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Nicolas Melissas

Instituto Tecnológico Autónomo de México

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Av. Camino a Santa Teresa # 930
Col. Héroes de Padierna
México, D.F. 10700
M E X I C O

The Trader, the Market Maker, his Guru and her Information*

Nicolas Melissas
Centro de Investigación Económica
Instituto Tecnológico Autónomo de México

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Abstract

This paper argues that a guru possessing a multi-dimensional informational advantage may want to truthfully report her opinion to the media to learn more out of the actions of other, sometimes better-informed traders.

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1 Introduction.

One can find many examples where a famous trader appears on television explaining her opinion concerning the future evolution of the stockmarket (or of a particular share or group of shares), concerning future interest rate changes, exchange rate adjustments, and so on.... What induces that guru to truthfully report her opinion? If everyone would believe what she says, she has an obvious incentive to manipulate the market price to buy for instance good shares at a bargain price. Theoretically, the issue is challenging: a guru only gives information to other stock market participants if she gains by doing so. She can only gain if her information influences share prices. In equilibrium she can only influence share prices when she speaks more often the truth than when she lies. Stockmarkets are a zero-sum game. Hence the gain of the guru must be offset by the loss of other traders. But how can other traders lose when they receive useful information from the guru?

*Correspondence can be sent to Nicolas Melissas (nmelissas@itam.mx), CIE - ITAM, Avenida Santa Teresa 930, México D.F. 10700, México. I am grateful to M. Dewatripont, M. Fernandes, F. Palomino, G. Palumbo and L.Gyongyi for helpful discussions.

Undoubtedly, gurus can by their words influence share prices and few economists would argue that they don't use their power to manipulate stock prices in their favour[1]. Quite surprisingly so far few papers analysed the origins of cheap talk in financial markets.

In this paper I use a variant of Glosten and Milgrom's (1985) set-up to show how a guru can gain by giving useful information to the public (i.e. to the market maker). Consider first the following example which helps the reader to grasp the intuition behind the basic set-up of my model. Think of the guru as a person who knows much about the firm (whose shares are being traded) and about the industry under which it operates. In "normal" times the future profitability of the firm depends mainly on its sales expectations which are closely related to the expected industry-wide growth rate. Hence, in "normal" times the guru is the best informed trader concerning the value of the asset. However sometimes the firm's future profitability is not very correlated with its expected industry-wide growth rate but depends more on an idiosyncratic shock. For example the firm may have plans to merge with another firm, or to replace her current CEO by another person, or to launch a new product on the market....These idiosyncratic shocks influence much more its future profitability than the expected industry-wide growth rate. The guru knows whenever such an event occurs[2] but as she specialised herself in acquiring and interpreting data to better predict the future industry-wide growth rate she is less strong in predicting the repercussions of the marketing of a new product on the firm's profitability. On the other hand, the trader is a person who is stronger in forecasting the consequences of these idiosyncratic shocks on the firm's future profitability (the trader need not be an insider nor does it always have to be the same person). Of course, the guru wants to know the value of a share whenever the firm is hit by an idiosyncratic shock. I show that by strategically revealing her private information she can induce the trader to act sincerely, i.e. to buy if the shares are good and to sell if the shares are worthless.

This example is modelled in the following way: a trader and a guru possess private information concerning the value of an asset. Their information is complementary: in the absence of an event (and with an "event" I mean that the firm is hit by an idiosyncratic shock), the guru knows the value of the asset and the trader knows nothing while in case an event happens the value of the asset is known to the trader and not to the guru. The market maker (i.e. the public) doesn't know the value of the

[1]For some colourful examples of stock-price manipulation via words see Benabou and Laroque (1992) and Sobel (1965).

[2]It is not crucial to assume that both the guru and the trader know about the occurrence of an event and not the market maker. What matters is that both the guru and the trader know about the occurrence of the event before the market maker does.

asset nor does he know whether an event occurred or not. I consider a three-period game: at time one the value of the asset becomes known to the trader or the guru and the trader can buy or sell one share. At time two the market maker, after observing the period-one buy-sell decision of the trader and after possibly having received information concerning the value of the asset from the guru, announces the second-period market price. The trader and the guru then both put their buy/sell orders and at time three the value of the asset becomes known to the market maker.

I first compute the equilibrium behaviour of the trader in the absence of a guru. If the guru doesn't have the possibility to express her opinion in the media there exists a unique equilibrium in which the trader, if an event happens, reveals his private information (and with this I mean that he buys (sells) a share in the first period if he possesses positive (negative) information) with a probability only equal to one half. This is because the trader wants to manipulate the market maker. If the market maker would believe that the trader acts in period one according to his private information with a probability greater than one half, then the trader can f.i. gain by selling one share at time one, to buy a lot of good shares at time two at a lower price (if the trader had some negative information concerning the value of the shares the situation would be completely opposite, he would then have an incentive to buy one share at time one to sell his worthless shares at time two at an inflated price).

