2\pi)^{-1}\int_{-\infty}^{\infty} k(\mathrm{x}) e^{i\xi x} \mathrm{dx}$.

ASSUMPTION 3. The bandwidth sequence satisfies

$$\lim_{\mathrm{n}\to\infty} \frac{1}{\ell} + \frac{\ell}{\mathrm{n}} = 0.$$

Then the following lemma establishes the consistency of the nonparametric estimator of $\tilde{\mathrm{C}}$.

LEMMA 3. *Under the ASSUMPTIONS 1-3 and the null hypothesis*

H_K, $\hat{\tilde{C}} \xrightarrow{p} \tilde{C}$.

Proof: Notice that $\hat{\tilde{\mathrm{C}}}$ and

$$\frac{1}{\mathrm{n}} \sum_{\mathrm{j}} \sum_{t} k(\frac{\mathrm{j}}{\ell}) \mathrm{w}_{\mathrm{t}} \mathrm{w}'_{\mathrm{t\text{-}j}}.$$

have the same probability limit and apply Theorem 2.1 in Davidson and De Jong (2000).

Hence, the following lemma establishes the asymptotic properties of the $\tilde{\mathrm{Q}}_{\mathrm{K}}$ test, namely its null asymptotic distribution and its consistency.

LEMMA 4. *Under the ASSUMPTIONS 1-3 and under the null hypothesis H_K, the test statistic $\tilde{Q}_K$ converges in distribution to a chi-square distribution with K degrees of freedom, and under $\rho \neq 0$, $\tilde{Q}_K$ diverges.*

Proof. The first part is obvious since $\hat{\tilde{\mathrm{T}}}$ is consistent for $\tilde{\mathrm{T}}$, that is, the covariance matrix of the sample autocorrelations r under H_{K}. In the second part, using the ergodic theorem, the estimator of $\tilde{\mathrm{T}}$ converges in probability to a positive definite matrix and at least one element of r is $O_P(1)$. Therefore, $\tilde{\mathrm{Q}}_{\mathrm{K}}$ diverges.

3. MONTE CARLO EXPERIMENTS UNDER THE NULL

This section reports estimates of the probability of making a Type I error for the Q_K, Q^*_K, GP_K and $\tilde{Q}_K$ tests based on critical values from the chi-square distribution with K degrees of freedom. The estimated rejection probabilities are computed for two MDS examples and two non-MDS examples. Three different hypotheses are considered: H_K: $\rho(1) = \ldots = \rho(K) = 0$, K = 1, 5, 10. Each hypothesis is tested at nominal levels 0.01,0.05 and 0.10. The estimates (empirical rejection probabilities) are calculated using 25,000 replications for sample sizes n = 200, 1,000 and 5,000. In the tables, an asterisk denotes that the empirical rejection probability is significantly different at the 0.01 level from the nominal rejection probability, where the significance is evaluated using a 0.01 level two-sided asymptotic test.

The $\tilde{Q}_K$ test requires a consistent estimator of the asymptotic covariance matrix of the sample autocorrelations. In this section, we explore the finite sample performance of two automatic data based covariance matrix estimation procedures that have been used in the literature. The first procedure (AUTO) employs AR(1) prewhitening/recoloring on each series and selects the bandwidth using formula (2.2) of Newey-West (1994) with weights equal to one and lag truncation equal to $2(n/100)^{2/9}$. The second procedure uses an autoregressive estimator of the covariance matrix where the order of each equation in the VAR is selected automatically. Phillips and Lee (1994) have employed an ARMA model in the scalar case and Den Haan and Levin (1997) a VAR in the vector case. We report results for the vector procedure (VAR) with the AIC (Akaike (1973)) and the SC (Schwarz (1978)) criteria. We set the maximum lag length as 3, 10 and 15 for sample sizes n = 200, 1000 and 5000, respectively, and, for any equation in the VAR, the same lag length is used for each element of the vector process.

3.1 MDS Examples

Monte Carlo experiments are conducted using two examples based on a GARCH (1,1) model, one with normal errors and one where the errors are centered chi-square with 3 degrees of freedom. The GARCH (1,1) model is $y_t = z_t \bullet \sigma_t$ where z_t is an iid sequence and $\sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta\sigma_{t-1}^2$, where α and β are constants such that $\alpha + \beta < 1$. This condition is needed in order that y_t is covariance stationary. He and Teräsvirta (1999) show that the unconditional fourth moment of y_t exists for GARCH (1,1) models if and only if $\beta^2 + 2\alpha\beta\nu_2 + \alpha\nu_4 < 1$ where $\nu_i = E|z_t|^i$. We set $\omega = 0.001$, $\alpha = 0.05$ and $\beta = 0.90$. With this parameter setting, the He and Teräsvirta (1999) condition for the existence of the fourth moment of y_t is satisfied for both models we consider. Estimates from stock return data suggest that $\alpha + \beta$ is close to 1 with β also close to 1; for example, see Bera and Higgins (1997).

