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Gurus, Opinion Polls and Social Learning

Nicolas Melissas

Instituto Tecnológico Autónomo de México

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Col. Héroes de Padierna
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M E X I C O

Gurus, Opinion Polls and Social Learning*

Nicolas Melissas
Centro de Investigación Económica
Instituto Tecnológico Autónomo de México

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Abstract

This paper analyzes cheap talk in an investment model with information externalities. In contrast to Gossner and Melissas (2006), I allow for (i) competition effects, (ii) positive network externalities and (iii) more than one interviewed player. In the presence of competition effects, a player will never truthfully reveal her information about the realized state of the world. In the presence of positive network externalities, however, there exists a parameter range where, under mild additional conditions, the unique equilibrium is the separating one. Finally, using numerical computations, I show that for a sufficiently large number of interviewed players there exists a separating equilibrium in my entire parameter range.

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*Correspondence can be sent to Nicolas Melissas (nmelissas@itam.mx), CIE - ITAM, Avenida Santa Teresa 930, México D.F. 10700, México. I am very grateful to M.Dewatripont for his many helpful suggestions. I also thank seminar participants at the ULB, Leicester, CIE-ITAM, UCD, and Nottingham for helpful comments and discussions. Finally I am also grateful to A.Rivière for her technical advice.

1 Introduction.

People often take actions or adopt strategies which are observed and can potentially be imitated by many other people. For example if a person buys equity in the stock-market then this signals to the other market participants that that person believes that the true value of the asset lies above its current price[1]. Similarly when bank depositors withdraw all their money this signals to other depositors that they possess some bad information concerning its repayment ability (Chari and Jagannathan (1997)). Technologies which have been successfully tested in one firm often get rapidly imitated by many other firms. This process of learning by observing others is called social learning, and it has recently been receiving considerable attention from members of the economics profession.

The first models to stress the inefficiencies associated with this form of learning are Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (BHW, 1992). In these models all agents possess some private information concerning the desirability of an investment action. People move in an exogenous order, and everyone observes the action of his predecessors. They find that if early movers decide to invest, then all subsequent movers will neglect their own private information and will inefficiently herd on the investment decisions of the few early movers. In these two papers people move in an exogenous order, i.e. they do not have the possibility to wait and see how many other optimists are present in the economy. Chamley and Gale (CG, 1994) introduced strategic waiting in an investment model similar to the one of BHW. They showed that if investors have the possibility to wait, social learning remains very inefficient because too little learning will occur in equilibrium.

These original models of social learning crucially rest on the assumption that information is only transmitted via actions and not via words. Typically, this assumption is defended by the claim that "actions speak louder than words". However this claim raises two objections: a practical and a theoretical one.

From a theoretical point of view we know from standard game-theoretical textbooks (see e.g. Gibbons (1992)) that cheap talk can be very informative as well. For example, if preferences are perfectly aligned, i.e. if low sender types prefer a low receiver's action and high sender types prefer a high receiver's action, then a lot of useful information can simply be transmitted via cheap talk.

In practice we observe that a lot of information is transmitted via words. For ex-

[1]For a model of herding in a financial market mechanism see Avery and Zemsky (1998)

ample every now and then a famous entrepreneur and/or economist appears in the news giving his opinion about the current and future economic prospects, whether the investment climate is favourable or not, etc... Similarly marketing agencies organise opinion polls in which a large number of persons are asked their opinions about existing and new products, advertisements, new laws passed by the parliament, etc... Some opinion polls also ask to a large number of investors how they believe the future economic environment will look like and whether they intend to invest or not and how much. Theoretically these information channels are puzzling. Even a famous entrepreneur (henceforth we call her a guru) whose opinion on economic matters is asked by the media is not sure whether her opinion is correct or not. This paper analyses her incentives to truthfully report her opinion, given that she has the possibility to wait and learn the other market participant's opinions before making her investment decision. In this context, since both a pessimistic as an optimistic guru wants to engage in social learning, it is not clear why a pessimistic guru would want to send a different signal to the remaining market participants as an optimistic one.

We analyse a two-stage communication waiting game similar to the one originally studied by Gossner and Melissas (2006). In the second stage investors play a waiting game similar to the one studied by CG. All players must make an investment decision and possess a private signal concerning the future state of the world. Investment is only profitable in the good state. In the good (bad) state of the world a majority of players are optimists (pessimists). Everyone can invest in both periods. In the second period, everyone observes how many players invested in the first period, and make their final investment decisions. As shown by Chamley (1997) this waiting game may exhibit two stable equilibria. In the low activity equilibrium all optimists play a waiting game and invest with a symmetric equilibrium probability λ^*. In the high activity equilibrium everyone (both the optimists and the pessimists) invest in the first period. This latter equilibrium only exists if the cost of the investment project is relatively low so that even pessimists are willing to invest. This equilibrium yields no social learning and is Pareto-dominated by the low activity one. In the low activity equilibrium the informational benefit of waiting is determined by the symmetric investment probability λ^*. The higher λ^*, the less "noise" (in the sense of Blackwell (1951)) is added to the public signal, the higher the informational benefit of waiting. As usual, optimists choose λ^* such that the informational benefit of waiting is equal to the cost of waiting.

It turns out that the low activity equilibrium is much more robust to the introduction of cheap talk than the high activity one. This is because in the former equilibrium both types share similar preferences in that they both want the optimists to randomise as much as possible, and therefore have an incentive to overreport their signals. Despite the fact that both types have similar preferences over the receiver's actions a separating equilibrium exists - for a certain range of parameter values - in the latter equilibrium because there a pessimist wants to refrain the other pessimists from investing in order to learn something from the optimists' investment actions.

We also introduce competition and positive network externalities in the model. In the presence of competition effects (or negative network externalities), the unique equilibrium in the communication game is the "babbling" one, i.e. no useful information is transmitted via words. However we show that our truthtelling equilibrium remains robust towards the introduction of positive network externalities. Moreover in that case we also show that in a limited parameter range we get, under mild additional assumptions, truthtelling as a unique equilibrium. This is due to our finding that pessimists strictly prefer to reveal their good signal because otherwise they don't learn anything, optimists strictly prefer to reveal their good signal because they want to be imitated. Finally we also introduce an opinion poll in CG's model. Prior to making their investment decisions J randomly drawn players are asked to report their signals. If our players don't know whether J is an even or an uneven number then for the entire range of parameter values (and if players focus on the high activity equilibrium (if it exists) in the continuation game) there exists a truthtelling equilibrium ($\forall J \geq 12$).

This paper belongs to three different strands of the literature: (i) the one which analyses the consequences of cheap talk in models of social learning, (ii) the one which analyses cheap talk in strategic contexts, and (iii) the one which introduces information externalities in investment models.

(i) Cheap talk in models of social learning:

Gossner and Melissas (2006) study a similar game to ours. However, they did not introduce competition effects and positive network externalities in their model. Nor did they tackle the case in which more than one player is being asked to divulge her signal prior to the waiting game. Banerjee and Fudenberg (BF,1994) and Smith and Sørensen (1997) are the first models to analyse the efficiency of social learning through costless communication[2]. In those models new agents must make a once-and-for-all decision between two technologies, the payoffs of which are independent of the technological choices of the other agents. Prior to making their investment

[2]In this line of research we also want to mention Banerjee's (1993) paper on how rumours can affect economic behaviour.

decision they ask a sample of incumbent agents which technology they use and how satisfied they are with it. Since these incumbents themselves adopted their technology after sampling other previous agents, one can think of their technological choice (and their subsequent payoffs) as an imperfect signal concerning the profitability of either technology. BF's results draws a less dark picture of social learning. In particular they find that if at least two other agents are sampled, asymptotically the dominant technology will supersede the inferior one. Those papers assume that all players truthfully report their signals. Given their exogenous queue framework it is quite natural to focus on truthful revelation. Their context is most realistic if one considers a consumer who asks to friends how happy they are with their brands. No doubt, word-of-mouth communication is an important information channel in everyday life, but it is by no means the only way through which cheap talk actually occurs in our society. This paper analyses cheap talk in an endogenous queue setting. Hence we do not take truthful communication for granted, instead its existence is endogenously derived from utility maximising agents. As BF we also find that costless communication improves matters in an endogenous queue setting provided that the investors focus on the less efficient equilibrium in the waiting game.

(ii) Cheap talk in strategic contexts:

As mentioned previously, in an abstract model of cheap talk Crawford and Sobel (1982) showed that costless communication can be very informative. Subsequently a number of papers appeared which analysed the existence of separating equilibria in different realistic contexts. For example, Mathews (1989) analyses how a Presidential veto threat can credibly convey information concerning the President's preferences to the Congress, Stein's (1989) model explains why imprecise policy announcements by the FED can be informative. As in the previous papers, I also prove the existence of a separating equilibrium. While cheap talk is informative in the previous papers due to an exogenous preference reversal, in this paper separating equilibria are driven by different (endogenous) outside options of the different players[3].

To the best of our knowledge this is the first paper which analyses what induces people to truthfully reveal their opinion when thay are asked to do so in an opinion poll. We show that the intuition why a player may want to truthfully reveal her opinion is not altered much by the presence of many other players at the communication stage. This is because in an opinion poll each player computes her best strategy under the assumption that she is the pivotal player whose report will infuence decisively the

[3]For a model which analyses the role of mechanism design to implement truthtelling see Glazer and Rubinstein (1997)

behaviour of the remaining players. This insight is similar to the one obtained in voting models (see e.g. Rosenthal (1993)) where each person in an election votes in the hope that her vote will overturn the election results. However in our set-up allowing for many senders improves matters because it doubles the range of parameter values in which a truthtelling equilibrium exists.

(iii) Information externalities in investment models:

CG argued that the aggregate investment level is, due to an information externality, too low from a social welfare point of view. On the basis of their model, one would want to subsidise investments to make the optimists invest more and increase the amount of information released. However, Chamley (1997) has shown that the issue is not so simple, because there exists another equilibrium in the waiting game which generates too much investments and too little learning. In his model it is not clear whether investments should be taxed or subsidised. This paper argues that not only does cheap talk overcome possible overinvestment problems, moreover it yields more information than the one which is released by having only optimists investing. Taxing investments is thus definitely not a good idea. In contradiction with Chamley (1997), CG is shown to be robust to the introduction of cheap talk. However this is also one of its weaknesses because CG preclude the existence of opinion polls which is at odds with what we observe in reality. This critique against CG and Chamley (1997) will be taken up more in detail in our seventh section. Finally this paper also provides a first attempt to incorporate (positive as well as negative) network externalities in CG's model[4].

This paper is organised as follows. In section two we present our two-stage communication waiting game. In the third section we solve our waiting game in the absence of any communication (this section merely represents a simplified version of CG's model). We explain that our waiting game can be characterised by multiple equilibria. We next introduce costless communication in the picture and we show that if our investors focus on the Pareto-dominated equilibrium in the waiting game, one player (which we call player i) has much more incentives to truthfully reveal her own signal. In section four we introduce competition effects in the model. Positive network externalities are introduced in the fifth section. In section six we provide the reader with some explanations concerning how to interpret player i. In section seven we analyse the case of opinion polls. A truthtelling equilibrium exists then for a much wider region of our exogenous variables. A discussion and final comments are

[4]For a simple model with two players which combines network and information externalities see Choi (1997).

summarized in the seventh and final section.

2 The model: the general framework

Assume we have a population of N risk neutral players who must decide whether to invest in a risky project or not. The revenues of the investment project, V, can take two values: $V \in \{1, 0\}$. The cost of the investment project is denoted by c. The state of the world is described by $\Theta \in \{H, L\}$. if $\Theta = H$ the good state prevails and $V = 1$. If $\Theta = L$, the economy is in a bad state and $V = 0$ ($P(\Theta = H) = P(\Theta = L) = P(H) = P(L) = \frac{1}{2}$). Each player possesses an imperfect private signal, s_i, concerning the realisation of Θ. $s_i \in \{h, l\}$, $i = 1, ..., N$. If $\Theta = H$, N_H individuals receive signal h. If $\Theta = L$, N_L individuals receive signal h, and $N - N_L$ individuals receive signal l. We define p as $\frac{N_H}{N}$ and we assume that $p > \frac{1}{2}$. For simplicity, we also assume that $\frac{N_L}{N} = 1 - p$. In this paper, unless stated differently, we assume that:

A1: $1 - p < c < p$

A1 implies that a player who received signal h is - a priori - willing to invest, because $P(H|h) = p > c$. Henceforth, we call a person having received at time - 1 (see below) a good (bad) signal an optimist (pessimist). A player who received a bad signal computes $P(H|l) = 1 - p$. As $1 - p < c$, in the absence of any other favourable news, pessimists abstain from investing. We also work under the simplifying assumption that N is very large. Under this assumption the $P(s_i = h|s_j = h, H) = P(s_i = h|H) = p, \forall j \neq i$ (where $j = 1, 2, ..., N$). In other words, if N is very large our model behaves as if signals were *iid*. Analysing our model in the case of a large population seems natural, because in our set-up N represents f.i. all the investors in the U.S.. In sections three, four and five we analyse the following two-stage game:

-1) The state of nature is realised and our N players receive their signals,
0) One randomly drawn player is asked her signal. Her report is made public to all N players,
1) All players make their investment decisions,
2) All players observe how many persons invested in period one, and those who haven't invested yet make their investment decisions. Payoffs are received and the game ends.

