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Elections as Targeting Contests

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M E X I C O

ELECTIONS AS TARGETING CONTESTS

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ABSTRACT. This paper develops a model of electoral turnout where parties compensate voters for showing up to the polls. Existence and uniqueness conditions are shown to impose substantial restrictions on the uncertainty about partisan support faced by the parties, and on the distribution of voting costs among citizens. The model predicts that voters in the minority will be more likely to vote, and that turnout increases with the importance of the election. The model can generate the observed correlation between election closeness and electoral turnout, although the cause of this correlation may depend on the distribution of voting costs.

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1. INTRODUCTION

Both abstention and participation rates are usually substantial in political elections around the world. Moreover, they exhibit considerable variability both within and across countries, well beyond what could be accounted for by compulsory voting laws, voter registration rules, and other such regulations.[1] Thus, explaining electoral participation is an interesting and important issue for political economy. At least since Palfrey and Rosenthal's [16] careful game-theoretic treatment of participation in large elections, it has become increasingly clear that explaining electoral participation patterns when voting is costly for individuals must somehow take into account parties' efforts in mobilizing voters or some other solution to the collective action problem faced by like-minded groups of voters.[2]

This paper develops a model of electoral participation where parties compensate voters for showing up to the polls. In the model, parties target accurately their favorable voters with lowest voting costs. The assumption that parties invest resources in mobilizing voters is similar to a recent contribution by Shachar and Nalebuff [17]. Our targeting model can also be interpreted as a reduced-form version of a model with rule-utilitarian voters. As shown by Feddersen and Sandroni [5], the equilibrium of an election with ethically motivated voters (in the sense of rule-utilitarianism borrowed from Harsanyi [7]) corresponds to a pure-strategy equilibrium of a two-party game of the sort modeled in this paper.

We provide conditions for existence and uniqueness that rely on quasi-concavity of the objective functions of the parties, as opposed to the concavity conditions proposed by Feddersen and Sandroni. This allows us to relax the assumptions of Feddersen and Sandroni with respect to the uncertainty about the supporters of either party, at the expense of imposing constraints on the distribution of voting costs. The advantages of relying on quasi-concavity rather than concavity are illustrated by means of examples. Existence and uniqueness considerations are not merely "technical" issues in the context of voter mobilization models. Conditions for existence and uniqueness impose severe restrictions on the functional forms that can be used in these models, as illustrated by the examples.

We obtain that, in general, the minority party targets in equilibrium a larger share of its favorable voters but obtains a smaller expected fraction of votes. This is equivalent to the "underdog effect" that has been observed in pivotal-voter models of turnout. Also, just as in pivotal-voter models of turnout, and consistent with the evidence discussed by Blais [1], turnout

[1] See Blais [1] for a recent summary of empirical evidence about turnout.

[2] Dhillon and Peralta [3], Feddersen [4] and Merlo [14] summarize from different perspectives the political economy literature on turnout.

increase with the importance of the election. Other comparative statics results depend on the assumptions about the distribution of voting costs. In particular, with uniformly distributed costs, (i) neither an increase in the importance of the election nor an increase in partisanship in favor of both parties has any effect on the difference of expected votes as a share of turnout, (ii) an increase in partisanship in favor of both parties results in an increase in turnout, and (iii) if the majority party loses its partisanship advantage then the difference in expected votes shares as a share of turnout decreases and expected turnout increases. This last prediction, in particular, generates the inverse relationship between turnout and election closeness that some researchers believe to see in the data.

More generally, assumptions about the distribution of voting costs play a key role in determining the sign of comparative static results. For instance, variations in the importance of the election can also generate an inverse relationship between turnout and election closeness for certain distributions of voting costs different from the uniform. In terms of estimating empirically mobilization models of turnout, assuming uniformly distributed voting costs may eliminate interesting sources of correlation between observable variables.

Besides the literature already mentioned, our paper is related to other recent contributions. Morton [15] presents an early model of group behavior. Coate and Conlin [2] estimate structurally a model with rule-utilitarian voters in a similar spirit to the model of Feddersen and Sandroni [5]. A model of targeting is discussed by Herrera et al. [8] in the context of specific functional forms. The emphasis there is in the effect on the policy positions of the parties of the anticipation of a targeting contest. Further afield, Meirowitz [13] develops a model of electoral contests in which parties increase their probability of winning the election by investing in "valence," which increases their attractiveness to supporters of either party.

The remainder of this paper is organized as follows. Section 2 presents a model of elections as targeting contests and develops existence and uniqueness conditions. Section 3 illustrates the model and the existence conditions with examples related to the previous literature. Section 4 is devoted to comparative statics results. Section 5 contains concluding remarks. All proofs are gathered in the Appendix.

2. An electoral contest

Two parties, D and R compete in an election. There is a continuum of voters. At the beginning of time, a random draw by nature determines the fraction of voters favoring each party and the fraction of voters favoring neither. The fraction of voters favoring party D and the fraction of voters

favoring party R are given, respectively, by the random variables $\tilde{d}$ and $\tilde{r}$. Let F be the distribution function of the random variable $\tilde{r}/\tilde{d}$. We assume that F is strictly increasing and continuously differentiable over $(0,\infty)$, with $\lim_{x\to 0} F(x) = 0$ and $\lim_{x\to\infty} F(x) = 1$.

Each voter has a random cost of participating in the election, distributed independently according to a distribution function H. We assume that H is strictly increasing and continuously differentiable over some interval $[0,\bar{c}]$ such that $0 < \bar{c}$, with $H(0) = 0$ and $H(\bar{c}) = 1$.

Voters are willing to vote for the party they favor if the party compensates them for their participation cost. Parties can target voters accurately; that is, if a fraction x_D of the voters favoring party D turns out to vote, the aggregate cost for party D is equal to

$$\tilde{d} \times \int_0^{m(x_D)} c H'(c)\, dc,$$

where $m(x) \equiv H^{-1}(x) \cap [0,\bar{c}]$ tells us the participation cost of the marginal type of voter who shows up to vote for D.

