Policy Reversals and Electoral Competition  
with Privately Informed Parties  

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Abstract: We develop a spatial model of competition between two policy-motivated parties. Parties know a state of the world which determines which policies are desirable for voters, while voters do not. The announced positions of the parties serve as signals to the voters concerning the parties’ private information. In all separating equilibria, when the left-wing party attains power, the policies it implements are to the right of the policies implemented by the right-wing party when it attains power. The intuition behind this result is that when right-wing policies become more attractive, the left party moves toward the right in order to be assured of winning, while the right-wing party stays put in a radical stance.

Key words: spatial models, party competition, asymmetric information, separating equilibria
1 Introduction

Elections aggregate information dispersed among the public by allowing voters to express their opinions about the policy positions adopted by the parties. However, the information available to the public might miss key facts in economic or foreign policy issues that are available only to policymakers. Indeed, while policymakers deal frequently with policy issues, for most other people it is irrational to become politically well-informed (Downs [1957]). In such a case, the welfare of society may increase if the policymakers convey their private information to voters before the election. Policy proposals, therefore, have two different roles. One is to announce the parties’ positions to the public so that voters can express their opinions and preferences. The other is to send signals to voters regarding the parties’ private information. The first role has long been recognized in the literature inspired by Condorcet’s [1785] Jury Theorem.1 The second role, on the other hand, has received much less attention.2 It is this aspect of electoral competition we investigate in the present paper.

If political parties are privately informed, there is no guarantee that they will reveal their information to the public. Strategic manipulation may arise when the preferences of parties and voters differ. In the model we propose, two parties with polarized preferences – a right-wing party and a left-wing party – obtain information unobservable by the public and compete for office, thus creating the possibility of such manipulation. It turns out that this model generates “policy reversals,” i.e., situations in which the right-wing


2Harrington [1992, 1993], Roemer [1994], Schultz [1996], and Cukierman and Tommasi [1998a,b] are a few exceptions.
party implements left-leaning policies, and vice versa.

It is interesting to note that in a number of episodes important policy shifts have been supported by parties or candidates whose traditional positions were to oppose such policies. The Nixon and Clinton administrations have been considered by the media to be examples of policy reversals.\(^3\) Cases in point are market-oriented reforms in Latin America. Throughout the region, radical trade liberalization and fiscal adjustment have been implemented by the parties or candidates that had proved in the past a penchant for populism and interventionism, such as Menem in Argentina, Fujimori in Peru, and Paz Estenssoro in Bolivia, as pointed by Rodrik [1993].\(^4\) While we do not claim that every large policy shift has been conducted by political parties having a record of opposing it. Clear counterexamples are the New Deal implemented under F. D. Roosevelt and the increase in the military spending under Reagan. But the episodes mentioned above raise the question of under which circumstances voters would end up supporting a political party to implement a set of policies that appear to be far from the party’s ideal policies rather than a political party whose ideology favors such policies.

Our model focuses on electoral competition between two equally informed parties. The political parties have better information than the voting public about the likely outcome of different policies. Political parties are represented as having distinct and polarized preferences on outcomes, and hence on policies.\(^5\) The utility of voters depends on the outcome of the policy adopted by

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\(^3\) For example, *The Economist* claims that Clinton governs like a Republican “just as Nixon governed like a Democrat.” (“A Democratic Nixon?” November 2, 1996.) The same parallel is drawn in *Newsweek*, November 16, 1998.

\(^4\) We do not claim that every large policy shift has been conducted by political parties having a record of opposing it. Clear counterexamples are the New Deal implemented under F. D. Roosevelt and the increase in the military spending under Reagan. But the episodes mentioned above raise the question of under which circumstances voters would end up supporting a political party to implement a set of policies that appear to be far from the party’s ideal policies rather than a political party whose ideology favors such policies.

\(^5\) The above assumption on parties’ preferences is not very realistic in two respects. First, it is extreme to assume that their preferences are polarized. However, the qualitative result would not change as long as the bliss points of the parties and the median voter are aligned in the same order in every state of the world, *i.e.*, the bliss point of the left-wing
the winning party. The voters know exactly their preferences over outcomes, but they do not know with certainty how policies relate to outcomes. The parties simultaneously announce and commit to their policy platforms, after obtaining some information about the correspondence between policies and outcomes. The voters do not have the information that the parties obtained. Instead, they observe the announced platforms and then decide which party to vote for. The policy announced by the winning party is implemented.

The paper characterizes all equilibria of the game in which parties play pure strategies. We distinguish two types of equilibria, pooling equilibria and separating equilibria. In pooling equilibria, voters are unable to infer the information held by the parties since neither party conditions its policy announcements on the received information. In separating equilibria, voters are able to infer the information held by the parties from their policy announcements, and use this information in order to decide which party to support.

In all equilibria, the platform of the right-wing party is located to the right of that of the left-wing party in every single election. That is, in every election, policy platforms are ordered as we would expect from the ideological positions of the parties. Separating equilibria, however, exhibit a paradoxical feature: the right-wing party implements policies that are to the left of the policies implemented by the left-wing party. Different parties win in different states of the world.

In separating equilibria the information shared by the parties serves as a correlation device for the parties’ policy platforms. There are two types (resp. right-wing) party is to the left (resp. right) of that of the median voter. Second, as discussed in Section 5, our results hold if parties care both about policies and about winning the election, as long as the policy motivation is strong compared to the desire to hold office.
of separating equilibria, according to whether policy platforms are positively correlated or negatively correlated with the “state of the world” (i.e. the median voter’s preferred policy platform if voters learn the information held by the parties). We focus the analysis on positive correlation equilibria. In all positive correlation equilibria, the median voter supports the left-wing party when the revealed information favors the adoption of right-leaning policies, and vice versa. This result, together with some additional observations, gives rise to the paradox of policy reversals.

Roughly speaking, the above result is supported as an equilibrium outcome by the following location choices of the parties and belief formation of the voters. Suppose that the signal favors the adoption of right-leaning policy (the other case is similarly taken care of). In this case, the policy position of the left-wing party is to the left of the median voter’s bliss point, while the position of the right-wing party is to the right, and they are equally distant from the bliss point. Although the median voter is indifferent, in equilibrium the left-wing party wins the election. If the right-wing party moves toward the median voter’s bliss point, then the voters’ beliefs are changed so that they put a positive probability on the other signal and still favor the position of the left-wing party. The movement in the opposite direction does not change the outcome. The left-wing party also has no incentive to move. If it moves to the left, then it loses the election, and the opponent implements the policy. It has no incentive to move to the right since it wins the election anyway.

The logic behind the nonexistence of other separating equilibria with positive correlation is complicated as there are many candidates for equilibria that should be eliminated. In this introduction, we provide intuition for why
the moderate wins for certain, i.e., the left-wing party wins with probability one when the signal favors the adoption of right-leaning policies, and the right-wing party wins in the opposite case.

Suppose, in an equilibrium with positive correlation, that the right-wing party wins with positive probability when the signal favors the right-leaning policies. We divide the argument into two cases. First, if this party’s policy position is located to the right of the median voter’s bliss point, then the left-wing party has an incentive to move towards the right just enough to win the election for certain. In doing so, it does not have to worry about an unfavorable shift of voters’ beliefs since they already have the worst beliefs for the party.

Second, if the right-wing party is located either at the median voter’s bliss point or to its left, then it wins the election for certain. In this case, the left-wing party has an incentive to mislead the public by switching its platform to the one that it would take when the realized signal favors the left-leaning policies. Thus, in an equilibrium with positive correlation it cannot be the case that the right-wing party wins with a positive probability if its favorite state is realized.

