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On Bid Disclosure in OCS Wildcat Auctions*

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Abstract

I study a game in which two players first bid for offshore tracts (below which oil and gas may be present) and next time their drilling decisions. High types bid more aggressively if the auctioneer discloses bids as this gives them useful information about the profitability of drilling. A low type fears that the disclosure of her "low" bid reduces the other player's incentive to drill. Hence, they bid more aggressively if the auctioneer does not disclose bids. If players are sufficiently patient, it is optimal to disclose bids. Otherwise, it may be optimal not to disclose them.

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1 Introduction

In recent years, some countries decided to put part of their offshore oil and gas reserves under the hammer. Brazil, Cuba, Libya, Nigeria, Russia, and the U.S., for example, organized offshore oil and gas auctions in the past decade. Those auctions often generate huge revenues and secure the supply of crucial energy resources. In a seminal paper, Hendricks and Porter (1996) argued that offshore drilling suffers from a public good problem: If a firm drills and finds oil (and gas), this is costlessly observed by other firms. They also argued that firms do *not* coordinate their drilling decisions, i.e. firms typically play a war of attrition to determine who will drill first. Comparing auctions in this context is thus a delicate matter as the post-auction war of attrition should influence bidding behavior (and vice versa).

In this paper I develop a two-unit, two-player bidding-drilling game to compare the performance of two different oil and gas auctions. My game tackles the following questions: Should oil and gas auctions be designed such that players learn other players' private information through their bids? Or should one use an auction in which a player's private information is only partially revealed after the auction? To understand my game, suppose both players end up owning a tract. Both tracts are assumed to possess the same (common) value, which is unknown to both players. Both players can then drill in two periods. If player i drills in the first investment period, both players observe whether oil and gas is present or not. In case player $-i$ waited (in the first investment period), she then takes a riskless drilling decision in the second investment period. Waiting thus yields an informational benefit but comes at the cost of discounting. Prior to this waiting game, players participate in an auction. I consider two different auction designs. In the first one (which closely matches the one used by the U.S. government), the auctioneer uses a first-price sealed bid auction and, prior to the waiting game, discloses both players' bids. In the second one, the auctioneer also uses a first-price sealed bid auction but does not disclose each player's bid. Instead, the auctioneer merely announces which tracts were won by which players.

I first show that my game with bid disclosure yields a unique equilibrium (within the class of the strongly symmetric strategies[1]) provided signals are sufficiently precise.[2] In that equilibrium, a high-type player[3] bids "high" (with probability one) while a low-type player bids "low"

[1]Bluntly stated, a strongly symmetric strategy is a symmetric strategy with the added requirement that if both players possess the same posterior at the start of the waiting game, they drill with the same probability (i.e. they play a war of attrition).

[2]In section 2, I will argue that recent developments in seismic technology considerably increased the precision of seismic tests. Hence, the "sufficiently-precise" signal case is not as unrealistic as one may a priori believe.

[3]A player is said to be "high-type" if she is "confident" about her prospects of finding oil and gas. A low-type

(with probability one). Next, I show that my game without bid disclosure also yields a unique equilibrium (within the class of the symmetric and monotone strategies[4]) provided signals are sufficiently precise. Next, I compare both auctions in terms of revenue and welfare when signals are sufficiently precise. It turns out that disclosing bids has one advantage and one disadvantage. The advantage is related to Blackwell's (1951) value of information theorem. Player one's observation that player two bid "low", for example, may convince her not to drill her tract. (Had bids not been disclosed, she might have drilled and would have incurred an expected loss.) Hence, player one, anticipating that she will receive useful information prior to drilling, values "winning" more and, thus, bids more aggressively. The disadvantage of disclosing bids is related to the information externality associated with any time-one drilling activity. To see this, consider player two's incentives to bid in my previous example. Anticipating that her low bid will destroy the other player's incentives to drill, player two is then not interested in acquiring a tract (i.e. she bids zero in the auction with bid disclosure). Had bids not been disclosed, her "low" bid would not have adversely affected the other player's incentives to drill. Anticipating this, she would have valued "winning" more and, thus, would have bid more aggressively. I show that, depending on the values of the parameters, either the advantage or the disadvantage can dominate. This insight implies that an open ascending auction possesses a previously unnoticed disadvantage in this context.

This is not the first paper to address issues of auction design when pre- or post-auction considerations are important. Haile (2000) considers a game in which players can resell after the auction took place.[5] He shows that the possibility of reselling affects bids in two opposing ways. On the one hand, some types have an incentive to bid more aggressively in order to extract seller's surplus at the reselling stage. On the other hand, some types have an incentive to bid less aggressively to extract buyer's surplus at the reselling stage. In general, either effect can dominate and an English auction (followed by resale) does not necessarily yield higher expected revenues than a second-, or a first price one (followed by resale). Goeree (2003) analyzes a game followed by some downstream interaction among all players. In contrast to this paper, downstream interaction is not modelled explicitly.[6] Instead he takes a reduced-form approach in

player is less confident about the probability of finding an oil and gas deposit.

[4]A strategy is said to be monotone if high-type players are (weakly) more likely to drill in the first investment period than low-type ones.

[5]Bikhchandani and Huang (1989) also compare the revenue properties of different auction designs in the presence of a resale market.

[6]Das varma (2003) models post-auction (Betrand and Cournot) competition explicitly and obtains essentially the same results as Goeree (2003).

which a player's payoff depends (i) on whether she won the object or not, (ii) on her true type and (iii) on her perceived type. He characterizes bidding strategies in first-price, second-price and English auctions and shows that an English auction may yield higher expected revenues than a sealed-bid auction when bidders have an incentive to understate their private information. (If bidders have an incentive to overstate their private information, the three auctions yield the same expected revenues.) In Haile and Goeree signaling motives are important at the auction stage. As incentives to signal are affected by the auction design, the revenue equivalence theorem fails to go through. In section 4, I show that if signals (instead of bids) were disclosed after the auction, both types would still bid the same amount as in the auction with bid disclosure. Furthermore, in the auction without bid disclosure, players trivially have no signaling motive. Signaling is thus not the driving force behind my results. Instead, my results are driven by my finding that in the auction with bid disclosure player i's signal becomes common knowledge after the auction. The high type does not mind her signal to be disclosed and prefers to know the other player's private information. The low type, however, prefers to hide her bad private information as this reduces the other player's incentives to drill. Arozamena and Cantillon (2004) analyze incentives to invest in a cost reducing technology prior to a procurement auction. They argue that a firm has more incentives to lower her costs if the procurement contract is offered via a second-price auction (as opposed to a first-price auction). Their results are driven by their finding that if a firm reduces her costs, this induces the other firms to bid more aggressively (in the subsequent first-price auction). The driving feature of their model is thus also different from the one operating in this paper.

2 Some institutional features

In this section I explain some important institutional features of the U.S. offshore leasing program. This will help the reader to understand better the game I will study. It also provides a justification for some of my simplifying assumptions.

I focus on wildcat tracts. Such a tract is situated in an offshore geographical area where no exploratory drilling has occurred in the past. Hence, in those auctions no firm should possess superior information about the value of a tract.[7] A tract covers an area not exceeding 5,760 acres ($\approx 23.3km^2$). Prior to bidding, firms perform seismic tests to assess the likelihood of finding oil (and gas). The seismic tests which prevailed 25 years ago only provided a noisy statistic about the

[7] The U.S. government sometimes auctions tracts that are situated next to already developed ones (those ones are called drainage tracts). Hendricks and Porter (1988) showed that the neighbor firm then possess an informational advantage over the value of those tracts.

value of the oil (and gas) underneath a tract. Fortunately, at the end of the 80' new technologies (such as 3D seismic mapping) were invented which permitted firms to drastically reduce the risk associated with offshore drilling. General Electric, for example, nowadays offer "intelligent drilling" technology which helps firms to locate oil and gas deposits and to extract them in the most economical way. Zampelli (2000) estimated that the introduction of those new technologies explains why the success rate (measured as $\frac{P}{1-P}$, where $P = \Pr(\text{Firm } i \text{ finds oil (and gas)} \mid \text{Firm } i \text{ drills})$) more than doubled between 1986 and 1995.

A bid is a dollar figure that the firm must pay if it wins the tract. Apart from the bid, firms must also pay a royalty fee which — depending on water depth — typically lies between one sixth and one eighth of the value of the extracted oil. Firms submit their bids simultaneously. Firms bid on a small subset of the tracts offered for sale. For example, between 1998 and 2005 (inclusive) the U.S. government organized 22 auctions. On average 3,145 tracts were offered in each one of them. On average only 305 of them received at least one bid.[8] Hence, in those auctions the number of tracts offered for sale by far exceeds total demand. As a result of this, few of the tracts offered for sale receive more than one bid. Summed over all those 22 auctions, for example, 6,705 tracts received at least one bid and 5,255 received exactly one bid. Stated differently, conditional on the event that a tract received at least one bid, there is a 78.4 % probability that that tract received only one bid.[9]

If a tract happens to possess only one bid, then the U.S. government decides whether or not to reject the bid. To do so, it estimates the "fair market value" of the tract. Henceforth, this fair-market-value estimate will be called the (government's) reservation price. A tract which received only one bid is sold if the bid exceeds the reservation price. The reservation price is computed after all bids were submitted. Hence, ex-ante bidders don't know what the realization of the reservation price will be. This insight, combined with my earlier finding that few tracts receive more than one bid, indicate that a player's bidding strategy is primarily determined by her desire to "beat" the reservation price rather than to "beat" a hypothetical competing bid. So far, only Hendricks, Porter and Spady (HPS, 1989) analyzed the government's rejection decision on offshore tracts. They focussed on drainage and development tracts that were sold during the period 1959 - 1979.[10] Wildcat tracts were unfortunately not included in their sample. They

[8]Source: own computations based on data taken from http://www.mms.gov/econ/EconHist.htm.

[9]Solo bidding, however, has not always been the norm in OCS auctions. In particular, Hendricks, Porter and Boudreau (1987) documented that Pr(tract i receives only one bid|tract i receives at least one bid) was approximately 32% for wildcat auctions held during the period 1954-1969.

[10]The definition of a drainage tract is provided in footnote 7. A development tract is a tract re-offered for sale as its past owner let her lease expire without drilling any well. See below for more details.

found that the rejection decision on drainage tracts was positively correlated with a tract's size, with the average wellhead price of offshore oil and with the identity of the highest bidder (i.e. the government was more likely to reject a given high bid submitted by a neighbor firm than by a non-neighbor one). The rejection decision was also negatively correlated with the value of the highest bid. The decision, however, was *not* significantly correlated with the amount of oil extracted *nor* with the bidding history of the neighboring tract. As the reservation price on drainage tracts did not depend on the expected quantity of oil (of the neighboring tract) nor on the neighbors' bids, there is no reason to assume that the contrary situation would prevail on wildcat tracts. After firms submitted their bids, but before the first drilling date, the government releases the identity of all bidders along with their bids. The government also releases the reservation price for tracts with a rejected high bid.[11] The reservation price for tracts with an accepted high bid, however, is not made public.