If the guru is allowed to express her opinion in the press she can elicit the traders' information by using the following strategy: whenever no event occurs (i.e. whenever the guru knows the value of the asset) and whenever the trader takes the "wrong" action (for instance buys a worthless share), then the guru (with a certain probability) honestly reports her information to the public. By doing so, she becomes credible whenever she tells the market maker not to "listen" to the trader but to follow her advice instead. In case an event happens, she uses her credibility by reporting the opposite of what the trader did. By doing so she removes the possibility from the trader to manipulate the market price in his favour. To understand this remember that a trader whenever he knows the value of the asset has two possibilities: (i) he acts sincerely at time one (i.e. he buys if he possesses good news and sells if he possesses bad news) or (ii) he tries to manipulate the second-period asset price in his favour. If he follows strategy (ii) then his payoff depends on the second-period price of the asset which depends on the information that was eventually given by the guru to the market maker. A trader only follows strategy (ii) if, for example, a period-one buy order increases the second-period price of the asset (because then he can sell his worthless shares at a higher second-period price). If the guru always says that the asset is worthless whenever the trader bought at time one (and if the

guru's information is credibly believed by the market maker) then the trader cannot hope to inflate the second-period price by buying at time one and strategy (i) remains the only available option for him. The market maker knows that the trader acts on the basis of his information, but he doesn't know when the trader possesses superior information. However, as the guru knows when an event happens, she is able to infer more information out of the trader's action than the market maker and that's why the guru can increase her informational advantage by strategically giving information to the market maker.

This paper mainly belongs to a branch of the market microstructure literature analysing the issue of strategic stock-price manipulation[3] by traders. The literature distinguishes among three different types of manipulation.

The bulk of all papers in this literature study how and when trade-based manipulation becomes possible. In Allen and Gale (1992) an uninformed manipulator mimics the behaviour of an informed trader in the hope that some positive information will reach the market in the future. In that case, uninformed investors believe that probably the manipulator is an informed trader, which induces them to buy and which allows the manipulator to sell his shares at a higher price. Allen and Gorton (1992) show how stock-price manipulation becomes possible when liquidity traders are more likely to sell than to buy. Brunnermeier (1997) considers a set-up quite similar to mine, where two traders possess complementary information concerning the value of an asset. They can both trade in two periods and prior to the second trading period one trader's information becomes known to the public. He shows that the trader whose information becomes common knowledge at date two has an incentive to trade aggressively in the first period to prevent the market maker to extract the other trader's information via technical analysis. To some extent, this paper is related to the former models in that the analysis focuses on how cheap talk can deter a trader to undertake price-manipulative actions. Nonetheless, the main idea of this paper is that a guru may want to truthfully report her information to induce other traders to act more informatively. In this paper "more informative acting" means "less manipulative acting", but I decided to focus on price-manipulative behaviour for analytical convenience. I believe my main idea to remain valid, even if the trader were not trying to manipulate the market maker, but would instead "hide" himself behind noise traders (as typically happens in strategic market order models (see Kyle (1985))).

[3]With price manipulation, I mean that some traders undertake actions which make stockprices move in their favour.

Another line of research analyses how stock prices can be manipulated via the strategic release of information. Benabou and Laroque (1992) assume that a guru can either be honest or dishonest. The honest guru always truthfully reports her opinion. They show how the presence of honest gurus induce in a repeated game setting the dishonest ones to not always misrepresent their signal, in order to exploit their good reputation in future trading rounds (where reputation is defined as the probability put by market participants that she is honest). No doubt, their paper sheds an important (and not counterintuitive) light on the issue of cheap talk in financial markets, however it need be mentioned that their argument crucially rests on the assumption that a (possibly substantial) fraction of gurus are inherently honest. This is a strong assumption which weakens their argument because they basically argue that cheap talk in financial markets exists because a (possibly substantial) fraction of gurus are honest. In this paper I offer an alternative explanation for the existence of cheap talk in financial markets which complements (instead of rivals) Benabou and Laroque's analysis.

Finally, stock prices can also be manipulated via actions. For example managers can sell short shares from their own company and then undertake an action which lowers its value (for a model of action-based manipulation see Vila (1998)).

This paper shows how the release of information at time two can induce a trader to act more informatively at time one. This result bears some close resemblance to the one derived earlier by Hirschleifer, Subrahmanyam and Titman (1994). In that paper, some investors receive payoff-relevant information before others. The authors show a.o. that early informed investors then trade more aggressively in the first period than if all traders had received their information simultaneously[4]. In their model information reaches the market exogenously, while in my model the amount of information which reaches the market at time two is endogenous.

This paper is organised as follows. In section 2, I first explain the basic assumptions underlying my model. Next (section 2.2) I compute the equilibrium strategy of the trader when the guru has no possibility to communicate her information to the market maker. In that case the trader engages in manipulative trading and the guru cannot infer his information out of his action. In section 2.3 the guru has the possibility to communicate her information to the market maker. I show that by appropriately choosing her lie-truth mix she can induce the trader to act sincerely and increase her

[4]In their model the increased aggressivity does not lead to more informative prices, but this is because they specified a competitive framework. As we learned from kyle (1985), in strategic contexts the more aggressive the trading behaviour, the more informative the price.

payoff. Final comments are summarised in the third and final section.

2 The Model

2.1 The general framework

Assume you have two traders, trader one (she) and trader two (he), who are both active in a market where a particular asset is traded. The price of the asset is set by a competitive market maker. The value of the asset, V, $\in \{0, 1\}$. e (ne) denotes the fact that an event occurred (did not occur). $Prob(V = 1|e) = Prob(V = 1|ne) = \frac{1}{2}$. The prior probability that an event happens is denoted by $\mu \in (0, \frac{1}{2})$.