Example 1. Gaussian GARCH. z_t is iid N(0, 1). For this process, $\gamma(0) = E(y_t - \mu)^2 = 0.02$, $E(y_t - \mu)^3/\gamma(0)^{3/2} = 0$, $E(y_t - \mu)^4/\gamma(0)^2 = 3.16$, and T is diagonal.

Example 2. Chi-square (3) GARCH. z_t is a demeaned and standardized iid chi-square random variable with 3 degrees of freedom. In this case (the skewness is an estimate), $\gamma(0) = E(y_t - \mu)^2 = 0.\ 02$, $E(y_t - \mu)^3/\gamma(0)^{3/2} = 1.72$, and $E(y_t - \mu)^4/\gamma(0)^2 = 8.27$ and T is no longer diagonal.

The empirical rejection probabilities for the GARCH (1,1) examples are reported in Table 1 for n = 200 and 1000. The main facts that emerge from these results are the following: (i) At n = 200, the GP_K test and the three versions of the $\tilde{Q}_K$ test tend to perform poorly, especially for $H_{10.}$ The exceptions are the VAR (AIC) version of the $\tilde{Q}_K$ test for Gaussian GARCH and the VAR (SC) version for the chi-square (3) GARCH. (ii)

At n = 1000, the GP_K test tends to work satisfactorily for both Gaussian and chi-square (3) GARCH and similarly for the VAR versions of the $\tilde{Q}_K$ test. AUTO tends to under-reject, especially for $H_{10.}$ The results for the GP_K test and the VAR (SC) version of the $\tilde{Q}_K$ test tend to be similar since the SC criterion tends to choose a zero lag, which, asymptotically, is the correct lag for the VAR.

The results are not reported for n = 5000 since at this sample size the asymptotic approximation to the finite sample distribution is accurate for the GP_K test and the three versions of the $\tilde{Q}_K$ test. Notice that the distortions in the rejection probabilities of the Q_K test are larger for n = 1000 than n = 200. In particular, the Q_K test suffers from substantial over-rejection for all hypotheses, especially for H_{10}.

3.2 Non-MDS Examples

This section investigates the finite sample performance of the tests for uncorrelated non-MDS processes, a bilinear model and an all-pass model. The bilinear model is described in Granger and Teräsvirta (1993) and the all-pass model in Breidt, Davis and Trindale (1999). Both these models can produce time series having similar properties to those of series produced by GARCH models, namely, where the autocorrelation function of the level of the series is flat, but the autocorrelation function of the squares (absolute values) of the series declines slowly. Neither the Q_K* test or the GP_K test is asymptotically valid for these two examples.

Example 3. Bilinear Model. Let $y_t = z_t + b\, z_{t-1}y_{t-2}$ where $\{z_t\}$ is a sequence of iid $N(0, \sigma^2)$ random variables. The y_t process is uncorrelated, but not independent and is covariance stationary provided that $b^2\, \sigma^2 < 1$. The fourth moment of this process exists if $3b^4\sigma^4 < 1$. We set b = 0.50 and $\sigma^2 = 1.0$. For this process, the first four moments are μ =

0, $\gamma(0) = E(y_t - \mu)^2 = \sigma^2/(1 - b^2\sigma^2) = 1.33$, $E(y_t - \mu)^3/\gamma(0)^{3/2} = 0$, $E(y_t - \mu)^4/\gamma(0)^2 = 3(1 - b^4\sigma^4)/(1 - 3b^4\sigma^4) = 3.46$. See Granger and Andersen (1978) for more details with respect to this example.