In section seven we consider the same game except that we will work under the assumption that at time zero the number of interviewed players is uncertain and is greater or equal than two. In other words sections three, four and five analyse the incentives of a single player while section seven analyses the case of an opinion poll. As usual the equilibrium strategies are computed by backwards induction. Therefore we first focus on our waiting game, afterwards we analyse the equilibrium strategies in the communication game.

3 Strategic waiting without network externalities.

In this section we skip time 0 in our game and we compute the equilibrium strategies in our waiting game. $\lambda \in [0,1]$ represents the strategy of an optimistic player, it denotes the (symmetric) probability with which all optimistic players invest in the first period. Similarly, $\lambda_p \in [0,1]$ represents the strategy of the pessimists, it denotes the probability with which all pessimists invest in period one. Throughout the paper endogenous variables and their equilibrium values are respectively denoted without and with a *.

Assume that $\lambda = 1$ and that $\lambda_p = 0$. In this case all the optimists invest in period one. In period two, the number of optimists (and thus the state of nature), becomes common knowledge. We assume that:

A2: $p - c < \delta p(1 - c)$

A2 implies that if an optimist believes all the other optimists will invest in the first period, then it's optimal for her to wait and to make her investment decision - based on superior information - in the second period. Stated differently, A2 puts a lower bound on the discount factor δ, such that there is a positive option value of waiting.

Call k $(= 0, 1, ..., N)$ the number of players who invest in the first period. $P(H|s_i, k, \lambda, \lambda_p)$ denotes player i's second-period posterior probability that $\Theta = H$ given player i's signal s_i, given that k players invested in the first period and given the investment probabilities λ and λ_p. From now on, whenever $P(\cdot|\cdot)$ does not explicitly contain λ_p as an argument, this means that λ_p is supposed to equal zero. As information in our game is incomplete, players' beliefs about other players' types must be specified as part of the equilibrium. In this model beliefs about other players' types ultimately matter because the presence of relatively many optimists indicates that $\Theta = H$. Therefore we define our equilibrium concept using the probability assessment $P(H|s_i, k, \lambda, \lambda_p)$ instead of working with each players' beliefs concerning the other players' types. A perfect Bayesian equilibrium (PBE) is a (p,λ, λ_p) such that:
(i) no player can gain by choosing a different strategy, given p and given that the

other players invest at time one with probabilities λ and λ_p
(ii) whenever possible, $P(H|s_i, k, \lambda, \lambda_p)$ is computed via Bayes'rule, given λ and λ_p.

The analysis of this waiting game is not original. A more general version of that waiting game has already been analysed by CG and by Chamley (1997). CG showed that the waiting game is characterised by a so-called low-activity equilibrium. In that equilibrium $\lambda_p^* = 0$ and $\lambda^* \in (0,1)$. In subsequent work Chamley (1997) showed that the waiting game may - provided that $c \leq 1-p$ - also be characterised by another symmetric equilibrium which he called the high activity equilibrium. In that equilibrium $\lambda_p^* = \lambda^* = 1$. We first present the train of thought which underlies CG's analysis. Next we analyse Chamley's high activity equilibrium.

3.1 The low activity equilibrium.

In this PBE $\lambda_p^* = 0$ and $\lambda^* \in (0,1)$. In this subsection we work under A1. Therefore it's a dominant strategy for each pessimist not to invest at time one and $\lambda_p^* = 0$. However it can be shown that the analysis remains unaffected if $c \leq 1-p$.

We first state our first lemma which is very useful in computing the ex ante gain of waiting of a player. In the lemma below k_1, $k_2 = 0, 1, ..., N_H - 1$.

Lemma 1: $P(H|s_i, k_2, \lambda) > P(H|s_i, k_1, \lambda)$, $\forall k_2 > k_1$, $\forall s_i$, $\forall \lambda \in (0,1)$
Proof: see appendix

Lemma one is very intuitive. Upon observing k investments, players compute their posteriors $P(H|s_i, k, \lambda)$. If k is high, then the other players conclude that probably many players are optimistic and that therefore $\Theta = H$. If an optimist waits, and if the other optimists invest with a probability λ, then her ex post payoff (discarding discounting costs) equals: $Max\{0, P(H|h, k, \lambda) - c\}$. Her ex ante payoff (net of discounting costs) of waiting equals:

$$W(h, \lambda) = \sum_{k \geq \underline{k}} [P(H|h, k, \lambda) - c] P(k|h, \lambda) \tag{1}$$

$\underline{k} = 0, 1, ..., N_H - 1$ represents the lowest integer such that $P(H|h, \underline{k}, \lambda) \geq c$. If $\lambda = 0$, $\underline{k} = 0$, if $\lambda = 1$, $\underline{k} = N_H - 1$. Similarly, we define $W(l, \lambda)$ as:

$$W(l, \lambda) = \sum_{k \geq \underline{k}_p} [P(H|l, k, \lambda) - c] P(k|l, \lambda)$$

where $\underline{k}_p (= 1, 2, ..., N_H)$ represents the lowest integer such that $P(H|h, \underline{k}_p, \lambda) \geq c$, whenever $\underline{k}_p$ exists[5]. Intuitively, $\underline{k}$ ($\underline{k}_p$) represents the minimal amount of information that an optimist (pessimist) must get in the second period to make her willing to invest given the symmetric investment probability λ. $\underline{k}$ and $\underline{k}_p$ are increasing in λ: observing f.i. 10 investments with a $\lambda = 0.01$ is better news than observing 10 investments with a $\lambda = 0.5$. Equation (1) represents thus one equation in two unknowns: λ and $\underline{k}$.

The higher λ the more precise an idea one can get about the number of optimists in the world and the higher the ex ante gain of waiting. This can best be understood by comparing player i's incentive to wait if λ were equal to zero to the one he faces if λ were equal to one. If $\lambda = 0$, then there cannot be any gain of waiting. For if an optimist were to wait, she wouldn't observe any first-period investment, she would compute $P(H|h, k = 0, \lambda = 0) = p$ and would have the same incentives to invest at time two as the ones she had at time one. On the other hand, if $\lambda = 1$ then there is a high gain of waiting. In that case the state of the world is perfectly revealed at time two and player i can make a riskless second-period investment decision. Therefore $W(\cdot, \lambda)$ is strictly increasing in λ[6] [7]. We know enough to state:

Theorem 1 *(Chamley and Gale (1994)) There exists a unique low activity equilibrium.*

Proof: In equilibrium the gain of waiting must be equal to the gain of investing, i.e. $p - c = \delta W(h, \lambda^*)$. By continuity, if $\lambda = 0$, $\delta W(h, 0) = \delta[p - c] < p - c$. If $\lambda = 1$, by A2 $\delta W(h, 1) > p - c$. Therefore there exists a unique symmetric PBE[8]. Q.E.D.

Note that the welfare consequences of this waiting game are drastic: in equilibrium there are no gains from information revelation! Indeed, as in equilibrium the (discounting) cost of waiting equals the (informational) benefit of waiting, optimists are indifferent between the two strategies and therefore the possibility of waiting does not affect their welfare. Pessimists benefit from the possibility of waiting because they can free-ride on the informational value of the first period investment decisions of the

[5] If $\lambda = 0$ (and hence $k = 0$) then a pessimist computes $P(H|l, 0, 0) = 1 - p < c$ and $\underline{k}_p$ doesn't exist. In all the cases where $\underline{k}_p$ doesn't exist, we assume that $W(l, 0) = 0$.

[6] For a more technical intuition, based on Blackwell's (1951) value of information theorem, why $W(\cdot, \lambda)$ is strictly increasing in λ see CG.

[7] $W(h, \lambda)$ is continuous in λ because once $\underline{k}$ is fixed $p(1-c)b(k; N_H - 1, \lambda) + (1-p)cb(k; N_L - 1, \lambda)$ is continuous in λ and because at the probability $\lambda^{\underline{k}}$ where $\underline{k}$ increases, the expected gain of investing equals zero. The same reasoning also applies to $W(l, \lambda)$.

[8] We computed λ^* under the assumption that players can only invest in two periods. As shown in CG this is without loss of generality. Indeed, they have shown that in equilibrium having the possibility to wait only one period or to wait an infinite number of periods leaves the equilibrium strategies unaffected. They coined this insight as the one-step property.

optimists. In this equilibrium the social returns to first-period investments exceed the private ones due to an information externality. Therefore a social planner will always implement a higher investment probability. Actually any probability higher than λ^* increases welfare in this model, because the investors get $p - c$, and the players who wait receive a more informative signal. On the basis of this model one would want to subsidise investments to induce investors to generate more information.

The lemma below will be very useful in the analysis of the equilibrium strategies in our communication game:

<u>lemma 2:</u> if $\delta < 1$, whenever $\lambda_p^* = 0$, λ^* is a strictly increasing function of p.
Proof: see appendix

More optimistic players face a higher gain of investing and are only indifferent if the gain of waiting also increases (and this only happens when λ^* increases).

3.2 High Activity Equilibrium.

As shown by Chamley (1997) if $c \leq 1 - p$ there also exists one (and only one) other symmetric PBE, which he called the high activity equilibrium. In that equilibrium no one randomises and optimists as well as pessimists invest in the first period (i.e. $\lambda^* = \lambda_p^* = 1$). To see that this constitutes an equilibrium, assume that player i is a pessimist and suppose he were to deviate by not investing at time one. At time two, he would observe $N - 1$ investments and he would compute $P(H|l, k = N - 1, \lambda = 1, \lambda_p = 1)$. This latter probability equals $P(H|l)$. This is logical: in the high activity equilibrium player i - independently of the realised state of the world - always observes $N - 1$ investments. As player i cannot infer the number of optimists out of these $N - 1$ investments, his prior remains unaffected and in the second period player i gets $\delta(P(H|l) - c)$. Hence, due to discounting, if player i expects that $\lambda = \lambda_p = 1$, he strictly prefers to invest as well in the first period (in which case he gets $1 - p - c \geq 0$).

The low activity equilibrium entails a lot of social learning and Pareto-dominates the high activity equilibrium. In the high activity equilibrium too little learning occurs due to too high an investment level. Therefore in this case one would want to tax investments.

For our purpose this issue of multiple equilibria is an important one. Even if $1 - p < c$, it is possible that - after the communication stage - pessimists become optimistic enough and face a positive gain of investing. We will see that the equilibrium strategy in the communication game depends strongly on which equilibrium will be selected in the subsequent waiting game. If our players expect the high activity equilibrium to be selected, then a separating equilibrium is more likely to happen in the communication game. In other words this paper shows that the low activity equilibrium is more robust to the introduction of cheap talk than the high activity one.

3.3 Strategic information transmission by one player without network externalities.

We now focus on the communication game. We analyse the case where one randomly chosen player (henceforth player i) is asked her opinion concerning the state of the world by the media. We analyse her incentives to truthfully reveal her signal given that she is also uncertain concerning the realised state of the world and given that she also wants to learn about the other's signals.

3.3.1 Some definitions

We assume that the reported signal $\hat{s}_i \in \{h, l\}$. Call x the strategy of an optimistic player, it represents the probability with which an optimist truthfully reports her good signal. y represents the strategy of a pessimistic player. It denotes the probability with which she strategically reports a good signal (even though she's a pessimist). A probability assessment β for player j is defined as a function $\beta : [0,1]^2 \times \{h,l\}^2 \to [0,1]$ with the interpretation that $\beta(x, y, s_j, \hat{s}_i) = P(H|s_j, \hat{s}_i, x, y)$. A perfect Bayesian equilibrium (in our communication game) is a (x, y, β)[9] such that:
(i) player i cannot achieve a strictly higher payoff by choosing a different x or y, given β and given that $(\beta, \lambda, \lambda_p)$ must form a PBE in the continuation game,
(ii) whenever possible, β must be computed via Bayes' rule, given x and y.