Parties D and R decide simultaneously the fraction of favorable voters x_D and x_R, respectively, each party will be compensated for voting without knowing the realization of the random variables $\tilde{d}$ and $\tilde{r}$. We can imagine party D committing to compensate any favorable voter with costs equal to or lower than $m(x_D)$ for casting a vote, and similarly for party R. Let d^e and r^e be the expected values of the random variables $\tilde{d}$ and $\tilde{r}$, respectively. The expected aggregate cost of attracting a fraction x_D of favorable voters is then

$$C_D(x_D) \equiv d^e \times \int_0^{m(x_D)} c H'(c)\, dc.$$

Similarly, the expected aggregate cost for party R of attracting a fraction x_R of favorable voters is

$$C_R(x_R) \equiv r^e \times \int_0^{m(x_R)} c H'(c)\, dc.$$

The party with more votes wins the election. Thus, given x_D and x_R, the probability of D winning the election is

$$P(x_D, x_R) \equiv \Pr\left[\tilde{d} x_D > \tilde{r} x_R\right].$$

Therefore,

$$P(x_D, x_R) = \begin{cases} F(x_D/x_R) & \text{if} \quad x_D > 0 \quad \text{and} \quad x_R > 0 \\ 0 & \text{if} \quad x_D = 0 \quad \text{and} \quad x_R > 0 \\ 1 & \text{if} \quad x_D > 0 \quad \text{and} \quad x_R = 0 \end{cases}.$$

For completeness, we assume $P(0,0) = 1/2$; that is, a tie occurs if turnout is negligible, and the tie is broken randomly. The exact value of $P(0,0)$ is

irrelevant, as it will become clear later. Note that P is continuous in x_D if $x_R > 0$ and it is continuous in x_R if $x_D > 0$.

Each party obtains a payoff of $G > 0$ in case of winning the election and of 0 in case of losing it. Thus, the expected utility of party D is

$$U_D(x_D, x_R) \equiv GP(x_D, x_R) - C_D(x_D),$$

and the expected utility of party R is

$$U_R(x_D, x_R) \equiv G(1 - P(x_D, x_R)) - C_R(x_R).$$

A *targeting equilibrium* is a pair $(x_D^*, x_R^*) \in [0,1]^2$ such that

$$U_D(x_D^*, x_R^*) \geq U_D(x_D, x_R^*) \text{ for all } x_D \in [0,1],$$
$$U_R(x_D^*, x_R^*) \geq U_R(x_D^*, x_R) \text{ for all } x_R \in [0,1].$$

Intuitively, we can expect that in equilibrium both parties have positive turnout: if either party targets a negligible fraction of voters, the other party can win the election for sure targeting an arbitrarily small (but positive) fraction of voters, so that in fact it lacks a best response.[3] Thus, we can expect that in equilibrium each party either equates "marginal benefit" and the "marginal cost" of an increase in the fraction of favorable voters who turn out to vote, or gets all the favorable voters. That is, we can expect

$$G \times \frac{\partial F(x_D^*/x_R^*)}{\partial x_D} \geq C_D'(x_D^*), \text{ with equality if } x_D^* < 1,$$
$$G \times \frac{\partial (1 - F(x_D^*/x_R^*))}{\partial x_R} \geq C_R'(x_R^*), \text{ with equality if } x_R^* < 1.$$

This system is equivalent to

$$\begin{aligned} GF'(x_D^*/x_R^*)/x_R^* &\geq d^e\, m(x_D^*), \text{ with equality if } x_D^* < 1, \\ GF'(x_D^*/x_R^*)(x_D^*/(x_R^*)^2) &\geq r^e\, m(x_R^*), \text{ with equality if } x_R^* < 1. \end{aligned} \tag{1}$$

The intuition turns out to be correct under the following assumptions:

Assumption 1. *For all $c \in [0, \bar{c}]$,*

$$H'(c)c/H(c) \leq 1.$$

Assumption 2. *F' es continuously differentiable. For all $x \in (0, \infty)$,*

$$-3 < F''(x)x/F'(x) < 1.$$

[3]Of course, as noted by Hirshleifer [9], the same reasoning applies to to any contest model where the probability of success is a function of the ratio of efforts invested by the conflicting parties.

Assumption 1 puts an upper bound on how fast the density of types can increase as parties increase the "maximum price" they are willing to pay for a vote. Equivalently, assumption 1 states that $H(c)/c$ is nonincreasing. Note that $H(c)$ does not need to be concave. Graphically, assumption 1 means that the curve $(c,H(c))$ is "visible" from the origin, in the sense that the straight line segment connecting $(0,0)$ with any point in $(c,H(c))$ is contained in the area $\{(c,y) : y \leq H(c)\}$.

The upper bound in assumption 2 provides a constraint on how fast the marginal probability of D winning the election can increase as party D increases its willingness to pay. The lower bound, in turn, provides a constraint on how fast the marginal probability of R winning the election can increase as party R increases its willingness to pay. Equivalently, assumption 2 states that $F'(x)/x$ is decreasing and $F'(x)x^2$ is increasing. Graphically, assumption 2 means that the curve $(x,F'(x))$ is (strictly) visible from the origin, and a similar condition with respect to the density of $\tilde{d}/\tilde{r}$.

Assumptions 1 and 2 together guarantee that the expected utilities of both parties are strictly quasiconcave. Thus, the "first order conditions" described by the system 1 are in fact necessary and sufficient for an equilibrium to exist.

We have

Theorem 1. *Under assumptions 1 and 2, there is a unique targeting equilibrium. The equilibrium is given by the solution to the system 1.*

The proof of this and other results is in the Appendix.

It is easy to check that $C_D(x)$ and $C_R(x)$ are strictly convex regardless of whether assumption 1 holds or not. Thus, an alternative to assumptions 1 and 2 would be to require concavity of both $F(x)$ and $1-F(1/x)$ (i.e. the distribution functions of the random variables $\tilde{r}/\tilde{d}$ and $\tilde{d}/\tilde{r}$) to ensure strict concavity of both parties' objective functions. Feddersen and Sandroni [5] propose this assumption in a similar context. This assumption is more restrictive than assumption 2. To see this, note that concavity of $F(x)$ implies $F''(x)x/F'(x) \leq 0$, which is stronger than the upper bound in assumption 2. Similarly, from concavity of $1-F(1/x)$,

$$-2F'(1/x)/x^3 + F''(1/x)/x^4 \leq 0$$

for all $x \in (0,\infty)$. But taking $y = 1/x$ this implies

$$2F'(y)y^3 + F''(y)y^4 \geq 0,$$

or equivalently, $F''(y)y/F'(y) \geq -2$ for all $y \in (0,\infty)$, which is stronger than the lower bound in assumption 2. Thus, as will be illustrated by means

of examples, our assumptions are more permissive with respect to the uncertainty about the supporters of either party, at the expense of imposing constraints on the distribution of voting costs.

We say that a targeting equilibrium (x_D^*, x_R^*) is *interior* if $(x_D^*, x_R^*) \in (0,1)^2$. From the previous discussion, we know that x_D^* and x_R^* are both positive. The next assumption guarantees that x_D^* and x_R^* are smaller than one:

Assumption 3. *Let $\underline{x}, \overline{x}$ solve*

$$\underline{x}m(\underline{x}) = \min\{r^e/d^e, 1\}\overline{c},$$
$$\overline{x}m(\overline{x}) = \min\{d^e/r^e, 1\}\overline{c}.$$

Then for all $x \in [\underline{x}, 1/\overline{x}]$,

$$GF'(x)x < \min\{r^e, d^e\}\overline{c}.$$

Assumption 3 provides a lower bound to the marginal cost of getting the most expensive voters to turn out in the election. Note that $\overline{c} = m(1)$. Moreover, $xm(x)$ is continuous, strictly increasing and goes from 0 to $m(1)$ as x goes from 0 to 1. Thus, $\underline{x}$ and $\overline{x}$ are well defined. In addition, $0 < \underline{x} \leq 1$ and $1 \leq 1/\overline{x} < \infty$. If $d^e = r^e$, we have $\underline{x} = \overline{x} = 1$, so that assumption 3 simply requires $GF'(1) < d^e\overline{c}$.