Example 4. All-Pass ARMA(1,1) Model. Let $(y_t - \mu) - \phi(y_{t-1} - \mu) = (z_t - \phi^{-1}z_{t-1})$ where $\{z_t\}$ is a sequence of iid random variables and where $\mu = 0$ and $\phi = 0.8$. The y_t process is uncorrelated, but not independent if z_t is nonnormal. The fourth moment of y_t is finite provided z_t has a finite fourth moment. In our example, z_t is Student t with 10 degrees of freedom. For this process, the first four moments are $\mu = 0$, $\gamma(0) = E(y_t - \mu)^2 = 1.95$, $E(y_t - \mu)^3/\gamma(0)^{3/2} = 0$, and $E(y_t - \mu)^4/\gamma(0)^2 = 3.44$. A special feature of this example (where the process is linear and uncorrelated) is that T is the identity matrix; see Bartlett (1946).

Table 2 reports the empirical rejection probabilities for the non-MDS examples samples of n = 1,000 and 5,000 since the asymptotic approximation is poor at n = 200. For the bilinear example, the empirical rejection probabilities are given in the first and second panel of Table 2. The Q_K test over-rejects by a large margin for the three hypotheses. The Q_K* and GP_K tests perform similarly for the three hypotheses; both tests substantially over-reject. The AUTO version of $\tilde{Q}_K$ test tends to be unsatisfactory. The VAR versions tend to over-reject when n = 1,000, but they are satisfactory for all three hypotheses when n = 5,000.

For the all-pass ARMA (1,1) examples, panels three and four of Table 2 show that the performance of the Q_K test is excellent. This is explained by the fact that the T matrix is the identity. The Q_K* and GP_K tests perform similarly; both under-reject the null. All three versions of the $\tilde{Q}_K$ test also tend to under-reject. The VAR (AIC) version, however, is noticeably better than the other two versions. And it only marginally under-rejects when n = 5000.

Computing. The random number generator used in the experiments was the very long period generator RANLUX with luxury level p = 3; see Hamilton and James (1997). Calculations were performed on a Silicon Graphics R10000 system and on a Sun Enterprise 3000 server using double precision Fortran 77. The program used for VARHAC was a version of the program by den Haan and Levin (http://weber.ucsd.edu/~wdenhaan/varhac.html) modified to run substantially faster. In order to mitigate the effect of occasional extreme estimates the program was also modified, using the procedure of Andrews and Monahan (1992), by setting the minimum singular value of the recoloring matrix to be 0.005.

4. POWERS

This section reports the empirical powers of the tests in a small Monte Carlo study. In the experiments, the times series are generated by an MA (1) process with uncorrelated errors: $y_t = u_t + \theta u_{t-1}$. The processes generating the u_t's are those specified in Examples 1 to 4. Under the null hypothesis, $\theta = 0$, y_t is uncorrelated. The values of θ are selected so that the alternative values of $\rho(1)$ range from 0.025 to 0.15. For the sake of brevity, results are reported only for the 0.05 tests of the null hypotheses H_1 and H_5 when n = 1000. To simplify the power comparisons, the critical values are adjusted so that empirical rejection probabilities of the tests under the null are exactly 0.05.

The first two panels in Table 3 reports the empirical powers for the GARCH (1,1) examples. The results show that the empirical powers are very similar for all the tests with the powers being essentially equal to one at $\rho(1) = 0.15$. Hence, for these examples there is no appreciable loss of power when the $\tilde{Q}_K$ test is employed. The last two panels present the empirical powers for three versions of the $\tilde{Q}_K$ for the bilinear and all-pass ARMA (1,1) examples. In each example, the empirical powers are again similar.

5. DISCUSSION

This paper presents an omnibus test of uncorrelatedness in the presence of statistical dependence. The proposed test statistic, $\tilde{Q}_K$, which can be viewed as an extension of the Box-Pierce Q_K statistic, is asymptotically chi-square distributed under the null. The finite sample performance of three automatic data-based versions of the $\tilde{Q}_K$ test are examined. The study first considers examples based on GARCH (1,1) models. For these examples, the comparison of interest is between the GP_K test and the $\tilde{Q}_K$ test since the GP_K test is designed for MDS processes. In this comparison, both tests tended to provide satisfactory control over the probability of making a Type I error when the sample size was n = 1000, but the control was unsatisfactory at n = 200.

The study next considered examples based on two non-MDS models, namely, a bilinear model and an all-pass ARMA (1,1) model. As expected, the GP_K test suffered from distortions in the Type I error at n = 1000 and 5000. The distortions are substantial for the bilinear example and relatively mild for the all-pass example. No version of the $\tilde{Q}_K$ test is satisfactory at n = 1000. The VAR (AIC) version works at n = 5000 for the bilinear example, but only marginally so for the all-pass ARMA (1,1). The message is that large samples are needed for asymptotic theory to provide a reasonable approximation to the distribution of the $\tilde{Q}_K$ statistic for non-MDS processes.