λ_o denotes the equilibrium strategy of the optimists in the waiting game if they possess one good and one bad signal and if $\lambda_p = 0$. λ^o denotes the equilibrium strategy of the optimists in the waiting game if they possess two favourable signals and if $\lambda_p = 0$. As before λ^* denotes the equilibrium strategy of the optimists if they don't possess any additional information (besides their own signals)[10] and if $\lambda_p = 0$.

Definition: A communication strategy is called informative if $P(H|s_i, \hat{s}_i, x, y) \neq P(H|s_i)$. A communication strategy is a truthtelling one whenever $x =$

[9] For the same reason as the one we mentioned before, we define our equilibrium concept using a probability assessment β instead of explicitly incorporating the players' beliefs concerning the other players' types into the definition.

[10] Even though λ^o and λ_o denote equilibrium probabilities, to economise on notation, we omit the * superscript.

1 and $y = 0$.

In other words a strategy is informative as soon as $x \neq y$. If $x = y$, then communication does not affect the players' posterior beliefs. This case corresponds to a pooling equilibrium in the communication game. $P(\cdot|s_j, \hat{s}_i = \cdot)$ denotes the posterior probability that $\Theta = \cdot$, given player j's signal, given that player i reported $\hat{s}_i = \cdot$, and given that player j revises his posterior under the assumption that player i truthfully reported her signal.

The bulk of this paper is devoted to proving the existence of a truthtelling equilibrium in the communication game. However, we will see that if positive network externalities are added to the model, we will obtain - in a certain parameter range - truthtelling as a unique equilibrium. Therefore we shall also use the following equilibrium refinement:

Equilibrium refinement (ER):[11] If player i reports a signal which she was supposed never to send, and if there exists one (and only one) type of player for which the following inequality is respected:

$$Max\{A(\cdot, \lambda^*, \lambda_p^*), \delta W(\cdot, \lambda^*, \lambda_p^*)\} \leq Max\{A(\cdot, \lambda, \lambda_p), \delta W(\cdot, \lambda, \lambda_p)\} \quad \forall \lambda, \lambda_p \in [0,1]$$

(where $A(\cdot, \lambda^*, \lambda_p^*)$ represents the ex ante payoff of investing in the first period when player i's signal equals $\cdot$, and when optimists (pessimists) invest in period one with the equilibrium probabilities λ^* (λ_p^*) given that no out-of-equilibrium information set was reached), then all players believe with probability one that player i's type is the one which respects the inequality above.

3.3.2 Strategic communication when players focus on the high activity equilibrium (if it exists) in the continuation game.

This case was already studied by Gossner and Melissas (2006). In particular they showed that:

PROPOSITION 1 *(Gossner and Melissas (2006))If players focus on the high activity equilibrium (if it exists) in the continuation game and if* $c \in (1-p, \frac{1}{2}]$ *there exists a truthtelling equilibrium.*

Proposition (1) is not trivial! Both sender's types share the same preferences over the receivers' actions in the sense that both of them want the optimists to invest more and the pessimists to remain quiet. Therefore one would expect that all equilibrium

[11]Note that this equilibrium refinement represents a very weak restriction. It is similar (but nonetheless different) to Cho and Kreps' (1987) intuitive criterion.

communication strategies be uninformative. For example, both in Stein's (1989) analysis of cheap talk by the FED and Mathews' (1989) analysis of a veto threat by a president[12], the existence of informative equilibria was driven by an exogenous preference reversal, i.e. different sender's types prefer different receiver's actions. In this paper a pessimist ultimately "prefers" to report l to h, not because of an exogenous preference reversal, but because both types are endowed with different outside options[13]. The outside option of a pessimist equals zero. A pessimist wants to avoid that the other pessimists destroy all informational gains from waiting and therefore truthfully reports her unfavourable signal. The outside option of the optimist equals $p - c$. As explained above, given this high outside option she cannot discriminate between the two strategies. Ideally she wants to report a bad signal to the pessimists and a good one to the optimists, but this is impossible. Therefore she's unable to achieve any higher utility level and is indifferent.

Proposition (1) highlights under which assumptions a truthtelling equilibrium exists. Of course it's not the unique informative equilibrium. One can find other informative equilibria in which $x^* \neq 1$ and $y^* = 0$. However, we don't want to enter into a detailed analysis of these semi seperating-pooling equilibria because none of these equilibria are robust towards the introduction of network externalities (see below).

4 Competition effects: the case of a first-mover advantage.

In this section we assume that our technology is characterised by negative network externalities and we analyse its consequence(s) on proposition (1). Introducing (positive as well as negative) network externalities in CG's framework constitutes a hard job. In this section, to simplify matters we work under the following specific assumption:

A3: If an investor is the first and the only one to invest in any of the two periods then she gets $\gamma > 1$ (if $\Theta = H$), independently of how many persons invest in the (eventual) next period.

A3 states that if an optimist is the only one to invest in period one then, indepen-

[12]In Mathews' paper an extremist president would not threaten to veto a large defence budget because if the congress responds to that threat by reducing the defence budget, he would (by assumption) be worse off.

[13]with this we mean the utility level obtained by a player if she doesn't receive any additional information.

dently of how many persons invest in the second period, she will get γ (if $\Theta = H$). However if at least one other player also invests in period one, they both get 1 (if $\Theta = H$). The same is true for second period investments except that a lonesome period two investor can never be imitated. There are two ways to interpret A3. First A3 can be interpreted as a typical first mover advantage, f.i. the first firm to enter into a market may benefit from some "brand name", or from temporary monopoly profits,... . Second, there also exists another way to interpret A3 which captures what one would call negative network externalities. Suppose that the investment yields a return of $\gamma > 1$ if only one firm invests (and if $\Theta = H$) and a return equal to one as soon as two firms invest (independently of when the second player invests). Suppose λ^* is chosen such that $P(H|h, k = 1, \lambda^*) < c$. Under these assumptions if in period one only one firm invests then that firm gets γ if $\Theta = H$ because no manager will be willing to invest in period two. So our assumption that a lonesome period one investor's payoff remains unaffected by subsequent investments is without loss of generality as long as $\underline{k}^* \geq 2$.

$A(h, \lambda)$ denotes the gain of investing in period one if $s_j = h$, if $\lambda_p = 0$ and if the other optimists invest with probability λ at time one. We also simplify the model by assuming that $\delta \to 1$. Note that $A(h, \lambda) \geq p - c\ \forall \lambda$. This is because if a player decides to invest in period one, he may end up being the only to have invested at time one in which case he gets γ instead of 1 (if $\Theta = H$).

In the appendix we prove that if time zero is omitted from our game, (i) if $c > 1 - p$, there exists a unique low activity equilibrium and (ii) if $c \leq 1 - p$ there also exists one (and only one) other symmetric equilibrium where $\lambda_p^* = \lambda^* = 1$. We also show that lemma 2 remains valid under A3.

Proposition (2) summarises the equilibrium communication strategies.

Proposition 2 *Under A1, A3, $\forall \gamma > 1$, independently of the chosen equilibrium in the continuation game, the unique equilibrium in the communication game is the babbling one.*

Proof (and intuition): We do not provide a proof of the equilibrium communication strategy if all players focus on the low activity equilibrium in the continuation game, nor do we prove the inexistence of informative communication strategies when $c > \frac{1}{2}$ and when players focus on the high activity equilibrium (if it exists) in the continuation game, because the proof parallels the one we developed for proposition (1). Intuitively, in both cases an informative communication strategy fails to exist in equilibrium because - as in our previous case without network externalities - a pessimist wants to pretend to be an optimist to increase the period-one symmetric investment probability λ^*. Moreover in this case - depending on the values of the parameters - even an optimist may not (not even partially) want to reveal her type because, with a first mover advantage, she may want to pretend to be a pessimist in the (vain) hope to decrease λ^*, and thus to increase her probability to be the only one to invest at time one[14].

More interestingly, assume that $c \leq \frac{1}{2}$, that all players revise their priors under the assumption that player i truthfully reveals her type, and that player i is an optimist. If she reports $\hat{s}_i = h$ then everyone (optimists as well as pessimists) invest and she gets $p - c$. However she can do better because by reporting $\hat{s}_i = l$ she gets $A(h, \lambda_o) > p - c$. Hence a truthtelling equilibrium does not exist because - even though a pessimist prefers to report her unfavourable signal - an optimist has an incentive to deviate. As previously one can easily show that there exists no other informative equilibrium in this context.Q.E.D.

Note that proposition (2) is true as soon as $\gamma > 1$. In other words even if the technology is characterised by very small negative network externalities no communication will ever take place[15]

5 Positive network externalities.

In this section we introduce positive network externalities in CG's model of social learning. We work under the following technological assumption:

A3': The investment generates a revenue equal to 1 if $\Theta = H$ and if at least two players invested. If only one player invests then the investment generates a revenue equal to $\gamma < 1$ (where $\gamma \to 1$) if $\Theta = H$.

Note that in this case $A(h, \lambda) \leq p - c$. This is because if an optimist invests at time one she may end up being the only one to have done so. In that case she gets γ instead of one (if $\Theta = 1$). In these paragraphs we do not give a detailed analysis of

[14] A formal proof is left to the reader as an exercise.

[15] Of course we do not look at the issue of report-contingent transfers. Apart from the fact that we do not observe them in practice they raise two theoretical objections. First there does not always exist a report-contingent transfer which implements the truthtelling equilibrium because the transfer may be higher than $W(l, \lambda_o)$ (meaning that the pessimist will prefer to report a high signal and pocket the transfer). Second if γ would depend on a firm-specific variable and each firm has some private information concerning its height, it's not clear how high the side transfer must be. In that case bargaining under asymmetric information yields inefficiencies which increases the cost of the side transfer.

the equilibrium behaviour in our waiting game (i.e. in the absence of time zero in our game)[16]. We just want to mention that under A3' the existence of a low activity equilibrium is not always guaranteed. However, if $\gamma \to 1$, (i) there almost always exists a low activity equilibrium and (ii) if a low activity equilibrium exists, all our previous insights remain unchanged. The intuition is simple: as $\gamma \to 1$, this model is almost identical to our benchmark case and all our previous results remain unchanged (see the appendix for a more complete and more formal exposition).

Before turning to the analysis of equilibrium communication strategies we assume that λ^*, λ^o and λ_o exist (which is quite natural as, once p and c are fixed, and, as $\gamma \to 1$, there almost always exists a unique low activity equilibrium). As before if all players focus on the low activity equilibrium in the continuation game, then an informative strategy doesn't exist in equilibrium because a pessimist wants to pretend to be an optimist to learn more.

More interestingly, assume that investors focus on the high activity equilibrium (if it exists) in the continuation game, assume that all players compute $\beta(\cdot)$ under the assumption that $x = 1$ and $y = 0$ and assume that $c \leq \frac{1}{2}$. We first consider the case of a pessimistic player. If she reports $\hat{s}_i = l$, she gets $\delta W(l, \lambda_o)$. Since $0 \leq \delta W(l, \lambda_o)$, a pessimist has no incentive to deviate. Assume player i is an optimist. If she reports $\hat{s}_i = l$, she gets $A(h, \lambda_0) \leq p - c$. If she reports $\hat{s}_i = h$, then - since $c \leq \frac{1}{2}$ and since we focus on the high activity equilibrium - everybody invests and she gets $p - c$. Hence in this case a truthtelling equilibrium exists.

Note that with positive network externalities if posteriors are computed under the assumption that player i truthfully reports her opinion, then an optimist may strictly prefers to report her favourable signal because she wants to be imitated. A pessimist also strictly prefers to report her unfavourable signal because she wants to learn about the others' signals. This finding that both types may strictly prefer to reveal their signals is not without any consequence. We can show that if $c \in (\frac{(1-p)^2}{(1-p)^2+p^2}, 1-p]$ and under some very weak additional assumptions that the unique equilibrium of the communication game is the truthtelling one. Our main results are summarised below.

Proposition 3 *If investors focus on the high activity equilibrium (if it exists) in the continuation game:*
(i) if $c \in (1-p, \frac{1}{2}]$, there exists a truthtelling equilibrium,

[16] This is a paper about cheap talk in an investment model with information externalities. We introduced positive and negative network externalities in our benchmark model because we were in the first place interested to check the robustness of proposition (1). In this and the previous section we do not want to make a complete analysis of all equilibria in a waiting game with information and network externalities.

(ii) if $c \in (\frac{(1-p)^2}{(1-p)^2+p^2}, 1-p]$, if there exists a probability $\epsilon_0 > 0$ that posteriors will be computed under the assumption that $x = 1$ and $y = 0$ and under ER, the unique equilibrium of our communication game is the truthtelling one.