We also analyze separating equilibria with negative correlation between platforms and the state of the world. These equilibria are even more paradoxical: both the left-wing party and the right-wing party change their platforms further to the left if the state of the world favors the adoption of right-leaning policies, a possibility for which we strongly doubt there is a real life counterpart. Indeed, we show that among separating equilibria only those with positive correlation survive a requirement similar in flavor to renegotiation-proofness, which we call credibility. Credibility requires that no party an-
nounces a platform which it would like to renege on after the election is over and the median voter, *a fortiori*, a majority, would be willing to go along with such a deviation from announced policy platforms.

We have clear-cut welfare implications when we measure welfare in terms of the expected payoff of the median voter. First, every separating equilibrium with positive correlation leads to a higher expected payoff for the median voter than does any pooling equilibrium, which in turn leads to a higher expected payoff than does any separating equilibrium with negative correlation. Moreover, among separating equilibria with positive correlation, the farther the two implemented platforms are apart, the higher is the expected payoff for the median voter. To the extent that they are best for voters, we can consider separating equilibria to be focal.

It is useful to compare our work with that of Schultz [1996]. In a similar model of competition between two informed parties, he obtains that separating equilibria necessarily entail full convergence of the parties’ policy platforms to the median voter’s desired policy in each state of the world. Separating equilibria only obtain if the preferences of at least one of the parties are not too different from those of the median voter, so that, say, even the left party favors the adoption of right-wing policies when the state of the world favors the adoption of such policies. The striking difference between his results and ours has two sources. First, unlike Schultz, we assume that there is some electoral uncertainty. This feature of our model, which we deem realistic and in line with the previous literature on spatial competition, precludes full convergence of the policy platforms in equilibrium as long as parties are primarily policy-motivated. Second, Schultz restricts voters’ beliefs to be resistant to unilateral deviations in separating equilibria.6 This

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6A similar refinement is proposed by Bagwell and Ramey [1991] in a related multisender model.
refinement precludes the existence of separating equilibria without full convergence such as those we study because at least one of the two parties is tempted to deviate toward the median voter’s optimal policy. In our setting, this refinement would select pooling equilibria. We remain agnostic on refinements, and consider both pooling and separating equilibria as plausible predictions.

It is also useful to compare our work with the work of Cukierman and Tommasi, who first built a model to explain policy reversals. Cukierman and Tommasi [1998a] explain policy reversals in a context in which the incumbent government, but not the challenger, has better information than voters about the relation between policies and outcomes. In a related paper, Cukierman and Tommasi [1998b] explain policy reversals in the context that the party in power must submit a policy proposal to a referendum. In both cases, the driving force of their result is that voters are willing to accept right-leaning policies only when they are proposed by a left-wing incumbent: “if even the left-wing party favors the rightward shift, we must favor it too.” For this argument to work, it must be the case that the policy reversal is observed only when there is an extreme shock so that even the incumbent prefers to move in the direction it normally dislikes. Cukierman and Tommasi note that the credibility of a policy (that is, how appropriate people think is a policy given their beliefs about the state of the world) depends on the ideological identity of the policy maker proposing it, as well as on the policy he proposes. The setup of our model is different from theirs in that we have two informed parties competing for the electorate. Both parties commit to policy platforms before the election, while in Cukierman and Tommasi [1998a] only the insignalling game.
formed party (the incumbent) has to commit and the competition between parties play no role. In our model, policy reversals are observed even when the most preferred policy of the left-wing party is unambiguously tilted to the left. Reversals occur not because everyone likes it, but because one of the two parties reluctantly moves toward the policies it dislikes in order to win the election. Cukierman and Tommasi’s account of policy reversals seems more appropriate to explain the case of a winning candidate who chooses a surprising policy, while our account is perhaps more appropriate when parties move their positions before an election in the direction most preferred by the eventual loser.

The rest of the paper is organized as follows. Section 2 builds a model with the features described above. The model is an extension of the Wittman-Calvert model in which parties, but not voters, are informed of an unknown state that affects the desirability of policies. Section 3 characterizes and discusses pooling equilibria. Section 4 does a similar analysis for separating equilibria. Section 5 presents some extensions. Section 6 concludes the paper.

2 Model

Consider a society with two parties, denoted by $L$ and $R$, and a number of voters. They play an election game. In the beginning, nature chooses one of two possible states, $-1$ and $1$. We assume that both states occur with the same probability. After observing the state, the two parties simultaneously propose a platform, given by a real number. In the next stage, voters vote for one of the two parties. Before they vote, they observe the platforms of the parties but not the state of nature. The party that obtains the majority of the votes wins the election and carries out the proposed platform.
If the state is $s = -1, 1$, and if the implemented policy is located at $x$, then the outcome $y$ is assumed to be given by

$$y = x - s.$$ 

Thus, the policy position $x = s$ always induces the outcome $y = 0$.

We assume that all voters have symmetric, single-peaked preferences on the outcome space. That is, for every voter there is an ideal outcome, and the utility of the voter is strictly decreasing in the distance from his or her ideal and the actual outcome. To keep matters simple, we assume that voters vote sincerely. Since the outcome space is one-dimensional and voters have single-peaked preferences, all that matters for the analysis is the vote of the median voter. The ideal outcome for the median voter is 0. Thus, if the proposed platform of the winning party is $x \in \mathbb{R}$, and if the state is $s$ ($s = -1, 1$), then the median voter’s payoff function $v(x, s)$ is single-peaked and symmetric around $s$. Voters are expected payoff maximizers.

Parties $L$ and $R$ have lexicographic preferences. They first care about the outcome of the actual implemented policy irrespective of which party is elected. If the payoffs from the chosen policies are identical, each party prefers winning the election to losing it. The (first-order) payoff functions for Party $L$ and Party $R$ are $u_L(x, s)$ and $u_R(x, s)$, respectively. For the sake of simplicity, we keep symmetry by assuming $u_L(-\delta, s) = u_R(\delta, s)$ for all $\delta$ for $s = -1, 1$. We assume further that $u_L(x, s)$ is strictly concave and strictly decreasing in $x$, a fortiori, $u_R(x, s)$ is strictly concave and strictly increasing in $x$. That is, Party $L$ always prefers outcomes to the left and Party $R$ always
prefers outcomes to the right. Moreover, both parties are risk-averse. A simple example would be \( u_L(x,s) = 1 - e^x \) and \( u_R(x,s) = 1 - e^{-x} \) for \( s = -1, 1 \).

Our equilibrium concept is a variant of perfect Bayesian equilibrium in which voters’ beliefs are required to satisfy Bayes’ law on the equilibrium path and each party’s action is required to be optimal not only against the voters’ equilibrium strategy, but also against strategies close to this equilibrium strategy. These trembles correspond to some (arbitrarily) small uncertainty about how voters would vote, which turns out to preclude complete convergence of policy platforms in equilibrium. We focus on the equilibria in which the parties use pure strategies but the median voter does not necessarily do so.

The above restrictions reduce our description of an equilibrium, or in general, a pair of a strategy profile and a belief system, to a tuple of the form \( ((x^-_L, x^+_L), (x^-_R, x^+_R), \mu, q) \). In this expression, \( x^-_P \) (respectively \( x^+_P \)) is the platform proposed by Party \( P = L, R \) when it receives the signal \( -1 \) (respectively \( 1 \)). Next, \( \mu : \mathbb{R}^2 \rightarrow [0,1] \) is a belief system which maps the proposed platforms of the two parties to the voters’ subjective probability of the true state’s being \( -1 \). Finally, \( q : \mathbb{R}^2 \rightarrow [0,1] \) is the strategy of the median voter: \( q(x_L, x_R) \) is the probability of Party \( L \)’s winning when Parties \( L \) and \( R \) propose \( x_L \) and \( x_R \), respectively.