In a small power study, the empirical powers of the GP_K test and the three versions of the $\tilde{Q}_K$ test are similar for the GARCH examples; the empirical powers of the three versions of the $\tilde{Q}_K$ test are also similar for each of the non-MDS examples.

REFERENCES

Akaike, H. (1973) Information theory and the extension of the maximum likelihood principle. In B. Petrov and F. Csaki (eds.) *Second International Symposium on Information Theory*, pp. 267-281. Budapest: Akailseoniai-Kiudo.

Andrews, D.W. K. and J. C. Monahan (1992) An improved heteroskedasticity and autocorrelation consistent covariance matrix estimator. *Econometrica* 60, 953-966.

Bartlett, M. S. (1946) On the theoretical specification and sampling properties of autocorrelated series. *Journal of the Royal Statistical Society*, supplement 8, 27-41, 85-97.

Bera, A.K. and M.L. Higgins (1997) ARCH and bilinearity as competing models for nonlinear dependence. *Journal of Business and Economic Statistics*, 15, 43-51.

Box, G.E.P. and D. A. Pierce (1970) Distribution of residual autocorrelations in autoregressive integrated moving average time series models. *Journal of the American Statistical Association* 65, 1509-1526.

Breidt, F.J., R.A. Davis and A. Trindade (1999) Least absolute deviation estimation for all-pass time series models, preprint, Colorado State University.

Davidson, J. and De Jong, R.M. (2000) Consistency of kernel estimators of heteroscedastic and autocorrelation covariance matrices. *Econometrica* 68, 407-424.

De Jong, R.M. and J. Davidson (1999) The functional central limit theorem and weak convergence to stochastic integrals I: weakly dependent processes, preprint, Cardiff University

Den Haan, W. J. and A. Levin (1997) A practitioner's guide to robust covariance matrix estimation. In G. S. Maddala and C.R. Rao (eds.) *Handbook of Statistics*: *Robust Inference*, vol 15, pp. 291-341. Amsterdam: North-Holland.

Diebold, F. X. (1986) Testing for serial correlation in the presence of heteroskedasticity. In *Proceedings of the American Statistical Association, Business and Economics Statistics Section*, pp.323-328.

Granger, C. W. J. and A. P. Andersen (1978) *An Introduction to Bilinear Time Series Models*. Gottingen: Vanenhoek and Ruprecht.

Granger, C. W. J. and T. Teräsvirta (1993) *Modelling Nonlinear Economic Relationships*. Oxford: University of Oxford Press.

Guo, B. B. and P.C. B. Phillips (1998) Testing for autocorrelation and unit roots in the presence of conditional heteroskedasticity of unknown form. Cowles Foundation for Research in Economics, Yale University.

Hamilton, K. G. and F. James (1997) Acceleration of RANLUX. *Computer Physics Communications* 101, 241-248.

He, C. and T. Teräsvirta (1999) Properties of moments of a family of GARCH processes. *Journal of Econometrics*, 92, 173-192

Lo, A.W. and A. C. MacKinlay (1989) The size and power of the variance ratio test in finite samples: a monte carlo investigation. *Journal of Econometrics* 40, 203-238.

Lobato, I., J.C. Nankervis and N.E. Savin (1999) Testing for autocorrelation using a modified Box-Pierce Q test, preprint, Department of Economics, University of Iowa.

Newey, W.K. and K.D. West (1994) Automatic Lag Selection in Covariance Matrix

Estimation. *Review of Economic Studies* 61, 631-653.

Phillips, P.C. B. and C.C. Lee (1994) An ARMA-prefiltered estimator of the long run variance with an application to unit root test, preprint, Yale University.

Schwarz, G. (1978) Estimating the dimension of a model. *Annals of Statistics* 6, 461-464.

Taylor, S. (1984) Estimating the variances of autocorrelations calculated from financial time series. *Applied Statistics* 33, 300-308.