Proof: the proof of point (i) appears above, the proof of point (ii) appears in the appendix

The intuition behind point (ii) is the following one. Assume it is common knowledge that with a probability $(1-\epsilon_0)$ posteriors will be computed at time one under the assumption that $x = y$. In that case both types are indifferent between reporting the two signals. However if there exists an arbitrarily small probability that all players will compute their posteriors under the assumption that $x = 1$ and $y = 0$ then both types are not indifferent anymore and strictly prefer to report their types[17]. If posteriors are computed under the assumption that $x = y = 1$ then a pessimist will strictly prefer to report her low signal because she knows that under ER, she will be credibly believed and this will discourage the other pessimists from investing.

6 Opinion Polls.

So far we only worked with one player in the communication game. In this section we analyse what happens if many players are simultaneously asked to report their signals. In proposition (1) and proposition (3) we showed that a truthtelling equilibrium exists if all players focus on the high activity equilibrium (if it exists) in the continuation game and if $c \leq \frac{1}{2}$. One may find this parameter range quite small. However, in this section we will see that if more than twelve players are simultaneously "interviewed", if our players don't know whether the number of interviewed players is even or uneven, and if our players focus on the high activity equilibrium (if it exists) in the continuation game, a truthtelling equilibrium exists for the whole parameter range $c \in (1-p, p)$. Unfortunately, but not unexpectedly, it was impossible to prove that result analytically. Instead we derived it using numerical computations. This section only focuses on the high activity equilibrium. In the light of our previous results this is logical because if players focus on the low activity equilibrium in the continuation game pessimists always wait, both types send as high a signal as possible and therefore all cheap talk is fully uninformative. There is no reason to suppose that this equilibrium behaviour would be affected by the mere presence of many players in the communication game. Finally, we assume that our technology is not characterised by any type of network externalities.

[17] Similarly, one can show that if $c \in (1-p, \frac{1}{2}]$ and if with a probability ϵ_0 posteriors are computed under the assumption that $x = 1$, $y = 0$, then no semi separating-pooling equilibrium exists in our communication game.

6.1 Some definitions.

We consider the same game as before except that at time zero J players are simultaneously asked to report their signals ($J = 2, 3, ..., \overline{J}$) ($\overline{J} < N_L$). $\hat{s}_{-i}$ denotes a vector ($1\text{x}(J-1)$) of the reported signals of the other players present in the opinion poll. $\hat{s}$ denotes a vector ($1\text{x}J$) containing the reports of the J players present in the opinion poll.

To fix ideas, suppose that $J = 4$. If $\hat{s}_{-i} = [h\ h\ h]$ this means that the three other players present in the opinion poll reported good signals. If $\hat{s}_{-i} = [h\ h\ l]$, this means that two out of the three other players present in the opinion poll reported signal h. From basic statistical textbooks we know that, if signals are identically and independently distributed, good signals inside $\hat{s}_{-i}$ cancel out bad ones. This insight allows us to simplify much the analysis of an opinion poll. To understand our train of thought, consider the following two configurations of $\hat{s}_{-i}$ (in the first one J is assumed to equal four, in the second example $J = 2$): $\hat{s}_{-i} = [h\ h\ l]$ and $\hat{s}_{-i} = [h]$. Our statistical property then implies that $P(\cdot|\hat{s}_i, \hat{s}_{-i} = [h\ h\ l]) = P(\cdot|\hat{s}_i, \hat{s}_{-i} = [h])$. In words, receiving two good and one bad signal is, statistically speaking, identical to receiving only one good signal.

This statistical property allows us to simplify our notations. $\hat{s}^a_{-i} = m\ h$ ($m\ l$) (where $m = 0, 1, ..., J+1$) denotes the aggregate informational value of $\hat{s}^a_{-i}$. Call n the number which is obtained after subtracting the number of bad signals from the number of good ones (contained in $\hat{s}_{-i}$). If $n < 0$, then we denote the aggregate informational value of $\hat{s}_{-i}$ by $|n|\ l$. If $n \geq 0$, then we denote the aggregate informational value of $\hat{s}_{-i}$ by $n\ h$. Similarly $\hat{s}^a$ denotes the aggregate informational value contained in $\hat{s}$. That value is computed in the same way the one we explained for $\hat{s}_{-i}$.

Note that, if $J = 4$, $\hat{s}^a_{-i}$ can only take four values: $3h, 1h, 1l$ and $3l$. $\hat{s}^a_{-i}$ cannot take the value $2h$ for example because in our example $J - 1$ is uneven. For example if J equalled five, then $J - 1$ would be an even number and $\hat{s}^a_{-i}$ could take the values: $4h, 2h, 0h, 2l$ and $4l$. As we will see this insight is not without any importance[18].

$\delta W(ml, \lambda)$ ($\delta W(mh, \lambda)$) denotes the ex ante gain of waiting given that $\hat{s}^a = m\ l$ ($m\ h$) and given that the other optimists randomise with probability λ. We first analyse the case where $c \in (1 - p, \frac{1}{2}]$. In our next subsection we analyse the other case, namely the one where $c \in (\frac{1}{2}, p)$. In these pages, we also restrict ourselves in proving the

[18] We still work under the assumption that $P(s_i = h|s^a_{-i} = J - 1h, H) = p$. Hence the reader should bear in mind that implicitly our analysis rests on the assumption that $\overline{J}$ is "small" relatively to N.

existence of a truthtelling equilibrium (we do not bother about the existence of other more complicated informative equilibria).

6.2 Strategic information transmission in opinion polls with a low investment cost.

Suppose that $c \in (1-p, \frac{1}{2}]$. Suppose that player i is a pessimist and that her signal is asked by a polling agency. We first analyse the case where J is an even number afterwards we consider the opposite case.

6.2.1 J is even.

First, it is important to note that in the vast majority of configurations of $\hat{s}^a_{-i}$ player i is indifferent between reporting h or reporting l. To see this suppose that $J = 10$. Suppose that $\hat{s}^a_{-i} = 9h$. In that case our player is indifferent between the two strategies, because - independently of her report - the remaining pessimists received too much positive news and invest in the first period. The same can be said of the configurations $\hat{s}^a_{-i} = 7h$, $\hat{s}^a_{-i} = 5h$, $\hat{s}^a_{-i} = 3h$. In all these configurations all players (optimists as well as pessimists) - independently of player i's report - end up investing after the result of the opinion poll was made public to the other players. Idem if $\hat{s}^a_{-i} = 9l$, i.e. if the nine other players present in the opinion poll are pessimists, - independently of player i's report - no one invests in the first period because they were all overwhelmed by too much bad news. The same happens in the configurations $\hat{s}^a_{-i} = 7l$, $\hat{s}^a_{-i} = 5l$ and $\hat{s}^a_{-i} = 3l$.

Assume that $\hat{s}^a_{-i} = 1h$, which means that five out of the nine other players present in the opinion poll are optimists. In that configuration player i is not indifferent between the two pure strategies. If she reports $\hat{s}_i = h$, then $\hat{s}^a = 2h$ and, after the announcement of the result of the opinion poll, pessimists compute $P(H|l, \hat{s}^a = 2h) = p > c$ and they all invest in the first period. Player i computes her posterior $P(H|l, \hat{s}^a_{-i} = 1h) = \frac{1}{2}$. Since $\frac{1}{2} > c$ she also invests in period one and ex ante she gets: $\frac{1}{2} - c$. However she can achieve a higher payoff by truthfully revealing her bad signal, because in that case $\hat{s}^a = 0h$ (as the five good signals cancel out the five bad ones the opinion poll contains no valuable information) and the players' posteriors remain unaffected. Hence pessimists do not invest in period one. Player i computes her posterior $P(H|l, \hat{s}^a_{-i} = 1h) = \frac{1}{2}$. Given that more optimistic players randomise it's optimal for her to wait and she gets $\delta W(0h, \lambda^*) > \frac{1}{2} - c$. So if $\hat{s}^a_{-i} = 1h$, the truthtelling strategy yields a payoff $\delta W(0h, \lambda^*) - (\frac{1}{2} - c)$ higher than the lying strategy.

Assume now that $\hat{s}^a_{-i} = 1l$. In this case a pessimist rather reports a high signal.

Suppose that $\hat{s}_i = l$. The informativeness of the opinion poll now equals two bad signals, $\hat{s}^a = 2l$. Optimists compute $P(H|h, \hat{s}^a = 2l) = 1 - p < c$ and no one invests in period one. Player i computes $P(H|l, \hat{s}^a_{-i} = 1l)$. Since $P(H|l, \hat{s}^a_{-i} = 1l) < 1-p < c$, she doesn't invest either and she gets zero. However strategic lying yields now a higher payoff. Suppose that $\hat{s}_i = h$. In that case the opinion poll is fully uninformative, $\hat{s}^a = 0h$, pessimists wait, optimists randomise with a probability λ^* and player i gets $\delta W(2l, \lambda^*) > 0$. To summarise a pessimistic player only truthfully reports her bad signal if and only if:

$$P(\hat{s}^a_{-i} = 1h|l, J)[\delta W(0h, \lambda^*) - (\frac{1}{2} - c)] > P(\hat{s}^a_{-i} = 1l|l, J)\delta W(2l, \lambda^*)$$

6.2.2 J is uneven.

In these paragraphs we assume that J is an uneven number. To illustrate what changes if J is uneven, suppose that J equals eleven. Then $J - 1$ equals 10. As previously $\hat{s}^a_{-i}$ can take many configurations, but only a few of them are worth studying. For example $\hat{s}^a_{-i} = 10h$, $\hat{s}^a_{-i} = 8h$, $\hat{s}^a_{-i} = 6h$, $\hat{s}^a_{-i} = 4h$ and $\hat{s}^a_{-i} = 2h$ are no interesting cases because - independently of player i's report - everyone invests in the first period. The only interesting configurations, if J is uneven, are the ones where $\hat{s}^a_{-i} = 0h$ and $\hat{s}^a_{-i} = 2l$.

Suppose first that $\hat{s}^a_{-i} = 0h$. Suppose our pessimistic player reports $\hat{s}_i = h$, In that event $\hat{s}^a = 1h$, pessimists compute $P(H|l, \hat{s}^a = 1h) = \frac{1}{2} > c$ and everyone invests in period one. Player i computes $P(H|l, \hat{s}^a = 0h) = 1 - p < c$ and does not invest. Therefore she gets zero. Suppose she reports $\hat{s}_i = l$. Then the value of the opinion poll's aggregate information equals one bad signal. Optimists compute $P(H|h, \hat{s}^a = 1l) = \frac{1}{2} > c$. Pessimists become, after hearing the result of the opinion poll, even more pessimistic than they already were and don't invest in period one. Optimists now face a lower gain of acting and randomise with a probability $\lambda_o < \lambda^*$. To summarise: if $\hat{s}^a_{-i} = 0h$, player i prefers to truthfully report her low signal because then she gets a payoff equal to $\delta W(l, \lambda_o) > 0$.