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7Our results will not change if we assume that Party \( L \) and Party \( R \) have bliss points to the left and to the right of the median voter’s, but we prefer to save on notation and length of the proofs.

8With polarized risk-neutral parties, information transmission from parties to voters effectively requires voters to have some information of their own; see e.g. the arbitration model of Gibbons [1988] and related models, such as Martinelli [1999].

9We use trembles only for the voting strategies. Note that trembling hand perfection defined by Selten [1975] cannot be applied to this game directly since there are a continuum of strategies for the parties.
A profile \(((x_L^-, x_L^+), (x_R^-, x_R^+), \mu, q)\) constitutes an *equilibrium* if the following three conditions hold:

- (optimality of parties’ strategies) there exists a sequence \(\{q^k\}\) of functions from \(\mathbb{R}^2\) into \((0, 1)\) that converges uniformly to \(q\) such that for each \(k\), for \(s = -1, 1\), and for any \(x_L' \in \mathbb{R}\),
  \[
  q^k(x_L^+, x_R^+)u_L(x_L^+, s) + (1 - q^k(x_L^+, x_R^+))u_L(x_R^+, s) \\
  \geq q^k(x_L^-, x_R^+)u_L(x_L^-, s) + (1 - q^k(x_L^-, x_R^+))u_L(x_R^-, s),
  \]
  and the equality implies \(q^k(x_L^-, x_R^-) \geq q^k(x_L^+, x_R^-)\), and for any \(x_R' \in \mathbb{R}\),
  \[
  q^k(x_L^+, x_R^-)u_R(x_L^+, s) + (1 - q^k(x_L^+, x_R^-))u_R(x_R^-, s) \\
  \geq q^k(x_L^-, x_R^-)u_R(x_L^-, s) + (1 - q^k(x_L^-, x_R^-))u_R(x_R^-, s),
  \]
  and the equality implies \(q^k(x_L^-, x_R^-) \leq q^k(x_L^+, x_R^-)\);

- (Bayesian updating) if \(x_L^- = x_L^+\) and \(x_R^- = x_R^+\) hold, then \(\mu(x_L^-, x_R^-) = 1/2\); otherwise, \(\mu(x_L^-, x_R^-) = 1\), and \(\mu(x_L^+, x_R^-) = 0\); and

- (optimality of median voter’s strategy)
  \[
  q(x_L, x_R) = \begin{cases} 
  1 & \text{if } E^\mu_L(x_L, x_R) > E^\mu_R(x_L, x_R), \\
  \in [0, 1] & \text{if } E^\mu_L(x_L, x_R) = E^\mu_R(x_L, x_R), \\
  0 & \text{if } E^\mu_L(x_L, x_R) < E^\mu_R(x_L, x_R)
  \end{cases}
  \]
  where \(E^\mu_L(x_L, x_R) = \mu(x_L, x_R)v(x_L, -1) + (1 - \mu(x_L, x_R))v(x_L, 1)\), and \(E^\mu_R(x_L, x_R) = \mu(x_L, x_R)v(x_R, -1) + (1 - \mu(x_L, x_R))v(x_R, 1)\).

In this definition, \(E^\mu_P(x_L, x_R)\) is the expected payoff of the median voter if Party \(P = L, R\) wins and implements \(x_P\). We will say that an equilibrium
is pooling if each party proposes the same platform irrespective of states, i.e., both $x_L^- = x_L^+$ and $x_R^- = x_R^+$ hold. Conversely, we will say that an equilibrium is separating if either $x_L^- \neq x_L^+$ or $x_R^- \neq x_R^+$ (or both) holds. On the equilibrium path, there is no updating of beliefs if the equilibrium is pooling, and there is a complete transmission of information from parties to voters if the equilibrium is separating. Note that no restriction is imposed on voters’ beliefs other than they are correct on the equilibrium path.

Before we go to each class of equilibria, it is worthwhile to mention the following lemma. It states that for each signal (i.e., in every election) the platform proposed by the left-wing party is to the left of the one proposed by the right-wing party. No policy reversal is observed in any single election.

**Lemma 1.** In any equilibrium $((x_L^-, x_L^+), (x_R^-, x_R^+), \mu, q)$, $x_L^- < x_R^-$ and $x_L^+ < x_L^-$ hold.

**Proof.** We prove the statement by contradiction. If $x_L^- > x_R^-$, then the party that wins with a positive probability has an incentive to deviate. Suppose that Party L wins with a positive probability. Then by setting its policy at $x_R^-$, it obtains $x_R^-$ for sure, which is better than a combination of $x_R^-$ and $x_L^- > x_R^-.$

If $x_L^- = x_R^-$, then we have

$$q^k(x_L^-, x_R^-)u_L(x_L^-, -1) + (1 - q^k(x_L^-, x_R^-))u_L(x_R^-, -1)$$

$$= u_L(x_L^-, -1) < q^k(x'_L, x_R^-)u_L(x'_L, -1) + (1 - q^k(x'_L, x_R^-))u_L(x_R^-, -1)$$

for any $q^k(\cdot, \cdot) \in (0, 1)$ and any $x'_L < x_L^-.$ Therefore, the first equilibrium condition (perfection) is violated, i.e., one cannot find a sequence $q^k$ for which incentive constraints are satisfied. Other cases are proven in the same
manner.

Note that, according to the lemma, no equilibrium in the model exhibits full convergence. The fact that uncertainty about voters’ responses leads to departures from full convergence when parties are policy-oriented has been well understood since the work of Wittman [1983] and Calvert [1985]. Our lemma extends their results to a situation in which voters’ beliefs about the correspondence between policies and outcomes are influenced by the policies proposed by the parties.

3 Pooling equilibria

In these equilibria, Parties $L$ and $R$ make different proposals from each other, say, $x_L$ and $x_R$, respectively, but each of them sticks to the same platform regardless of the state. Hence, no information is revealed to voters on the equilibrium path. Parties stick to a given platform because voters interpret any deviation as evidence against the deviating party. Thus, no information is transmitted from parties to voters. The existence of uninformative equilibria is standard in the context of signaling games, and it does not depend on the particular features of the model we propose, other than the fact that we do not impose any restriction on out-of-equilibrium beliefs.

We claim the following:

**Proposition 1.** $((x_L, x_L), (x_R, x_R), \mu, q)$ is an equilibrium for some $\mu$ and $q$ if and only if $x_L < x_R$ and one of the following three conditions is satisfied:

(i) $-1 \leq (x_L + x_R)/2 \leq 1$, and $x_R \leq 1$;

(ii) $-1 \leq (x_L + x_R)/2 \leq 1$, and $x_L \geq -1$;
(iii) $x_L = -1 - \delta, x_R = 1 + \delta$ for some $\delta > 0$, and, for $s = -1, 1$,

$$u_L(1 - \delta, s) \leq \frac{1}{2} [u_L(x_L, s) + u_L(x_R, s)].$$

A typical pooling equilibrium outcome in terms of policy positions is described in Figure 1. Note that in all pooling equilibria the left-wing party’s platform is to the left of the median voter’s ideal point, given that no information is revealed to voters, while the right-wing party’s platform is to the right.