We have

Proposition 1. *Under assumptions 1, 2 and 3, the (unique) targeting equilibrium is interior.*

Conveniently, in an interior equilibrium the system 1 holds with equality.

3. Examples

3.1. **Beta uncertainty.** Suppose that the fraction of voters supporting party D is a beta random variable with parameters $\alpha > 0$ and $\beta > 0$, and that all voters support either party D or party R. Then the density of the random variable $\tilde{d}$ is

$$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} d^{\alpha-1}(1-d)^{\beta-1}$$

for $d \in (0,1)$, where Γ is the gamma function and $\tilde{r} = 1 - \tilde{d}$. The density of the random variable $\tilde{r}/\tilde{d}$ (or equivalently $1 - 1/\tilde{d}$) is

$$F'(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{x^{\beta-1}}{(1+x)^{\alpha+\beta}}$$

for $x \in (0,\infty)$, known as Pearson type VI density or as beta density of the second kind with parameters α and β (see e.g. Johnson et al. [10] p. 248). We can easily obtain

$$F''(x)x/F'(x) = \beta - 1 - \frac{x}{1+x}(\alpha+\beta).$$

Thus, since $x \in (0,\infty)$,

$$-1-\alpha < F''(x)x/F'(x) < \beta - 1.$$

Assumption 2 is verified if and only if $\alpha \leq 2$ and $\beta \leq 2$. For instance, if $\tilde{d}$ is uniformly distributed on $[0,1]$, it has a beta density with parameters $\alpha = \beta = 1$ so that 2 is verified.

It is easy to check that $F(x)$ and $1-F(1/x)$ (that is, the distribution functions of the random variables $\tilde{r}/\tilde{d}$ and $\tilde{d}/\tilde{r}$) are both concave functions if and only if $\alpha \leq 1$ and $\beta \leq 1$. That is, assumption 2 is less demanding than concavity of both $\tilde{r}/\tilde{d}$ and $\tilde{d}/\tilde{r}$.

In a related model, Coate and Conlin [2] assume a beta distribution of $\tilde{d}$ as in this example. They also assume a uniform distribution of voting costs with support $[0,\bar{c}]$, which satisfies our assumption 1. Thus, a sufficient condition for the existence of a targeting equilibrium in Coate and Conlin's model is $\alpha \leq 2$ and $\beta \leq 2$.[4] With respect to assumption 3, suppose that $\beta \leq \alpha$. Note that, with beta uncertainty,

$$d^e = \frac{\alpha}{\alpha+\beta} \text{ and } r^e = \frac{\beta}{\alpha+\beta}.$$

Note also that, with uniform costs, $m(x) = \bar{c}x$. Thus,

$$\underline{x} = \sqrt{\beta/\alpha}.$$

Finally, note that, with beta uncertainty, $F'(x)x$ is decreasing for all $x > \beta/\alpha$. Since $\beta \leq \alpha$ implies $\sqrt{\beta/\alpha} > \beta/\alpha$, we get that assumption 3 is equivalent to

$$GF'(\sqrt{\beta/\alpha})\sqrt{\beta/\alpha} < \frac{\beta}{\alpha+\beta}\bar{c}.$$

Simplifying, we get that assumption 3 is equivalent to

$$G < \frac{\Gamma(\alpha)\Gamma(\beta+1)}{\Gamma(\alpha+\beta+1)} \frac{(\sqrt{\alpha}+\sqrt{\beta})^{\alpha+\beta}}{(\sqrt{\alpha})^{\alpha}(\sqrt{\beta})^{\beta}}\bar{c}.$$

3.2. **Dirichlet uncertainty.** In the example above that there is only one source of uncertainty, the fraction of voters supporting either party. This is unlike the examples discussed by Feddersen and Sandroni [5], where there is also uncertainty about the fraction of voters who are not willing to support either party (unethical voters in their framework). This example has some of that flavor.

[4] This condition implies that the second order conditions identified by Coate and Conlin [2] (Proposition 2) are satisfied. We are ignoring here the assumption of their model that parties may not be able to target voters with too high costs.

Suppose that the joint density of $\tilde{d}$ and $\tilde{r}$ is the Dirichlet density

$$\frac{\Gamma(\alpha+\beta+\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)} d^{\alpha-1} r^{\beta-1} (1-d-r)^{\gamma-1}$$

for $d, r \in (0,1)$, where α, β and γ are positive. Here, $1-d-r$ represents voters who favor neither party. As it follows from well-known results about the Dirichlet distribution (see e.g. Kotz et al. [11]), the density of the random variable $\tilde{r}/\tilde{d}$ is again a type VI Pearson density

$$F'(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{x^{\beta-1}}{(1+x)^{\alpha+\beta}}.$$

Thus, as in the previous example, assumption 2 is verified if $\alpha \leq 2$ and $\beta \leq 2$.

3.3. **Quadratic aggregate costs.** Although we take as primitive of our model the distribution of voters' costs of participating in the election and derive from it the aggregate cost of parties, we can also interpret a given aggregate cost as coming from some underlying distribution of voters' costs. For instance, consider the aggregate cost function

$$C_p(x_P) = p^e \times \left(x_P^2/2 + \eta x_P\right)$$

for $P = D, R$, where η is a positive constant. It is easy to establish in our framework

$$C_P'(x_P)/p^e = m(x_P) = c \text{ for } x_P = H(c).$$

Thus, the cost function above can be generated by the distribution of voters' costs $H(c) = m^{-1}(c) = c - \eta$ with support $[\underline{c}, \overline{c}] = [\eta, 1+\eta]$.

Of course, if $C_P(x_P)$ is not strictly convex, we cannot reinterpret it as coming from accurately targeting voters with increasing costs of participation. For instance, Shachar and Nalebuff [17] assume that the fraction of favorable voters each party is able to attract to the polls is an exponential function of some underlying effort variable, and that the cost of effort is quadratic. But then the aggregate cost function is quadratic in the logarithm of the fraction of favorable voters, which implies its second derivative is negative for small values of the fraction of favorable voters. Thus, Shachar and Nalebuff's [17] model cannot be reinterpreted as a targeting contest.

4. Comparative statics

In what follows, we maintain the assumptions 1, 2 and 3, so that there is a unique equilibrium and it is interior.