Assume now that $\hat{s}^a_{-i} = 2l$. In this configuration our player rather reports a high signal. If she truthfully reports her bad signal, then everyone is overwhelmed by too much bad news and no one invests. If she strategically lies, she reduces the amount of bad news contained in the opinion poll and this will induce some optimists to invest and to produce information externalities. More formally, if $\hat{s}_i = h$, $\hat{s}^a = 1l$ and optimists randomise with probability λ_o. Player i possesses now three bad signals. Therefore , if $\hat{s}^a_{-i} = 2l$, the lying strategy yields her a payoff of $\delta W(3l, \lambda_o) > 0$. To summarise: if J is uneven player i only truthfully reports her bad signal if and only if:

$$P(\hat{s}^a_{-i} = 0h|l, J)\delta W(l, \lambda_o) > p(\hat{s}^a_{-i} = 2l|l, J)\delta W(3l, \lambda_o)$$

6.2.3 Uncertainty concerning the number of sampled players.

So far we assumed that all players knew how many players were "interviewed" in the opinion poll. This assumption is not a very realistic one. In many contexts the interviewed players have no precise idea concerning the number of interviewed players. Even marketing agencies which organise opinion polls don't know how many persons will return them their questionnaires. We introduce this kind of uncertainty because we do not want our results to hinge on J being even or uneven. Moreover if J is even we do not get the existence of a truthtelling equilibrium for our entire parameter range. In this section we analyse numerically the incentives player i faces if she's uncertain whether J is an even or an uneven number. Without loss of generality assume that J is an even number and assume that it is commonly known that, with equal probability, J or $J+1$ players are present in the opinion poll. From what we have seen above it should be clear that player i truthfully reports her bad signal if and only if:

$$\frac{1}{2}[P(\hat{s}^a_{-i} = 1h|l, J)[\delta W(0h, \lambda^*) - (\frac{1}{2} - c)] - P(\hat{s}^a_{-i} = 1l|l, J)\delta W(2l, \lambda^*)]$$

$$+\frac{1}{2}[P(\hat{s}^a_{-i} = 0h|l, J+1)\delta W(l, \lambda_o) - p(\hat{s}^a_{-i} = 2l|l, J+1)\delta W(3l, \lambda_o) > 0 \tag{2}$$

Or if and only if:

$$\delta W(0h, \lambda^*) - (\frac{1}{2} - c) - \frac{P(\hat{s}^a_{-i} = 1l|l, J)}{P(\hat{s}^a_{-i} = 1h|l, J)}\delta W(2l, \lambda^*) + \frac{P(\hat{s}^a_{-i} = 0h|l, J+1)}{P(\hat{s}^a_{-i} = 1h|l, J)}\delta W(l, \lambda_o)$$

$$-\frac{P(\hat{s}^a_{-i} = 2l|l, J+1)}{P(\hat{s}^a_{-i} = 1h|l, J)}\delta W(3l, \lambda_o) > 0 \tag{3}$$

The lhs of this last inequality was computed for a wide range of parameter values (remind that we still work under the assumption that $c \in (1-p, \frac{1}{2}])$. In appendix two a numerical example is detailed where our way of working is explained to the reader. The numerical computations were performed with the aid of a little program which runs in MATLAB which computes the equilibrium strategy of the optimists and the ex ante gain of waiting of player i[19]. The results are shown below:

[Insert here Table One]

[19]The program is not provided in this paper but can be obtained from the author by simple request.

We see that, for all our numerical computations, a pessimistic player prefers to adopt the truthtelling strategy to the lying one.

To understand the intuition of this result it is useful to go back to equation (2). Suppose that J players are interviewed. First note that $\delta W(2l, \lambda^*)$ represents the cost of wrongly reporting l (i.e. reporting l if $\hat{s}^a_{-i} = 1l$). This cost is quite low because if $\hat{s}^a_{-i} = 1l$, player i is already very pessimistic and doesn't expect to learn much out of the waiting game. $\delta W(0h, \lambda^*) - (\frac{1}{2} - c)$ represents the gain of rightly reporting l (i.e. reporting l if $\hat{s}^a_{-i} = 1h$). This gain is often quite substantial because if $\hat{s}^a_{-i} = 1h$, our (former) pessimist expects to hear with probability $\frac{1}{2}$ good news out of the waiting game. In 80% of our simulations $\frac{1}{2}[P(\hat{s}^a_{-i} = 1h|l, J)[\delta W(0h, \lambda^*) - (\frac{1}{2} - c)] - P(\hat{s}^a_{-i} = 1l|l, J)\delta W(2l, \lambda^*)]$ is a positive number. In the other 20% of our simulations, c is relatively low (close to $1 - p$), therefore $(\frac{1}{2} - c)$ is relatively high and the gain of rightly reporting l quite low. Suppose now that J is uneven. As before $\delta W(l, \lambda_o)$ represents the gain of rightly reporting l and $\delta W(3l, \lambda_o)$ represents the cost of wrongly reporting l. In all our numerical simulations $P(\hat{s}^a_{-i} = 0h|l, J+1)\delta W(l, \lambda_o) - p(\hat{s}^a_{-i} = 2l|l, J+1)\delta W(3l, \lambda_o)$ is strictly positive, thereby compensating the eventual negative gain of reporting l to h in the event when J is even.

Table one merely shows that if a pessimist believes that the other players will truthfully reveal their signals, it will be optimal for her to do so too. We still must check whether an optimist - knowing that all the other players will truthfully reveal their signals - doesn't have an incentive to deviate. As previously the reader can easily check that for many configurations of $\hat{s}^a_{-i}$ an optimistic player is indifferent between her two pure strategies. Therefore we directly focus on the same cases as the ones we analysed when player i was a pessimist.

First assume that $\hat{s}^a_{-i} = 1h$. If she reports $\hat{s}_i = h$, pessimists possess too many positive signals, everyone invests in the first period and our player gets $P(H|2h) - c$. If she reports a low signal then pessimists wait and optimists randomise with probability λ^*. However our player possesses two positive signals and is more optimistic than the remaining optimists (who are indifferent between investing and waiting). From lemma 2, we know that she cannot be indifferent between reporting h or l, instead she rather invests in the first period and gets $P(H|2h) - c$. So if $\hat{s}^a_{-i} = 1h$ she is indifferent between reporting a favourable or an unfavourable signal. Next assume that $\hat{s}^a_{-i} = 0h$. If $\hat{s}_i = h$, then $\hat{s}^a = 1h$ and everyone (pessimists as well as optimists) invest in the first period. Player i remains (of course) an optimist, therefore she also invests in period one and she gets $p - c$. If she were to report a low signal, then

$\lambda_p = 0$ and optimists randomise with a probability λ_o. However our player remains an optimist and she rather acts immediately in period one and she gets $p - c$. Hence if $\hat{s}^a_{-i} = 0h$ an optimistic player is also indifferent between her two pure strategies.

Suppose now that $\hat{s}^a_{-i} = 1l$. If she also reports a bad signal, then optimists possess two bad signals along with their good one and don't want to invest anymore. However player i's posterior equals $\frac{1}{2}$ (since she only possesses one bad piece of evidence against her good one). Therefore she invests and gets (in expected terms) $\frac{1}{2} - c$. If she reports her good signal, then the opinion poll is uninformative and optimists randomise with a probability λ^*. In this case she waits and gets $\delta W(0h, \lambda^*) > \frac{1}{2} - c$. Finally we consider the case where $\hat{s}^a_{-i} = 2l$. In this case an optimist also strictly prefers to report her favourable signal because then she gets $\delta W(l, \lambda_o)$ which is higher than zero. So if many players are simultaneously "interviewed" in the communication game an optimist strictly prefers to report her good signal. It's interesting to compare this finding with the one we obtained with only one interviewed guru. In that case the optimist was indifferent between $\hat{s}_g = h$ and $\hat{s}_g = l$. The intuition is similar as the one we explained previously and is also based on the insight that the different players possess different outside options. For example, if $\hat{s}^a_{-i} = 1h$, our player can (just as in the previous case) strategically lie to discourage the pessimists from investing. However this would lead the other optimists to randomise with too low an investment probability. If $\hat{s}^a_{-i} = 1l$, pessimists - independently of player i's report - don't want to invest anyway. In such a case, player i's outside option is lower than the one of the other optimists. Since she doesn't have to bother about the pessimists' actions, she rather reports h to l to let the other optimists invest more.

So far we have shown that, if it is as likely that J is an even or an uneven number a truthtelling equilibrium exists if $c \in (1 - p, \frac{1}{2}]$. This result shows that proposition (1) and proposition (3) are robust to the introduction of many players at the communication stage.

6.3 Strategic information transmission in opinion polls with a high investment cost.

In this section we work under the assumption that $c \in (\frac{1}{2}, p)$. We will see that - in contrast to our previous case where only one player was considered in the communication game - a truthtelling equilibrium also exists in this parameter range.

6.3.1 J is even.

As before for many configurations of $\hat{s}^a_{-i}$ player i will be indifferent between her two pure strategies. Player i will not be indifferent between her two pure strategies if $\hat{s}^a_{-i} = 1h$ or if $\hat{s}^a_{-i} = 1l$.

Assume that $\hat{s}^a_{-i} = 1h$. Assume that player i strategically lies and reports a high signal. The opinion poll contains then two good signals. Pessimists compute $P(H|l, \hat{s}^a = 2h) = p > c$ and everyone invests in period one. Player i computes $P(H|l, \hat{s}^a_{-i} = 1h) = \frac{1}{2}$. Since $\frac{1}{2}$ is lower than c, our player does not invest and she gets zero. Assume now that our player truthfully reports her low signal. In that case the opinion poll is uninformative, $\hat{s}^a = 0h$, our players' posteriors are not affected, optimists randomise with a probability λ^* and player i gets $\delta W(0h, \lambda^*) > 0$.

Suppose now that $\hat{s}^a_{-i} = 1l$. If $\hat{s}_i = l$, the opinion poll contains two bad signals, optimists compute $P(H|l, \hat{s}^a = 2l) = 1 - p < c$ and don't invest. Hence player i gets zero. Suppose she reports $\hat{s}_i = h$. In that case $\hat{s}^a = 0h$ and optimists randomise with probability λ^*. Player i now possesses two unfavourable signals and ex ante she gets $\delta W(2l, \lambda^*) > 0$. To summarise if J is even and if $c \in (\frac{1}{2}, p)$ our player only truthfully reports her low signal if and only if:

$$P(\hat{s}^a_{-i} = 1h|l, J)\delta W(0h, \lambda^*) > P(\hat{s}^a_{-i} = 1l|l, J)\delta W(2l, \lambda^*)$$

6.3.2 J is uneven

In this case only the configurations $\hat{s}^a_{-i} = 2h$ and $\hat{s}^a_{-i} = 0h$ are worth considering. By now the reader must be able to check easily that in all the other configurations of $\hat{s}^a_{-i}$ player i is indifferent between the two pure strategies.

If $\hat{s}^a_{-i} = 2h$, our player rather reports truthfully her low signal. If she reports $\hat{s}_i = h$, everyone invests and she doesn't learn anything. She computes $P(H|l, \hat{s}^a_{-i} = 2h) = p > c$. Hence she invests and she gets $p - c$. If she were to report $\hat{s}_i = l$, then $\hat{s}^a = 1h$, pessimists compute $P(H|l, \hat{s}^a = 1h) = \frac{1}{2} < c$ and refrain from investing. Optimists now possess two favourable signals and invest with an equilibrium probability $\lambda^o > \lambda^*$. Player i possess two favourable and one unfavourable signal. Since $\lambda^o > \lambda^*$ it's optimal for her to wait and ex ante she gets $\delta W(h, \lambda^o) > p - c$.

Suppose that $\hat{s}^a_{-i} = 0h$. If she reports her unfavourable signal, then $\hat{s}^a = 1l$, optimists compute $P(H|h, \hat{s}^a = 1l) = \frac{1}{2} < c$ and don't invest. Since player i does not receive any additional information, she remains a pessimist, she doesn't invest and she gets zero. If she reports $\hat{s}_i = h$, the aggregate informational value of the opinion poll equals one good signal, pessimists compute $P(H|l, \hat{s}^a = 1h) = \frac{1}{2} < c$ and don't invest and optimists invest with probability λ^o. Therefore player i gets $\delta W(l, \lambda^o) > 0$. To summarise: if J is uneven and if $c \in (\frac{1}{2}, p)$ player i only truthfully reports her low signal if and only if:

$$P(\hat{s}^a_{-i} = 2h|l, J)[\delta W(h, \lambda^o) - (p - c)] > p(\hat{s}^a_{-i} = 0h|l, J)\delta W(l, \lambda^o)$$

6.3.3 Uncertainty concerning the number of sampled players.

Suppose it's as likely that J as $J+1$ players are present in the opinion poll. Without loss of generality assume that J is even. From what precedes it should be clear that a pessimistic player only truthfully reports her bad signal if and only if:

$$\delta W(0h, \lambda^*) - \frac{P(\hat{s}^a_{-i} = 1l|l, J)}{P(\hat{s}^a_{-i} = 1h|l, J)}\delta W(2l, \lambda^*) + \frac{P(\hat{s}^a_{-i} = 2h|l, J+1)}{P(\hat{s}^a_{-i} = 1h|l, J)}[\delta W(h, \lambda^o) - (p - c)]$$

$$-\frac{P(\hat{s}^a_{-i} = 0h|l, J+1)}{P(\hat{s}^a_{-i} = 1h|l, J)}\delta W(l, \lambda^o) > 0 \tag{4}$$

The lhs of this last inequality was computed for a wide range of parameter values. The results are shown below:

[Insert here Table Two]

We see again that over our entire parameter range a pessimist always strictly prefers to report her unfavourable signal. The reader can check that - for the same reason as the one we explained previously - an optimist also (strictly) prefers to report her favourable signal.