![Figure 1: Pooling equilibrium (i)](image)

**Proof.** To show the if-part, we assume the condition and find appropriate $\mu$ and $q$. First, let $\mu(x_L, x_R) = 1/2$ and

$$q(x_L, x_R) = \begin{cases} 
1 & \text{if } x_L + x_R > 0, \\
1/2 & \text{if } x_L + x_R = 0, \\
0 & \text{if } x_L + x_R < 0.
\end{cases}$$

As for out-of-equilibrium-path beliefs, we assume\(^{10}\)

$$\mu(x'_L, x'_R) = \begin{cases} 
1 & \text{if } x'_L = x_L, x'_R \neq x_R, \\
0 & \text{if } x'_L \neq x_L, x'_R = x_R, \\
1/2 & \text{otherwise.}
\end{cases}$$

\(^{10}\)This is an extreme belief system. In general, we can construct more realistic belief systems in which beliefs change “gradually” as platforms change.
Values of $q(\cdot, \cdot)$ for out-of-equilibrium platforms are easily obtained from the assumed beliefs.

In case (i), whenever Party $L$ deviates, the outcome will be either $x_R$ or something that is to the right of $x_R$. Thus, Party $L$ has no incentive to deviate. Party $R$ has no incentive to deviate, either. Indeed, if the party deviates, voters believe that the state is $-1$, and in order to beat Party $L$, Party $R$ has to move to the left if it is to win with positive probability in the equilibrium, in which case it is worse off. The proof of perfection is relegated to Appendix 1.

Case (ii) is the mirror image of case (i). In case (iii), an alternative for Party $L$ is to set its platform just to the right of $1 - \delta$ to obtain the median voter’s support with probability one. The condition stated in (iii) implies that this deviation does not give the party a higher payoff. The same argument holds for Party $R$.

To show the only-if-part, note first that $x_L < x_R$ is guaranteed by the lemma. Suppose that $(x_L + x_R)/2 < -1$. Then Party $R$ has an incentive to switch to $x_R + \varepsilon$ for a sufficiently small $\varepsilon > 0$. For any belief, $x_R + \varepsilon$ is strictly preferred to $x_L$ by the median voter. Therefore, Party $R$ does not fail to win and obtains something better than before. The case of $(x_L + x_R)/2 > 1$ is similarly taken care of.

Next, suppose that $x_R > 1$ and $x_L < -1$ without satisfying $x_L + x_R = 0$. Consider the case $x_L + x_R < 0$ first. In this case, $x_R$ is preferred to $x_L$ by the median voter. Then Party $L$ has an incentive to move to a platform between 1 and $x_R$. By this movement, it can capture the median voter under any belief, and it is better off. In case $x_L + x_R > 0$, Party $R$ has an incentive to deviate for a similar reason.
Finally, suppose that $x_L = -1 - \delta$, $x_R = 1 + \delta$ for some $\delta > 0$, but we have the reverse of the strict inequality of (iii), i.e.,

$$u_L(1 - \delta, s) > \frac{1}{2}[u_L(x_L, s) + u_L(x_R, s)]$$

for some $s$. In this case, in state $s$ Party $L$ has an incentive to set its platform just right of $1 - \delta$ to obtain the median voter’s support with probability one. The above inequality now implies that this deviation gives the party a higher payoff.

4 Separating equilibria

These equilibria exhibit a paradoxical phenomenon: the policy platform carried out by the right-wing party is located to the left of the policy platform carried out by the left-wing one. Note that Lemma 1 is still valid, namely, in each state of the world, Party $R$ proposes a platform which is to the right of the one proposed by Party $L$ at each state, i.e., $x_L < x_R$ and $x_L^+ < x_R^+$. When we compare the platforms of the winning parties across time, Party $L$’s platform is located to the right of Party $R$’s platform.

The set of separating equilibria can be divided into two subclasses, depending on whether the implemented policy and the state of nature are positively or negatively correlated. We consider the two subclasses in that order. For reasons that will become apparent later on, our main focus is on equilibria with positive correlation.

4.1 Separating equilibria with positive correlation

In these equilibria, Party $L$ wins the election with probability one if $s = 1$, i.e., if the state of the world favors the adoption of right-leaning policies,
and Party $R$ wins if $s = -1$. In other words, the “wrong” party wins every election. Voters, however, benefit from the fact that the policy platforms of both parties are positively correlated with the state of the world. The discussion in Section 5 shows that these equilibria are the best in terms of the median voter’s welfare.

It might be useful to provide some intuition about why separating equilibria with positive correlation necessarily exhibit such policy reversals. In positive correlation equilibria, the party that most dislikes the policies implied by the state of the world is forced to play as a moderate because of the knowledge that the other party will play as a radical, and vice versa, the party that plays as a radical will do so because of the knowledge that its opponent will be a moderate. That is, both parties’ policy positions move in the same direction, left or right, because the state of the world serves as a correlation device for their strategies.

As is usually the case in models of electoral competition, we can imagine that the attempt to win the election (thus, ensuring the adoption of more desirable policies than those espoused by their opponents) will push both parties in a separating equilibrium to converge toward the optimal policy for the median voter – that is, the state of the world. In our model, however, there is a countervailing force: voters use policy positions to make inferences about the state of the world. Thus, if Party $L$ moves towards the right this is interpreted as evidence of the state of the world being 1, and if Party $R$ moves toward the left this is interpreted as evidence on the contrary. For the party playing as a radical, moving towards the true state of the world risks convincing voters that the opposite state of the world is more likely. However, for the party playing as a moderate, voters’ beliefs are already the
worst possible ones: that is why it is behaving as a moderate in the first place. Hence, it is “less costly” for the moderate to converge to the optimal policy for the swing voter. In equilibrium, the moderate party will move its policy position close enough to the state of the world to win the election, knowing that at some point the radical party will give up on getting closer: before convincing voters that the state of the world is more likely to be on the opposite side of the political spectrum.

**Proposition 2.** Suppose that \((x_L^-, x_L^+)\) and \((x_R^-, x_R^+)\) satisfy the following for some \(q^* \in (0, 1)\):

(i) \(x_L^- < x_R^- < x_L^+ < x_R^+\),

(ii) \((x_L^- + x_R^-)/2 = -1\), and \((x_L^+ + x_R^+)/2 = 1\),

(iii) \(u_L(x_L^+, 1) > q^* u_L(x_L^-, 1) + (1 - q^*) u_L(x_R^+, 1)\),

(iv) \(u_R(x_R^-, -1) > q^* u_R(x_L^-, -1) + (1 - q^*) u_R(x_R^+, -1)\).

Then there exist \(\mu\) and \(q\) such that

(v) \(q(x_L^-, x_R^-) = 0, q(x_L^+, x_R^-) = 1,\) and \(q(x_L^-, x_R^+) = q^*\)

for which \(((x_L^-, x_L^+), (x_R^-, x_R^+), \mu, q)\) is an equilibrium.

A typical situation looks like the one in Figure 2. Appendix 3 shows the set of (symmetric) separating equilibria with positive correlation for the case \(u_L(x, s) = 1 - e^x\) and \(u_R(x, s) = 1 - e^{-x}\) for \(s = -1, 1\).