After winning her tract, a firm is given five years to initiate an exploratory drilling program. If after five years it has not initiated such a program, its lease expires and the tract is returned to the government which may decide to resell it in some future auction. The tracts are usually smaller than the sizes of the deposits. Lin (2007), for instance, documents that the largest petroleum field in the Gulf of Mexico spans 23 tracts. Depending on water depth, 57% to 67% of all productive tracts had to share their deposits with at least one neighboring firm. Furthermore, even if two adjacent tracts do not share the same deposit, this does not mean that their tract values are uncorrelated. As adjacent tracts possess almost the same geological characteristics, their values should still be significantly correlated. More generally, one would expect the correlation of tract values gradually to decrease with distance.

Drilling an exploratory well can be very costly. According to Zampelli (2000) in 1996 the average exploratory well had a depth of 11,203 feet (3,414 meters) and cost 3.3 million USD. This cost, however, dramatically increases with well depth: A 15,000 feet (4,572 meters) exploratory well cost 10 million USD. As tract values are correlated and as drilling is costly, a firm has an incentive to postpone its exploratory drilling in the hope that a (not-too-distant) neighbor drills first. This plausible strategic behavior is not inconsistent with the available empirical evidence. Hendricks and Porter (1996) documented that the hazard rate of drilling (i.e. the probability to drill at time t given that the tract has not been drilled before) features a U-shaped pattern. A tract is most likely to be drilled at the start or at the end of her lease term. In years 2, 3 and 4, however, the hazard rate is significantly lower. If a firm drills its tract during the final year of her lease, this indicates that it must hold sufficiently optimistic beliefs about her prospects of finding

[11] Those reservation prices can be downloaded from http://www.mms-gov/econ/EconHist.htm.

oil. The fact that it postponed its drilling decision indicates that there was a positive option value of waiting. A plausible explanation behind this option value of waiting is that the firm hoped to learn from its (not-too-distant) neighbor's drilling outcomes. Furthermore, Hendricks and Porter also found that the probability to drill during the second and the third year of the lease term is positively influenced by the number of past successful drilling outcomes.

3 The general set-up

Two risk-neutral players are interested in acquiring one of two offshore tracts. The seller offers them in two sealed-bid simultaneous first-price auctions. Each of the players bid in one of the two auctions.[12] The value of both tracts depends on the realized state of the world. In particular, I assume that the state of the world is either high (H) or low (L). If the state of the world is high (low), then the value of the oil (underneath both tracts) is equal to one (zero).[13] The probability that the state of the world is high is equal to $\frac{1}{2}$.

Both players possesses an informative, but imperfect signal concerning the realized state of the world. Formally, if the state of the world is H, a player receives signal h with probability $p \in (\frac{1}{2}, 1)$, and signal l with probability $(1-p)$. Similarly, if the state of the world is L, a player receives signal h with probability $(1-p)$, and signal l with probability p. Signals are (conditionally) independent. I denote the common drilling cost by c. I assume that

Assumption 1 $1 - p < c < p$.

The assumption implies that a player who received signal h is - a priori - willing to drill ($\Pr(H|h) = p > c$), and that a player who received a signal l is a priori not willing to drill ($\Pr(H|l) = 1 - p < c$).

[12]Implicitly, I am making two assumptions here. First, I assume that bidders have unit demand. Second, I assume that there is only one bidder per tract. The first assumption can be defended on the grounds that firms may not want to bid on all the tracts offered for sale (recall that in the period 1998-2005 on average 3,145 tracts were simultaneously offered for sale!) either because of bidding constraints, or limited refining capacity, or because of a bottleneck in the supply of drilling rigs or because of risk-aversion. None of those reasons, however, are explicitly modeled here. Next, because of the information externality, a firm's valuation of a particular tract is nondecreasing in the number of neighboring tracts it wins in the auction. Recall, however, that I study how the information externality at the drilling stage affects some issues of auction design. Introducing supermodular utility functions in the analysis would therefore unnecessarily complicate matters. The second assumption considerably simplifies computations and is consistent with the recent U.S. experience as explained in section 2.

[13]I assume that both tracts possess the same value. This is a simplifying rather than a crucial assumption. My main results should go through if both tracts possess (imperfectly) correlated values.

The (nominal) value of the oil is equal to PQ, where P and Q respectively denote the price and quantity of oil. As $Q \in \{0, 1\}$, the real value of the oil is either equal to zero or equal to one. Furthermore, suppose the government's (nominal) reservation price on tract i, is given by: $R_i = f(P) + \epsilon_i$, where $\epsilon_i \sim U[\underline{\epsilon}, \bar{\epsilon}]$ and where f denotes an arbitrary function. This is consistent with the empirical findings of HPS which showed that the government's rejection decision was only correlated with (i) the tract size, (ii) the winning bid, (iii) the identity of the winning bidder and (iv) the price of oil. Recall that the quantity of oil and a neighbor's bid were *not* significant in their regression equation. Hence, there is no reason to assume that R_i is contingent on the bid on tract $-i$ or on Q. Nor is there any reason to assume that r_i is correlated with r_{-i}. In my model both tracts have the same size and both bidders do not own a neighboring tract. Perform the following normalizations: $r_i \equiv \frac{R_i}{P}$, $\underline{\epsilon} \equiv -f(P)$ and $\bar{\epsilon} \equiv P - f(P)$. Then, $r_i \sim U[0, 1]$. Finally, players discount the future at a rate $\delta \in (0, 1)$.

4 The auction with bid disclosure

In this section, I consider the following sequencing of events:

-1 Nature draws the state of the world, the reservation prices and players receive their signals.

0 Player one bids on tract one, player two bids on tracts two.

$\frac{1}{2}$ The auctioneer publicly announces all bids and whether they were higher or lower than the reservation price.[14]

1 If player i won her tract, she decides whether to drill or wait.

2 In case player $-i$ drilled, player i observes the state of the world. If player i waited, she decides whether or not to drill.

3 Players receive their payoffs and the game ends.

4.1 Equilibrium

Let $\mathtt{h}_t (t = 0, 1, 2)$ denote the history of the game at time t. Thus, $\mathtt{h}_0 = \{\emptyset\}$, $\mathtt{h}_1 = (b_i, b_{-i}, \alpha_i, \alpha_{-i})$, where $\alpha_i \in \{r_i > b_i, r_i < b_i\}$. $\mathtt{h}_2 = (h_1, a_{i,1}, a_{-i,1}, \xi)$ where $a_{i,1} \in \{drill, wait\}$ represents player

[14] As mentioned in section 2, the reservation price for tracts with a rejected high bid can be downloaded (after some time) from the Minerals Management Service website (which is the division in the Department of the Interior responsible for organizing those auctions). The reservation price for tracts with an accepted high bid, however, is not made public.

i's time-one action and $\xi = \{\emptyset\}$ if $a_{i,1} = a_{-i,1} = wait$ and is equal to the state of the world if at least one of the two players drilled at time one. H_t denotes the set of all possible histories at time t. Let $H \equiv \bigcup_{t=1}^{2} H_t$. A symmetric behavioral strategy is a (β, λ) where $\beta : \{h, l\} \to \Delta[0, 1]$ and $\lambda : \{h, l\} \times H \to [0, 1]$. $\beta(s_i)$ represents a distribution function over player i's possible bids. $\lambda(s_i, \mathtt{h}_1)$ and $\lambda(s_i, \mathtt{h}_2)$ represent the probabilities with which player i will respectively drill at times one and two. If $r_i > b_i$ (i.e. if player i does not own tract i), then player i can never drill and, thus, $\lambda(s_i, \mathtt{h}_1) = \lambda(s_i, \mathtt{h}_2) = 0$. A player can only drill once. Therefore, $\lambda(s_i, \mathtt{h}_2) = 0$ if $a_1^i = drill$.

Henceforth, $E_t(U|\cdot)$ denotes player i's expected utility at time t. In particular, $E_{\frac{1}{2}}(U|s_i)$ denotes i's expected utility conditional on her signal *and on the event that she won her tract.* $E_{\frac{1}{2}}(U|s_i)$, however, is not conditioned upon α_{-i}, i.e. player i computes $E_{\frac{1}{2}}(U|s_i)$ after learning that $r_i < b_i$ but *before* finding out whether the other player won her tract or not. Similarly, $E_0(U|s_i, b_i) = \Pr(r_i < b_i|b_i)(E_{\frac{1}{2}}(U|s_i) - b_i)$. As $r_i \sim U[0, 1]$, $\Pr(r_i < b_i|b_i) = b_i$.

When solving my game, I rely on two equilibrium selection criteria. First, I require a candidate equilibrium to belong to the class of the perfect Bayesian equilibria. In a perfect Bayesian equilibrium (PBE) strategies and beliefs (concerning the other player's type) must be such that (i) player i cannot gain by choosing a $\beta \neq \beta^*$ and a $\lambda \neq \lambda^*$ given her beliefs and (ii) beliefs must be computed using Bayes's rule whenever possible. Second, I restrict attention to the class of the *strongly symmetric* strategies. A strategy is said to be strongly symmetric if it satisfies the following two restrictions: (i) the strategy must be symmetric and (ii) if both players won their tracts and if i believes that she possesses the same time-one posterior as $-i$, then she computes her time-one drilling probability under the assumption that $-i$ will drill with the same probability as herself. To illustrate this assumption, suppose beliefs are updated under the assumption that high-type players always bid y while low-type players always bid z ($\neq y$). Suppose player one is a high-type player while player two is a low-type player. Suppose player one bids y while player two bids z. At time one, player one's posterior ($= \Pr(H|h, b_2 = z)$) is then equal to the one of player two ($= \Pr(H|l, b_1 = y)$). As both players possess different private information, a symmetric strategy does not put any restriction on their time-one drilling behavior. However, as both players possess the same time-one posterior, a strongly symmetric strategy prescribes them to drill at time one with the same probability. Stated differently, this assumption implies that — for a sufficiently high discount factor — both players focus on the mixed-strategy Nash equilibrium of the continuation game, i.e. player one drills with some probability to make player two indifferent between drilling and waiting, and vice versa.[15]

[15] Pre-auction cooperation is problematic: Prior to bidding, firms must invest in geophysical surveys and in a team of experts to interpret the seismic data. Understandably, firms are reluctant to share this private information

Some $\lambda^*(\cdot)$'s are easy to compute. Consider, for example, the case in which player $-i$ did not win her tract (i.e. $b_{-i} < r_{-i}$). Then, $\lambda^*(s_i, \mathtt{h}_1) = 1$ if and only if $\Pr(H|s_i, b_{-i}) \geq c$. Similarly, suppose player i won her tract but did not drill at time one. Player i's time-two equilibrium strategy is then also easy to compute: Either player $-i$ drilled at time one (in which case i drills if and only if the state of the world is high), or player two did not drill at time one and $\lambda^*(s_i, \mathtt{h}_2) = 1$ if and only if $\Pr(H|s_i, \mathtt{h}_2) \geq c$. Hence, from now on I restrict attention to computing (i) optimal bidding strategies and (ii) time-one drilling decisions when both players own their tracts.