4.1. **The underdog effect and the upset probability.** We say that party R is the minority party and party D the majority party if $d^e > r^e$, and viceversa if the inequality is reversed. It is simple to verify the following result:

Proposition 2. *The minority party targets in equilibrium a larger share of its favorable voters but obtains a smaller expected fraction of votes. If $d^e = r^e$, both parties target in equilibrium the same share of favorable voters.*

Thus, as in pivotal-voter models of turnout, in electoral contests there is an *underdog effect*: supporters of a minority party are more likely to turn out to vote (see e.g. Levine and Palfrey [12]). However, the fact that minority voters are more likely to vote does not fully compensate for the fact that there are fewer of them in terms of expected vote.

For "well-behaved" distributions of $\tilde{d}$ and $\tilde{r}$, the party that expects to have fewer votes will also be less likely to win the election. For instance, in the context of the beta or the Dirichlet example, the probability of an upset (that is, the minority party winning the election) is below 1/2 if $\alpha > 1$ and $\beta > 1$. To see this, suppose that R is the minority party. Recall that the probability that R wins the election is $1 - F(x_D^*/x_R^*)$. Using Corollary 2 we have

$$x_D^*/x_R^* > r^e/d^e = \beta/\alpha.$$

Moreover,

$$\begin{aligned} F(\beta/\alpha) &= \int_{x=0}^{\beta/\alpha} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{x^{\beta-1}}{(1+x)^{\alpha+\beta}}\, dx \\ &= \int_{y=0}^{\beta/(\alpha+\beta)} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\beta-1}(1-y)^{\alpha-1}\, dy, \end{aligned}$$

where we use the change of variables $y = x/(1+x)$. Note that the last expression is equal to a beta distribution function with parameters β and α evaluated at its mean. Since $\beta < \alpha$ by the assumption that D is favored by voters, this beta distribution is positively skewed. Since $\alpha > 1$ and $\beta > 1$, this implies that the mean is larger than the median, that is $F(\beta/\alpha) > 1/2$ (Groeneveld and Meeden [6]). But this implies $1 - F(x_D^*/x_R^*) < 1/2$.

In the reminder of this section we explore the effect of changes in the underlying parameters of the model on the expected turnout, the probability of each party winning the election, and the expected closeness of the election. In equilibrium, expected turnout is

$$\mathrm{E}(\tilde{d}x_D^* + \tilde{r}x_R^*) = d^e x_D^* + r^e x_R^*.$$

The difference between expected votes for party D and expected votes for party R, as a share of expected turnout, is

$$\frac{|\mathrm{E}(\tilde{d}x_D^* - \tilde{r}x_R^*)|}{\mathrm{E}(\tilde{d}x_D^* + \tilde{r}x_R^*)} = \left| \frac{d^e x_D^*}{r^e x_R^*} - 1 \right| \Big/ \left(\frac{d^e x_D^*}{r^e x_R^*} + 1 \right).$$

From Corollary 2, if party D is the majority party, $(d^e x_D^*)/(r^e x_R^*) > 1$. Thus, both the probability of this party winning the election and the difference in expected votes increase if the ratio x_D^*/x_R^* increases.

4.2. **The importance of the election.** We have

Proposition 3. *If the payoff G of winning the election increases, then (i) expected turnout increases, and (ii) the probability that the majority party wins the election and the difference in expected votes as a share of turnout decrease, remain constant, or increase, respectively, if $H'(c)c/H(c)$ is increasing, constant, or decreasing for all c.*

Increasing the "prize" associated with the election leads both parties to target a larger fraction of voters. If $H'(c)c/H(c)$ is decreasing, the increase is disproportionately larger for the majority party. This is because the minority party, which is targeting a larger fraction of voters, finds it more difficult to target voters at the margin. Thus, the difference in expected votes between the parties increases. This is the case, for instance, in the example with quadratic aggregate costs.

If $H'(c)c/H(c)$ is increasing, on the other hand, the increase in the fraction of voters targeted is disproportionately larger for the minority party, so that elections become closer in the sense of a smaller difference in expected votes.

Finally, if $H'(c)c/H(c)$ is constant for all c–or equivalently, if $H(c)$ is uniform–we get that the difference in expected votes as a percentage of turnout remains constant, so that the effect of changes in G is to scale up or down turnout for both parties.

4.3. **Partisanship.** We study here the effect of an *increase in partisanship*, defined as a proportional increase in the expected fraction of voters favoring both parties that leaves the ratio r^e/d^e and the distribution of the random variable $\tilde{r}/\tilde{d}$ unchanged. In terms of the Dirichlet example, this would happen if and only if γ decreases but α and β remain constant. We have

Proposition 4. *If partisanship increases, then (i) the fraction of favorable voters targeted by party D and by party R (respectively x_D and x_R) decrease,(ii) the probability that the majority party wins the election and the difference in expected votes as a share of turnout increase, remain constant, or decrease, respectively, if $H'(c)c/H(c)$ is increasing, constant, or decreasing for all c, and (iii) turnout increases if $H'(c)c/H(c)$ is constant for all c.*

As in the previous subsection, the sign of the effect of an increase in partisanship on election closeness is determined by the distribution of voters' costs. In particular, with quadratic aggregate costs, elections with more partisanship are more closely fought.

While an increase in partisanship leads both parties to target a smaller fraction of favorable voters, the sign of its effect on turnout is not clear except if $H'(c)c/H(c)$ is constant. For instance, in Coate and Conlin's [2] model, an increase in partisanship will lead to an increase in turnout but will have no effect on election closeness.

4.4. **Losing the Advantage.** Suppose in the initial situation party D is the majority party, and it loses its advantage in the sense that r^e is increased until it becomes equal to d^e. From Proposition 2, we have that $(d^e x^D)/(r^e x^R)$ is larger than one in the initial situation and equal to one in the final situation; that is, the election becomes closer in terms of expected vote shares. Moreover, x^D/x^R is smaller than one in the initial situation and equal to one in the final situation; that is, party D increases its effort in relative terms. Manipulating the interior equilibrium conditions as in the proof of Proposition 3, we get that party D must also increase its effort in absolute terms. What is the effect on turnout? Using the expression above for expected turnout, we get that expected turnout increases with a marginal increase in r^e if

$$\partial x_D > -\partial x_R r^e/d^e.$$

Again, manipulating the interior equilibrium conditions as in the proof of Proposition 3 we get that $m(x_D)x_R$ must increase in going from the initial to the final situation. In marginal terms, this implies

$$\partial x_D > -\partial x_R r^e/d^e \times \frac{x_D d^e/(x_R r^e)}{m'(x_D)x_D/m(x_D)}.$$

The numerator in the rightmost fraction is larger than or equal to one on account of Proposition 2. The denominator is larger than or equal to one on account of assumption 1. In particular, if $H'(c)c/H(c)$ is constant for all c–or equivalently, if $H(c)$ is uniform–we get that the denominator is exactly equal to one. Thus, with uniformly distributed costs, if the majority party loses its advantage the result is an increase in turnout and a reduction in the expected difference in vote shares.