**Proof.** First, let \(\mu(x_L^-, x_R^-) = 1\) and \(\mu(x_L^+, x_R^+) = 0\) be the beliefs on the outcome path. Since the swing voter is indifferent between \(x_L^-\) and \(x_R^-\), and
between $x^+_L$ and $x^+_R$, we can set $q(x^-_L, x^-_R) = 0$ and $q(x^+_L, x^+_R) = 1$. Next, let $\mu$ be given by

$$
\mu(x'_L, x'_R) = \begin{cases} 
1 & \text{if } x'_L = x^-_L \text{ or } x'_L = x^+_L, \\
0 & \text{if } x'_L \neq x^-_L \text{ or } x'_L = x^+_L, \\
\mu^* & \text{if } (x'_L, x'_R) = (x^-_L, x^+_R), \\
1/2 & \text{otherwise},
\end{cases}
$$

where $\mu^*$ satisfies

$$(1) \quad \mu^* v(x^-_L, -1) + (1 - \mu^*) v(x^-_R, 1) = \mu^* v(x^+_R, -1) + (1 - \mu^*) v(x^+_R, 1),$$

or equivalently,

$$
\mu^* = \frac{v(x^+_L, 1) - v(x^-_L, 1)}{v(x^+_L, 1) - v(x^-_L, 1) + v(x^-_R, -1) - v(x^+_R, -1)}.
$$

Such a $\mu^*$ exists between zero and one due to Conditions (i) and (ii) and the assumption that $v(x, s)$ is single-peaked at $s$. We will prove that Party $L$ does not have an incentive for a unilateral deviation. The incentive constraint for Party $R$ is checked in the same manner. Suppose first that the state is $-1$. In this state, if Party $L$ moves to $x'_L \leq x^-_R$, then $\mu(x'_L, x^-_R) = 0$ holds, and $x^-_R$ is still chosen. If, on the other hand, it moves to the right of $x^-_R$, there will be no gain, either. Suppose next that the state is 1. There is no incentive to move further right. If it moves to $x'_L < x^+_L$, then we have two
possibilities, $x_L' \neq x_L^-$ and $x_L' = x_L^-$. If $x_L' \neq x_L^-$ holds, then $\mu(x_L', x_L^-) = 0$, and therefore, $|x_L' - 1| > |x_L^- - 1|$ implies $x_R^+$ is chosen. If $x_L' = x_L^-$ holds, then we have $\mu(x_L', x_R^+) = \mu(x_L^-, x_R^+) = 4$, and equation (1) implies the median voter is indifferent between the two alternatives. So we are allowed to let $q(x_L^-, x_R^+) = q^*$. Then, Condition (iii) implies that Party $L$ has no incentive to choose $x_L^-$ if the state is 1. The proof of perfection is relegated to Appendix 2.

Next, we show that there exists at least some separating equilibria characterized above. To find one, it is sufficient to prove that there exist $(x_L^-, x_L^+), (x_R^-, x_R^+), q$ and $q^*$ that jointly satisfy (i)-(v). Suppose that $x_R^-$ and $x_L^+$ are given by

$$x_R^- = -\varepsilon, x_L^+ = \varepsilon,$$

respectively, where $\varepsilon > 0$ will be chosen to be sufficiently small, and that $x_L^-$ and $x_R^+$ are given by

$$x_L^- = -2 + \varepsilon, x_R^+ = 2 - \varepsilon,$$

respectively. Note that Conditions (i) and (ii) are satisfied for any sufficiently small $\varepsilon > 0$. Let $q^* = 1/2$. Condition (v) is simply assumed. It now suffices to show that (iii) and (iv) hold for a sufficiently small $\varepsilon > 0$. Due to the strict concavity of $u_L$, we have

$$u_L(0, 1) > \frac{1}{2}u_L(-2, 1) + \frac{1}{2}u_L(2, 1).$$

The continuity of $u_L$ (which is a direct consequence of strict concavity on $\mathbb{R}$) implies that

$$u_L(\varepsilon, 1) > \frac{1}{2}u_L(-2 + \varepsilon, 1) + \frac{1}{2}u_L(2 - \varepsilon, 1)$$

holds for a sufficiently small $\varepsilon > 0$. Condition (iii) is proven. Condition (iv) is the mirror image of (iii) and satisfied for the same $\varepsilon$. 20
4.2 Separating equilibria with negative correlation

These equilibria are even more paradoxical than those with positive correlation. In these equilibria, the right-wing party wins the election when the state of the world favors right-leaning policies, and the left-wing party when the state of the world favors left-leaning policies. However, both parties shift their policy platforms to the left if the state of the world favors the adoption of right-leaning policies. Hence, the “correct” party wins the election, but it does so with the “wrong” policy platform. As a result, the median voter may end up being worse off than in an equilibrium in which no information held by the parties is revealed (see Section 5). The discussion in Section 5 shows that these equilibria do not satisfy a requirement related to the credibility of parties’ commitment to their electoral platforms.

**Proposition 3.** Suppose that \( (x^-_L, x^+_L) \) and \( (x^-_R, x^+_R) \) satisfy the following for some \( q^{**} \in (0, 1) \):

(i)’ \( x^+_L < x^+_R < x^-_L < x^-_R \),

(ii)’ \( (x^-_L + x^-_R)/2 \leq 1 \), and \( (x^+_L + x^+_R)/2 \geq -1 \),

(iii)’ \( u_L(x^-_L, -1) > q^{**}u_L(x^+_L, -1) + (1 - q^{**})u_L(x^-_R, -1) \),

(iv)’ \( u_R(x^+_R, 1) > q^{**}u_R(x^+_L, 1) + (1 - q^{**})u_R(x^-_R, 1) \).

Then there exist \( \mu \) and \( q \) such that

(v)’ \( q(x^-_L, x^-_R) = 1, q(x^+_L, x^+_R) = 0, \) and \( q(x^+_L, x^-_R) = q^{**} \).

for which \( ((x^-_L, x^+_L), (x^-_R, x^+_R), \mu, q) \) is an equilibrium.
Proof. First let $\mu(x_L^+, x_R^-) = 1$, and $\mu(x_L^+, x_R^+) = 0$, be the beliefs on the outcome path. For other beliefs, let $\mu$ satisfy:

$$
\mu(x'_L, x'_R) = \begin{cases} 
  1 & \text{if } x'_L = x_L^- \text{ or } x_L^+, \text{ and } x'_R \neq x_R^-, x_R^+; \\
  0 & \text{if } x'_L \neq x_L^-, x_L^+, \text{ and } x'_R = x_R^- \text{ or } x_R^+; \\
  \mu^{**} & \text{if } (x'_L, x'_R) = (x_L^+, x_R^-), \\
  1/2 & \text{otherwise,}
\end{cases}
$$

where $\mu^{**}$ satisfies

$$(2) \quad \mu^{**}v(x_L^+, -1) + (1 - \mu^{**})v(x_L^+, 1) = \mu^{**}v(x_R^-, -1) + (1 - \mu^{**})v(x_R^-, 1),$$

or equivalently,

$$\mu^{**} = \frac{v(x_R^-, 1) - v(x_L^+, 1)}{v(x_R^-, 1) - v(x_L^+, 1) + v(x_L^+, -1) - v(x_R^-, -1)}. $$

This $\mu^{**}$ is between zero and one due to Conditions (i)' and (ii)' above and the assumption that $v(x, s)$ is single-peaked at $s$. For this value of $\mu^{**}$, the median voter is indifferent between $x_L^+$ and $x_R^-$. Therefore, any probability $q(x_L^+, x_R^-)$, in particular, $q^{**}$, is consistent with the equilibrium condition. As in the previous case, it is verified that there exists at least some separating equilibria as characterized above.
4.3 Nonexistence of other separating equilibria

Proposition 4. There are no separating equilibria other than those described by Propositions 2 and 3.

Proof. Suppose that \(((x^-_L, x^+_L), (x^-_R, x^+_R), \mu, q^*)\) is a separating equilibrium. We know from Lemma 2.1 that \(x^-_L < x^-_R\) and \(x^+_L < x^+_R\). We divide our analysis into two cases. First, let us assume that \(x^-_L < x^+_R\) holds. This corresponds to the case of positive correlation equilibria. Recall its equilibrium conditions (i)-(v). We show that each condition is necessary.