Both traders possess some information concerning the value of the asset. Trader one receives a private certifiable signal, s_1, concerning the realisation of V. The precision of V depends on whether or not an event had taken place. Formally:

$$Prob(s_1 = 0|V = 0, ne) = Prob(s_1 = 1|V = 1, ne) = 1$$

$$Prob(s_1 = 0|V = 0, e) = Prob(s_1 = 1|V = 1, e) = \frac{1}{2}$$

Thus in the absence of an event, she knows the true value of the asset for sure, while in case an event happened she has no idea concerning the value of the asset. I also assume that her private information is certifiable, i.e. she can "prove" to an outsider that $s_1 = 0$ or that $s_1 = 1$. This assumption is made for the sake of simplicity, the main idea present in this paper would remain valid if information were uncertifiable but it would complicate the market maker's updating process. Trader two also receives some private information, s_2, concerning the value of the asset. s_2 is generated by the following process:

$$Prob(s_2 = 0|V = 0, ne) = Prob(s_2 = 1|V = 1, ne) = p > \frac{1}{2}$$

$$Prob(s_2 = 0|V = 0, e) = Prob(s_2 = 1|V = 1, e) = 1$$

For simplicity, I assume that $p \to \frac{1}{2}$. So in the absence of an event trader two possesses an almost completely uninformative signal. The market maker possesses no information concerning V, he doesn't know whether an event occurred or not, nor does he know the value of the asset. He must rely on his priors to compute the probability that $V = 1$. There are no trading costs. I assume that $\mu < \frac{1}{2}$. This entails that trader one "on average" is better informed about V than trader two. Therefore from now on trader one and trader two are referred to as the "guru" and the "trader".

Note that the guru and the trader have "perfect complementary" information: in the absence of an event, the guru knows the value of the asset and the trader knows almost nothing, while in case an event occurred one obtains the contrary situation. This complementarity is crucial: this model shows how the guru can "extract" the trader's private information by (sometimes) giving her information to the market maker. If the guru were always the best informed party, she would never reveal her private information. Note also that if an event occurred the guru possesses a private, certifiable piece of information which is completely irrelevant for estimating the value of a share, but which she can nonetheless decide to give to the public (i.e. to the market maker) (without telling the public that her information is irrelevant). Henceforth, I say that the guru "lies" whenever she certifies her private information when an event occurred. In the other case the guru "truthfully" certifies her signal.

The trader possesses an initial endowment of half a unit of money and one share. The guru possesses no money and no shares at the start of the game. The trader seeks to maximise his final wealth W by trading shares in two periods. In the first period, the trader is not allowed to buy or sell more than one share, while in the second period he can buy or sell k ($k \in I\!R^+$) shares. These restrictions on the traders' buy-sell strategy are imposed because otherwise the informativeness of the trader's strategy would depend on his initial wealth. For simplicity, I only allow the guru to buy or sell one share at time two and to speculate on second-, and third-period price differences. For example if the guru knows that $V = 1$, then she can only make money by buying one share at time two and selling it back at time three. Call G the gain of the guru when she puts her buy-sell (or sell-buy) decision.

The trader features state-dependent preferences: if an event occurs and if $V = 1$, then he's infinitely risk averse if his final wealth W is (strictly) lower than two. If an event happens and if $V = 0$, he's infinitely risk-averse if his final wealth is (strictly) lower than one. If his final wealth is greater or equal to two ($V = 1$) or one ($V = 0$), then he's risk-neutral[5]. If no event occurred, then the trader is risk neutral. The guru features state-independent preferences. She is infinitely risk averse for any gain G (strictly) lower than zero. If her gain is nonnegative, then she's risk neutral. These restrictions on the utility functions simplify the computation of the equilibrium strategies of the trader and the guru. In the next sections, I will explain how the analysis would be affected if we were using more realistic utility functions. Finally, the market maker is risk neutral and "simply" computes the expected value of the asset on the basis of all the available information. The timing of the game is explained below:

[5]More formally, if an event happens and if $V = 1$, $U(W < 2) = -\infty$ and $U(W \geq 2) = W$.

0) Nature decides upon the value of V and the two traders receive their information.
1) At time one:
a) The market maker sets his price on the basis of his priors.
b) The trader may submit a buy- or sell order.
2) At time two:
a) The market maker and the guru observe the first-period action of the trader. The guru has the opportunity to certify her information to the market maker. The market maker sets his second-period price.
b) The guru and the trader may submit their buy-sell orders. The trader is allowed to trade k shares, while the guru may only buy (sell) one share in this period.
3) In the third period the market maker learns the state of the world. If the guru bought (sold) at time two, then she sells (buys) her share back to (from) the market maker.

Note that the market maker is losing money because he trades against people who are better informed than him. There are two ways to remediate this problem. The first one consists of entering liquidity traders in the picture. As shown by Smith and Sørensen (1996), this class of models is robust towards the introduction of noise. Hence it must be possible for the market maker to recoup his loss at the expense of the liquidity traders without altering the insights of my model. Second, one may allow the market maker to set different bid-ask prices[6]. However, this should not affect the analysis (except of making everything harder to compute) and it shouldn't provide us with additional insights.

P_i $(i = 1, 2, 3)$ denotes the price of the asset in period i. Given our assumptions above it's immediate that $P_1 = \frac{1}{2}$. P_2 is more complicated to compute and depends on the action of the trader in the first period, his strategy, the information given by the guru and on her strategy. In the third period, the value of the asset becomes common knowledge, so $P_3 \in \{0, 1\}$.