The intuition is the same as the one we explained when the investment cost is low. Assume first that $J + 1$ players are interviewed. In appendix 2 we show that $\frac{P(\hat{s}^a_{-i}=0h|l,J+1)}{P(\hat{s}^a_{-i}=1h|l,J)} = 1$ and that $\frac{P(\hat{s}^a_{-i}=2h|l,J+1)}{P(\hat{s}^a_{-i}=1h|l,J)} = \frac{J}{J+2}$. Assume that $J = 12$ and thus that $\frac{J}{J+2} = \frac{6}{7}$. A pessimistic player only truthfully reveals her bad signal if $\frac{6}{7}[\delta W(h, \lambda^o) - (p - c)] - \delta W(l, \lambda^o) > 0$. As before $\delta W(l, \lambda^o)$ represents the cost of wrongly reporting l (i.e. reporting l if $\hat{s}^a_{-i} = 0h$). $\delta W(h, \lambda^o) - (p - c)$ represents the gain of rightly reporting l (i.e. reporting l if $\hat{s}^a_{-i} = 2h$). Analytically it's easy to see that $\delta W(h, \lambda^o) > \delta W(l, \lambda^o)$ (since an optimistic player expects to hear more good news out of the waiting game, she faces an ex ante higher value of waiting). If c is close to p then $(p - c)$ is low and the gain of rightly reporting l outweighs its cost. It turns out that in $\pm 50\%$ of our simulations $(p - c)$ is relatively high and thus that $\frac{6}{7}[\delta W(h, \lambda^o) - (p - c)] - \delta W(l, \lambda^o)$ is a negative number. Assume now that J

players are interviewed. In appendix 2 it is shown that $\frac{P(\hat{s}^a_{-i}=1l|l,J)}{P(\hat{s}^a_{-i}=1h|l,J)} = \frac{p^2+(1-p)^2}{2p(1-p)}$. For all our numerical computations $\delta W(0h,\lambda^*) - \frac{p^2+(1-p)^2}{2p(1-p)}\delta W(2l,\lambda^*)$ is a strictly positive number. The gain of truthtelling if J is even is that high that it always compensates the possible loss of truthtelling if J is uneven. Our main conclusion is summarised below:

PROPOSITION 4 *When $J \geq 12$ players are simultaneously asked to report their signals and if it's unknown whether J or $J+1$ players are present in the opinion poll, in the high activity equilibrium and for all our numerical computations with $c \in (1-p,p)$ there exists a truthtelling equilibrium.*

7 Discussion and conclusions.

In this paper we introduced cheap talk in an investment model with information externalities. If investors focus on the Pareto-dominated equilibrium in the waiting game (if it exists), then one player has a lot of incentives to truthfully reveal her signal. This insight is robust if many investors are asked their signals in an opinion poll because everyone acts in the hope to be the "pivotal" player who influences the investment behaviour of the other players. In the presence of competition effects, no information can be transmitted via cheap talk. In the presence of small positive network externalities under mild additional assumptions and in a certain parameter range the unique equilibrium is the truthtelling one. We believe proposition (1), (3) and result (4) to be interesting because they show the existence of a truthtelling equilibrium despite the fact that both senders' types share similar preferences over the receivers' actions. This paper also provides a first attempt to introduce competition effects and positive network externalities in a model of investment with information externalities. We believe this still constitutes an avenue for future research.

Finally, we also want to discuss some important implications of result (4). CG assumed that $1-p<c<p$. Given this assumption it is natural to focus on the low activity equilibrium in the waiting game. However this paper shows that even if at the onset $1-p<c$, one may not analyse the efficiency of cheap talk by focusing only on the low activity equilibrium. For, it is possible that, after the communication stage, even pessimists have incentives to invest. Section seven shows then that $\forall c \in (1-p,p)$ the mere threat that the high activity equilibrium will be triggered suffices to give everyone enough incentives to truthfully reveal their type. If the opinion poll is large (excluding extremely unlikely sampling errors) opinion polls achieve first best. We can draw two lessons out of this finding.

First, it remains to be seen whether Chamley (1997) is robust to the introduction of cheap talk. That paper considers a continuum of agents endowed with a continuum of signals and contains interesting insights concerning the rate of convergence of beliefs under both equilibria. Admittedly, we obtain a truthtelling equilibrium because we only allow reports to be $\in \{h, l\}$. For example we should allow the pessimist to report no signal, i.e. it should be possible for her to say that she has no opinion on that matter. Clearly, if beliefs are updated under the assumption of truthtelling, a pessimist prefers to report no signal to her low one, as this leaves the equilibrium strategies of the optimists unaffected (and pessimists remain pessimists and don't want to invest). However we cannot enrich the action space of player i without enriching our waiting game either! How can a message $\hat{s}_i = \phi$ be believed if we only allow for optimists and pessimists in the waiting game? Nonetheless we believe that even in Chamley's context a lot of useful information can be generated via cheap talk (at least for a large range of parameter values). Future research may shed some light on this issue.

Second, CG is robust to the introduction of cheap talk. However in a lot of countries opinion polls exist which ask to entrepreneurs how favourable they think the investment climate is and whether they intend to invest or not. The mere existence of this "market for signals" proves its informativeness. Obviously, CG have a hard time explaining where these opinion polls come from. This paper shows that in CG's model people have much more incentives to truthfully reveal their signals than one might a priori expect. Hence we believe that CG are focusing on the wrong equilibrium when they describe investment behaviour there where we also observe the existence of opinion polls.

Appendix

Proof of lemma one:

We present the proof under the assumption that $s_i = h$. The other case where $s_i = l$ is fully symmetric and is left to the reader by means of an exercise. First note that:

$$P(H|h, k, \lambda) = \frac{pb(k; N_H - 1, \lambda)}{pb(k; N_H - 1, \lambda) + (1-p)b(k; N_L - 1, \lambda)}$$

where $b(k; N_H - 1, \lambda)$ represents the probability of k investments in $N_H - 1$ independent Bernoulli trials with a probability λ. It follows that $P(H|h, k_2, \lambda) > P(H|h, k_1, \lambda)$ iff:

$$\frac{b(k_2; N_H - 1, \lambda)}{b(k_2; N_L - 1, \lambda)} > \frac{b(k_1; N_H - 1, \lambda)}{b(k_1; N_L - 1, \lambda)}$$

Replacing all $b(\cdot)$ by their analytical counterparts[20], after simplification, we obtain that the above inequality is always true iff:

$$(N_H - 1 - k_1)(N_H - 2 - k_1)...(N_L - k_1) > (N_H - 1 - k_2)(N_H - 2 - k_2)...(N_L - k_2)$$

Both the rhs and the lhs count $N_H - N_L - 1$ terms. As $k_2 > k_1$, the above inequality is always respected, which proves our lemma. Q.E.D.

<u>Proof of lemma two:</u>

First note that if $\delta < 1$, $\underline{k}^* > 0$. By contradiction assume that $\underline{k}^* = 0$. This implies that $A(h, \lambda) = W(h, \lambda)$. But then $A(h, \lambda) > \delta W(h, \lambda)$. Hence $\underline{k}^*$ cannot be equal to zero. Out of (1) we know that:

$$p - c = \delta p \sum_{k \geq \underline{k}^*} [b(k; N_H - 1, \lambda^*)(1 - c) + b(k; N_L - 1, \lambda^*)c] - \delta \sum_{k \geq \underline{k}^*} b(k; N_L - 1, \lambda^*)c$$

For λ^* to be increasing with p it must be that $\frac{\delta}{\delta p}\delta W(h, \lambda^*) < 1$. From this last equality we can easily compute that:

$$\text{(5)} \qquad \delta W(h, \lambda^*)' = \delta[(1 - c) \sum_{k \geq \underline{k}^*} b(k; N_H - 1, \lambda^*) + c \sum_{k \geq \underline{k}^*} b(k; N_L - 1, \lambda^*)]$$

Since $\sum_k b(k; N_H - 1, \lambda) = 1$ and since $\underline{k}^* > 0$, we know that $\sum_{k \geq \underline{k}} b(k; N_H - 1, \lambda) < 1$. Therefore (5) represents a weighted average of two numbers strictly lower than one. Therefore $\delta W(h, \lambda^*)' < 1$ $\forall \lambda^*$ and $\forall \delta < 1$. Q.E.D.

<u>Formal exposition of our equilibria under A3.</u>

We first compute $W(h, \lambda)$. Suppose an optimistic player doesn't invest in period one and observes $k = 0$. Our player then computes $P(H|h, 0, \lambda)$. If our player anticipates that no other player will invest in period two, then, if she invests, she gets $P(H|h, 0, \lambda)\gamma - c$. There are two possibilities: $P(H|h, 0, \lambda)\gamma \leq c$ and $P(H|h, 0, \lambda)\gamma > c$. If $P(H|h, 0, \lambda)\gamma \leq c$, everyone, after observing $k = 0$, gets zero. Consider the other case $P(H|h, 0, \lambda)\gamma > c$. The analysis of that case is simplified by the following lemma:

<u>lemma 3:</u> $P(H|h, 0, \lambda^*) < c$.
Proof: by contradiction, assume that $P(H|h, 0, \lambda^*) \geq c$. Then $A(h, \lambda^*)$ can impossibly be equal to $\delta W(h, \lambda^*)$, because under that assumption:

$$A(h, \lambda^*) = P(k = 0|h, \lambda^*)[P(H|h, 0, \lambda^*)\gamma - c] + \sum_{k>0} [P(H|h, k, \lambda^*) - c]P(k|h, \lambda^*)$$

[20]Remind that, f.i., $b(k_2; N_H - 1, \lambda) = \frac{(N_H-1)!}{k_2!(N_H-1-k_2)!}\lambda^{k_2}(1 - \lambda)^{N_H-1-k_2}$.

$$W(h,\lambda^*) = P(k=0|h,\lambda^*)[P(H|h,0,\lambda^*)\gamma - c] + \sum_{k>0}[P(H|h,k,\lambda^*) - c]P(k|h,\lambda^*)$$

As $\gamma > 1$ and $\delta < 1$, $A(h,\lambda^*) > \delta W(h,\lambda^*)$. Q.E.D.

As, in equilibrium, $P(H|h,0,\lambda) < c$ we know that in the event that $k=0$, all optimists play a mixed-strategy Nash equilibrium in the second period and invest with a symmetric probability. In a mixed-strategy Nash equilibrium everyone must be indifferent between the two pure strategies and therefore all optimists, after observing $k=0$, whether they decide to invest in period two or not, get zero. Therefore $W(h,\lambda)$ remains identical to the one of our previous case.

The ex ante payoff of acting given that the other optimists randomise with probability λ equals:

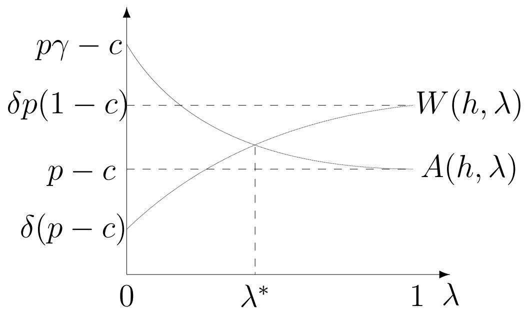
(6) $$A(h,\lambda) = p - c + p(1-\lambda)^{N_H - 1}(\gamma - 1)$$

which is continuous and (strictly) decreasing in λ. This is logical: the lower λ, the higher the probability that a period-one investor will end up being the only one to enter the market, the higher the ex ante gain of investing.

$A(h,0) = p\gamma - c > W(h,0) = p - c$ and $A(h,1) < W(h,1)$. By continuity there exists a unique symmetric equilibrium strategy in which $\lambda_p^* = 0$ and $\lambda^* \in (0,1)$.

Graphically, the low activity equilibrium can be represented as follows:

Graph 1: Low activity equilibrium with a first mover advantage



Using a reasoning similar to the one we did previously, the reader can also check that if $c \le 1 - p$, there also exists a high activity equilibrium where $\lambda_p^* = \lambda^* = 1$.

Finally, note that lemma 2 remains valid here as long as $A(h,\lambda^*)' > W(h,\lambda^*)'$. From (6) one can easily see that $A(h,\lambda^*)' > 1 = W(h,\lambda^*)'$ (from lemma 2). Therefore, whenever $\lambda_p^* = 0$, λ^* remains an increasing function of p. Q.E.D.

Formal exposition of our equilibria under A3'.

Call $\Xi = \{(p, c, \delta)$:A1 and A2 are satisfied$\}$. $\forall$ (p,c) which satisfy A1, we define $\Delta(p, c) = (\frac{p-c}{p(1-c)}, 1]$. $\mu(\Delta)$ represents the Lebesgue measure of Δ. With positive network externalities it is no longer true that an equilibrium strategy λ^* always exists. Therefore we also introduce the following definition: $\Xi^{\phi} = \{(p, c, \delta) \in \Xi : \lambda^*$ does not exist$\}$. $\Xi^{-\phi} = \{(p, c, \delta) \in \Xi : \lambda^*$ exists$\}$.