4.2 Bidding behavior with bid disclosure

PROPOSITION 1 *If signals are sufficiently precise or if both tracts are marginal and δ sufficiently small, there exists a unique equilibrium (within the class of the strongly symmetric strategies) in which player i bids*

$$b^*_{s_i} = \frac{1}{2} \sum_{s_{-i}} \Pr(s_{-i}|s_i) \max\{\Pr(H|s_i, s_{-i}) - c, 0\}. \tag{1}$$

Proof: See Melissas (2008). Observe that the right-hand side of 1 is higher if $s_i = h$ than if it were equal to l. Hence, The proposition states (a.o) that if signals are sufficiently precise, my game possess a unique equilibrium (within the class of the strongly symmetric strategies) in which a high-type player bids "high" (with probability one) while a low-type player bids "low" (with probability one). In particular, there exists no equilibrium in which some player's type randomizes her bids.[16] To understand the proposition above, it is useful to consider first the hypothetical case in which signals instead of bids are revealed at time $\frac{1}{2}$. Suppose player i submits bid b_i and that she wins her tract. Either player $-i$ also won her tract or player $-i$ submitted a bid lower than

with firms which are uninterested in drilling (and bidding) in their geographical area. (Hendricks and Porter (1992) provide empirical evidence which is consistent with this explanation.) If firms were to know the identity of their (not-too-distant) neighbor, they might decide to create a joint venture prior to bidding. Unfortunately, the identity of a firm's (not-too-distant) neighbor only becomes available after the auction. Post-auction cooperation is also problematic. Both firms could, for instance, exchange their seismic information after the auction and then bargain over a course of action. This "solution", however, still leaves scope for disagreement. First, firms need to decide on when to drill. (Firms may posses different private information about future oil and gas prices.) Second, firms need to agree on where to drill. Third, firms need to reveal how they interpreted the seismic data. This may increase other firms' expertise knowledge in interpreting seismic data and allow them to bid more aggressively in future auctions. Fourth, firms need to agree on how to share costs and benefits (if any). Hence, even if firms were to disclose their seismic data, firms still possess private information along other dimensions which impede efficient post-auction bargaining.

[16] In Melissas (2008), I analyzed the same game except that I did not solely focus on the "sufficiently-precise" signal case. I then showed that this game is characterized by many equilibria, some of which involving randomization of bids.

the government's reservation price. In the latter case, $E_1(U|s_i, b_i) = \max\{\Pr(H|s_i, s_{-i}) - c, 0\}$. Suppose the former case prevails. As signals are revealed at time $\frac{1}{2}$, both players possess the same time-one posterior. As explained in my previous subsection, in a strongly symmetric equilibrium this implies that $E_1(U|s_i, b_i)$ is also equal to $\max\{\Pr(H|s_i, s_{-i}) - c, 0\}$. At the start of time $\frac{1}{2}$, player i does not know player $-i$'s signal. Therefore,

$$E_{\frac{1}{2}}(U|s_i) = \sum_{s_{-i}} \Pr(s_{-i}|s_i) \max\{\Pr(H|s_i, s_{-i}) - c, 0\}.$$

Hence, at time zero player i chooses b_i to maximize $b_i(E_{\frac{1}{2}}(U|s_i, b_i) - b_i)$. This is a very simple strictly concave problem: if player i increases her bid, she increases her chances of winning her tract. This benefit, however, comes at a cost of having to put more money on the table. The solution to this maximization problem is given in 1.

Suppose now that bids instead of signals are disclosed. Proposition 1 states that if signals are sufficiently precise there exists no (strongly symmetric) equilibrium in which players bid differently than 1. To understand this uniqueness result, consider candidate equilibrium strategies in which both types of players randomize their bids according to some distribution functions. Call $\underline{b}_h$, the lowest bid that may be submitted by a high-type player in a candidate equilibrium strategy. If signals are sufficiently precise, this lower bound will be "high". This is intuitive: a high-type player is then very confident about her prospects of finding oil. Hence, she would never agree to submit a "low" bid not even if this guaranteed her the right to free-ride with probability one. Call $\bar{b}_l$, the highest bid that may be submitted by a low-type player in a candidate equilibrium strategy. If signals are sufficiently precise, this upper bound will be "low": As a low-type player is very skeptical about the existence of oil, she would never bid "high", not even if this were to make the other player drill with probability one. Hence, for sufficiently precise signals (or if both tracts are marginal ones and if δ is sufficiently small) in any candidate equilibrium $\bar{b}_l < \underline{b}_h$, which implies that any bid will perfectly reveal a player's type. Thus, at time one both players will possess the same posterior. If $\Pr(H|s_i, b_{-i}) < c$, both players never drill. If $\Pr(H|s_i, b_{-i}) \geq c$ and if only i won her tract, then i drills at time one with probability one. If $\Pr(H|s_i, b_{-i}) \geq c$ and if both players won their tracts, two cases arise. In the first one the discount factor is "low" and both players drill at time one with probability one. In the second case, the discount factor is not "low" and i drills with probability $\lambda^*(\cdot)$ to make player $-i$ indifferent between drilling and waiting (and vice versa). In the latter case, i's time-one payoff also equals the one she would get if she were to drill at time one with probability one. Hence, if signals are sufficiently precise in any candidate equilibrium strategy, at time zero player i faces the following problem:

$$\max_{b_i} b_i \left[\sum_{s_{-i}} \max \left\{ 0, \Pr(H|s_i, s_{-i}) - c \right\} - b_i \right].$$

As she will be able to perfectly infer $-i$'s signal out of her bid, it is without loss of generality to sum over $-i$'s possible types. Observe that this is the same objective function as above, and, thus, yields the same solution.

5 The auction without bid disclosure

In this section I consider the same game as above, except that at time $\frac{1}{2}$ the auctioneer only announces whether $b_i > r_i$ or whether $b_i < r_i$, i.e. the auctioneer does not disclose bids.

5.1 Equilibrium

The formal description of this game is equal to my previous one, except that time-one histories now become $\mathtt{h}_1 = (b_i, \alpha_i, \alpha_{-i})$ where $\alpha_i \in \{r_i < b_i, r_i > b_i\}$. As above, some $\lambda^*(\cdot)$'s are easy to compute. Therefore, I restrict attention to computing (i) optimal bidding strategies and (ii) time-one drilling decisions when both players own their tracts. With a slight abuse of notation, let $\lambda(s_i, b_i)$ denote player i's time-one drilling probability given her signal, her bid, and given that both players won their tracts. I rely on two equilibrium selection criteria. First, every candidate equilibrium must belong to the class of the symmetric perfect Bayesian equilibria (which is defined in section 4.1). Second, I restrict attention to the class of the *monotone* strategies. A strategy is said to be monotone if, $\lambda(l) \leq \lambda(h)$, where $\lambda(s_i)$ denotes player i's ex-ante time-one probability of drilling conditional on the event that both players won their tracts, i.e. $\lambda(s_i) = \int_0^1 \lambda(s_i, b_i) d\beta(s_i)$.

Lemma 1 *Suppose player $-i$ randomizes her bid in the support $[\underline{b}_{s_{-i}}, \bar{b}_{s_{-i}}]$, according to an arbitrary c.d.f. (denoted by $\beta(s_{-i})$). Suppose also that player $-i$, conditional on both players having won their tracts, drills (at time one) with probability $\lambda(s_{-i}, b_{-i})$. i's best response is then to bid b_l^* (with probability one) if $s_i = l$ and to bid b_h^* (with probability one) if $s_i = h$. Furthermore, $b_l^* < b_h^*$.*

To understand the intuition behind this result, suppose player $-i$ follows the strategy described in the lemma. i chooses her bid to maximize $\max_{b_i} b_i(E_{\frac{1}{2}}(U|s_i) - b_i)$. One has:

$$\begin{aligned} E_{\frac{1}{2}}(U|s_i) &= \Pr(r_{-i} < b_{-i}|s_i) \max \left\{ \Pr(H|s_i, r_{-i} < b_{-i}) - c, \delta W(s_i, \lambda(l), \lambda(h)) \right\} \\ &+ \Pr(r_{-i} > b_{-i}|s_i) \max \left\{ 0, \Pr(H|s_i, r_{-i} > b_{-i}) - c \right\}, \end{aligned} \quad (2)$$

where $W(\cdot)$ represents i's gain of waiting (net of discounting costs), conditional on her signal, on the time-one drilling probabilities, and on the event that both players won their tracts. Formally,

$$\begin{aligned} W(s_i, \lambda(l), \lambda(h)) &= \Pr(H, a_{-i,1} = drill | s_i, r_{-i} < b_{-i}; \lambda(l), \lambda(h))(1-c) \\ &+ \Pr(a_{-i,1} = wait | s_i, r_{-i} < b_{-i}; \lambda(l), \lambda(h)) \\ &\times \max\{0, \Pr(H | s_i, r_{-i} < b_{-i}, a_{-i,1} = wait) - c\}. \end{aligned}$$

At the risk of stating the obvious, observe that $-i$'s "complicated" strategy does not impede i to "easily" compute all the above probabilities. For example, I showed above how to compute $\lambda(s_i)$ and $\Pr(r_{-i} < b_{-i} | s_i) = \sum_{s_{-i}} \Pr(s_{-i}|s_i) \int_{\underline{b}_{s_{-i}}}^{\bar{b}_{s_{-i}}} b_{-i} d\beta(s_{-i}) = E(b_{-i}|s_i)$. Furthermore, as r_1 is independently drawn from r_2,[17] and as bids are not disclosed, all the above probabilities are independent of b_i. It then follows from i's time-zero maximization problem that she should bid $\frac{1}{2} E_{\frac{1}{2}}(U|s_i)$ with probability one, i.e. i's best response to any arbitrary strategy of her rival is to bid according to a deterministic bidding function. In the Appendix, I show that $E_{\frac{1}{2}}(U|l) < E_{\frac{1}{2}}(U|h)$, thereby establishing the last claim of the lemma. The intuition is straightforward: As the high-type player is more confident about the value of her tract, she bids more aggressively than a low-type player. This result does not hinge on my restriction that players use monotone strategies. Even if $\lambda(l)$ were allowed to be greater than $\lambda(h)$, b_h^* would still be greater than b_l^*.[18].