4.5. **Targeting accuracy.** We have assumed that parties target accurately the voters with lowest costs. More generally, we can think of the marginal cost of the fraction of favorable voters attracted to the polls, $m(x_P)$, as being decreasing in the accuracy of targeting. Immediately from system 1, the analysis of an increase in targeting accuracy is similar to that of an increase in the importance of the election.

5. Conclusions

This paper presents a model of electoral participation built on three ideas: First, voting is costly for individual voters. Second, parties composed of like-minded voters solve the collective action problem raised by costly voting by compensating voters who show up to the polls. Third, parties manage to do so efficiently; that is, they compensate voters with the lowest costs. Of course, in reality, parties do not hand out cash to their supporters in exchange for voting, or at least this does not happen too frequently in established democracies. But parties do reduce the cost of voting by handing information to supporters, by helping with registration, etc. They also invest in the attractiveness of the act of voting for their candidates by publicizing personal characteristics of those candidates that may not be relevant for the office contested in the election but make voters feel better about voting for the candidate, not unlike the effect of endorsements in the publicity of consumer goods. While efficient targeting is a strong assumption, we believe it is a useful benchmark for a voter mobilization model of elections.

Modeling elections as targeting contests implies that the cost for parties of mobilizing voters is convex in the fraction of supporters attracted to the polls, at least under the assumption of efficient targeting. Existence of equilibrium imposes some additional constraints on the distribution of voting costs and the uncertainty regarding partisan support for either party, guaranteeing the quasi-concavity of the objective functions of both parties.

In terms of comparative statics, the model can replicate the correlation between turnout and closeness of election observed by many researchers (see e.g. Blais [1] p. 59). How the correlation may arise depends on the assumptions about the distribution of voting costs. If voting costs are uniformly distributed, the reason for the observed correlation may be variations in the difference between the support of the majority party and the support of the minority party. For certain other distributions, the reason may simply be variations in the importance of the election as perceived by the parties.

APPENDIX

Theorem 1. *Under assumptions 1 and 2, there is a unique targeting equilibrium. The equilibrium is given by the solution to the system 1.*

Proof. A difficulty we face in the proof of *existence* is that the payoff functions $U_D(x_D,x_R)$ and $U_R(x_D,x_R)$ are discontinuous near $(0,0)$. To deal with this, we perturb slightly these functions as in Feddersen and Sandroni's [5] existence proof. Let $y_D \equiv (1-\varepsilon)x_D+\varepsilon$ and $y_R \equiv (1-\varepsilon)x_R+\varepsilon$ for some $0<\varepsilon<1$, and let

$$U_D^{\varepsilon}(x_D,x_R) \equiv U_D(y_D,y_R) \text{ and } U_R^{\varepsilon}(x_D,x_R) \equiv U_R(y_D,y_R).$$

Note that under assumptions 1 and 2, the functions U_D^{ε} and U_R^{ε} are twice continuously differentiable. In particular, the first derivatives with respect to x_D and x_R, respectively, are

$$U_{D1}^{\varepsilon}(x_D,x_R) = (1-\varepsilon)\left(GF'\left(\frac{y_D}{y_R}\right)\frac{1}{y_R} - d^e m(y_D)\right)$$

and

$$U_{R2}^{\varepsilon}(x_D,x_R) = (1-\varepsilon)\left(GF'\left(\frac{y_D}{y_R}\right)\frac{y_D}{y_R^2} - r^e m(y_R)\right).$$

The second derivatives with respect to x_D and x_R, respectively, are

$$U_{D11}^{\varepsilon}(x_D,x_R) = (1-\varepsilon)\left(GF''\left(\frac{y_D}{y_R}\right)\frac{1}{y_R^2} - \frac{d^e}{H'(m(y_D))}\right)$$

and

$$U_{R22}^{\varepsilon}(x_D,x_R) = \\ (1-\varepsilon)\left(-GF''\left(\frac{y_D}{y_R}\right)\frac{y_D^2}{y_R^4} - 2GF'\left(\frac{y_D}{y_R}\right)\frac{y_D}{y_R^3} - \frac{r^e}{H'(m(y_R))}\right).$$

We claim that if for some pair x_D,x_R we have $U_{D1}^{\varepsilon}(x_D,x_R) \le 0$, then we also have $U_{D11}^{\varepsilon}(x_D,x_R) < 0$. (This implies that U_D^{ε} is strictly quasiconcave in x_D.) To see this, using

$$\frac{U_{D1}^{\varepsilon}(x_D,x_R)}{y_D(1-\varepsilon)} = GF'\left(\frac{y_D}{y_R}\right)\frac{1}{y_D y_R} - \frac{d^e m(y_D)}{y_D},$$

we get

$$\begin{aligned}\frac{U^{\varepsilon}_{D11}(x_D,x_R)}{1-\varepsilon} &= \left(GF''\left(\frac{y_D}{y_R}\right)\frac{1}{y_R^2}-GF'\left(\frac{y_D}{y_R}\right)\frac{1}{y_Dy_R}\right)\\ &\quad+\left(-\frac{d^e}{H'(m(y_D))}+\frac{d^em(y_D)}{y_D}\right)\\ &\quad+\frac{U^{\varepsilon}_{D1}(x_D,x_R)}{y_D(1-\varepsilon)}.\end{aligned}$$

The first term in the RHS is negative on account of the upper bound in assumption 2, while the second term is negative on account of 1 and the identity $y_D = H(m(y_D))$.

Similarly, we claim that $U^{\varepsilon}_{R2}(x_D,x_R) \leq 0$ implies $U^{\varepsilon}_{R22}(x_D,x_R) < 0$. (That is, U^{ε}_R is strictly quasiconcave in x_R.) To see this, using

$$\frac{U^{\varepsilon}_{R2}(x_D,x_R)}{y_R(1-\varepsilon)} = GF'\left(\frac{y_D}{y_R}\right)\frac{y_D}{y_R^3}-\frac{r^em(y_R)}{y_R},$$

we get

$$\begin{aligned}\frac{U^{\varepsilon}_{R22}(x_D,x_R)}{1-\varepsilon} &= \left(-GF''\left(\frac{y_D}{y_R}\right)\frac{y_D^2}{y_R^4}-3GF'\left(\frac{y_D}{y_R}\right)\frac{y_D}{y_R^3}\right)\\ &\quad+\left(-\frac{r^e}{H'(m(y_R))}+\frac{r^em(y_R)}{y_R}\right)\\ &\quad+\frac{U^{\varepsilon}_{R2}(x_D,x_R)}{y_R(1-\varepsilon)}.\end{aligned}$$

The first term in the RHS is negative on account of the lower bound in assumption 2, while the second term is non-positive on account of assumption 1 and the identity $y_R = H(m(y_R))$.