Suppose \((x^+_L + x^-_R)/2 \neq 1\). If \((x^+_L + x^-_R)/2 > 1\) holds, then when the signal is 1, L has an incentive to deviate to \(x^+_L - \varepsilon\) for a small \(\varepsilon > 0\) and win the election with a more favorable platform. If \((x^+_L + x^-_R)/2 < 1\) holds, then R will win the election with its platform \(x^+_R\) when \(s = 1\). In this case, in order for L not to deviate to \(x^-_L < x^+_R\), its probability of winning at \((x^-_L, x^+_R)\) must be zero. However, if this is the case, then R would have an incentive to deviate from \(x^-_R\) to \(x^+_R\) when \(s = -1\). Thus, \((x^+_L + x^-_R)/2 < 1\) cannot be the case. Similarly, \((x^-_L + x^-_R)/2 = -1\) should hold. Condition (ii) is established. Under (ii), if \(q(x^+_L, x^-_R) < 1\) (respectively \(q(x^-_L, x^-_R) > 0\)) holds, then L (respectively R) has an incentive to move its platform toward the right (respectively left) by a sufficiently small \(\varepsilon > 0\) and win the election for sure. This establishes Condition (v). If (iii) is violated, then L switches to \(x^-_L\) when it gets \(s = 1\). Similarly, if (iv) is violated, then R switches to \(x^+_R\) when it gets \(s = -1\). Finally, if \(x^-_R > x^+_L\) holds, if \(q(x^+_L, x^-_R) < 1\), then R switches to \(x^-_R\) when it gets \(s = 1\). On the other hand, if \(q(x^-_L, x^-_R) > 0\), then L switches to \(x^+_L\) when it gets \(s = -1\). Thus, at least one of them has an incentive to deviate, which establishes Condition (i). Hence, all Conditions
(i)-(v) are necessary if $x^-_L < x^+_R$ holds.\footnote{Note that we require the inequalities in (iii) and (iv) to hold strictly. Some slack between the LHS and the RHS of the inequalities is required in order to satisfy the first equilibrium condition, as it is clear from Appendix 2. The same holds true with respect to Conditions (iii)' and (iv)'.

Next, assume $x^-_L > x^+_R$ holds. This corresponds to the negative correlation case. First of all, Condition (i)' holds by the assumption and the lemma. To check Condition (ii)', suppose $(x^+_L + x^+_R)/2 < -1$ holds. Then, $R$ has an incentive to move its platform toward the right by a sufficiently small $\varepsilon > 0$. If, on the other hand, $(x^-_L + x^-_R)/2 > 1$ holds, then $L$ has an incentive to move its platform toward the left by $\varepsilon > 0$. If (iii)' does not hold, then $L$ has an incentive to switch to $x^+_L$ when the signal is $-1$. Similarly, if (iv)' does not hold, then $R$ switches to $x^-_R$ when the signal is 1. Finally, if Condition (v)' is violated, it implies that the median voter does not vote for his favorite platform. Hence, Conditions (i)'-(v)' are necessary.

5 Further Remarks

5.1 Welfare

If we identify welfare with the payoff to the median voter, we can state the following for the special but important case in which the median voter has Euclidean preferences (i.e. $v(x, s) = -|x - s|$):

**Proposition 5.** Welfare is higher in separating equilibria with positive correlation than in any other equilibrium.

Indeed, in separating equilibria with positive correlation the median voter’s expected payoff is given by

$$-\frac{1}{2}|x^-_R + 1| - \frac{1}{2}|x^+_L - 1|.$$
Since in this subclass of equilibria we have $-1 < x_R^- < x_L^+ < 1$, the median voter’s expected payoff is greater than $-1$. In separating equilibria with negative correlation, the median voter’s expected payoff is given by
\[-\frac{1}{2}|x_R^- - 1| - \frac{1}{2}|x_L^+ + 1|.
\]
Since in this subclass of equilibria we have $-1 < x_R^+ < x_L^- < 1$, the expected payoff is smaller than $-1$. Finally, if a pooling equilibrium satisfies either (i) or (ii) of Section 3, the expected payoff is $-1$, while if it satisfies (iii), the expected payoff is $-1 - \delta$ where $\delta$ is positive. Thus, if one of the best equilibria is played, policy reversals are necessarily observed. We can consider separating equilibria with positive correlation to be “focal” to the extent that they are the best for voters.

5.2 Credibility of commitment

In separating equilibria, the information shared by the parties about the state of nature serves as a signal leading to correlated play by the parties. However, a subclass of separating equilibria exhibit the seemingly unnatural feature of a negative correlation between policy platforms and the state of nature. Note that negative correlation equilibria require the left-wing party, if it wins, to pursue a policy platform that is to the right of the state of the world. (The opposite happens if the right-wing party wins.) Hence, both the winning party and the median voter could be made better off if the party in office were allowed to renegotiate its policy platform. Our commitment assumption becomes suspect in these equilibria because it does not seem reasonable to assume that voters would punish a party for a move that would benefit them. This leads one to think that a sensible requirement to ask of an equilibrium is that commitment should be credible in the sense that in case of winning.
a party would not modify its policy platform even if the median voter were willing to go along with such a decision.

In particular, we will say that an equilibrium \( ((x_L^-, x_L^+), (x_R^-, x_R^+), \mu, q) \) is credible if for \( s = -1, 1 \), and for \( P = L, R \), there is no \( x' \) such that \( u_P(x') > u_P(x_P^s) \) and

\[
\mu(x_L^s, x_R^s)v(x', -1) + (1 - \mu(x_L^s, x_R^s))v(x', 1) > \mu(x_L^s, x_R^s)v(x_P^s, -1) + (1 - \mu(x_L^s, x_R^s))v(x_P^s, 1).
\]

In words, we say that an equilibrium is credible if, after the election is over and regardless of the result of the election, there is no policy platform different from the one prescribed by the equilibrium that makes both the winning party and the median voter better off. In separating equilibria, the credibility requirement boils down to \( x_L^- \leq -1, x_L^+ \leq 1, x_R^- \geq -1, x_R^+ \geq 1 \). It is easy to see that

**Proposition 6.** Separating equilibria with positive correlation are credible, while separating equilibria with negative correlation are not.

(With respect to pooling equilibria, credibility imposes the further constraint on Conditions (i) and (ii) that \( x_L \leq 0, x_R \geq 0 \).)

### 5.3 A larger number of states

We think that a model with two possible signal values is a good representation of the situations in which parties (and especially voters) can only get a rough idea of the direction in which policy should be moving, for instance, low state intervention in the economy versus pervasive intervention (Harrington [1993] makes a similar point). Moreover, it is possible to extend some of our results to a situation with a larger number of states. Suppose that there is a number
of intermediate states between $-1$ and $1$, and keep the assumption that the probability that the state is less than zero is $1/2$. Define a semiseparating equilibrium as one in which the right-wing party and the left-wing party adopt the policy platforms $x^-_R, x^-_L$, respectively, whenever the state is less than 0, and the policy platforms $x^+_R, x^+_L$, respectively, whenever the state is larger than or equal to 0. Following the steps of Proposition 2, it is possible to show that such semiseparating equilibrium exists if $(x^-_L, x^+_L)$ and $(x^-_R, x^+_R)$ satisfy the Conditions (i) to (iv) for some $q^* \in (0, 1)$. As in section 4.1, an example is $x^-_L = -2 + \varepsilon, x^+_L = \varepsilon, x^-_R = -\varepsilon, x^+_R = 2 + \varepsilon$ with $q^* = 1/2$, where $\varepsilon$ is chosen to be sufficiently small. Note that if there are intermediate states then the median voter is no longer indifferent between the two parties in this equilibrium but strictly prefers to vote for the left party if the parties propose $(x^+_R, x^+_L)$, and for the right party if the parties propose $(x^-_R, x^-_L)$. (If there are intermediate states, semiseparating equilibria other than those described by Conditions (i) to (iv) of Proposition 2 and even completely separating equilibria might exist.)