5.2 Bidding and drilling behavior without bid disclosure

The proposition below summarizes equilibrium behavior in this game.

PROPOSITION 2 *If signals are sufficiently precise, there exists a unique equilibrium (within the class of the symmetric and monotone strategies) in which* $b_h^* = \frac{1}{2}(p-c)$ *and in which:*

1. *If* $\delta < \delta_1^c$ *(where* $0 \leq \delta^{c_1}$*),* $\lambda^*(l) = \lambda^*(h) = 1$*, and* $b_l^* = b_l^d > 0$.

2. *If* $\delta \in (\delta^{c_1}, \delta^{c_2})$ *(where* $\delta^{c_1} \leq \delta^{c_2}$*),* $\lambda^*(l) \in (0,1)$*,* $\lambda^*(h) = 1$*, and* $b_l^* = b_l^d$.

[17] If, for example, r_1 were equal to r_2, then $\Pr(H|s_i, r_{-i} < b_{-i})$ would depend on b_i. To see this, suppose both players bid b_l if they are low-type players and b_h ($> b_l$) if they are high-type players. Then $\Pr(H|s_i, r_{-i} < b_{-i}) = \Pr(H|s_i)$ if $b_i = b_l$ while $\Pr(H|s_i, r_{-i} < b_{-i}) > \Pr(H|s_i)$ if $b_i = b_h$. In the former case player i knows that player $-i$ won her tract because $r_1 = r_2$ was lower than b_l. In the latter case, she believes that perhaps player $-i$ won her tract because $b_{-i} = b_h$. Remember that my independence assumption is consistent with HPS's empirical findings (see section 2).

[18] To understand this result, suppose that $\lambda(h) = 0$ and that $\lambda(l) = 1$. As $\Pr(s_{-i} = l | s_i = h) < \Pr(s_{-i} = l | s_i = l)$, a low-type player thinks it is more likely that the other player will drill. This, however, does not induce her to bid more aggressively, as she also believes the high state of the world to be less likely. Formally, $\Pr(H, s_{-i} = l | l) = \Pr(H, s_{-i} = l | h) = p(1-p)$ (see the Appendix for more details.)

3. *If* $\delta \in (\delta^{c_2}, \delta^{c_3})$ *(where* $\delta^{c_2} < \delta^{c_3} < 1$*),* $\lambda^*(l) = 0$*,* $\lambda^*(h) = 1$*, and* $b_l^* = b_l^w(\delta, 0, 1)$*.*

4. *If* $\delta \in (\delta^{c_3}, 1)$*,* $\lambda^*(l) = 0$*,* $\lambda^*(h) \in (0, 1)$*, and* $b_l^* = b_l^w(\delta, 0, \lambda^*(h))$*.*

To understand a high-type player's bidding strategy, suppose first that $c < \Pr(H|h, l)$.[19] In this case a high-type player faces a positive gain of drilling (at time one) both when $r_{-i} < b_{-i}$ and when $r_{-i} > b_{-i}$. As I focus on the class of the monotone and symmetric strategies, a high-type player either drills at time one with probability one (in case $r_{-i} > b_{-i}$ or in case δ is "very low") or she is indifferent between both actions. Hence, conditional upon winning the tract, she always gets a payoff equal to the one she would get if she were to drill with probability one. Equation 2 therefore boils down to[20]

$$\begin{aligned} E_{\frac{1}{2}}(U|h) &= \Pr(r_{-i} < b_{-i}|h)[\Pr(H|h, r_{-i} < b_{-i}) - c] \\ &+ \Pr(r_{-i} > b_{-i}|h)[\Pr(H|h, r_{-i} > b_{-i}) - c] \\ &= p - c. \end{aligned}$$

Hence, at time zero, she chooses her bid to maximize $b_h(p - c - b_h)$, which yields as unique solution $b_h^* = \frac{1}{2}(p - c)$. Suppose now that $c > \Pr(H|h, l)$. In that case either $r_{-i} < b_{-i}$ or $r_{-i} > b_{-i}$. In the former case player i faces a positive gain of drilling[21] and, as argued above, gets a payoff equal to the one she would get if she were to drill at time one with probability one. In the latter case, she computes $\Pr(H|h, r_{-i} > b_{-i})$. As $b_l^* < b_h^*$, she revises her posterior probability downwards. The higher b_h^*, the lower $\Pr(H|h, r_{-i} > b_{-i})$. This is intuitive: if b_h^* were equal to one, for example, then i would know that $-i$ did not win her tract because she is a low-type player. $\Pr(H|h, r_{-i} > b_{-i})$ is then equal to $\Pr(H|h, l)$ which, by assumption, is less than c. If b_h^* were equal to b_l^*, then high-types are as likely to "beat" the random reservation price than the low types. In that case $\Pr(H|h, r_{-i} > b_{-i}) = \Pr(H|h)$, which, by assumption, is bigger than c. Suppose i possesses a high signal and that player $-i$ bids one if she also possesses a high signal. This implies that

$$\begin{aligned} E_{\frac{1}{2}}(U|h) &= \Pr(r_{-i} < b_{-i}|h)[\Pr(H|h, r_{-i} < b_{-i}) - c] \\ &> \Pr(r_{-i} < b_{-i}|h)[\Pr(H|h, r_{-i} < b_{-i}) - c] \\ &+ \Pr(r_{-i} > b_{-i}|h)[\Pr(H|h, r_{-i} > b_{-i}) - c] \\ &= p - c. \end{aligned}$$

[19] Henceforth, whenever two signals realizations appear as conditioning variables, it is assumed that the first one denotes i's signal while the second one denotes $-i$'s.

[20] In line with my earlier notation, the first conditioning variable always denotes i's signal.

[21] As $b_l^* < b_h^*$, observing the other player winning her tract is good news, i.e. $c < \Pr(H|h) < \Pr(H|h, r_{-i} < b_{-i})$.

As i's time-$\frac{1}{2}$ expected payoff, is "extremely high", it is a best response for her to bid $\frac{1}{2}E_{\frac{1}{2}}(U|h)$, which is "very high", but less than one. i's "very high" bid, induces $-i$'s time-$\frac{1}{2}$ expected gain to be "high". This, in its turn, induces the other player to bid "moderately high". This "moderately high" bid, in its turn, induces i to bid even lower, etc... In the Appendix I prove that both best responses only cross once and that, in equilibrium, $\Pr(H|h, r_{-i} > b_{-i}) > c$, which explains why b_h^* always equals $\frac{1}{2}(p-c)$.

Suppose that $s_i = l$. Her bidding strategy then depends on her behavior in the continuation game. If i anticipates that she will drill at time one (provided both players won their tracts), then her equilibrium bid is denoted by b_l^d. If i anticipates that she will wait at time one (provided both players won their tracts), then her equilibrium bid is denoted by b_l^w. Observe that b_l^w depends on δ, on $\lambda(l)$ and on $\lambda(h)$. Ceteris paribus, an increase in δ reduces i's cost of waiting and induces her to bid more aggressively. Ceteris paribus an increase in either $\lambda(l)$ or $\lambda(h)$ increases her gain of waiting (as it becomes more likely that she will then learn the state of the world) which induces her to bid more aggressively as well. To stress this dependence, in the proposition I wrote $b_l^w(\delta, \lambda^*(l), \lambda^*(h))$ instead of b_l^w.

In the Appendix, I show that if $c > \Pr(H|h, l)$, $\delta^{c_1} = \delta^{c_2} = 0$, while if $c < \Pr(H|h, l)$, $0 < \delta^{c_1} < \delta^{c_2}$. This is intuitive: if c is greater than $\Pr(H|h, l)$, a low-type player never drills at time one, not even if she were to learn that the other player possesses a high signal. Suppose now that $c < \Pr(H|h, l)$ and that δ is strictly positive but "very low". There are then two possibilities: Either $r_{-i} > b_{-i}$, or $r_{-i} < b_{-i}$. As $b_l^* < b_h^*$, $\Pr(H|l, r_{-i} > b_{-i}) < \Pr(H|l) < c$ and a low-type player never drills. In the latter case, however, she revises her posterior probability of finding oil upwards. The lower b_l^*, the higher her posterior probability $\Pr(H|l, r_{-i} < b_{-i})$. In the limit (i.e. when b_l^* tends to zero), $\Pr(H|l, r_{-i} < b_{-i}) = \Pr(H|l, h)$ which, by assumption, is greater than c. In the Appendix, I prove the existence of an equilibrium in which

$$b_l^* = \frac{1}{2}\Pr(r_{-i} < b_{-i}|l)[\Pr(H|l, r_{-i} < b_{-i}) - c] \equiv b_l^d,$$

and in which both types of players drill at time one (with probability one). The intuition should be clear: As b_l^d is "very low", $\Pr(H|l, r_{-i} < b_{-i}) - c > 0$. Furthermore, as δ is "very low", no player wants to postpone her drilling plans.

Suppose now that $\delta \in (\delta^{c_1}, \delta^{c_2})$ (which, as argued above, implies that $c < \Pr(H|h, l)$). As a high-type player is confident about her prospects of finding oil, and as the discount factor is quite low, her (opportunity) cost of waiting is too high: Even if she were to anticipate the other player to drill with probability one, she would still prefer to drill (at time one) with probability one. More interestingly, Suppose i is a low-type player. As she is less confident about her prospects of finding oil, she faces a much lower (opportunity) cost of waiting. If she were to anticipate the other player to drill with probability one, she would prefer to wait. If she were to anticipate the other player to drill (with probability one) if $s_{-i} = h$ (and to wait otherwise), she would prefer to drill (as the discount factor is "low"). It is relatively straightforward (see Lemma 3 in the Appendix) to prove that i's gain of waiting is increasing in the other player's drilling probability (the higher the probability that $-i$ will drill, the higher i's chances to observe the state of the world and, thus, the higher her gain of waiting). By continuity, there exists then a unique equilibrium in which low-type players drill with probability $\lambda^*(l) \in (0,1)$ (where $\lambda^*(l)$ is computed to ensure the indifference of a low-type player between her two time-one actions given that $\lambda^*(h) = 1$). As a low-type player is indifferent between drilling and waiting, she bids as if she were to drill at time one with probability one. Therefore $b_l^* = b_l^d$.