$W(h, \lambda)$ remains unaffected in this case. To see this suppose all optimists randomise in period one with the symmetric equilibrium probability λ and consider player j who is an optimist and who waited in period one. Suppose $k = 0$. In that case there are two possibilities: (i) $P(H|h, 0, \lambda)\gamma \geq c$ and (ii) $P(H|h, 0, \lambda)\gamma < c$. In (i) investing (in period two) is a dominant strategy for every single investor. Therefore all optimists invest in the second period and they all get $P(H|h, 0, \lambda) - c$. In other words in (i) an optimist knows that she is never going to invest alone in the second period. Therefore she doesn't have to bother about network externalities, and her choice is identical to the one she would have made in our benchmark model. In (ii) there are also two possibilities: (a) $P(H|h, 0, \lambda) \geq c$ and (b) $P(H|h, 0, \lambda) < c$. In (b) no optimist who waited in period one invests in period two. In (a) there are two stable equilibria: one where all optimists invest and one where they all abstain[21] This issue of multiple equilibria is not an important one, what really matters is that all optimists always act together in the second period, thereby realising all gains from network externalities. Therefore a waiting optimist never bothers about network externalities and her payoff is identical to the one of our benchmark case.

Henceforth we call $A_1(h, \lambda)$ the gain of acting given that $\underline{k} \leq 1$. $A_2(h, \lambda)$ denotes the gain of acting given that $\underline{k} \geq 2$. $A(h, \lambda) = A_1(h, \lambda) \cup A_2(h, \lambda)$. Assume $\underline{k} \leq 1$ (remind that by definition this means that $P(H|h, k = 1, \lambda) \geq c$). Suppose player j decides to act in period one and suppose he's the only one to do so. The other optimists (who all waited) observe one investment and compute $P(H|h, 1, \lambda) \geq c$. Therefore they all invest in period two and player j gets $A_1(h, \lambda) = p - c$. Suppose now that $\underline{k} \geq 2$. In that case $A_2(h, \lambda)$ becomes:

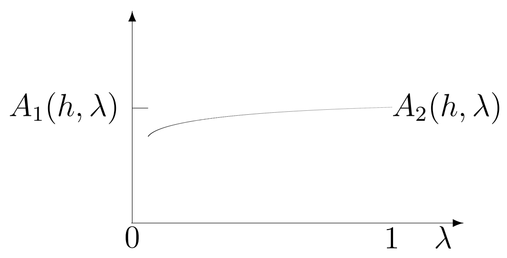
$$A_2(h, \lambda) = \sum_{i=0}^{N_H - 1} P(H, k = i|h, \lambda) + P(H, k = 0|h, \lambda)(\gamma - 1) - c$$

[21]There also exists a third equilibrium where the optimists randomise such as to be indifferent between the two pure strategies. However that third equilibrium is characterised by strategic complementarity and is unstable.

Note that both $A_1(h, \lambda)$ and $A_2(h, \lambda)$ are continuous in λ. So *keeping $\underline{k}$ fixed*, $A(h, \lambda)$ is a continuous function $\forall \underline{k}$. However at the probability λ^2 where $P(H|h, 2, \lambda^2) = c$ (or in words at the probability λ^2 where $\underline{k}$ switches from 1 to 2) $A(h, \lambda)$ is not a continuous function because at that probability $lim_{\lambda \to \lambda^2} A_1(h, \lambda) \neq lim_{\lambda^2 \leftarrow \lambda} A_2(h, \lambda)$. Intuitively this discontinuity is due to the fact that $\underline{k}$ is increasing in λ. If λ is very low, then an optimist knows that if he invests, this will be interpreted as good news by the other optimists who will then be induced to invest in the second period as well. However if λ is not very low then an optimist knows that if he happens to be the only player to invest at time one, that this will not suffice to induce the remaining optimists to invest in the second period as well. In that case the optimist "only" gets γ and not one if $\Theta = H$.

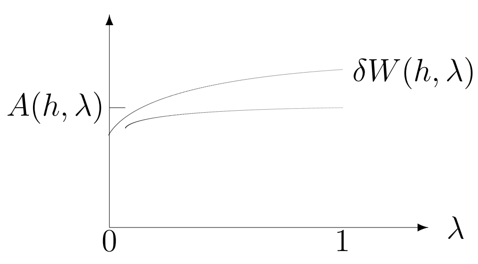
Graphically we obtain the following:

Graph 2: Gain of investing with positive network externalities



This form of $A(h, \lambda)$ is cumbersome because it entails three difficulties. First it is not sure whether a symmetric low activity equilibrium still exists. As we showed in our third section, Chamley and Gale's existence and uniqueness theorem basically rests on a continuity argumentation. If the functions are not continuous anymore we might run into inexistence-type problems as the graph below shows:

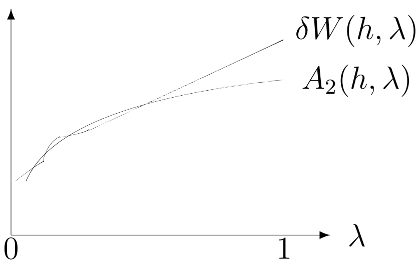
Graph 3: Inexistence of a low activity equilibrium with positive network externalities



Second, with positive network externalities it is no longer sure whether lemma two still holds. Lemma two basically states that more optimistic players produce more information (i.e. invest with a higher equilibrium probability λ^* at time one). This was proven by showing that $p-c$ increases faster in p than $\delta W(h,\lambda)$. For high enough a λ, the gain of investing equals $A_2(h,\lambda)$ and $A_2(h,\lambda)' < 1$. It is then not sure whether optimistic players want to produce more information.

Finally, one cannot rule out the issue of multiple equilibria. As both $A_2(h,\lambda)$ and $\delta W(h,\lambda)$ are increasing in λ, we may a priori not rule out the case where $A_2(h,\lambda)$ and $\delta W(h,\lambda)$ cross each other several times as the graph below shows:

Graph 4: Multiple low activity equilibria with positive network externalities



However, our next proposition shows that, as $\gamma \to 1$, (i) there almost always exists a PBE, (ii) if a PBE exists, it is unique and (iii) lemma two remains valid.

Proposition 5 *Under A1, A2 and A3':*
(i) $\exists \tilde{\Delta}(\tilde{p},\tilde{c}) \subset \Delta(\tilde{p},\tilde{c}) : \forall \delta \in \tilde{\Delta}(\tilde{p},\tilde{c}), (\tilde{p},\tilde{c},\delta) \in \Xi^{\phi}$*. If* $\gamma \to 1$, $\mu(\tilde{\Delta}) \to 0$,
(ii) If $\gamma \to 1$*, if an equilibrium strategy* λ^* *exists, it is unique,*
(iii) If $\gamma \to 1$*, if* $\delta < 1$*, if* λ^* *exists, it is also increasing in* p*.*

Proof: Before proving these three claims, we first rewrite $\gamma \to 1$ as $\gamma = 1-\epsilon$ where $\epsilon > 0$.
We first tackle point (ii):

Suppose there exists an equilibrium with $\underline{k}^* \leq 1$, which means that $p-c = \delta W(h,\lambda^*)$. From CG we know that in that case the equilibrium is unique. Suppose there exists an equilibrium with $\underline{k}^* \geq 2$, which means that $A_2(h,\lambda^*) = \delta W(h,\lambda^*)$. Equation $A_2(h,\lambda)$ as well as $\delta W(h,\lambda)$ are increasing in λ. A sufficient condition (though it's not a necessary one) for issues of multiple equilibria not to arise is that $\forall (p,c,\delta) \in \Xi, \forall \lambda \in [0,1]$, $\frac{\delta}{\delta\lambda} A_2(h,\lambda) < \frac{\delta}{\delta\lambda}\delta W(h,\lambda)$. Call max the maximal value take by $\frac{\delta}{\delta\lambda} A_2(h,\lambda)$, $\forall (p,c,\delta) \in \Xi, \forall \lambda \in [0,1]$. Call min the minimal value taken by $\frac{\delta}{\delta\lambda}\delta W(h,\lambda)$, $\forall (p,c,\delta) \in$

$\Xi, \forall\lambda \in [0,1]$. If max < min then an equilibrium strategy λ^*, if it exists, is unique. We know that:

$$\frac{\delta}{\delta\lambda}\delta A_2(h,\lambda) = (N_H - 1)p(1-\lambda)^{N_H-2}(1-\gamma) > 0$$

As $\lambda = 0$ and as $p \to 1$, max $= (N_H - 1)(1-\gamma)$. Unfortunately we cannot analytically compute $\frac{\delta}{\delta\lambda}\delta W(h,\lambda)$, we only know (from Blackwell's Theorem) that it is strictly positive. max is a function of γ. If $\gamma = 1$, max < min. Call $1 - \epsilon_1$ the value of γ such that min=max. If $\epsilon < \epsilon_1$, then max<min and issues of multiple equilibria do not arise.

We next tackle point (iii):

If $\underline{k}^* \leq 1$, the proof is identical to the one we saw in lemma 2. Suppose that $\underline{k}^* \geq 2$. In lemma 2 we have shown that if $\delta < 1$, $\delta W(h,\lambda^*)' < 1, \forall\lambda^*$. We also know that:

$$\frac{\delta}{\delta p}A_2(h,\lambda) = 1 - (1-\lambda)^{N_H-1}(1-\gamma) < 1$$

A sufficient (though not necessary) condition for λ^* to be increasing in p is that $\forall(p,c,\delta) \in \Xi$, $\forall\lambda \in [0,1]$, $\forall\delta < 1$[22], $\frac{\delta}{\delta p}\delta W(h,\lambda) < \frac{\delta}{\delta p}A_2(h,\lambda)$. Unfortunately we cannot analytically compute the first derivative as λ influences both $W(h,\lambda)$ and $\underline{k}$. But we know that if $\delta < 1$, $\delta W(h,\lambda)' < 1$ (over our entire parameter range). Call max' the greatest value taken by $\delta W(h,\lambda)'$ over the parameter range $(p,c,\delta) \in \Xi$, $\forall\lambda \in [0,1]$, $\forall\delta < 1$. Call min' the minimal value taken by $A_2(h,\lambda)$, $\forall(p,c,\delta) \in \Xi$, $\forall\lambda \in [0,1]$. min' is a function of γ. If $\gamma = 1$, min' = 1 > max'. Call $1 - \epsilon_2$ the value of γ such that: min' = max'. So if ϵ is strictly lower than ϵ_2, min' > max'.

Finally we consider point (i):

First we assume that $\epsilon < min\{\epsilon_1, \epsilon_2\}$. Fix any arbitrary $(\tilde{p},\tilde{c})$ which satisfy A1. Note that $\forall\delta \in \Delta(\tilde{p},\tilde{c}), \delta W(h,0) \leq A(h,0)$ and $\delta W(h,1) > A(h,1)$. Moreover $\delta W(h,\lambda)$ is continuous in λ. If $A(h,\lambda)$ were also a continuous function in λ, then this would automatically prove our existence theorem. However $A(h,\lambda)$ is characterised by one discontinuity point at λ^2. From (ii) we know that if $\gamma \to 1$, λ^* does not exist if and only if $A_1(h,\lambda^2) > \delta W(h,\lambda^2) > A_2(h,\lambda^2)$. Note that $\delta W(h,\lambda)$ shifts continuously upwards with every increase in δ. If $\delta = \frac{\tilde{p}-\tilde{c}}{\tilde{p}(1-\tilde{c})}, \lambda^* = 1$ and $\delta W(h,\lambda^2) < A_2(h,\lambda^2)$. If $\delta = 1$, $\lambda^* = 0$ and $\delta W(h,\lambda^2) > A_1(h,\lambda^2)$. By continuity $\forall(\tilde{p},\tilde{c})$ which satisfy A1, $\mu\tilde{\Delta}(\tilde{p},\tilde{c}) \neq 0$. As $\gamma \to 1$, $A_2(h,\lambda^2) \to A_1(h,\lambda^2)$ and $\mu(\tilde{\Delta}) \to 0$. Q.E.D.

[22] If we would allow δ to equal one then $\delta W(h,0)' = 1$ which is higher than $A_2(h,0)' = \gamma$.