On account of strict quasiconcavity, for any given x^R, the function U^{ε}_D has a unique maximizing choice of x^D or best-response, say $B_D(x_R)$. Similarly, for any given x^D, the function U^{ε}_R has a unique maximizing choice of x^R or best-response, say $B_R(x_D)$. On account of quasiconcavity and differentiability of U^{ε}_D and U^{ε}_R, best-response functions are continuous. Since x_D and x_R are elements of a compact set, it follows from Brouwer's fixed point theorem that there is an equilibrium of the perturbed contest, i.e. a pair $(x^{\varepsilon}_D, x^{\varepsilon}_R)$ such that

$$\begin{aligned}U^{\varepsilon}_D(x^{\varepsilon}_D,x^{\varepsilon}_R) &\geq U^{\varepsilon}_D(x_D,x^{\varepsilon}_R) \text{ for all } x_D \in [0,1],\\ U^{\varepsilon}_R(x^{\varepsilon}_D,x^{\varepsilon}_R) &\geq U^{\varepsilon}_R(x^{\varepsilon}_D,x_R) \text{ for all } x_R \in [0,1].\end{aligned}$$

Next, consider any converging subsequence of equilibria of perturbed contests $(x^{\varepsilon}_D, x^{\varepsilon}_R)$ as ε goes to zero. Let (x^0_D, x^0_R) be the limit point of that sequence.

We claim that x_D^0 and x_R^0 are positive. To see this, suppose $x_D^0 = 0$ and $x_R^0 > 0$. Then for small enough ε we must have $x_D^\varepsilon < 1$ and $x_R^\varepsilon > 0$, which implies $U_{D1}^\varepsilon(x_D^\varepsilon, x_R^\varepsilon) \leq 0$ and $U_{R2}^\varepsilon(x_D^\varepsilon, x_R^\varepsilon) \geq 0$. However, if $F'(0) < \infty$ we get $\lim_{\varepsilon\to 0} U_{R2}^\varepsilon(x_D^\varepsilon, x_R^\varepsilon) = -r^e m(x_R^0) < 0$, a contradiction, and if $F'(0) = \infty$ we get $\lim_{\varepsilon\to 0} U_{D1}^\varepsilon(x_D^\varepsilon, x_R^\varepsilon) = +\infty$, a contradiction.

Suppose instead that $x_D^0 > 0$ and $x_R^0 = 0$. Then for small enough ε we must have $x_D^\varepsilon > 0$ and $x_R^\varepsilon < 1$, which implies $U_{D1}^\varepsilon(x_D^\varepsilon, x_R^\varepsilon) \geq 0$ and $U_{R2}^\varepsilon(x_D^\varepsilon, x_R^\varepsilon) \leq 0$. However, if $\lim_{x\to\infty} F'(x)x = 0$ we get $\lim_{\varepsilon\to 0} U_{D1}^\varepsilon(x_D^\varepsilon, x_R^\varepsilon) = -d^e m(x_D^0) < 0$, a contradiction, and if $\lim_{x\to\infty} F'(x)x = +\infty$ we get $\lim_{\varepsilon\to 0} U_{R2}^\varepsilon(x_D^\varepsilon, x_R^\varepsilon) = +\infty$, a contradiction.

Suppose, finally, that $x_D^0 = x_R^0 = 0$. Then for small enough ε we must have $x_D^\varepsilon < 1$ which implies $U_{D1}^\varepsilon(x_D^\varepsilon, x_R^\varepsilon) \leq 0$. However $\lim_{\varepsilon\to 0} U_{D1}^\varepsilon(x_D^\varepsilon, x_R^\varepsilon) = +\infty$, a contradiction.

Having established that x_D^0 and x_R^0 are positive, on account of continuity of P we obtain that (x_D^0, x_R^0) is in fact a targeting equilibrium and must satisfy the system 1.

For the proof of *uniqueness*, suppose that (x_D^*, x_R^*) is an equilibrium such that $x_D^* < 1$ and $x_R^* < 1$. We claim first that there is no other equilibrium $\hat{x}_D, \hat{x}_R$ such that $\hat{x}_D \leq x_D^*$ and $\hat{x}_R \geq x_R^*$. To see this, from the system 1 we get

$$d^e x_D^* m(x_D^*) = r^e x_R^* m(x_R^*)$$

and

$$d^e \hat{x}_D m(\hat{x}_D) \geq r^e \hat{x}_R m(\hat{x}_R).$$

But this is impossible since $xm(x)$ is strictly increasing, which implies

$$d^e x_D^* m(x_D^*) \geq d^e \hat{x}_D m(\hat{x}_D)$$

and

$$r^e \hat{x}_R m(\hat{x}_R) \geq r^e x_R^* m(x_R^*),$$

with at least one of the two last inequalities being strict.

Next, we claim that there is no other equilibrium $(\hat{x}_D, \hat{x}_R)$ such that $\hat{x}_D \geq x_D^*$ and $\hat{x}_R \leq x_R^*$. The proof is analogous to the previous claim.

Third, we claim that there is no other equilibrium $(\hat{x}_D, \hat{x}_R)$ such that $\hat{x}_D \geq x_D^*$ and $\hat{x}_R \geq x_R^*$. To see this, suppose that $\hat{x}_D/\hat{x}_R > x_D^*/x_R^*$. From the system 1 we get

$$GF'(x_D^*/x_R^*)(x_R^*/x_D^*) = d^e(m(x_D^*)/x_D^*)x_R^{*2}$$

and

$$GF'(\hat{x}_D/\hat{x}_R)(\hat{x}_R/\hat{x}_D) \geq d^e(m(\hat{x}_D)/\hat{x}_D)\hat{x}_R^2.$$

On account of the upper bound in assumption 2, $F'(x)/x$ is strictly decreasing. Thus $\hat{x}_D/\hat{x}_R > x_D^*/x_R^*$ implies

$$GF'(x_D^*/x_R^*)(x_R^*/x_D^*) > GF'(\hat{x}_D/\hat{x}_R)(\hat{x}_R/\hat{x}_D).$$

However, on account of assumption 1, $m(x)/x$ is non decreasing. (To verify this, simply write $m(x)/x = m(x)/H(m(x))$.) This and $x_R^* \leq \hat{x}_R$ imply

$$d^e(m(x_D^*)/x_D^*)x_R^{*2} \leq d^e(m(\hat{x}_D)/\hat{x}_D)\hat{x}_R^2,$$

which leads to a contradiction. Next, suppose $\hat{x}_D > x_D^*$ and $\hat{x}_R > x_R^*$ and $\hat{x}_D/\hat{x}_R = x_D^*/x_R^*$. A similar argument leads to a contradiction. Finally, suppose that $\hat{x}_D/\hat{x}_R < x_D^*/x_R^*$. From the system 1 we get

$$GF'(x_D^*/x_R^*)(x_D^*/x_R^*)^3 = r^e(m(x_R^*)/x_R^*)x_D^{*2}$$

and

$$GF'(\hat{x}_D/\hat{x}_R)(\hat{x}_D/\hat{x}_R)^3 \geq r^e(m(\hat{x}_R)/\hat{x}_R)\hat{x}_D^2.$$

On account of the lower bound in assumption 2, $F'(x)x^3$ is strictly increasing. Thus $\hat{x}_D/\hat{x}_R \geq x_D^*/x_R^*$ implies

$$GF'(x_D^*/x_R^*)(x_D^*/x_R^*)^3 > GF'(\hat{x}_D/\hat{x}_R)(\hat{x}_D/\hat{x}_R)^3.$$

However, $m(x)/x$ is non decreasing. This and $x_D^* \leq \hat{x}_D$ imply

$$r^e(m(x_R^*)/x_R^*)x_D^{*2} \leq r^e(m(\hat{x}_R)/\hat{x}_R)\hat{x}_D^2,$$

which leads to a contradiction.