If $c < \Pr(H|h,l)$, δ^{c_2} is constructed such that a low-type player is indifferent between drilling and waiting given that $\lambda^*(l) = 0$ and given that $\lambda^*(h) = 1$. δ^{c_3} is constructed such that a high-type player is indifferent between drilling and waiting given that $\lambda^*(l) = 0$ and given that $\lambda^*(h) = 1$.[22] As high-type players face a higher opportunity cost of waiting, $\delta^{c_2} < \delta^{c_3}$. If $c > \Pr(H|h,l)$, $\delta^{c_2} = 0$. Hence, if $\delta \in (\delta^{c_2}, \delta^{c_3})$ a low-type player (anticipating that $\lambda^*(l) = 0$ and that $\lambda^*(h) = 1$) strictly prefers to wait (as $\delta > \delta^{c_2}$) while a high-type player strictly prefers to drill (as $\delta < \delta^{c_3}$). As a low-type player waits, conditional on winning her tract, she gets $E_{\frac{1}{2}}(U|l) = \Pr(r_{-i} < b_{-i}|l)\delta W(l,0,1)$. It then follows from her time-zero maximization problem that $b_l^* = \frac{1}{2}\Pr(r_{-i} < b_{-i}|l)\delta W(l,0,1) \equiv b_l^w(\delta,0,1)$, where the last two arguments of b_l^w respectively refer to $\lambda^*(l)$ and $\lambda^*(h)$.

Suppose that $s_i = h$ and that both players won their tracts. As δ increases, player i has *more* incentives to wait as her (opportunity) cost decreases. An increase in δ, however, also induces low-type players to increase their bids. This, in turn, reduces $\Pr(s_{-i} = h|h, r_{-i} < b_{-i})$. This is intuitive: as low-type players bid more aggressively, player i infers less good news (about $-i$'s type) upon observing that the other player won her tract. i therefore revises downwards her probability that the other player will drill, which *reduces* her incentives to wait. Observe, however, that with precise signals, $\Pr(s_{-i} = h|h, r_{-i} < b_{-i})$ is not very sensitive to changes in b_l. This is also intuitive: If signals are perfectly informative (i.e. $p = 1$), for example, then both players possess the same type and $\Pr(s_{-i} = h|h, r_{-i} < b_{-i})$ is independent of b_l. Hence, if signals are sufficiently precise, player i's incentives to wait are increasing in δ (this result is proven in Lemma 8). In lemma 9, I prove that if signals are sufficiently precise, i's incentives to wait are

[22]Proving the existence and uniqueness of such a discount factor is not straightforward: A high-type player's incentives to drill (as opposed to waiting) depends on her posterior probability $\Pr(H|h, r_{-i} < b_{-i})$. That probability depends on b_l. As a low-type player waits, her bid depends on the discount factor. Hence, δ^{c_3} depends on b_l, and vice versa (See Lemma 14 for more details).

also strictly increasing in $\lambda(h)$: The higher $\lambda(h)$, the higher the likelihood that i will learn the state of the world if she waits.[23] It then follows from Lemma 8 that if $\delta > \delta^{c_3}$, if signals are sufficiently precise, and if i anticipates the other player to drill (with probability one) if $s_{-i} = h$ and to wait (with probability one) if $s_{-i} = l$, then she prefers to wait. From Lemma 9, we know that there exists then a unique $\lambda^*(h) \in (0, 1)$ which makes high-type players indifferent between drilling and waiting. Suppose now that $s_i = l$. As high-type players are indifferent, and as low-type players possess a lower (opportunity cost) of waiting, i strictly prefers to wait and bid $b_l^* = b_l^w(\delta, 0, \lambda^*(h))$.

The following lemma will be useful in our next section.

Lemma 2 *If* $\delta = 1$, $\lambda^*(l) = \lambda^*(h) = 0$ *and* $b_l^* = b_l^w(1, 0, 0) = b_l^d$.

Suppose that $\delta = 1$, that both players won their tracts and that there exists a symmetric equilibrium in which $0 < \lambda^*(h)$. Observe that $\lambda^*(h) > 0$ only if her gain of drilling is at least as large as her gain of waiting, i.e.

$$\Pr(H|h, r_{-i} < b_{-i}) - c \geq \delta W(h, \lambda^*(l), \lambda^*(h)).$$

However, if $\delta = 1$ and if $\lambda^*(h) > 0$, the inequality above cannot be satisfied as waiting yields a positive gain (i.e. with a probability no lower than $\Pr(L, s_{-i} = h|s_i = h)\lambda^*(h)$ she will learn that she should not drill at time two) at no cost. For an identical reason, $\lambda^*(l) = 0$.

As $b_l^* < b_h^*$, $\Pr(H|l, r_{-i} > b_{-i}) < \Pr(H|l) < c$. This result, combined with our earlier insight that $\lambda^*(l) = \lambda^*(h) = 0$, implies that at time-$\frac{1}{2}$ an l-type player gets $\Pr(r_{-i} < b_{-i}|l) \max\{0, \Pr(H|l, r_{-i} < b_{-i}) - c\}$. Hence, an l-type player faces the same time-zero maximization problem when $\delta = 1$ as when it is equal to zero. The only difference being that if $\delta = 0$, an l-type player makes a once-and-for-all decision at time zero, while if $\delta = 1$, she faces the same decision problem but at time two. Therefore, $b_l^* = b_l^w(1, 0, 0) = b_l^d$.

6 Revenue and welfare comparisons.

Before comparing both auction designs, it is necessary to introduce some additional notations. As should be clear from above, bids and time-one drilling probabilities may differ in both auctions. For example, if $c > \Pr(H|h, l)$, Proposition 1 states that high-type players bid $\frac{1}{2}\Pr(s_{-i} = h|s_i =$

[23] An increase in $\lambda(h)$, however, also induces low-type players to bid more aggressively. This reduces i's posterior probability that the other player possesses signal h (conditional upon the event that $r_{-i} < b_{-i}$), and reduces i's incentives to wait. If signals are sufficiently precise, however, this effect is dominated.

$h)[\Pr(H|h,h)-c]$[24], while Proposition 2 states that they bid $\frac{1}{2}(p-c)$. To avoid confusion, I will sometimes use the symbol $\mathcal{D}$ ($\mathcal{ND}$) to refer to bids, time-one drilling probabilities, expected utilities and auction design when the auctioneer discloses (does not disclose) bids.

It follows from my discussion after Proposition 1 that in the auction with bid disclosure:

$$E_{\frac{1}{2}}(U|s_i;\mathcal{D}) = \sum_{x^{\mathcal{D}}} \Pr\left(x^{\mathcal{D}}|s_i\right) \max\left\{0, \Pr\left(H|s_i, x^{\mathcal{D}}\right) - c\right\}, \tag{3}$$

where $x^{\mathcal{D}} \in \{(s_{-i}=h),(s_{-i}=l)\}$. The equation above teaches us that at time $\frac{1}{2}$ (i.e. conditional on the event that i won her tract) the auction with bid disclosure boils down to the following decision problem: Player i first observes a statistic $x^{\mathcal{D}}$ which reveals the other player's type. Next, she decides whether to drill or not.

It follows from Proposition 2 that in the auction without bid disclosure:

$$\begin{aligned} E_{\frac{1}{2}}(U|s_i;\mathcal{ND}) &= \Pr(r_{-i}<b_{-i}|s_i)\max\Big\{\Pr(H|s_i,r_{-i}<b_{-i})-c, \delta W(s_i,\lambda(l),\lambda(h))\Big\} \\ &+ \Pr(r_{-i}>b_{-i}|s_i)\max\Big\{0,\Pr(H|s_i,r_{-i}>b_{-i})-c\Big\}, \end{aligned} \tag{4}$$

Note that in a symmetric monotone equilibrium $\Pr(H|h,r_{-i}<b_{-i})-c \geq \delta W(h,\lambda^*(l),\lambda^*(h))$. To see this, first observe that $c < \Pr(H|h) < \Pr(H|h,r_{-i}<b_{-i})$, where the second inequality follows from my finding (stated in Lemma 1) that $b_l^*(\mathcal{ND}) < b_h^*(\mathcal{ND})$. (In words, observing the other player wining her tract is "good" news.) Next, suppose that $\Pr(H|h,r_{-i}<b_{-i})-c < \delta W(h,\lambda^*(l),\lambda^*(h))$, which implies that $\lambda^*(h)=0$. As I focus on the class of the monotone strategies, $\lambda^*(l)=0$. However,

$$\delta W(h,0,0) = \delta(\Pr(H|h,r_{-i}<b_{-i})-c) < \Pr(H|h,r_{-i}<b_{-i})-c,$$

a contradiction. Hence, if $s_i=h$, I can, without loss of generality, rewrite 4 as:

$$E_{\frac{1}{2}}(U|h;\mathcal{ND}) = \sum_{x_1^{\mathcal{ND}}} \Pr\left(x_1^{\mathcal{ND}}|h\right) \max\left\{0, \Pr\left(H|h, x_1^{\mathcal{ND}}\right) - c\right\}, \tag{5}$$

where $x_1^{\mathcal{ND}} \in \{r_{-i}<b_{-i}, r_{-i}>b_{-i}\}$. Suppose now that $s_i=l$, that both players won their tracts and that $\delta \leq \delta^{c_2}$. From Proposition 2, we know that if the auctioneer does not disclose bids, $\max\{0,\Pr(H|l,r_{-i}<b_{-i})-c\} \geq \delta W(l,\lambda^*(l),\lambda^*(h))$. Intuitively, if the discount factor is "low" and if $c < \Pr(H|l,h)$, a low-type player either strictly prefers to drill at time one, or she is indifferent between drilling and waiting.[25] If $r_{-i}>b_{-i}$, she refrains from drilling and gets zero. Hence, in this case I can rewrite 4 as:

$$E_{\frac{1}{2}}(U|l;\mathcal{ND},\delta\leq\delta^{c_2}) = \sum_{x_1^{\mathcal{ND}}} \Pr\left(x_1^{\mathcal{ND}}|l\right) \max\left\{0, \Pr\left(H|l, x_1^{\mathcal{ND}}\right) - c\right\}. \tag{6}$$

[24]Henceforth, to economize on notations I will write "$\Pr(s_{-i}|s_i)$" instead of "$\Pr(s_{-i}=\cdot|s_i=\cdot)$".

[25]From the discussion which follows Proposition 2, we know that if $c > \Pr(H|h,l)$, $\delta^{c_2}=0$. In that case, $\max\{0,\Pr(H|l,r_{-i}<b_{-i})-c\}=0$ and $\delta=\delta^{c_2}=0$.