Terdik, G. (1999*) Bilinear Stochastic Models and Related Problems of Nonlinear Time Series Analysis.* New York: Springer Verlag

Table 1. Rejection Probabilities (Percent) of Tests: MDS Examples

	H_1			H_5			H_{10}		
	1	5	10	1	5	10	1	5	10
	GARCH (1,1) with Normal Errors, n = 200								
Q	1.4*	6.0*	11.3*	1.7*	7.0*	12.5*	1.9*	6.9*	12.4*
Q*	1.0	4.9	10.1	1.0	4.9	9.8	1.1	5.0	9.3*
GP	1.0	4.9	10.1	0.6*	4.1*	8.9*	0.4*	3.0*	7.4*
AUTO	0.7*	4.8	10.1	0.5*	3.6*	8.1*	0.2*	1.9*	5.3*
VAR(AIC)	0.6*	4.4*	9.7	0.9	4.9	10.1	1.0	4.4*	9.4*
VAR(SC)	0.7*	4.4*	9.7	0.7*	4.4*	9.2*	0.4*	3.3*	7.7*
	GARCH (1,1) with Normal Errors, n = 1000								
Q	1.6*	6.5*	12.2*	2.1*	7.9*	14.4*	2.3*	8.5*	15.4
Q*	1.0	4.9	9.9	1.0	4.8	9.9	1.1	4.8	9.7
GP	1.0	4.9	9.9	0.9	4.7	9.8	0.8	4.3*	9.2*
AUTO	1.0	4.9	9.7	0.8	4.4*	9.5	0.6*	3.6*	8.1*
VAR(AIC)	0.9	4.7	9.7	0.9	5.0	10.2	0.9	4.6	9.8
VAR(SC)	1.0	4.9	9.9	1.0	4.8	10.0	0.8	4.4*	9.4*
	GARCH (1,1) with Chi-square (3) Errors, n = 200								
Q	1.9*	7.4*	13.6*	2.9*	9.2*	15.7*	3.4*	9.8	16.4*
Q*	1.1	5.4	11.0*	1.2	5.7*	11.2*	1.5*	6.1*	11.7*
GP	1.1	5.4	11.0*	0.6*	4.5*	9.9	0.5*	3.7*	8.3*
AUTO	0.8	4.8	10.0	0.7*	3.6*	8.0*	0.5*	2.2*	5.2*
VAR(AIC)	0.6*	4.4*	10.1	1.0	4.7	10.0	1.8*	5.8*	10.9*
VAR(SC)	0.7*	4.5*	10.1	0.9	4.9	10.4	1.1	4.8	9.6
	GARCH (1,1) with Chi-square (3) Errors, n = 1000								
Q	2.8*	9.5	15.8*	5.7*	14.8*	22.6*	7.0*	17.0*	25.9*
Q*	1.1	5.1	9.9	1.4	5.9*	11.2*	1.3*	5.7	10.9*
GP	1.1	5.1	9.9	1.2	5.6*	10.7	0.9	4.7	9.9
AUTO	0.8	4.8	9.8	0.8	4.4*	9.2*	0.5*	3.2*	7.5*
VAR(AIC)	0.8	4.7	9.7	0.9	4.8	10.1	0.8	4.3*	9.3*
VAR(SC)	0.9	4.8	9.8	1.2	5.5*	11.0*	1.2	5.4	11.0*

Notes: The number of replications is 25,000. An asterisk denotes that the empirical rejection probability is significantly different at the 0.01 level from the nominal rejection probability, where the significance is evaluated using a 0.01 level two-sided asymptotic test.