Finally, I also assume that the guru can commit herself to time-inconsistent strategies. In this model I show that the guru can extract the perfect signal from the trader by threatening him to reveal her signal to the market maker in the second period. This strategy is not subgame perfect because in the absence of an event she has an incentive not to reveal any information to the market maker. Of course, this time-inconsistency stems from the fact that I consider a one shot-game. If I would have considered a repeated game, then trader two would care about her reputation and then she would

[6]This is how economists usually compute these models. One notable exception (of a Glosten-Milgrom type of model without a bid-ask spread) is the paper by Lee(1999).

have an incentive to commit herself to certifying her signal in case no event occurred. Instead of analysing a more complicated repeated game, I decided to work under this simplifying assumption.

2.2 A benchmark case: no communication.

In this section I assume that the guru doesn't have the possibility to communicate her signal to the market maker in the second period. I show that she can then impossibly infer the trader's signal out of his action, because he wants to manipulate the market maker. Call $\lambda \in [\frac{1}{2}, 1]$ the probability with which the trader trades on the basis of his information when an event occurred. λ^* denotes the equilibrium strategy of the trader. If $\lambda = 1$ this means that the trader acts "sincerely", i.e. buys if $s_2 = 1$, and sells if the share is worthless. $a_1 \in \{b, s\}$ denotes the action of the trader. $a_1 = b$ (s) means that the trader buys (sells) one share at time one. A perfect Bayesian equilibrium (PBE) is a (λ^*, P_2) such that:
(i) the trader cannot gain by deviating, and
(ii) P_2 is computed according to Bayes' law.
In this case I obtain the following result:

PROPOSITION 1 *In the absence of communication, the unique PBE is the one where* $\lambda^* = \frac{1}{2}$.

Proof: suppose first that the market maker computes P_2 under the assumption that $\lambda^* = 1$. Suppose an event happened and suppose (without loss of generality) that $s_2 = 1$. $\lambda^* = 1$ cannot be an equilibrium because the trader has then an obvious incentive to sell a share today to buy tomorrow a lot of shares at a bargain price. Formally, $W(a_1 = b) = 2 < W(a_1 = s) = \frac{1}{P_2(a_1=s)}$ (where $P_2(a_1 = s) = (1-\mu)\frac{1}{2}$). Suppose next that the market maker revises his prior under the assumption that $\lambda^* \in (\frac{1}{2}, 1)$. suppose an event happens and suppose (without loss of generality) that $s_2 = 1$. The trader is only willing to randomise with a probability λ^* if he's indifferent between the two pure strategies. But this is impossible because if he buys he gets $W(a_1 = b) = 2$, if he sells he gets: $W(a_1 = s) = \frac{1}{p_2(a_1=s)}$ (he only sells in period one to buy k shares in period two at a (hopefully) lower price). If the trader sells his (only) share at time one at a price of $\frac{1}{2}$, then he possesses one unit of money at time two[7] with which he can buy $1/p_2(a_1 = s)$ shares (which are all worth one unit of money). Now $p_2(a_1 = s) = Prob(V = 1|a_1 = s) = \mu(1-\lambda^*) + (1-\mu)\frac{1}{2}$. If $\lambda^* \neq \frac{1}{2}$, this means that $P_2(a_1 = s) < \frac{1}{2}$ and thus that $W(a_1 = s) > 2$. But then the trader cannot be indifferent between the two pure strategies, a contradiction. Q.E.D.

[7] He already possessed an initial endowment of half a unit of money on top of which P_1 must be added.

As $\lambda^* = \frac{1}{2}$ the guru can impossibly infer the traders' information out of his action. This is because she cannot know whether the trader sold because the shares are worthless or because he wants to buy cheap shares in period two. Note also that as the market maker fully realises that the trader wants to "mislead" him, P_2 always equals $\frac{1}{2}$ whatever action was taken by the trader. Without communication whenever an event happens, the guru doesn't trade and gets a payoff of zero.

2.3 Communication in a financial market mechanism.

In this section the guru has the possibility to certify her private information to the market maker at the beginning of time two. I show that - if the trader is infinitely risk-averse below a certain point - by making her communication strategy contingent on s_1 and a_1 - she can extract the private information held by the trader at almost no cost.

It turns out that the optimal certification strategy of the guru is directly influenced by how s_1 and a_1 *compare to each other*. Therefore, I define the trader's type t as a function $t : \{1, 0\} \times \{b, s\} \rightarrow \{o, s\}$ with the interpretation that $t = o$ if (i) $s_1 = 1$ and $a_1 = s$ or (ii) $s_1 = 0$ and $a_1 = b$ and $t = s$ if (i) $s_1 = 1$ and $a_1 = b$ or (ii) $s_1 = 0$ and $a_1 = s$. In words $t = o$ (s) means that the trader possesses the opposite (same) information as the one possessed by the guru. The market maker realises that the guru strategically certifies her signal. Therefore his posterior probability that an event occurred depends on whether the guru certifies her information or not. Call β (β') the market makers' posterior probability that an event occurred given that the guru certifies (doesn't certify) her signal. The strategy of the guru, σ, is represented by a vector (1x4) containing the function $x : \{e, ne\} \times \{o, s\} \rightarrow [0, 1]$ with the interpretation that $x(e, o)$ ($x(ne, o)$) is the probability with which the guru certifies her information given that an event (no event) occurred and that the trader's type $t = o$. $x^*(\cdot, \cdot)$ denotes the equilibrium probability of $x(\cdot, \cdot)$. $P_2(a_1 = \cdot, s_1 = \cdot)$ denotes the equilibrium second-period price when the trader undertook action $\cdot$ and when the guru certified her private her private information. Similarly, $P_2(a_1 = \cdot)$ denotes the second-period equilibrium price when the guru did not certify her information. A perfect Bayesian equilibrium is a $(P_2(\cdot), \sigma, \lambda, \beta, \beta')$ such that:
(i) no agent can gain by deviating, and
(ii) whenever possible, $P_2(\cdot)$, β and β' are computed according to Bayes' law, given σ and λ.