It follows from 5 and 6 that if $r_i < b_i$ and if $\delta \leq \delta^{c_2}$, the auction without bid disclosure is equivalent to the following decision problem: Player i first observes a statistic $x_1^{\mathcal{ND}}$ which reveals some information about the the other player's type. Next, she decides whether to drill or not. Observe that if

$$x^{\mathcal{D}} = (s_{-i} = h), \text{ then } x_1^{\mathcal{ND}} = \begin{cases} r_{-i} < b_{-i} & \text{with prob. } b_h^*(\mathcal{ND}), \text{ and} \\ r_{-i} > b_{-i} & \text{with prob. } 1 - b_h^*(\mathcal{ND}). \end{cases} \tag{7}$$

Similarly, if

$$x^{\mathcal{D}} = (s_{-i} = l), \text{ then } x_1^{\mathcal{ND}} = \begin{cases} r_{-i} < b_{-i} & \text{with prob. } b_l^*(\mathcal{ND}), \text{ and} \\ r_{-i} > b_{-i} & \text{with prob. } 1 - b_l^*(\mathcal{ND}). \end{cases} \tag{8}$$

Stated differently, $x_1^{\mathcal{ND}}$ is created by "adding noise" to $x^{\mathcal{D}}$. It then follows from Blackwell's (1951) theorem that

$$\begin{aligned} E_{\frac{1}{2}}(U|h;\mathcal{D}) &\geq E_{\frac{1}{2}}(U|h;\mathcal{ND}) \text{ and that} \\ E_{\frac{1}{2}}(U|l;\mathcal{D}) &\geq E_{\frac{1}{2}}(U|l;\mathcal{ND}, \delta \leq \delta^{c_2}). \end{aligned} \tag{9}$$

The intuition behind my last two inequalities should be clear: As players do not coordinate their drilling decisions, in both auction designs a high-type player is — at best — indifferent between drilling and waiting. Hence, her time-$\frac{1}{2}$ payoff is equal to the one she gets if she were to take a once-and-for-all decision at time one. Stated differently, at time-$\frac{1}{2}$ a high-type player can be thought of as a decision maker who faces a static time-1 investment problem. The analysis for a low-type player is less clear-cut. If the discount factor is "very low", however, she will never wait. Hence, one can then also think about her as a decision maker who faces a static time-1 investment problem. Blackwell's (1951) value of information theorem then implies that both types' time-$\frac{1}{2}$ payoffs do not decrease in the amount of information they receive.

If $c > \Pr(H|h,l)$, $E_{\frac{1}{2}}(U|h;\mathcal{D}) > E_{\frac{1}{2}}(U|h;\mathcal{ND})$. This is easy to understand: In my previous section, I showed that if bids are not disclosed and if $c > \Pr(H|h,l)$, a high-type player gets the same payoff as the one she would get if she were to drill at time one with probability one (both when $r_{-i} > b_{-i}$ and when $r_{-i} < b_{-i}$). With probability $\Pr(l|h)$, however, she then incurs an expected loss equal to $c - \Pr(H|h,l)$, which she would have avoided in the auction with bid disclosure. If $c < \Pr(H|h,l)$, $E_{\frac{1}{2}}(U|l;\mathcal{D}) > E_{\frac{1}{2}}(U|l;\mathcal{ND}, \delta \leq \delta^{c_2})$. This is also intuitive: with probability $\Pr(h|l)(1 - b_h^*(\mathcal{ND}))$, she refrains from drilling in the auction without bid disclosure, and foregoes a gain of $\Pr(H|l,h) - c$ which she would have obtained in the auction with disclosure. Similarly, with probability $\Pr(l|l)b_l^*(\mathcal{ND})$, she drills at time one (in the auction without bid disclosure) and loses $c - \Pr(H|l,l)$. Had bids been disclosed, she would not have drilled and would have avoided that loss. We now know enough to state and prove:

Proposition 3 *If* $c \neq \Pr(H|h,l)$, $\exists\ \underline{\delta} > 0$ *such that* $\forall\ \delta < \underline{\delta}$, *disclosing bids (strictly) raises revenues and welfare.*

Proof: (Expected) welfare in auction k ($k \in \{\mathcal{D}, \mathcal{ND}\}$), $W(k)$, is defined as

$$W(\mathcal{ND}; \delta \leq \delta^{c_2}) = \Pr(l)E_0(U|l; \mathcal{ND}, \delta \leq \delta^{c_2}) + \Pr(h)E_0(U|h; \mathcal{ND}), \text{ and,}$$

$$W(\mathcal{D}) = \Pr(l)E_0(U|l; \mathcal{D}) + \Pr(h)E_0(U|h; \mathcal{D}).$$

My two last equalities imply that:

$$(E_0(U|l; \mathcal{ND}, \delta \leq \delta^{c_2}), E_0(U|h; \mathcal{ND})) < (E_0(U|l; \mathcal{D}), E_0(U|h; \mathcal{D})) \Rightarrow W(\mathcal{ND}; \delta \leq \delta^{c_2}) < W(\mathcal{D}).$$

In my previous section, I argued that $b_{s_i}(k) = \frac{1}{2}E_{\frac{1}{2}}(U|s_i; k)$. Plugging this equality into $E_0(U|s_i; k) = b_{s_i}(k)(E_{\frac{1}{2}}(U|s_i; k) - b_{s_i}(k))$, yields: $E_0(U|s_i; k) = \frac{1}{4}E_{\frac{1}{2}}(U|s_i; k)^2$. Hence:

$$(E_{\frac{1}{2}}(U|l; \mathcal{ND}, \delta \leq \delta^{c_2}), E_{\frac{1}{2}}(U|h; \mathcal{ND})) < (E_{\frac{1}{2}}(U|l; \mathcal{D}), E_{\frac{1}{2}}(U|h; \mathcal{D})) \Rightarrow W(\mathcal{ND}; \delta \leq \delta^{c_2}) < W(\mathcal{D}).$$

These results, combined with the explanations above, prove that if $\delta = 0$, $W(\mathcal{ND}; \delta = 0) < W(\mathcal{D})$. By continuity, there exists a $\underline{\delta} > 0$ such that $\forall \delta < \underline{\delta}$, $W(\mathcal{ND}) < W(\mathcal{D})$. ■

Observe that the proof of the proposition does not rely on my assumption that $\Pr(s_i = h) = \Pr(s_i = l) = \frac{1}{2}$.

Suppose now that $s_i = l$, that $\delta > \delta^{c_2}$ and that $r_{-i} < b_{-i}$. From Proposition 2, we know that in the auction without bid disclosure, she prefers to wait. Hence, her expected time-$\frac{1}{2}$ payoff can be written as:

$$\begin{aligned} E_{\frac{1}{2}}(U|l; \mathcal{ND}, \delta > \delta^{c_2}) &= \Pr(H, r_{-i} < b_{-i}, a_{-i,1} = drill|l)\delta(1-c) \\ &+ \Pr(r_{-i} < b_{-i}, a_{-i,1} = wait|l)\delta \max\left\{0, \Pr(H|l, r_{-i} < b_{-i}, a_{-i,1} = wait) - c\right\} \\ &+ \Pr(r_{-i} > b_{-i}|l)\delta \max\left\{0, \Pr(H|l, r_{-i} > b_{-i}) - c\right\}. \end{aligned} \quad (10)$$

From my previous section we know that if $\delta = 1$, $\lambda^*(l; \mathcal{ND}) = \lambda^*(h; \mathcal{ND}) = 0$. Furthermore, Proposition 2 and Lemma 2 also state that $b_l^*(\mathcal{ND}, \delta = 0) = b_l^*(\mathcal{ND}, \delta = 1) = b_l^d$ and that $b_h^*(\mathcal{ND}, \delta = 0) = b_h^*(\mathcal{D}, \delta = 1) = \frac{1}{2}(p - c)$. It is easy to check that those insights imply that

$$E_{\frac{1}{2}}(U|l; \mathcal{ND}, \delta = 1) = E_{\frac{1}{2}}(U|l; \mathcal{ND}, \delta = 0).$$

This result, combined with our earlier insight (explained in the first paragraph after Proposition 2) that

$$E_{\frac{1}{2}}(U|h; \mathcal{ND}, \delta = 1) = E_{\frac{1}{2}}(U|h; \mathcal{ND}, \delta = 0) = p - c,$$

allows me to conclude that if $\delta = 1$, player i gets the same payoff as the one she gets when $\delta = 0$. This is intuitive: If $\delta = 0$ the waiting game boils down to a static problem in which all players take a once-and-for-all decision at time one. Prior to taking this time-one decision, a player observes whether the other player won her tract or not. If $\delta = 1$, no one drills at time one. The waiting game then boils down to a static problem in which all players take a once-and-for-all decision at time two. Prior to taking this time-two decision, a player observes whether the other one won her tract or not. As both types bid the same amount when $\delta = 0$ as when $\delta = 1$, i's posterior probability $\Pr(H|s_i, r_{-i} < b_{-i})$ (and hence her expected gain from drilling) is the same in both cases. As above, in the auction with bid disclosure a player receives a more informative statistic about the other player's type. The following result (whose proof is omitted as it is similar to the one of Proposition 3) then follows.

PROPOSITION 4 *If* $c \neq \Pr(H|h,l)$, $\exists\ \bar{\bar{\delta}} < 1$ *such that* $\forall \delta > \bar{\bar{\delta}}$, *disclosing bids increases revenues and welfare.*

I now prove that — for some parameter values — the auctioneer raises welfare and revenues by not disclosing bids. It follows from the proof of Proposition 3 that $W(\mathcal{D}) < W(\mathcal{ND})$ if

$$(E_{\frac{1}{2}}(U|l;\mathcal{D}), E_{\frac{1}{2}}(U|h;\mathcal{D})) < (E_{\frac{1}{2}}(U|l;\mathcal{ND}), E_{\frac{1}{2}}(U|h;\mathcal{ND})). \tag{11}$$

I have shown that a high-type player always (i.e. for all parameter values) weakly prefers the auction with bid disclosure. Hence, inequality 11 is satisfied if

$$E_{\frac{1}{2}}(U|h;\mathcal{D}) = E_{\frac{1}{2}}(U|h;\mathcal{ND}) \text{ and if } E_{\frac{1}{2}}(U|l,\mathcal{D}) < E_{\frac{1}{2}}(U|l,\mathcal{ND},\delta). \tag{12}$$

It follows from 9 that the latter inequality is satisfied only if $\delta > \delta^{c_2}$. Observe that equality 10 can be rewritten as:[26]

$$\begin{aligned} E_{\frac{1}{2}}(U|l;\mathcal{ND},\delta > \delta^{c_2}) &= \Pr(r_{-i} < b_{-i}, a_{-i,1} = drill|l)\delta \max\left\{0, \Pr(H|l,h) - c\right\} \\ &+ \Pr(r_{-i} < b_{-i}, a_{-i,1} = drill|l)\delta \left[\Pr(H|l,h)(1-c) - \max\{0, \Pr(H|l,h) - c\}\right] \\ &+ \Pr(r_{-i} < b_{-i}, a_{-i,1} = wait|l)\delta \max\left\{0, \Pr(H|l, r_{-i} < b_{-i}, a_{-i,1} = wait) - c\right\} \\ &+ \Pr(r_{-i} > b_{-i}|l)\delta \max\left\{0, \Pr(H|l, r_{-i} > b_{-i}) - c\right\}. \end{aligned} \tag{13}$$

Consider now the following decision problem (denoted by $\mathcal{P}^{\mathcal{ND}}$): Player i must decide whether to drill or not. Prior to making her decision, she observes a statistic $x_2^{\mathcal{ND}} \in \{(r_{-i} < b_{-i}, a_{-i,1} = drill), (r_{-i} < b_{-i}, a_{-i,1} = wait), (r_{-i} > b_{-i})\}$ which provides her with some information about the other player's type. Suppose $s_i = l$. The expected value of this decision problem is:

$$E(U|l;\mathcal{P}^{\mathcal{ND}}) = \sum_{x_2^{\mathcal{ND}}} \Pr\left(x_2^{\mathcal{ND}}|l\right) \max\left\{0, \Pr\left(H|l, x_2^{\mathcal{ND}}\right) - c\right\}. \tag{14}$$

[26]From Proposition 2 we know that player $-i$ only drills if she is a high-type player. Hence, in the right-hand side of equality 13 it is without loss of generality to replace $\Pr(H|l, a_{-i,1} = drill)$ by $\Pr(H|l,h)$.

