INSTITUTO TECNOLÓGICO AUTÓNOMO DE MÉXICO

CENTRO DE INVESTIGACIÓN ECONÓMICA

Discussion Paper Series

Follow the Leader: Theory and Evidence on Political Participation: A Comment

César Martinelli

Instituto Tecnológico Autónomo de México

November 2005

Discussion Paper 05-06

Av. Camino a Santa Teresa # 930
Col. Héroes de Padierna
México, D.F. 10700
M E X I C O

FOLLOW THE LEADER: THEORY AND EVIDENCE ON POLITICAL PARTICIPATION: A COMMENT

CÉSAR MARTINELLI

In a very influential article, Ron Shachar and Barry Nalebuff (SN 1999) develop and estimate empirically a *pivotal-leader model* of political participation with the following characteristics. The leaders of party $p = D, R$ decide how much effort E_p expend to get followers to vote. The proportion of followers of party p that turn out to vote on election day is equal to $\psi^p(E_p, x, \varepsilon)$, where x represents observable factors influencing participation and ε is some random term with density $f(\varepsilon)$. The fraction of voters favorable to party D is a random variable $\tilde{d}$ with density $h(\cdot)$ and distribution function $H(\cdot)$. The realizations of ε and d are unknown to leaders at the moment of deciding on effort levels. Thus, the probability of party D winning the election is

$$P(E_D, E_R) = \int_{-\infty}^{\infty} \left[1 - H\left(\frac{\psi^R}{\psi^R + \psi^D} \right) \right] f(\varepsilon)\, d\varepsilon. \tag{1}$$

The cost of effort is $C(E_p, N)$, where N is the electorate size. Letting V denote the value of winning office, the expected utility of the leaders of parties D and R as a function of effort levels are, respectively,

$$U^D(E_D, E_R) = VP(E_D, E_R) - C(E_D, N), \tag{2}$$

$$U^R(E_D, E_R) = V(1 - P(E_D, E_R)) - C(E_R, N). \tag{3}$$

SN adopt the following functional forms

$$\begin{aligned} \psi^p(E_p, x, \varepsilon) &= \exp(\rho E_p + \beta_p + x \cdot \beta + \varepsilon), \\ C(E_p, N) &= E_p^2/2 + \eta \cdot N \cdot E_p. \end{aligned} \tag{4}$$

SN claim that, given the functional forms in (4), the objective functions of parties D and R are concave with respect to the parties' own effort, and therefore they focus on the solution to the first-order conditions derived from equations (2) and (3). In this note, (I) I prove that the claim is incorrect, (II) provide conditions under which the objective functions are concave, and attempt to verify whether these conditions hold for the parameter values estimated by SN.

Address: CIE-ITAM, Camino Santa Teresa 930, 10700 Mexico, D.F., MEXICO. Email: `martinel@itam.mx`. *Date*: November 15th 2005.

I. A FAILURE OF CONCAVITY: AN EXAMPLE

SN assume for the structural estimation of their model that $\tilde{d}$ is normally distributed. This is, of course, not very convenient for a random variable with support $(0,1)$, but can be taken as an approximation if the variance is small enough. Suppose, then, that $\tilde{d}$ is approximated in the interval $(0,1)$ by a normal random variable with mean μ and variance σ. (In the next section more general forms of uncertainty are considered.)

The derivative of the objective function of party $p = D, R$ with respect to its own effort is

$$U_p^p(E_D, E_R) = V \cdot h(\alpha(E_D, E_R))\alpha(E_D, E_R)(1 - \alpha(E_D, E_R))\rho - E_p - \eta \cdot N,$$

where

$$\alpha(E_D, E_R) \equiv \frac{\exp(\rho E_R + \beta_R)}{\exp(\rho E_R + \beta_R) + \exp(\rho E_D + \beta_D)}.$$

From the first-order conditions, *if* there is an interior equilibrium, it must satisfy

$$E_D = E_R = V \cdot h(\overline{\alpha})\overline{\alpha}(1 - \overline{\alpha})\rho - \eta \cdot N,$$

where $\overline{\alpha} = \exp(\beta_R)/(\exp(\beta_R) + \exp(\beta_D))$ (cf. Proposition 2 in SN). Under the normality assumption we have

$$E_D = E_R = V \cdot \frac{1}{\sigma}\phi\left(\frac{\overline{\alpha} - \mu}{\sigma}\right)\overline{\alpha}(1 - \overline{\alpha})\rho - \eta \cdot N,$$

where ϕ is the standard normal density.