### 5.4 Downsian parties

Note that our payoff specification for the parties downplays the traditional Downsian motivation for getting elected: holding office *per se*. If parties were primarily office-motivated in this sense, there would be some pooling equilibria with full convergence in which both parties would have a positive probability of winning the election as long as both played according to the equilibrium strategy.

Separating equilibria with full convergence in either or both states, however, would remain impossible: they would imply either $x^-_L = x^-_R = -1$ or
If, for instance, \( x_L^- = x_R^- = -1 \), Party \( R \) would have an incentive to “mislead” voters by adopting policy \( x_R^+ \) whenever the true state of the world is \(-1\), and this should not decrease its probability of winning the election; for if not, Party \( L \) would increase its probability of winning the election by adopting policy \( x_L^- \) whenever the true state of the world is \(1\).

If parties are exclusively office-motivated, then separating equilibria disappear. To see this, assume that the parties try to maximize their respective winning probabilities. Take a separating equilibrium of the original game. We know that (i) Party \( R \) wins at \( s = -1 \), (ii) Party \( L \) wins at \( s = 1 \), and if, with some deviation, \((x_R^-, x_L^+)\) is an announced pair, either one of them (or both), say, \( L \) wins with a positive probability. Then Party \( L \) has an incentive to deviate to \( x_L^+ \) at state \( s = -1 \).

More generally, it is shown that there exist some separating equilibria if policy-orientation is relatively strong compared to power-orientation. To see this point, let \( u_i(\cdot, \cdot) \) and \( w_i(\cdot, \cdot) \) be the payoffs of Party \( i = L, R \) when it wins the election and when it loses the election, respectively. Assume symmetry between \( L \) and \( R \) as before. Also, assume the continuity of these functions with respect to their first argument. In order to destroy the incentive for Party \( L \) to deviate at \( s = -1 \), Party \( L \) has to prefer \( x_R^- \) implemented by Party \( R \) to a mixture between that and \( x_L^+ \) implemented by itself. The former gives Party \( L \) the payoff of \( w_L(x_R^-, -1) \), while the latter gives the payoff of

\[
q(x_R^-, x_L^+)w_L(x_R^-, -1) + [1 - q(x_R^-, x_L^+)]u_L(x_L^+, -1)
\]

Therefore, we need to have

\[
w_L(x_R^-, -1) \geq u_L(x_L^+, -1)
\]
for $x_R^-, x_L^+$ to be equilibrium actions. Thus, in order for some separating equilibrium to exist, we must have

$$w_L(-1, -1) > u_L(1, -1).$$

It is easy to verify that other incentives for deviations can be eliminated by appropriately choosing the belief system $\mu$. For instance, we can specify $\mu(x, x_R^-) = 1$ for $x > x_R^-$ so that it never pays for the left party to deviate to the right of $x_R^-$, regardless of its love for office. Therefore, the above condition turns out to be a necessary and sufficient condition for the existence of separating equilibria with policy reversals when parties are office-motivated as well as policy-motivated.

### 6 Conclusion

We present a spatial model of electoral competition with asymmetric information between parties and voters. We describe the complete set of equilibria in which parties follow pure strategies, and show that, when parties get to observe one of two possible signals, one favoring the adoption of left-leaning policies and the other favoring the adoption of right-leaning policies, all separating equilibria exhibit policy reversals. That is, the policies implemented by the left-wing party when it wins the election are located to the right of the policies implemented by the right-wing party. Reversals are observed only across elections: in each election, the platform proposed by the left party is located to the left of that proposed by the right party. The model shows, then, that policy reversals are possible whenever: (i) there is uncertainty among voters about which is the best policy course, and adopting the best policy course may involve a substantial policy shift, (ii) political parties share
some relevant information that voters lack, which may serve as a correlating
device for their policy platforms, and (iii) parties are policy-oriented and
have polarized ideological preferences.

Formally, the model we develop is a multi-sender signaling game. Ap-
plications of such games are still relatively sparse. Our paper illustrates
that they might be useful to analyze electoral competition, with potentially
surprising results.
Appendix 1

This appendix establishes that the pooling equilibria described by Proposition 1 satisfy the first equilibrium condition. We consider explicitly case (i), with $x_L + x_R < 0$ (see Figure 1). All other cases are analogous.

According to the specified beliefs, if both parties propose their equilibrium actions, voters believe that the two states are equally likely (and vote for Party $R$). If only Party $L$ deviates, voters will believe that the state of the world is 1. Hence,

$$q(x_L', x_R) = \begin{cases} 
0 & \text{if } x_L' < x_R \text{ or } x_L' > 2 - x_R, \\
1/2 & \text{if } x_L' = x_R \text{ or } x_L' = 2 - x_R, \\
1 & \text{if } x_R < x_L' < 2 - x_R.
\end{cases}$$

(To save on notation, we let $q(\cdot, \cdot) = 1/2$ if the voters are indifferent between the two parties and the equilibrium does not require them to randomize in a particular way.) To check for perfection, consider

$$q^k(x_L', x_R) = \begin{cases} 
\varepsilon^k \min_s \left( \frac{u_L(x_L', s) - u_L(x_L, s)}{u_L(x_L, s) - u_L(x_R, s)} \right) & \text{if } x_L' < x_L, \\
\varepsilon^k & \text{if } x_L \leq x_L' < x_R \text{ or } x_L' > 2 - x_R, \\
1/2 & \text{if } x_L' = x_R \text{ or } x_L' = 2 - x_R, \\
1 - \varepsilon^k & \text{if } x_R < x_L' < 2 - x_R.
\end{cases}$$

for $k = 1, 2, \cdots$ and some $\varepsilon \in (0, 1)$. The sequence $q^k(\cdot, \cdot)$ converges uniformly to $q(\cdot, \cdot)$ since $u_L(x_L', s) - u_L(x_R, s)$ is bounded away from zero. With this sequence, it is verified that for each $s$, for each $k$ and for any $x_L' \in \mathbb{R}$

$$q^k(x_L, x_R)u_L(x_L, s) + (1 - q^k(x_L, x_R))u_L(x_R, s) \geq q^k(x_L', x_R)u_L(x_L', s) + (1 - q^k(x_L', x_R))u_L(x_R, s),$$

and the equality implies $q^k(x_L, x_R) \geq q^k(x_L', x_R).$
Similarly, if only Party $R$ deviates, voters will believe that the state of the world is $-1$. Hence,

\[
q(x_L, x'_R) = \begin{cases} 
0 & \text{if } x'_R = x_R \text{ or } |x'_R + 1| < |x_L + 1|, \\
1/2 & \text{if } x'_R = x_L \text{ or } x'_R = -2 - x_L, \\
1 & \text{if } x'_R \neq x_R \text{ and } |x'_R + 1| > |x_L + 1|.
\end{cases}
\]

To check for perfection, consider

\[
q^k(x_L, x'_R) = \begin{cases} 
\varepsilon^k & \text{if } x'_R = x_R \text{ or } |x'_R + 1| < |x_L + 1|, \\
1/2 & \text{if } x'_R = x_L \text{ or } x'_R = -2 - x_L, \\
1 - \varepsilon^k \min_s \left( \frac{u_R(x'_R, s) - u_R(x_L, s)}{u_R(x'_R, s) - u_R(x_L, s)} \right) & \text{if } x'_R < x_R \text{ and } |x'_R + 1| > |x_L + 1|, \\
1 - \varepsilon^k \min_s \left( \frac{u_R(x'_R, s) - u_R(x_L, s)}{u_R(x'_R, s) - u_R(x_L, s)} \right) & \text{if } x'_R > x_R.
\end{cases}
\]

for $k = 1, 2, \cdots$ and some $\varepsilon \in (0, 1/2)$. The sequence $q^k(x_L, \cdot)$ converges uniformly to $q(x_L, \cdot)$. With this sequence, it is verified that for each $s$, for each $k$ and for any $x'_R \in \mathbb{R}$

\[
q^k(x_L, x'_R)u_R(x_L, s) + (1 - q^k(x_L, x'_R))u_R(x_R, s) > q^k(x_L, x'_R)u_R(x_L, s) + (1 - q^k(x_L, x'_R))u_R(x'_R, s).
\]

Finally, when both parties deviate, voters believe that the two states are equally likely. Hence, we can take $q^k(x'_L, x'_R) = q(x'_L, x'_R) = 1/2$. 