The guru faces a four-dimensional maximisation problem because she chooses $x(e, o)$, $x(ne, o)$, $x(e, s)$ and $x(ne, s)$ such as to maximise her utility. Her maximisation problem is non-trivial in the sense that different configurations of σ may induce different

equilibrium behaviours by the trader, which in their turn influence the second-period equilibrium price. Given the many simplifying assumptions I set out above, I am able to compute her optimal strategy using standard mathematical instruments. I first state two lemma's which reduce her maximisation problem to a two-dimensional one. Next I use linear programming techniques to compute her optimal certification strategy.

LEMMA 1 *The guru only engages in speculative trading after an event occurred iff* $\lambda^* = 1$.

Proof (and intuition): Suppose an event occurs. If the guru does not trade at time two, she gets a zero gain and a zero utility. Since she gets a utility of $-\infty$ whenever her gain is negative, she is only willing to engage in speculative trading whenever she makes a riskless buy-sell (or sell-buy) decision. Her trade decision is riskless whenever she knows the prevailing state of the world. This is the case whenever $\lambda^* = 1$. Q.E.D.

Before turning to my second lemma, I first explain how the restrictions on the traders' utility function affect his behaviour. For this purpose, assume that an event occurs and that $s_2 = 1$[8]. We know that the trader is infinitely risk-averse if his final wealth is strictly lower than two. If he buys one share at time one, then his final wealth equals two. Consider what happens if he were to sell one share. We know that $\forall \sigma \in [0,1]^4$, $P_2(a_1 = s, s_1 = 0) \leq P_2(a_1 = s, s_1 = 1)$. The trader only sells in the hope to buy cheap shares in the second period. However as $U(W < 2) = -\infty$, he acts sincerely as soon as $P_2(a_1 = s, s_1 = 1) > \frac{1}{2}$. For if $P_2(a_1 = s, s_1 = 1) > \frac{1}{2}$ then by strategic selling there is a (maybe extremely small but nonetheless strictly positive) probability that in the next period the guru will give some good news to the market maker which will push the price above $\frac{1}{2}$. Then the trader can buy less than two shares in the second period and gets a final wealth strictly lower than two! I can now state and prove my second lemma.

LEMMA 2 *If the guru trades whenever an event happened, then* $x^*(\cdot, s) = 0$

Proof: Suppose the guru trades after the occurrence of an event. From lemma one we know that the trader must act sincerely. Suppose an event occurred and without loss of generality suppose that $s_2 = 1$. Suppose the vector of strategies $\tilde{\sigma} = [\tilde{x}(ne, o)\ \tilde{x}(e, o)\ \tilde{x}(ne, s)\ \tilde{x}(e, s)]$ achieves sincere acting. From the discussion above we know that if the guru adopts strategy $\tilde{\sigma}$, $P_2(a_1 = s, s_1 = 1) \geq \frac{1}{2}$. Now:

$$P_2(a_1 = s, s_1 = 1) = 1 - \beta = \frac{(1-\mu)x(ne, o)}{\mu x(e, o) + (1-\mu)x(ne, o)} \tag{1}$$

[8]The reader can easily check the other case where $s_2 = 0$.

So $P_2(a_1 = s, s_1 = 1)$ is only determined by the probabilities $x(ne, o)$ and $x(e, o)$. Hence if $\tilde{\sigma}$ achieves sincere acting, so does the strategy $\tilde{\sigma}' = [\tilde{x}(ne, o)\ \tilde{x}(e, o)\ 0\ 0]$. In other words setting $x(e, s)$ and/or $x(ne, s)$ different from zero does not induce the trader to act sincerely. But this implies that $x^*(e, s)$ and $x^*(ne, s)$ must be equal to zero. For, assume (without loss of generality) that $a_1 = s$ and that $s_1 = 0$. Given sincere acting this means that, whether an event occurred or not, the value of the asset equals zero. If the guru were to certify her bad information (either because $x^*(ne, s)$ and/or $x^*(e, s)$ differ from zero) then she would get zero while otherwise she would have got $P_2(a_1 = s) > 0$. Q.E.D.

The intuition behind lemma two is the following one. Assume (without loss of generality) that the trader sold a share at time one and that the guru gave a bad piece of information to the market maker. If no event occurred then the market maker knows that $V = 0$ because by assumption the guru only possesses certifiable information concerning s_1. If an event occurred then, by lemma one, the market maker also knows that $V = 0$ because $\lambda^* = 1$. Hence if the guru trades after the occurrence of an event and if she certifies the "same" information as the one given by the trader (and thereby I mean that she certifies $s_1 = 1$ when $a_1 = b$ or $s_1 = 0$ when $a_1 = s$), then the value of V is perfectly revealed to the market maker at the beginning of the second period. But then the guru loses money (without gaining in credibility) because she could have used her informational advantage to trade a share in the second period.

On the basis of (1) it's easy to see that in equilibrium the following inequality must hold, otherwise the trader doesn't act sincerely.