Suppose, for instance, $V = 1$, $\beta_D = \beta_R$, $\mu = 1/2$, $\sigma = 0.0378$, $\rho = 0.6$, $\eta = 0.15$ and $N = 6\frac{2}{3}$.[1] For these parameter values, in an interior equilibrium,

$$E_D = E_R \approx 0.5831.$$

Figures 1 and 2 below show the objective function of party D for $E_R = 0.5831$. The objective function achieves a local maximum at approximately 0.5831 (Figure 1) but it achieves a global maximum at 0 (Figure 2).

[1] The values of σ, ρ and η are those estimated by SN, while the others are adopted for illustration purposes. SN seem to measure electorate size in millions (see the first paragraph in page 542).

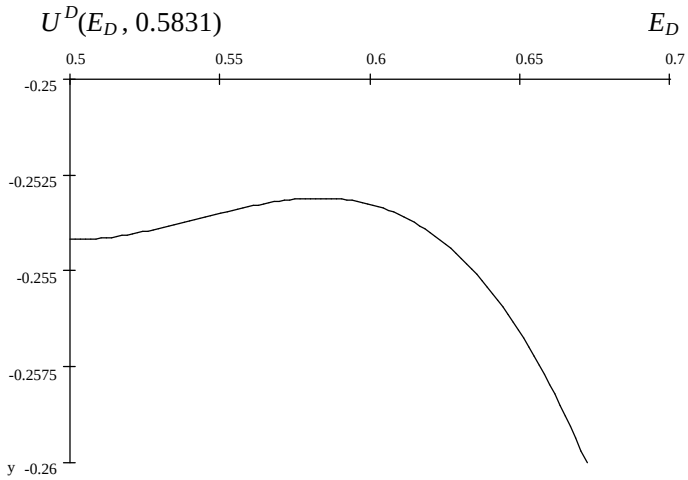

FIGURE 1: A LOCAL MAXIMUM…

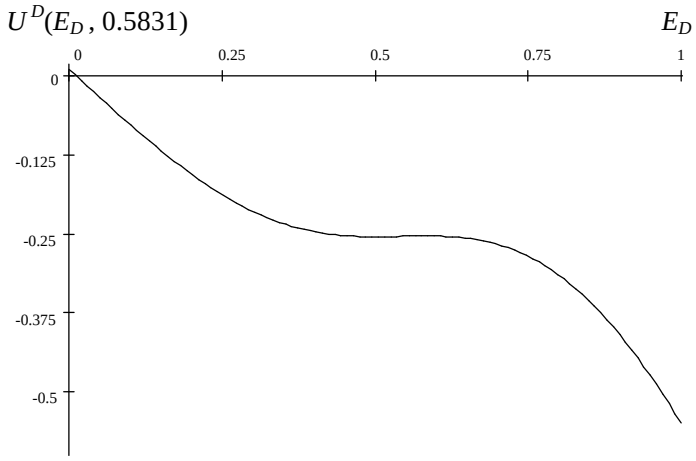

FIGURE 2: BUT NOT A GLOBAL MAXIMUM

It is possible to check that the corner point $E_D = E_R = 0$ is not an equilibrium either. Figure 3 shows the objective function of party D for $E_R = 0$. Thus, there is no (pure strategy) equilibrium in this example. First-order conditions can be seriously misleading in the SN model.

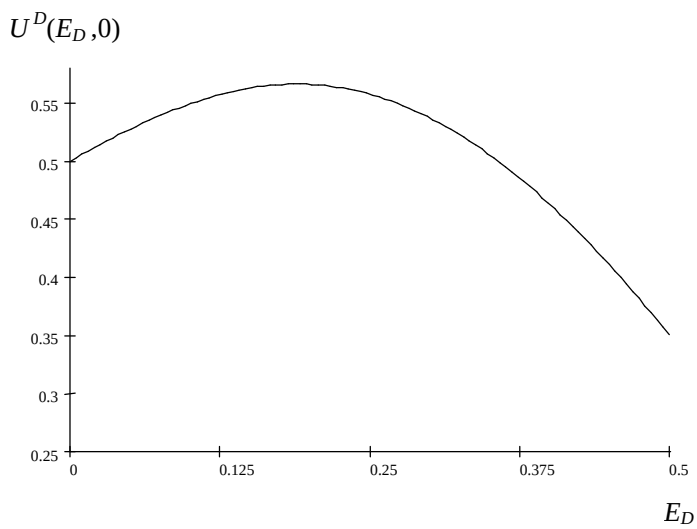

FIGURE 3: THERE IS NO CORNER EQUILIBRIUM

II. CONDITIONS FOR CONCAVITY

The second derivative of the objective function of parties D and R with respect to their own effort are, respectively,

$$U^D_{DD}(E_D,E_R) = V\rho^2\alpha(1-\alpha)\left(-h'(\alpha)\alpha(1-\alpha)+2h(\alpha)(\alpha-1/2)\right)-1$$

and

$$U^R_{RR}(E_D,E_R) = V\rho^2\alpha(1-\alpha)\left(h'(\alpha)\alpha(1-\alpha)+2h(\alpha)(1/2-\alpha)\right)-1.$$