An analogous argument shows that there is no other equilibrium $(\hat{x}_D, \hat{x}_R)$ such that $\hat{x}_D \leq x_D^*$ and $\hat{x}_R \leq x_R^*$. Thus, if there is an interior targeting equilibrium, it is the unique equilibrium.

Now, suppose that (x_D^*, x_R^*) is an equilibrium such that $x_D^* < 1$ and $x_R^* = 1$. We claim that there is no other equilibrium $(\hat{x}_D, \hat{x}_R)$ such that $\hat{x}_D = 1$ and $\hat{x}_R < 1$. To see this, from the system 1 we get

$$d^e x_D^* m(x_D^*) \geq r^e x_R^* m(x_R^*)$$

and

$$r^e \hat{x}_R m(\hat{x}_R) \geq d^e \hat{x}_D m(\hat{x}_D).$$

But this is impossible since $xm(x)$ is strictly increasing, which implies

$$d^e \hat{x}_D m(\hat{x}_D) > d^e x_D^* m(x_D^*)$$

and

$$r^e x_R^* m(x_R^*) > r^e \hat{x}_R m(\hat{x}_R).$$

Similarly, we claim that if $(x_D^*, x_R^)*$ is an equilibrium such that $x_D^* < 1$ and $x_R^* = 1$, then $(1,1)$ is not an equilibrium. To see this, from the system 1 we get

$$GF'(x_D^*)/x_D^* = d^e m(x_D^*)/x_D^*$$

and

$$GF'(1)/1 \geq d^e m(1)/1.$$

However, $F'(x)/x$ is strictly decreasing while $m(x)/x$ is non decreasing.

Finally, suppose that (x_D^*, x_R^*) is an equilibrium such that $x_D^* = 1$ and $x_R^* < 1$. We claim that $(1,1)$ is not an equilibrium. To see this, from the system 1 we get

$$GF'(1/x_R^*)/(x_R^*)^3 = r^e m(x_R^*)/x_R^*$$

and

$$GF'(1)/1 \geq r^e m(1)/1.$$

However, on account of the lower bound in assumption 2, $F'(1/x)/x^3$ is strictly decreasing while $m(x)/x$ is non decreasing.

(From the previously considered cases, it follows that if $(1,1)$ is an equilibrium, there are no other equilibria.) □

Proposition 1. *Under assumptions 1, 2 and 3, the (unique) targeting equilibrium is interior.*

Proof. From Theorem 1, we know that under assumptions 1 and 2 the equilibrium is unique and satisfies system 1.

Suppose that $(1,1)$ is an equilibrium. Then, from system 1, we get

$$GF'(1) \geq d^e \bar{c},$$

which violates assumption 3, since $\underline{x} \leq 1 \leq 1/\bar{x}$.

Suppose that (x_D^*, x_R^*) is an equilibrium such that $x_D^* = 1$ and $x_R^* < 1$. Then, from system 1, we get

$$GF'(1/x_R^*)(1/x_R^*) \geq d^e \bar{c} \text{ and } GF'(1/x_R^*)(1/x_R^*) = r^e m(x_R^*) x_R^*.$$

This implies

$$m(x_R^*) x_R^* \geq (d^e/r^e)\bar{c}.$$

Thus, $x_R^* \geq \bar{x}$, or equivalently, $1/x_R^* \leq 1/\bar{x}$. But then the equilibrium condition $GF'(1/x_R^*)(1/x_R^*) \geq d^e \bar{c}$ violates assumption 3.

The remaining case is analogous. □

Proposition 2. *The minority party targets in equilibrium a larger share of its favorable voters but obtains a smaller expected fraction of votes.*

Proof. Using the system 1, we have

$$d^e x_D^* m(x_D^*) = r^e x_R^* m(x_R^*),$$

so that

$$d^e \gtreqless r^e \iff x_R^* \gtreqless x_D^*.$$

But then, since $(d^e x_D^*)/(r^e x_R^*) = m(x_R^*)/m(x_D^*)$ and m is a strictly increasing function,

$$d^e \gtreqless r^e \iff \frac{d^e x_D^*}{r^e x_R^*} \gtreqless 1.$$

□

Proposition 3. *If the payoff G of winning the election increases, then (i) expected turnout increases, and (ii) the probability that the party favored by voters wins the election and the difference in expected votes decrease, increase or remain constant, respectively, if $H'(c)c/H(c)$ is decreasing, increasing or constant for all c.*

Proof. Let G^1 be the initial value of the payoff of winning the election, and let $G^2 > G^1$ be the final value. (Interior) equilibrium conditions 1 require

$$G^t F'(x_D^t/x_R^t)(x_R^t/x_D^t) = d^e (x_R^t)^2 m(x_D^t)/x_D^t \tag{2}$$

and

$$G^t F'(x_D^t/x_R^t)(x_D^t/x_R^t)^3 = r^e (x_D^t)^2 m(x_R^t)/x_R^t \tag{3}$$

for $t = 1, 2$. From these two equations we get

$$r^e x_R^t m(x_R^t) = d^e x_D^t m(x_D^t). \tag{4}$$

Thus, either (a) $x_D^2 < x_D^1$, $x_R^2 < x_R^1$, or (b) $x_D^2 = x_D^1$, $x_R^2 = x_R^1$, or (c) $x_D^2 > x_D^1$, $x_R^2 > x_R^1$. We claim that (a) is not possible. To see this, suppose first that (a) holds and $x_D^2/x_R^2 \leq x_D^1/x_R^1$. Recall that $F'(x)/x$ is strictly decreasing under assumption 2 and $m(x)/x$ is strictly increasing under assumption 1. Thus, the left-hand side of equation 2 increases in going from $t = 1$ to $t = 2$, but the right-hand side decreases. Suppose that (a) holds and $x_D^2/x_R^2 > x_D^1/x_R^1$. Recall that $F'(x)x^3$ is strictly increasing under assumption 2. Thus, the left-hand side of equation 3 increases in going from $t = 1$ to $t = 2$, but the right-hand side decreases. By simple inspection of equation 2, (b) is not possible. Thus, we get $x_D^2 > x_D^1$, $x_R^2 > x_R^1$. Using the formula for expected turnout in the text we get that expected turnout increases.