Corollary: $x^*(ne, o) > \frac{\mu}{1-\mu} x^*(e, o)$

To grasp the intuition behind this inequality assume that no event occurred, that $s_1 = 1$ and that the trader sells a share at time one. If the guru truthfully certifies her information, then as her informational advantage with the market maker decreases, she loses money. However by truthfully certifying her information whenever the trader took a wrong action, her "reports" become credible. In equilibrium she builds strong enough a reputation such that whenever the market maker gets two opposed signals, he tends to believe the guru more than the trader (and this happens whenever $x^*(ne, o) > \frac{\mu}{1-\mu} x^*(e, o)$). In that case $P_2(a_1 = s, s_1 = 1) > \frac{1}{2}$ which implies that if the trader strategically sells at time one (i.e. sells at time one in the hope to buy a lot of good shares at time two) he gets a final wealth lower than two.

We now know enough to state and prove our most interesting finding (ϵ_y represents

an arbitrarily small strictly positive number and $\epsilon_x > 1$, $\epsilon_x \to 1$):

PROPOSITION 2 *The unique PBE is the one where* $x^*(ne, o) = \frac{\mu}{1-\mu} x^*(e, o)\epsilon_x$ *and where* $x^*(e, o) = \epsilon_y$. *The guru is then able to extract the private information held by the trader.*

Proof: First, to economise on notations (and with a slight abuse of notation) I redefine $x(ne, o)$ as x and $x(e, o)$ as y. Next, I compute the guru's gain of trading as a function of x and y. Her gain depends on whether she certifies or not. $G(C)$ ($G(NC)$) denotes her gain, given that she certifies (does not certify) her information:

$$G(C) = \beta P_2(a_1 = s, s_1 = 1) + (1-\beta)P_2(a_1 = s, s_1 = 1) = \frac{2\mu(1-\mu)xy}{(\mu y + (1-\mu)x)^2}$$

$$G(NC) = \beta' P_2(a_1 = s) + (1-\beta')[\tfrac{1-x}{2-x}(1 - P_2(a_1 = s)) + (1 - \tfrac{1-x}{2-x})P_2(a_1 = s)]$$
$$= \frac{2(1-\mu)(1-x)(1+\mu(1-y))}{(\mu(2-y)+(1-\mu)(2-x))^2}$$

Note that in the expressions of $G(C)$ and $G(NC)$ above $P_2(a_1 = s)$ and $P_2(a_1 = s, s_1 = 1)$ were computed under the assumption of sincere acting (this is normal given the insights we obtained out of lemma (1)). $G(C)$ was computed under the assumption that $a_1 = s$ and that $s_1 = 1$. As by construction the problem is symmetric, this is without loss of generality. $\frac{1-x}{2-x}$ represents the probability that both traders possess opposite pieces of information given that she did not certify her information and given that no event occurred. The ex ante probability that the guru certifies her information equals $\frac{1}{2}(\mu y + (1-\mu)x)$. The probability that she doesn't certify her information equals $\frac{1}{2}(\mu(2-y) + (1-\mu)(2-x))$. The guru chooses x and y such as to maximise $Prob(C).G(C) + Prob(NC)G(NC)$ subject to some inequality constraints. More formally her problem can be written as:

$$\underset{x,y}{Max} \qquad \frac{\mu(1-\mu)xy}{\mu y + (1-\mu)x} + \frac{(1-\mu)(1-x)(1+\mu(1-y))}{\mu(2-y) + (1-\mu)(2-x)}$$

$$s.t. \;\; (i) \;\; y > 0 \quad (ii) \;\; x > \frac{\mu}{1-\mu}y \quad (iii) \;\; 1 - x \geq 0 \quad (iv) \;\; 1 - y \geq 0$$

In the appendix, I show that the unique solution of her maximisation problem is obtained when $x = \frac{\mu}{1-\mu} y\epsilon_x$ and $y = \epsilon_y$. Q.E.D.

To understand the intuition behind this result, first note that given sincere acting every time the guru truthfully certifies her private information, she loses money because her informational advantage disappears. Therefore she will not truthfully certify her information more than what is necessary. Therefore $x^* = \frac{\mu}{1-\mu} y^*\epsilon_x$. $y^* = \epsilon_y$ because I assumed that the trader (if an event occurs) is infinitely risk-averse below two (if

$V = 1$) and one (if $V = 0$). Therefore his behaviour is very much driven by his fear that $P_2(a_1 = s, s_1 = 1) > \frac{1}{2}$ (in which case he gets $-\infty$). In case of an event, the trader knows that the guru lies with an ex ante probability of $\frac{1}{2}\epsilon_y$. But then there exists a (albeit small but nonetheless) strictly positive probability that he will get $-\infty$ if he doesn't act sincerely.

3 Discussion and conclusions.

This paper explains why, in the presence of multi-dimensional informational advantages, a guru sometimes wants to truthfully report her opinion to the media. We depicted a situation in which one would expect to learn very little out of the actions of a (maybe informed) trader. We showed how a guru by sometimes reporting the opposite of what the trader did in the first period (f.i. reporting good news when the trader sold at time one) can induce an informed trader to act sincerely at time one. To conclude, we want to mention some limitations of our analysis and some thoughts about where we could generalise our results.