Table 2. Rejection Probabilities (Percent) of Tests: Non-MDS Examples

	H_1			H_5			H_{10}		
	1	5	10	1	5	10	1	5	10
	Bilinear, n = 1000								
Q	5.6*	14.9*	22.4*	6.4*	16.3*	25.0*	4.5*	12.8*	20.6*
Q*	2.0*	7.9*	14.3*	2.1*	7.8*	14.1*	1.7*	6.7*	12.5*
GP	2.0*	7.9*	14.3*	1.8*	7.4*	13.6*	1.2	5.8*	11.6*
AUTO	1.1	5.8*	11.5*	1.0	5.0	10.3	0.6*	4.0*	8.7*
VAR(AIC)	0.9	4.9	10.0	1.2	5.5*	11.1*	1.2	5.4	10.8*
VAR(SC)	1.2*	5.9*	11.4*	1.5*	6.6*	12.3*	1.2	5.9*	11.4*
	Bilinear, n = 5000								
Q	6.1*	15.1*	23.0*	6.6*	17.0*	26.2*	4.7*	13.8*	22.4*
Q*	2.1*	8.1*	14.2*	1.9*	7.9*	14.2*	1.7*	6.5*	13.3*
GP	2.1*	8.1*	14.2*	1.8*	7.7*	14.1*	1.5*	6.5*	12.8*
AUTO	1.3*	5.9*	11.2*	1.2	5.7*	10.8*	1.0	4.8	10.0
VAR(AIC)	1.0	4.9	10.0	1.1	5.2	10.4	1.1	5.3	10.4
VAR(SC)	1.0	4.7	9.7	1.1	5.3	10.4	1.2	5.9*	11.2*
	All-Pass ARMA (1,1), n =1000								
Q	0.9	4.8	9.8	0.9	4.7	9.7	0.9	4.8	9.6
Q*	0.7*	3.9*	8.5*	0.7*	3.8*	8.4*	0.8	4.2*	8.7*
GP	0.7*	3.9*	8.5*	0.7*	3.8*	8.3*	0.6*	3.9*	8.3*
AUTO	0.7*	4.3*	9.2*	0.6*	4.0*	8.5*	0.5*	3.6*	8.1*
VAR(AIC)	0.7*	4.4*	9.3*	0.8	4.2*	9.0*	0.8	4.4*	9.4*
VAR(SC)	0.7*	4.1*	8.7*	0.7*	4.0*	8.5*	0.6*	4.0*	8.4*
	All-Pass ARMA (1,1), n = 5000								
Q	1.1	5.1	10.0	0.9	4.8	9.6	1.0	4.9	9.8
Q*	0.8	4.1*	8.4*	0.7*	3.9*	8.2*	0.8	4.2*	8.7*
GP	0.8	4.1*	8.4*	0.7*	3.9*	8.2*	0.7*	4.1*	8.6*
AUTO	1.0	4.7	9.4	0.8	4.4*	8.9*	0.8	4.3*	9.0*
VAR(AIC)	0.9	4.7	9.4*	0.7*	4.5*	9.2*	0.9	4.6	9.4*
VAR(SC)	0.8	4.3*	8.7*	0.7*	4.1*	8.5*	0.7*	4.1*	8.7*

Notes: See Table 1.

Table 3. Empirical Powers (Percent) of 0.05 Adjusted Tests, n = 1000

	H_1							H_5					
$\rho(1)$	.025	.050	.075	.100	.125	.150		.025	.050	.075	.100	.125	.150
	GARCH (1,1) with Normal Errors												
Q*	11.1	30.6	59.6	83.8	96.0	99.3		7.32	16.3	35.0	60.2	82.3	95.2
GP	11.1	30.6	59.6	83.8	96.0	99.3		7.22	16.2	35.1	60.8	83.2	95.4
AUTO	10.9	30.4	59.1	83.7	95.9	99.4		7.16	15.8	33.7	59.1	81.9	94.9
VAR(AIC)	10.9	30.4	59.0	83.3	95.7	99.2		7.10	15.5	32.8	57.2	79.9	93.7
VAR(SC)	10.9	30.6	59.4	83.7	96.0	99.4		7.37	16.3	35.0	60.4	82.8	94.9
	GARCH (1,1) with Chi-square (3) Errors												
GP	7.63	23.4	50.4	78.2	94.0	99.0		5.76	11.3	25.4	49.1	74.7	91.8
AUTO	7.84	23.9	51.4	78.4	93.7	98.9		5.80	11.6	25.8	48.9	73.8	90.2
VAR(AIC)	7.96	23.3	50.2	76.8	92.6	98.1		6.02	12.1	26.0	48.5	71.8	88.0
VAR(SC)	7.74	23.2	50.2	77.4	92.9	98.4		5.98	12.1	26.1	49.5	73.8	90.4
	Bilinear												
AUTO	7.29	18.6	38.6	62.9	82.7	94.3		5.94	11.5	23.6	42.6	64.7	83.0
VAR(AIC)	7.12	18.0	37.6	61.8	82.1	93.8		6.01	10.9	21.5	38.8	60.4	79.9
VAR(SC)	7.06	18.0	37.5	60.8	81.1	93.2		6.28	12.3	24.5	43.9	66.3	84.2
	All-Pass ARMA (1, 1)												
AUTO	11.6	33.8	64.7	87.7	97.7	99.8		7.71	17.6	38.1	65.2	86.7	97.1
VAR(AIC)	11.7	33.4	64.4	87.4	97.6	99.8		8.05	18.3	38.9	65.9	87.2	97.2
VAR(SC)	11.9	34.0	65.0	88.0	97.7	99.8		7.81	18.3	39.4	66.6	87.7	96.9

Notes: The number of replications is 25,000. The critical values are adjusted so that empirical rejection probabilities of the tests under the null are exactly 0.05.