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Appendix 2

This appendix establishes that the separating equilibria described by Propositions 2 and 3 satisfy the first equilibrium condition. We consider explicitly a separating equilibrium with positive correlation as described by Proposition 2 (see Figure 2). The proof for a separating equilibrium with negative correlation is analogous.

For the sake of brevity, we restrict our attention to Party $L$. Suppose first that the state is $s = 1$. To check the first equilibrium condition for the present case, let $q(x'_L, x^+_R)$ be given by

$$q(x'_L, x^+_R) = \begin{cases} 
1 & \text{if } x'_L \leq x'_L < x^+_L, \\
1/2 & \text{if } x'_L = x^+_L, \\
q^* & \text{if } x'_L = x^-_L, \\
0 & \text{if } x'_L \neq x^-_L \text{ and } x'_L < x^+_L \text{ or } x'_L > x^+_L.
\end{cases}$$

Note that this is consistent with other equilibrium conditions. To check the perfection of the equilibrium, consider

$$q^k(x'_L, x^+_R) = \begin{cases} 
1 - \varepsilon^k & \text{if } x'_L \leq x'_L < x^+_R, \\
1/2 & \text{if } x'_L = x^+_R, \\
q^* & \text{if } x'_L = x^-_L, \\
\varepsilon^k (u_L(x^+_L, 1) - u_L(x^-_R, 1)) & \text{if } x'_L \neq x^-_L \text{ and } x'_L < x^+_L, \\
\varepsilon^k & \text{if } x'_L > x^+_L,
\end{cases}$$

for $k = 1, 2, \cdots$ and some $\varepsilon > 0$ satisfying

$$\varepsilon \leq 1 - q^*(u_L(x^-_L, 1) - u_L(x^-_R, 1))/(u_L(x^+_L, 1) - u_L(x^+_R, 1)).$$

(Condition (iii) implies that the RHS of the above inequality is strictly positive). The sequence $q^k(\cdot, x^+_R)$ converges uniformly to $q(\cdot, x^+_R)$. Moreover, it verifies the Party L’s optimality condition.
The case in which the state is $s = -1$ is simpler. Let $q(x'_L, x^-_R) = 0$ for all $x'_L$, and to check the perfection of the equilibrium consider

$$q^k(x'_L, x^-_R) = \begin{cases} 
\varepsilon^k & \text{if } x'_L \geq x^-_L, \\
\varepsilon^k \left( \frac{u_L(x^-_L, -1) - u_L(x^-_R, -1)}{u_L(x^+_L, -1) - u_L(x^-_R, -1)} \right) & \text{if } x'_L < x^-_L, 
\end{cases}$$

for $k = 1, 2, \ldots$ and some $\varepsilon \in (0, 1)$. The sequence $q^k(\cdot, x^+_R)$ converges uniformly to $q(\cdot, x^-_R)$. Moreover, it verifies the Party L’s optimality condition.

In the spirit of (trembling hand) perfection, we just offer one possible sequence of strategies for the median voter converging to his equilibrium strategy. Of course, there are other sequences of strategies for the median voter that verify the optimality of Party L strategy, a requirement being that

$$q^k(x'_L, x^-_R) \leq \left( \frac{u_L(x^-_L, -1) - u_L(x^-_R, -1)}{u_L(x^+_L, -1) - u_L(x^-_R, -1)} \right) q^k(x^+_L, x^-_R) \quad \text{if } x'_L \neq x^-_L \text{ and } x'_L \leq x^+_L,$$

$$q^k(x'_L, x^-_R) \leq \left( \frac{u_L(x^-_L, -1) - u_L(x^-_R, -1)}{u_L(x^+_L, -1) - u_L(x^-_R, -1)} \right) q^k(x^-_L, x^-_R) \quad \text{if } x'_L \leq x^-_L.$$

That is, if Party L deviates toward the left of its equilibrium platform, the median voter is very unlikely to vote for it. This makes sense: the further to the left is Party L’s platform, the more costly it is for the median voter to vote for Party L by mistake, if the median voter believes that the state of the world is 1 in case $x_R = x^+_R, x_L \neq x^-_L, x_L < x^+_L$, and for any given beliefs in case $x_R = x^-_R, x_L < x^-_L$. A simple example of trembles is provided in Appendix 3.
Appendix 3

We calculate here the set of symmetric separating equilibria with positive correlation for the example \( u_L(x, s) = 1 - e^x \) and \( u_R(x, s) = 1 - e^{-x} \) for \( s = -1, 1 \). From Conditions (i) to (iv) in Proposition 2, with \( q^* = 1/2 \), we obtain that the policy platforms corresponding to this set of equilibria are given by \( x^-_L = -1 - \alpha \), \( x^-_R = -1 + \alpha \), \( x^+_L = 1 - \alpha \), and \( x^+_R = 1 + \alpha \) for \( \alpha \in (\frac{1}{2} \ln(2 - e^{-2}), 1) \), where \( \frac{1}{2} \ln(2 - e^{-2}) \approx 0.31154 \). These equilibria are supported by \( q(\cdot, \cdot) \) as given in Appendix 2. From \( u_L(x, s) < 1 \) for \( s = -1, 1 \) and \( \alpha > \frac{1}{2} \ln(2 - e^{-2}) \) we can obtain that for any symmetric separating equilibrium with positive correlation

\[
\frac{u_L(x^-_L, 1) - u_L(x^+_L, 1)}{u_L(x'_L, 1) - u_L(x^-_R, 1)} > \frac{e^2 - 1}{2e^2 - 1} \approx 0.46371.
\]

Thus, the optimality of Party L’s equilibrium strategy in any such equilibrium is verified by a sequence of strategies for the median voter of the following form:

\[
q^k(x'_L, x^-_R) = \begin{cases} 
1 - \varepsilon^k & \text{if } x^+_L \leq x'_L < x^+_R, \\
1/2 & \text{if } x'_L = x^+_R, \\
q^* & \text{if } x'_L = x^-_L, \\
K \varepsilon^k & \text{if } x'_L \neq x^-_L \text{ and } x'_L < x^+_R, \\
\epsilon^k & \text{if } x'_L > x^+_R,
\end{cases}
\]

\[
q^k(x'_L, x^-_R) = \begin{cases} 
\varepsilon^k & \text{if } x'_L \geq x^-_L, \\
K \varepsilon^k & \text{if } x'_L < x^-_L,
\end{cases}
\]

for \( k = 1, 2, \cdots \), some \( K \leq (e^2 - 1)/(2e^2 - 1) \), and \( \varepsilon \) small enough to satisfy equation (3). We can deal similarly with the optimality of Party R’s equilibrium strategy.
References