Proposition (2) implies that a guru almost never appears in the media to certify her information. If the trader were risk-neutral, then $x^* = \frac{\mu}{1-\mu} y^* \epsilon_x$ and $y^* = \epsilon_y$ cannot be an equilibrium, because then the trader doesn't want to act sincerely anymore. This is easy to understand: the market maker knows that the trader acts sincerely at time one. As the guru certifies her private information with a probability equal to $\frac{1}{2}\epsilon_y$, the trader knows that almost surely if he buys this will increase the second-period price. However as we assumed that he was infinitely risk-averse he did not want to take any risk and preferred to act sincerely. However if the trader were risk-neutral then, given that he can influence the second-period price and given that the guru almost never certifies her private information, it will be optimal for him to f.i. buy at time one to sell his worthless shares at a higher second-period price. In that case the guru will have to appear more often in the media to induce the trader to behave properly[9]. I believe that analytical results with a risk-neutral trader can be obtained.

The analysis in this paper heavily (though not crucially) rests on my assumption that the trader can only trade one share at time one and $k(\in \mathbb{R}^+)$ shares at time two. Due to that assumption it's hard to learn the trader's information out of his period-one buy/sell order because he has the incentive to f.i. buy a lot of good shares at a lower

[9] I performed a numerical simulation which confirms that intuition. Assume that the guru is still infinitely risk-averse below zero, that $\mu = 0.1$ and that the trader is risk-neutral. Then the gurus' optimal strategy is the one where $x(ne, o) = 0.13$ and $x(e, o) = 0.64$. This strategy increases her payoff by 8% (!) relative to her alternative strategy of never certifying information.

price (even though in equilibrium this never happens because the market maker is not stupid). My context could be reproduced in a kyle-type model by assuming that the liquidity of the market varies over time and that sometimes the market is not liquid at all (this case corresponds to our first period) while sometimes the market is infinitely liquid (which corresponds to our second period). Alternatively, my context can also be reproduced by focusing on another context where the trader can buy shares in a completely illiquid spot market and then buy options on these shares in an infinitely liquid options market. In future research it might be interesting to endogenise this $1 - k$ assumption. I also believe that some analytical results can be obtained along this line.

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Appendix

The lagrangian of her maximisation problem equals:

$$L = \frac{\mu xy}{(1-\mu)x+\mu y} + \frac{(1-x)(1+\mu-\mu y)}{(1-\mu)(2-x)+\mu(2-y)} + \lambda_1(x-\epsilon_x) + \lambda_2(y-\epsilon_y) + \lambda_3(x - \frac{\mu}{1-\mu}y)$$
$$+\lambda_4(1-x) + \lambda_5(1-y)$$

The first-order conditions are:

$$\frac{\delta L}{\delta x} \equiv \frac{\mu^2 y^2}{((1-\mu)x+\mu y)^2} - \frac{(1+\mu-\mu y)^2}{((1-\mu)(2-x)+\mu(2-y))^2} + \lambda_1 + \lambda_3 - \lambda_4 = 0$$

$$\frac{\delta L}{\delta y} \equiv \frac{\mu(1-\mu)x^2}{((1-\mu)x+\mu y)^2} - \frac{\mu(1-\mu)(1-x)^2}{((1-\mu)(2-x)+\mu(2-y))^2} + \lambda_2 - \frac{\mu}{1-\mu}\lambda_3 - \lambda_5 = 0$$

$$\lambda_1(x-\epsilon_x) = \lambda_2(y-\epsilon_y) = \lambda_3(x - \frac{\mu}{1-\mu}y) = \lambda_4(1-x) = \lambda_5(1-y) = 0$$

Assume the first three constraints are binding (implying that $\lambda_4 = \lambda_5 = 0$). The first-order conditions then become:

$$\frac{\delta L}{\delta x} \equiv 4\lambda_1 + 4\lambda_3 = \mu(2+\mu) \tag{2}$$

$$\frac{\delta L}{\delta y} \equiv \frac{4}{\mu}\lambda_2 - \frac{4}{1-\mu}\lambda_3 = -\mu \tag{3}$$

It is easy to see that there exists $\lambda_1,\ \lambda_2,\ \lambda_3 \geq 0$ s.t. (2) and (3) are respected. For example, assume that $\lambda_2 = 0$. Then $\lambda_3 = \frac{\mu(1-\mu)}{4} > 0$. Insert this last expression for λ_3 in (2) and one sees that $\lambda_1 = \frac{\mu(1+2\mu}{4} > 0$. Hence, the Kuhn-Tucker necessary conditions[10] are satisfied in the point $x = \epsilon_x$ and $y = \epsilon_y$. The reader can easily verify that no other point in the feasible region satisfies the KT-conditions so I'm left with only one candidate equilibrium.

I still must check whether L is concave in the feasible region in the point (ϵ_x, ϵ_y). Therefore I take the second-order derivatives:

$$\frac{\delta^2 L}{\delta x^2} = \frac{-2\mu^2(1-\mu)y^2}{((1-\mu)x+\mu y)^3} - \frac{2(1-\mu)(1+\mu-\mu y)^2}{((1-\mu)(2-x)+\mu(2-y))^3} < 0$$

$$\frac{\delta^2 L}{\delta y^2} = -\frac{2\mu^2(1-\mu)x^2}{((1-\mu)x+\mu y)^3} - \frac{2\mu^2(1-\mu)(1-x)^2}{((1-\mu)(2-x)+\mu(2-y))^3} < 0$$

$$\frac{\delta^2 L}{\delta x \delta y} = \frac{2\mu^2(1-\mu)xy}{((1-\mu)x+\mu y)^3} + \frac{2\mu(1-\mu)(1-x)(1+\mu-\mu y)}{((1-\mu)(2-x)+\mu(2-y))^3} > 0$$

[10] The constraints are linear and the feasible region is a convex set, hence the constraint qualification is met and the Kuhn-Tuker conditions are indeed necessary.