It follows from 13 and 14 that

$$\begin{aligned} E_{\frac{1}{2}}(U|l;\mathcal{ND},\delta>\delta^{c_2}) &= E(U|l;\mathcal{P}^{\mathcal{ND}}) \\ &+ \Pr(r_{-i}<b_{-i},a_{-i,1}=drill|l)\delta\left[\Pr(H|l,h)(1-c)-\max\{0,\Pr(H|l,h)-c\}\right]. \end{aligned} \tag{15}$$

In words, in decision problem $\mathcal{P}^{\mathcal{ND}}$ if $x_2^{\mathcal{ND}}=(r_{-i}<b_{-i},a_{-i,1}=drill)$, i knows that $-i$ won her tract and drilled at time one. She does not observe, however, $-i$'s drilling outcome. In equation 13, however, she observes $-i$'s drilling outcome and only drills if the other player found oil. This accounts for the difference between 13 and 14 as shown in my last equation. It follows from 15 that the second inequality reported in 12 can be rewritten as:

$$\begin{aligned} E_{\frac{1}{2}}(U|l,\mathcal{D})-E_{\frac{1}{2}}(U|l,\mathcal{P}^{\mathcal{ND}}) &< \Pr(r_{-i}<b_{-i},a_{-i,1}=drill|l) \\ &\times \delta\left[\Pr(H|l,h)(1-c)-\max\{0,\Pr(H|l,h)-c\}\right]. \end{aligned} \tag{16}$$

Observe that if

$$x^{\mathcal{D}}=(s_{-i}=h),\text{ then } x_2^{\mathcal{ND}}=\begin{cases}(r_{-i}<b_{-i},drill) & \text{with prob. } b_h^*(\mathcal{ND})\lambda^*(h;\mathcal{ND}),\text{ and}\\ (r_{-i}<b_{-i},wait) & \text{with prob. } b_h^*(\mathcal{ND})(1-\lambda^*(h;\mathcal{ND})),\text{ and}\\ (r_{-i}>b_{-i}) & \text{with prob. } 1-b_h^*(\mathcal{ND}).\end{cases}$$

Similarly, if

$$x^{\mathcal{D}}=(s_{-i}=l),\text{ then } x_2^{\mathcal{ND}}=\begin{cases}(r_{-i}<b_{-i},drill) & \text{with prob. } 0,\text{ and}\\ (r_{-i}<b_{-i},wait) & \text{with prob. } b_l^*(\mathcal{ND}),\text{ and}\\ (r_{-i}>b_{-i}) & \text{with prob. } 1-b_l^*(\mathcal{ND}).\end{cases}$$

Thus, $x^{\mathcal{D}}$ is a sufficient statistic for $x_2^{\mathcal{ND}}$. It then follows from Blackwell's (1951) theorem that the left-hand side of inequality 16 is non-negative. The right-hand side of the inequality above, measures the advantage of not disclosing bids (as opposed to disclosing them): with probability $\Pr(r_{-i}<b_{-i},s_{-i}=h|s_i=l)\lambda^*(h;\mathcal{ND})$ player $-i$ drills in which case i gets $\Pr(H|l,h)(1-c)$. Had bids not been disclosed, she would have got $\max\{0,\Pr(H|l,h)-c\}$. The term between square brackets thus represents the additional gain (when bids are not disclosed as opposed to the disclosure case) i is making in the event the other player drills.

Observe that if $c=\Pr(H|l,h)$, $\Pr(H|l,l)<\Pr(H|l,h)=c$. Thus,

$$E_{\frac{1}{2}}(U|l,\mathcal{D};c=\Pr(H|l,h))=0,$$

which, combined with the insight present in my previous paragraph, implies that the left-hand side of 16 is zero when $c=\Pr(H|l,h)$. The right-hand side of 16, however, is strictly positive. Moreover, if $c=\Pr(H|h,l)$, a high-type player always (i.e $\forall\, x^{\mathcal{D}}$ and $\forall\, x_2^{\mathcal{ND}}$) faces a non-negative gain of drilling. Stated differently, no news can make a high-type player refrain from drilling.

As is well known, this implies that the first equality present in 12 is satisfied. Hence, if $c = \frac{1}{2}$, and if $\delta \in (\delta^{c_2}, 1)$, $W(\mathcal{D}) < W(\mathcal{ND})$. The Proposition below (whose proof can be found in the Appendix) states that this revenue and welfare result holds generically.

PROPOSITION 5 *$\forall\ \delta \in (\delta^{c_2}, 1)$, $\exists(\underline{c}(\delta), \bar{c}(\delta))$ (with $\underline{c}(\delta) < \Pr(H|h,l) < \bar{c}(\delta)$) such that $\forall\ c \in (\underline{c}(\delta), \bar{c}(\delta))$, not disclosing bids increases revenues and welfare.*

Intuitively, if $c \geq \Pr(H|h,l)$ a low-type player knows that the disclosure of her low type destroys the other player's incentives to drill. She therefore prefers the auctioneer not to disclose bids. If $c \leq \Pr(H|h,l)$ a high-type player is indifferent between both auctions as she always gets a payoff equal to the one she would get if she were to drill at time one. Hence, for c sufficiently close to $\Pr(H|h,l)$ the gain of the low types (when bids are not disclosed) exceeds the (possibly zero) loss of the high types.

7 Final Remark

My results are *not* driven by my assumption that bids are disclosed between the auction and the drilling stage. Instead, they are driven by the fact that bids are disclosed *before* the drilling game. An open ascending auction should thus suffer from the same weakness as the auction with bid disclosure: If i observes that $-i$ released her button early, this affects her incentives to drill negatively. Anticipating this, $-i$ may bid less aggressively in an open ascending auction than in a first-price sealed bid one (not followed by bid disclosure). More research is needed to address this and related questions.

Appendix

Proof of Lemma 1

One has:

$$\Pr(H, r_{-i} < b_{-i}|l)(1-c) - \Pr(L, r_{-i} < b_{-i}|l)c \leq \Pr(H, r_{-i} < b_{-i}|h)(1-c) - \Pr(L, r_{-i} < b_{-i}|h)c \tag{17}$$

$$\Leftrightarrow 0 \leq (p-(1-p))\Pr(r_{-i} < b_{-i}|H)(1-c) + (p-(1-p))\Pr(r_{-i} < b_{-i}|L)c,$$

which is satisfied as $p > \frac{1}{2}$. Using an identical reasoning, one can also prove that

$$\Pr(H, r_{-i} > b_{-i}|l)(1-c) - \Pr(L, r_{-i} > b_{-i}|l)c \leq \Pr(H, r_{-i} > b_{-i}|h)(1-c) - \Pr(L, r_{-i} > b_{-i}|h)c. \tag{18}$$

I now show that

$$\Pr(r_{-i} < b_{-i}|l)W(l, \lambda(l), \lambda(h)) \leq \Pr(r_{-i} < b_{-i}|h)W(h, \lambda(l), \lambda(h)). \tag{19}$$

Observe that

$$\Pr(r_{-i} < b_{-i}|s_i)W(s_i, \lambda(l), \lambda(h)) = \sum_{s_{-i}} \Pr(H, s_{-i}, r_{-i} < b_{-i}|s_i)\lambda(s_{-i})(1-c)$$

$$+ \max\{0, \Pr(H, a_{-i,1} = wait, r_{-i} < b_{-i}|s_i) - \Pr(a_{-i,1} = wait, r_{-i} < b_{-i}|s_i)c\}.$$

The equality above, combined with my earlier insight that $\Pr(r_{-i} < b_{-i}|s_{-i}) = b^*_{s_{-i}}$, allows me to rewrite 19 as

$$\begin{aligned} 0 &\leq (p-(1-p))p\ b^*_h\ \lambda(h)(1-c) + (p-(1-p))(1-p)\ b^*_l\ \lambda(l)(1-c) \\ &+ \max\{0, pA-(1-p)B\} - \max\{0, (1-p)A - pB\}, \end{aligned} \tag{20}$$

where

$$\begin{aligned} A &= [p\ b^*_h\ (1-\lambda(h)) + (1-p)\ b^*_l\ (1-\lambda(l))](1-c), \text{ and} \\ B &= [(1-p)\ b^*_h\ (1-\lambda(h)) + p\ b^*_l\ (1-\lambda(l))]c. \end{aligned}$$

Inequality 20 is satisfied as $p > \frac{1}{2}$ (which implies the non-negativity of the first two terms and which also implies that if $(1-p)A - pB > 0$ then $pA - (1-p)B > 0$). Inequalities 17, 18 and 19, combined with the fact that $b^*_{s_i} = \frac{1}{2}E_{\frac{1}{2}}(U|s_i)$, imply that $b^*_l \leq b^*_h$.