Next, note that $H'(c)c/H(c) = m(x)/(xm'(x))$ for $m(x) = c$. Thus

$$\partial(H'(c)c/H(c))/\partial c \gtreqless 0 \text{ for all } c \iff \partial(m'(x)x/m(x))/\partial x \lesseqgtr 0 \text{ for all } x.$$

In what remains of the proof, we assume that party D is favored by voters, so that, as argued in the text, $x_D < x_R$. Similar arguments hold if party R is favored by voters.

Suppose $H'(c)c/H(c)$ is increasing. From equation 4 we get that $x_D^2/x_R^2 \geq x_D^1/x_R^1$ implies $m(x_D^2)/m(x_R^2) \leq m(x_D^1)/m(x_R^1)$. Consider a marginal increase in G. Differentiating totally x_D/x_R, we get that x_D/x_R increases weakly if and only if $\partial x_D/\partial x_R \geq x_D/x_R$. Differentiating totally $m(x_D)/m(x_R)$, we get that $m(x_D)/m(x_R)$ decreases weakly if and only if

$$\frac{m'(x_D)}{m(x_D)} \frac{\partial x_D}{\partial x_R} \leq \frac{m'(x_R)}{m(x_R)}.$$

Thus, if x_D/x_R increases weakly and $m(x_D)/m(x_R)$ decreases weakly, we get

$$\frac{m'(x_D)x_D}{m(x_D)} \leq \frac{m'(x_R)x_R}{m(x_R)}.$$

But since $m'(x)x/m(x)$ is decreasing and $x_D < x_R$ we get a contradiction. Thus, an increase in G must lead to a reduction in x_D/x_R. An analogous arguments shows that if $H'(c)c/H(c)$ is decreasing, an increase in G must lead to an increase in x_D/x_R, and if $H'(c)c/H(c)$ is constant, the ratio x_D/x_R must not change with changes in G. □

Proposition 4. *If partisanship increases, then (i) the fraction of favorable voters targeted by party D and by party R (respectively x_D and x_R) decrease,(ii) the probability that the party favored by voters wins the election and the difference in expected votes increase, remain constant, or decrease, respectively, if $H'(c)c/H(c)$ is increasing, constant, or decreasing for all c, and (iii) turnout increases if $H'(c)c/H(c)$ is constant for all c.*

Proof. As in the previous proof, (interior) equilibrium conditions 1 require

$$GF'(x_D^t/x_R^t)(x_R^t/x_D^t) = d^{et}(x_R^t)^2 m(x_D^t)/x_D^t \tag{5}$$

and

$$GF'(x_D^t/x_R^t)(x_D^t/x_R^t)^3 = r^{et}(x_D^t)^2 m(x_R^t)/x_R^t \tag{6}$$

for $t = i, f$. From these two equations we get

$$(r^e)^t x_R^t m(x_R^t) = (d^e)^t x_D^t m(x_D^t). \tag{7}$$

Thus, since the ratio r^e/d^e remains constant, either (a) $x_D^f < x_D^i$, $x_R^f < x_R^i$, or (b) $x_D^f = x_D^i$, $x_R^f = x_R^i$, or (c) $x_D^f > x_D^i$, $x_R^f > x_R^i$. We claim that (c) is not possible. To see this, suppose first that (c) holds and $x_D^f/x_R^f \geq x_D^i/x_R^i$. Recall that $F'(x)/x$ is strictly decreasing under assumption 2 and $m(x)/x$ is strictly increasing under assumption 1. Thus, the left-hand side of equation 5 decreases in going from $t = i$ to $t = f$, but the right-hand side increases. Suppose that (c) holds and $x_D^f/x_R^f < x_D^i/x_R^i$. Recall that $F'(x)x^3$ is strictly increasing under assumption 2. Thus, the left-hand side of equation 6 decreases in going from $t = i$ to $t = f$, but the right-hand side increases. By simple inspection of equation 5, (b) is not possible. Thus, we get $x_D^f < x_D^i$, $x_R^f < x_R^i$. That is, the fraction of voters targeted by either party decreases. This proves part (i) of the Lemma.

Next, recall from the previous proof that

$$\partial(H'(c)c/H(c))/\partial c \gtreqless 0 \text{ for all } c \iff \partial(m'(x)x/m(x))/\partial x \lesseqgtr 0 \text{ for all } x.$$

In what remains of the proof, we assume that party D is favored by voters, so that, as argued in the text, $x_D < x_R$. Similar arguments hold if party R is favored by voters.

Suppose $H'(c)c/H(c)$ is increasing. From equation 7 we get that $x_D^f/x_R^f \leq x_D^i/x_R^i$ implies $m(x_D^f)/m(x_R^f) \geq m(x_D^i)/m(x_R^i)$. Consider a marginal increase in partisanship. Differentiating totally x_D/x_R, and using $\partial x_D < 0$ and $\partial x_R < 0$, we get that x_D/x_R decreases weakly if and only if $\partial x_D/\partial x_R \geq x_D/x_R$. Differentiating totally $m(x_D)/m(x_R)$, we get that $m(x_D)/m(x_R)$ increases weakly if and only if

$$\frac{m'(x_D)}{m(x_D)}\frac{\partial x_D}{\partial x_R} \leq \frac{m'(x_R)}{m(x_R)}.$$

Thus, if x_D/x_R decreases weakly and $m(x_D)/m(x_R)$ increases weakly, we get

$$\frac{m'(x_D)x_D}{m(x_D)} \leq \frac{m'(x_R)x_R}{m(x_R)}.$$

But since $m'(x)x/m(x)$ is decreasing and $x_D < x_R$ we get a contradiction. Thus, an increase in partisanship must lead to an increase in x_D/x_R and therefore in $(d^e x_D)/(r^e x_R)$. Analogous arguments hold if $H'(c)c/H(c)$ is decreasing or constant. This proves part (ii) of the Lemma.

With respect to part (iii), we can rewrite equations 5 and 6 as

$$GF'(x_D^t/x_R^t)(x_R^t/x_D^t) = (d^{et}x_D^t)m(x_D^t)(x_R^t/x_D^t)^2$$

and

$$GF'(x_D^t/x_R^t)(x_D^t/x_R^t)^3 = (r^{et}x_R^t)m(x_R^t)(x_D^t/x_R^t)^2$$

for $t = i, f$. From parts (i) and (ii) of the Lemma, if $H'(c)c/H(c)$ is constant and partisanship increases, x_D and x_R decrease but x_D/x_R remains constant. Thus, from the two equations above we get that $d^e x_D$ and $r^e x_R$ must increase. □

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