If (in the feasible region) $d^2L < 0$ (evaluated at the point (ϵ_x, ϵ_y)), then we know (sufficient condition) that our candidate equilibrium represents a maximum in the feasible region. From basic mathematical textbooks we know that $d^2L < 0$ can be rewritten as:

$$2\frac{\delta^2 L}{\delta x \delta y} < -\frac{\delta^2 L}{\delta x^2}\frac{dx}{dy} - \frac{\delta^2 L}{\delta y^2}\frac{dy}{dx} \tag{4}$$

Call $-\frac{\delta^2 L}{\delta x^2} = a > 0$, $-\frac{\delta^2 L}{\delta y^2} = b > 0$ and $\frac{\delta^2 L}{\delta x \delta y} = c > 0$. Call R the RHS of this last inequality.

LEMMA 3 *In the feasible region $a\frac{dx}{dy} + b\frac{dy}{dx}$ is minimal when $dx = \frac{\mu}{1-\mu}dy$.*

Proof: Take the two following couples in a two-dimensional space $(\tilde{dx}, \tilde{dy})$ and $(\frac{\mu}{1-\mu}dy, dy)$. Assume both couples lie in the feasible region. By definition of R we know that:

$R(\frac{\mu}{1-\mu}dy, dy) = a\frac{\mu}{1-\mu} + b\frac{1-\mu}{\mu}$ and that $R(\tilde{dx}, \tilde{dy}) = a\frac{\tilde{dx}}{\tilde{dy}} + b\frac{\tilde{dy}}{\tilde{dx}}$

It follows that:

$$R(\frac{\mu}{1-\mu}dy, dy) < R(\tilde{dx}, \tilde{dy}) \qquad \Leftrightarrow \qquad a(\frac{\tilde{dx}}{\tilde{dy}} - \frac{\mu}{1-\mu}) > b(\frac{1-\mu}{\mu} - \frac{\tilde{dy}}{\tilde{dx}})$$

Since $(\tilde{dx}, \tilde{dy})$ lie in the feasible region $\tilde{dx} > \frac{\mu}{1-\mu}\tilde{dy}$. Hence, $\frac{\tilde{dx}}{\tilde{dy}} > \frac{\mu}{1-\mu}$ and $\frac{\tilde{dy}}{\tilde{dx}} > \frac{1-\mu}{\mu}$. So, $R(\frac{\mu}{1-\mu}dy, dy) < R(\tilde{dx}, \tilde{dy})$ iff $\frac{a}{b} > \frac{1-\mu)\tilde{dy}}{\mu\tilde{dx}}$. Note that if $\tilde{dx}$ were equal to $\frac{\mu}{1-\mu}\tilde{dy}$ then $\frac{1-\mu)\tilde{dy}}{\mu\tilde{dx}} = \frac{(1-\mu)^2}{\mu^2}$. As $\tilde{dx} > \frac{\mu}{1-\mu}\tilde{dy}$, it follows that $\frac{1-\mu)\tilde{dy}}{\mu\tilde{dx}} < \frac{(1-\mu)^2}{\mu^2}$. I now prove that $\frac{a}{b} > \frac{(1-\mu)^2}{\mu^2}$. This last inequality must be respected at the point (ϵ_x, ϵ_y). At the optimum $x = \frac{\mu}{1-\mu}y$. Therefore I can rewrite a and b as:

$$a = \frac{1-\mu}{4\mu y} + \frac{(1-\mu)(1+\mu-\mu y)^2}{4(1-\mu y)^3}$$

$$b = \frac{\mu}{4(1-\mu)y} + \frac{(\mu^2(1-\mu-\mu y)^2}{4(1-\mu y)^3(1-\mu)}$$

The question becomes:

$$\frac{a}{b} = \frac{(1-\mu)^2(1-\mu y)^3 + (1-\mu)^2(1+\mu-\mu y)^2\mu y}{\mu^2(1-\mu y)^3 + \mu^3(1-\mu-\mu y)^2 y} > \frac{(1-\mu)^2}{\mu^2}$$

This last inequality is always verified because $(1+\mu-\mu y)^2 > (1-\mu-\mu y)^2$. Q.E.D.

Lemma (3) has shown that in the feasible region $a\frac{dx}{dy} + b\frac{dy}{dx}$ is minimal at the point where $dx = \frac{\mu}{1-\mu}dy$. Replacing dx by $\frac{\mu}{1-\mu}dy$ in (4) I can rewrite my sufficient condition as:

$$2c < a\frac{\mu}{1-\mu} + b\frac{1-\mu}{\mu} \tag{5}$$

Keeping into account the fact that at the optimum $x = \frac{\mu}{1-\mu}y$, we can rewrite c as:

$$c = \frac{1}{4y} + \frac{(1+\mu-\mu y)\mu(1-\mu-\mu y)}{4(1-\mu y)^3}$$

Condition (5) then becomes:

$$\frac{2}{4y} + \frac{(1+\mu-\mu y)\mu(1-\mu-\mu y)}{2(1-\mu y)^3} < \frac{2}{4y} + \frac{\mu(1+\mu-\mu y)^2 + \mu(1-\mu-\mu y)^2}{4(1-\mu y)^3}$$

This last condition boils down to checking the inequality $4\mu^2 > 0$ which is verified $\forall \mu \in (0, \frac{1}{2})$.