Proof of point (ii) of proposition 3

Assume that with a probability $(1-\epsilon_0)$, $\beta(\cdot)$ is computed under the assumption that $x = y$ (where $x \in (0,1)$). However, with a probability ϵ_0, $\beta(\cdot)$ is computed under the assumption that $x = 1$ and $y = 0$. Think of ϵ_0 as an arbitrarily small strictly positive number. This cannot constitute an equilibrium communication strategy. To see this assume that player i is a pessimist. If she were to report signal h with probability $x \in (0,1)$ then all players compute $P(H|s_j = \cdot, \hat{s}_i = \cdot, x = y) = P(H|s_j)$. As $c \leq 1-p$, by assumption everyone invests and player i gets $1-p-c$. However, if she were to report $\hat{s}_i = l$ with probability one then ex ante with a probability $(1-\epsilon_0)$ she would also get $1-p-c$, while with a probability ϵ_0 she would get $\delta W(l, \lambda_o) > 1-p-c$.

Assume now that with a probability $(1-\epsilon_0)$, $\beta(\cdot)$ is computed under the assumption that $x = y = 1$ (with a probability ϵ_0, $\beta(\cdot)$ is still computed under the assumption that $x = 1$ and $y = 0$). Assume player i is a pessimist. Suppose she reports $\hat{s}_i = l$. In that case $P(H|\cdot, \hat{s}_i = l, x = y = 1)$ doesn't exist. Under ER all players believe that she is a pessimist and player i gets $\delta W(l, \lambda_o) > 1-p-c$. Hence $x = y = 1$ can't be an equilibrium strategy either. For similar reasons $x = y = 0$ can also not be an equilibrium strategy because the pessimist has now an incentive to report signal h.

Hence, under ER and if there exists a probability ϵ_0 that all players will revise their priors under the assumption that $x = y = 1$, no pooling equilibria in the communication game exist. We now show by contradiction that an informative communication strategy where with a probability equal to one $\beta(\cdot)$ is computed under the assumption that $x \geq y$, and where $x \neq 1$ and/or $y \neq 0$ cannot be an equilibrium strategy either. We know that in equilibrium $P(H|l, \hat{s}_i = h, x \geq y) \geq c$ otherwise a pessimist strictly prefers to report h to l. If a pessimist reports $\hat{s}_i = l$, optimists compute $P(H|h, \hat{s}_i = l, x \geq y)$ and randomise with probability , say $\hat{\lambda}$ $(0 < \hat{\lambda} < \lambda^*)$. If she were to report $\hat{s}_i = h$, then everyone would invest and she would get $1-p-c$. As $1-p-c < \delta W(l, \hat{\lambda})$ a pessimist cannot be indifferent reporting h or l. Hence $y^* = 0$. $x^* < 1$ iff optimists are willing to randomise between reporting h or l. But this contradicts our finding that $p - c > A_2(h, \lambda_o)$.

Appendix Two

In this appendix we detail our way of working when a pessimist's gain of truthfully revealing her bad signal is numerically computed. We first simplify equation (3) by computing analytically the different weights. We start with $\frac{P(\hat{s}_{-i}=1l|l,J)}{P(\hat{s}_{-i}=1h|l,J)}$.

$$P(\hat{s}_{-i} = 1l|l, J) = pC_{J-1}^{\frac{J}{2}} p^{\frac{J}{2}} (1-p)^{\frac{J}{2}-1} + (1-p) C_{J-1}^{\frac{J}{2}} p^{\frac{J}{2}-1} (1-p)^{\frac{J}{2}}$$

$$= C_{J-1}^{\frac{J}{2}} p^{\frac{J}{2}-1}(1-p)^{\frac{J}{2}-1}[p^2+(1-p)^2]$$

$$P(\hat{s}_{-i}=1h|l,J) = (1-p)C_{J-1}^{\frac{J}{2}} p^{\frac{J}{2}}(1-p)^{\frac{J}{2}-1} + pC_{J-1}^{\frac{J}{2}} p^{\frac{J}{2}-1}(1-p)^{\frac{J}{2}}$$

$$= 2C_{J-1}^{\frac{J}{2}} p^{\frac{J}{2}}(1-p)^{\frac{J}{2}}$$

Therefore, $\frac{P(\hat{s}_{-i}=1l|l,J)}{P(\hat{s}_{-i}=1h|l,J)} = \frac{p^2+(1-p)^2}{2p(1-p)}$ and is independent of J. Next we compute $\frac{P(\hat{s}_{-i}=0h|l,J+1)}{P(\hat{s}_{-i}=1h|l,J)}$:

$$P(\hat{s}_{-i}=0h|l,J+1) = (1-p)C_J^{\frac{J}{2}} p^{\frac{J}{2}}(1-p)^{\frac{J}{2}} + pC_J^{\frac{J}{2}} p^{\frac{J}{2}}(1-p)^{\frac{J}{2}}$$

Therefore,

$$\frac{P(\hat{s}_{-i}=0h|l,J+1)}{P(\hat{s}_{-i}=1h|l,J)} = \frac{C_J^{\frac{J}{2}}}{2C_{J-1}^{\frac{J}{2}}} = \frac{\frac{J!}{\frac{J}{2}!\frac{J}{2}!}}{\frac{2(J-1)!}{\frac{J}{2}!(\frac{J}{2}-1)!}} = 1$$

which is also independent of J. Finally we compute $\frac{P(\hat{s}_{-i}=2l|l,J+1)}{P(\hat{s}_{-i}=1h|l,J)}$:

$$P(\hat{s}_{-i}=2l|l,J+1) = pC_J^{\frac{J}{2}+1} p^{\frac{J}{2}+1}(1-p)^{\frac{J}{2}-1} + (1-p){C_J}^{\frac{J}{2}+1} p^{\frac{J}{2}-1}(1-p)^{\frac{J}{2}+1}$$

$$= {C_J}^{\frac{J}{2}+1} p^{\frac{J}{2}-1}(1-p)^{\frac{J}{2}-1}[p^3+(1-p)^3]$$

Therefore,

$$\frac{P(\hat{s}_{-i}=2l|l,J+1)}{P(\hat{s}_{-i}=1h|l,J)} = \frac{{C_J}^{\frac{J}{2}+1}[p^3+(1-p)^3]}{2C^{\frac{J}{2}}{}_{J-1}p(1-p)} = \frac{J[p^3+(1-p)^3]}{(J+2)p(1-p)}$$

We replace $\frac{P(\hat{s}_{-i}=2l|l,J+1)}{P(\hat{s}_{-i}=1h|l,J)}$ in (3) by its heighest possible value namely by $\frac{p^3+(1-p)^3}{p(1-p)}$. A sufficient (though not necessary) condition for truthtelling $\forall J \geq 2$ is thus that the following inequality must be respected:

$$\delta W(0h,\lambda^*) - (\frac{1}{2}-c) - \frac{p^2+(1-p)^2}{2p(1-p)}\delta W(2l,\lambda^*) + \delta W(l,\lambda_o) - \frac{p^3+(1-p)^3}{p(1-p)}\delta W(3l,\lambda_o) > 0$$

The lhs of this inequality was numerically computed for different values of our exogenous parameters. For example assume that $p = 0.6$, $c = 0.42$ and $\delta = 0.8$. We computed the different equilibrium probabilities and the different ex ante gains of waiting. In this example $\lambda^* = 0.17$ and $\lambda_o = 0,0383$. $\delta W(l,\lambda_o) = 0,03548$, $\delta W(3l,\lambda_o) = 0,003836$, $\delta W(0h,\lambda^*) = 0,1289$ and $\delta W(2l,\lambda^*) = 0,05277$. If J is even then: $0,1289 - 0,08 - 1.08333.0,05277 = -0,00827$. This number is negative which means that in this example if J is commonly known to be even there is no truthtelling equilibrium. However if it is equiprobable that J or $J+1$ players are interviewed, then a pessimistic player must add $0,03548 - 1,1666.0,003838 = 0,031$ to that negative number. $0,031 - 0,00827 = 0,023$ is a positive number. Hence in this example pessimists want to truthfully reveal their bad signals.

We now detail our way of working when $c \in (\frac{1}{2}, p)$. Equation (4) also represents a weighted average of different gains of waiting. We first simplify $\frac{P(\hat{s}_{-i}=2h|l,J+1)}{P(\hat{s}_{-i}=1h|l,J)}$. We know that $P(\hat{s}_{-i} = 2h|l, J+1) = C_J{}^{\frac{J}{2}-1} p^{\frac{J}{2}} (1-p)^{\frac{J}{2}}$. Therefore:

$$\frac{P(\hat{s}_{-i} = 2h|l, J+1)}{P(\hat{s}_{-i} = 1h|l, J)} = \frac{C_J{}^{\frac{J}{2}-1}}{2C_{J-1}^{\frac{J}{2}}} = \frac{J}{J+2}$$

Assume that $J = 12$[23]. In that case a truthtelling equilibrium $\forall J \geq 12$ if $c \in (\frac{1}{2}, p)$ exists if the following inequality is respected:

$$\delta W(0h, \lambda^*) - \frac{p^2 + (1-p)^2}{2p(1-p)} \delta W(2l, \lambda^*) + \frac{6}{7}[\delta W(h, \lambda^o) - (p-c)] - \delta W(l, \lambda^o) > 0$$

This inequality was computed for different values of our exogenous parameters. The results are summarized in table two. For example if $p = 0,7$, $c = 0,65$ and $\delta = 0,9$, then $\lambda^* = 0,0066$ and $\lambda^o = 0,055$. Equation (4) then becomes: $0,007443 - 1,381.0,000049 + \frac{6}{7}[0,125 - 0,05] - 0,023 \simeq 0,0487$ which is the number reported in the table. This last number is positive which indicates that a pessimist strictly prefers to report her low signal.

[23] If J would be less than 12, then we do not obtain the existence of a truthtelling equilibrium for our entire parameter range.

Table 1:

Computation of the gain of truhtful reporting for a pessimist in an opinion poll (in procentual terms).

	$\delta = 0.9$					$\delta = 0.8$				
c^p	0.55	0.6	0.7	0.8	0.9	0.55	0.6	0.7	0.8	0.9
0.48	1.6	3.3	7.4	12.4	0.34	1.5	3.5	8	13	(**)
0.42	(*)	2.7	5.9	10.2	(**)	(*)	2.3	6.1	(**)	(**)
0.35	(*)	(*)	5.1	8.59	(**)	(*)	(*)	4.6	(**)	(**)
0.25	(*)	(*)	(*)	(**)	(**)	(*)	(*)	(**)	(**)	(**)

	$\delta = 0.7$					$\delta = 0.6$				
c^p	0.55	0.6	0.7	0.8	0.9	0.55	0.6	0.7	0.8	0.9
0.48	1.3	3.4	7.8	(**)	(**)	1.2	3	(**)	(**)	(**)
0.42	(*)	1.8	3.7	(**)	(**)	(*)	0.92	(**)	(**)	(**)
0.35	(*)	(*)	(**)	(**)	(**)	(*)	(**)	(**)	(**)	(**)
0.25	(**)	(**)	(**)	(**)	(**)	(**)	(**)	(**)	(**)	(**)

If a number in the table is strictly positive, this means that a pessimist strictly prefers to truthfully report her signal if $J \geq 2$ and if she believes the other players will do so too for the given set of exogenous variables.

(*) A1 not respected,

(**) A2 not respected.

Table 2:

Computation of the gain of truhtful reporting for a pessimist in an opinion poll (in procentual terms).

	$\delta = 0.9$					$\delta = 0.8$				
c^{p}	0.55	0.6	0.7	0.8	0.9	0.55	0.6	0.7	0.8	0.9
0.75	(*)	(*)	(*)	6.33	7.74	(*)	(*)	(*)	6.14	6.39
0.65	(*)	(*)	4.87	6.46	10.2	(*)	(*)	4.47	6.03	9.67
0.58	(*)	2.82	4.44	7.84	12.7	(*)	2.6	4.15	7.16	(**)
0.52	1.02	2.06	4.83	9.01	14.8	0.89	1.71	3.95	7.6	(**)
	$\delta = 0.7$					$\delta = 0.6$				
c^{p}	0.55	0.6	0.7	0.8	0.9	0.55	0.6	0.7	0.8	0.9
0.75	(*)	(*)	(*)	5.05	5.95	(*)	(*)	(*)	4.04	(**)
0.65	(*)	(*)	4.07	5.42	(**)	(*)	(*)	3.25	4.33	(**)
0.58	(*)	2.45	3.46	5.67	(**)	(*)	1.91	2.47	(**)	(**)
0.52	0.77	1.07	2.69	(**)	(**)	0.4	0.15	0.95	(**)	(**)

If a number in the table is strictly positive, this means that a pessimist strictly prefers to truthfully report her signal if $J \geq 12$ and if she believes the other players will do so too for the given set of exogenous variables.
(*) A1 not respected,
(**) A2 not respected.

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