(The argument of the function α is omitted for briefness.) Thus, the objective functions of parties D and R are strictly concave with respect to their own effort if and only if

$$-\frac{1}{\rho^2 V} < x(1-x)\left(-h'(x)x(1-x)+2h(x)(x-1/2)\right) < \frac{1}{\rho^2 V} \quad \text{for all } x \in (0,1). \tag{5}$$

Feddersen and Sandroni (FS 2001) develop an *ethical-voter model* of political participation with micro foundations that differ from those of SN, but arrive at a problem similar to that described by the objective functions

(2) and (3), with E_p interpreted now as the share of supporters of party p that turns out to vote.[2] To obtain concavity, FS assume that P is strictly concave in E_D and $(1-P)$ is strictly concave in E_R. (FS foundations guarantee that C is a convex function.) Unfortunately, this assumption would not work in the SN model because it implies that $-h'(x)x(1-x)+2h(x)(x-1/2)$ is simultaneously positive and negative.

Coate and Conlin (CC 2004) develop and estimate another ethical-voter model along the lines of FS. CC assume that h is a beta density; that is,

$$h(x) = \frac{\Gamma(u+v)}{\Gamma(u)\Gamma(v)} x^{v-1}(1-x)^{u-1},$$

where $v > 0$, $u > 0$. Using (5), this assumption leads to the following condition for strict concavity in the SN model:

$$(6) \quad -\frac{1}{\rho^2 V}\frac{\Gamma(u)\Gamma(v)}{\Gamma(u+v)} < x^v(1-x)^u\left((2+v-u)x-v\right) < \frac{1}{\rho^2 V}\frac{\Gamma(u)\Gamma(v)}{\Gamma(u+v)} \quad \text{for all } x \in (0,1).$$

This condition is easily checked since the first order condition for computing the maximum and the minimum of the middle expression is a quadratic equation. For instance, if h is uniform ($v = u = 1$), condition (6) implies $\rho^2 V < 6\sqrt{3}$.

Under the normal distribution assumption of SN, condition (5) becomes

$$(7) \quad -\frac{1}{\rho^2 V} < x(1-x)\phi\left(\frac{x-\mu}{\sigma}\right)\left(\frac{(x-\mu)x(1-x)}{\sigma^3}+\frac{2x-1}{\sigma}\right) < \frac{1}{\rho^2 V} \quad \text{for all } x \in (0,1).$$

For instance, if $\sigma = 0.0378$ and $\rho = 0.6$ (the values estimated by SN) and $\mu = 0.46$ (the average Democratic vote share in the data used by SN, state-by-state data for eleven US presidential elections, 1948-1988),[3] concavity requires $V < 0.262$. In SN model, $V = S + R_{jt}$, where S is the component of the value of winning the state' presidential vote that is independent from winning the national election, and R_{jt} is the chance that state j is pivotal in presidential election t. SN estimate $S = 0.0790$, so that concavity requires $R_{jt} < 0.183$. In their estimates of pivotal probabilities (cf. Table 8B), this upper bound is violated only in three cases: New York in 1960 and Pennsylvania and Texas in 1948.

[2] In the FS setup, the maximization of U^D and U^R with respect to E_D and E_R is not a problem explicitly solved by political leaders but rather an equilibrium condition for participation rules adopted by ethically-motivated supporters of each party.

[3] SN estimate (though not report) different values of μ_{jt} for each state and election year. Ideally, those estimates should be used to compute different upper bounds for R_{jt}.

III. CONCLUSION

The objective functions (2) and (3) are not necessarily concave under the functional forms given in (4), as opposed to what is claimed by SN. A necessary and sufficient condition for concavity is provided by equation (5) in this note. Assuming normal uncertainty, as SN do, the concavity condition is given by equation (7). The parameters estimated by SN using state-by-state data for presidential elections seem to satisfy equation (7) for most states and election years, with very few exceptions corresponding to states with an exceptionally high pivotal probability. In those few cases, it remains to be verified that the solution to the first-order conditions derived from (2) and (3) actually corresponds to a global maximum of the estimated objective functions of the parties. If the solution to the first-order conditions does not maximize the objective functions, it does not seem appropriate to use those data points in an estimation relying on an equilibrium model.

REFERENCES

[1] Coate, Stephen and Michael Conlin (2004), "A Group-Utilitarian Approach to Voter Turnout: Theory and Evidence," *American Economic Review* 94, 1476-1504.

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[3] Shachar, Ron and Barry Nalebuff (1999), "Follow the Leader: Theory and Evidence on Political Participation," *American Economic Review* 89, 525-547.