Suppose there exists an equilibrium in which $b^*_l = b^*_h$. From above, we know that such an equilibrium only exists if $E_{\frac{1}{2}}(U|l) = E_{\frac{1}{2}}(U|h)$. If $b^*_l = b^*_h$, then

$$\begin{aligned} \Pr(H|l, r_{-i} < b_{-i}) - c &= \Pr(H|l, r_{-i} > b_{-i}) - c = 1-p-c < 0, \text{ and,} \\ \Pr(H|h, r_{-i} < b_{-i}) - c &= \Pr(H|h, r_{-i} > b_{-i}) - c = p - c > 0. \end{aligned} \tag{21}$$

Hence, $\lambda^*(l) = 0$. Observe also that $\delta W(l, 0, \lambda^*(h)) < \delta W(h, 0, \lambda^*(h))$. Therefore,

$$\begin{aligned} E_{\frac{1}{2}}(U|l) &= \Pr(r_{-i} < b_{-i}|l)\delta W(l, 0, \lambda^*(h)) \\ &< \Pr(r_{-i} < b_{-i}|h) \max\{p-c, \delta W(h, 0, \lambda^*(h))\} \\ &+ \Pr(r_{-i} > b_{-i}|h)(p-c) \\ &= E_{\frac{1}{2}}(U|h), \end{aligned} \tag{22}$$

which contradicts my earlier assumption. ■

Proof of Proposition 2

Let $W(s_i, \lambda(l), \lambda(h))$ denote player i's gain of waiting, net of discounting costs, given that both players won their tracts, given her signal and drilling probabilities. Formally,

$$\begin{aligned} W(s_i, \lambda(l), \lambda(h)) &= \sum_{s_{-i}} \Pr(H, s_{-i}|s_i, r_{-i} < b_{-i})\lambda(s_{-i})(1-c) \\ &+ \Pr(a_{-i,1} = wait|s_i, r_{-i} < b_{-i}) \max\{0, \Pr(H|s_i, r_{-i} < b_{-i}, a_{-i,1} = wait) - c\}. \end{aligned} \tag{23}$$

Lemma 3 *If* $(\lambda(l), \lambda(h)) < (\lambda'(l), \lambda'(h))$*, then* $W(s_i, \lambda(l), \lambda(h)) < W(s_i, \lambda'(l), \lambda'(h))$.

Proof: Observe that $W(\cdot)$ can be rewritten as

$$\begin{aligned} W(s_i, \lambda(l), \lambda(h)) &= \mathcal{I}[\Pr(H|s_i, r_{-i} < b_{-i})(1-c) - \Pr(L, a_{-i,1} = wait|s_i, r_{-i} < b_{-i})c] \\ &+ (1-\mathcal{I}) \Pr(H, a_{-i,1} = drill|s_i, r_{-i} < b_{-i})(1-c), \end{aligned}$$

where $\mathcal{I} = 1$ if $\Pr(H|s_i, r_{-i} < b_{-i}, a_{-i,1} = wait) \geq c$ and $\mathcal{I} = 0$ otherwise. The lemma then follows from the fact that

$$\Pr(L, a_{-i,1} = wait|s_i, r_{-i} < b_{-i}; (\lambda(l), \lambda(h))) > \Pr(L, a_{-i,1} = wait|s_i, r_{-i} < b_{-i}; (\lambda'(l), \lambda'(h))),$$

and that

$$\Pr(H, a_{-i,1} = drill|s_i, r_{-i} < b_{-i}; (\lambda(l), \lambda(h))) < \Pr(H, a_{-i,1} = drill|s_i, r_{-i} < b_{-i}; (\lambda'(l), \lambda'(h))).$$

■

Let $q(s_i) \equiv \Pr(H|s_i, r_{-i} < b_{-i})$ and let

$$\Delta(s_i, \lambda(l), \lambda(h)) \equiv \Pr(H|s_i, r_{-i} < b_{-i}) - c - \delta W(s_i, \lambda(l), \lambda(h)). \tag{24}$$

Lemma 4 $\Delta(l, \lambda(l), \lambda(h)) < \Delta(h, \lambda(l), \lambda(h))$.

Proof: Observe that $\Delta(\cdot)$ can be rewritten as

$$\begin{aligned} \Delta(\cdot) &= q(s_i) - c - \delta \sum_{s_{-i}} q(s_i) \Pr(s_{-i}|\theta = H, r_{-i} < b_{-i}) \lambda(s_{-i})(1-c) \\ &- \delta \max\{0, q(s_i) \Pr(a_{-i,1} = wait|H, r_{-i} < b_{-i})(1-c) - (1-q(s_i)) \Pr(a_{-i,1} = wait|L, r_{-i} < b_{-i})c\}. \end{aligned}$$

Observe also that $\Pr(s_{-i}|\theta = H, r_{-i} < b_{-i})$, $\lambda(s_{-i})$, $\Pr(a_{-i,1} = wait|H, r_{-i} < b_{-i})$, $\Pr(a_{-i,1} = wait|L, r_{-i} < b_{-i})$ do not depend on s_i. Let $\mathcal{I} = 1$ if the term between curly brackets is strictly positive, otherwise $\mathcal{I} = 0$. One has,

$$\begin{aligned} \frac{\partial \Delta(\cdot)}{\partial q} &= 1 - \delta \sum_{s_{-i}} \Pr(s_{-i}|H, r_{-i} < b_{-i}) \lambda(s_{-i})(1-c) \\ &\quad -\delta \mathcal{I}[\Pr(a_{-i,1} = wait|H, r_{-i} < b_{-i})(1-c) + \Pr(a_{-i,1} = wait|L, r_{-i} < b_{-i})c] \\ &\geq 1 - \delta(1-c) - \delta \Pr(a_{-i,1} = wait|L, r_{-i} < b_{-i})c \\ &> 0. \end{aligned}$$

The lemma then follows from the fact that $q(l) < q(h)$. ■

Corollary 1 $W(l, \lambda(l), \lambda(h)) < W(h, \lambda(l), \lambda(h))$.

Proof: From the proof of Lemma 4 we know that

$$\begin{aligned} W(s_i,\cdot) &= \sum_{s_{-i}} q(s_i)\Pr(s_{-i}|\theta=H, r_{-i}<b_{-i})\lambda(s_{-i})(1-c) \\ &+ \max\{0, q(s_i)\Pr(wait|H, r_{-i}<b_{-i})(1-c)-(1-q(s_i))\Pr(wait|L, r_{-i}<b_{-i})c\}. \end{aligned}$$

Hence,

$$\begin{aligned} \frac{\partial W(\cdot)}{\partial q(\cdot)} &= \sum_{s_{-i}} \Pr(s_{-i}|\theta=H, r_{-i}<b_{-i})\lambda(s_{-i})(1-c) \\ &+ \mathcal{I}[\Pr(wait|H, r_{-i}<b_{-i})(1-c)+\Pr(wait|L, r_{-i}<b_{-i})c], \end{aligned} \tag{25}$$

where $\mathcal{I}$ is defined in Lemma 4. Observe that $\frac{\partial W(\cdot)}{\partial q(\cdot)}>0$ if $(\lambda(l),\lambda(h))>(0,0)$. Furthermore, if $\lambda(l)=\lambda(h)=0$, then

$$W(h,0,0)=\Pr(H|h,r_{-i}<b_{-i})-c>\max\{0,\Pr(H|l,r_{-i}<b_{-i})-c\}=W(l,0,0).$$

This result, combined with the fact that $q(l)<q(h)$, proves the corollary. ■

Lemma 5 *In any symmetric monotone equilibrium,* $b_h^*=\frac{1}{2}(p-c)$.

Proof: One has,

$$\begin{aligned} E_{\frac{1}{2}}(U|h) &= \Pr(r_{-i}<b_{-i}|h)\max\{\Pr(H|h,r_{-i}<b_{-i})-c, \delta W(h,\lambda(l),\lambda(h))\} \\ &+ \Pr(r_{-i}>b_{-i}|h)\max\{0,\Pr(H|s_i,r_{-i}>b_{-i})-c\}. \end{aligned}$$

In the first paragraph which follows Proposition 2 I argued that — within the class of the symmetric and monotone strategies — in equilibrium $\Pr(H|h,r_{-i}<b_{-i})-c\geq\delta W(h,\lambda(l),\lambda(h))$. Hence, I can, without loss of generality, rewrite $E_{\frac{1}{2}}(U|h)$ as

$$\max\left\{p-c,[p^2(1-c)-(1-p)^2c]b_h-p(1-p)(2c-1)b_l\right\}\equiv\max\left\{p-c,G(b_h,b_l)\right\},$$

where $E_{\frac{1}{2}}=p-c$ if $\Pr(H|h,r_{-i}>b_{-i})\geq c$ and $E_{\frac{1}{2}}=G(b_h,b_l)$ otherwise. As $r_i\sim U[0,1]$, $b_h^*=\frac{1}{2}\max\{p-c,G(b_h^*,b_l^*)\}$. Call LHS (RHS) the left-hand (respectively right-hand) side of this last equality. Observe that $\frac{\partial RHS}{\partial b_h}\leq\frac{1}{2}(p^2(1-c)-(1-p)^2c)<1=\frac{\partial LHS}{\partial b_h}$. Furthermore, if $b_h=b_l$, $\max\{p-c,G(b_l,b_l)\}=\max\{p-c,(p-c)b_l\}=p-c$ and $G(b_l,b_l)\leq b_l=LHS$. Those insights prove the lemma. ■

Let

$$Z(b_l,\delta)\equiv b_l-\frac{1}{2}\delta\Pr(r_{-i}<b_{-i}|l,b_l)W(l,\lambda(l),\lambda(h),b_l). \tag{26}$$

Lemma 6 *If* $\lambda(h)>0$*, there exists a unique* $b_l^w>0$ *such that* $Z(b_l^w,\delta)=0$*. Moreover,* $0<\frac{db_l^w}{d\delta}$.

Proof: Observe that $\Pr(r_{-i} < b_{-i}|l)W(l,\cdot)$ can be rewritten as:

$$\begin{aligned}\Pr(\cdot|l)W(l,\cdot) &= \sum_{s_{-i}} \Pr(s_{-i}, r_{-i} < b_{-i}|l)\Pr(H, drill|l, s_{-i})(1-c) \\ &+ \mathcal{I}\sum_{s_{-i}} \Pr(s_{-i}, r_{-i} < b_{-i}|l)[\Pr(H, wait|l, s_{-i})(1-c) - \Pr(L, wait|l, s_{-i})c],\end{aligned}$$

where $\mathcal{I} = 1$ if $\Pr(H|l, r_{-i} < b_{-i}, a_{-i,1} = wait) \geq c$ and $\mathcal{I} = 0$ otherwise. One has, $\frac{\partial \Pr(\cdot|l)W(\cdot)}{\partial b_l} = A + \mathcal{I}B - \mathcal{I}C$, where $A = \Pr(l|l)\Pr(H, drill|l, l)(1-c)$, $B = \Pr(l|l)\Pr(H, wait|l, l)(1-c)$, and $C = \Pr(l|l)\Pr(L, wait|l, l)c$. Observe that $\frac{\partial \Pr(\cdot|l)W(\cdot)}{\partial b_l} < 1$ both when $\mathcal{I} = 1$ and when $\mathcal{I} = 0$. Hence, $\frac{\partial Z(b_l,\delta)}{\partial b_l} > 0$. Observe also that if $\lambda(h) > 0$, $W(l,\cdot) > 0$ and thus that $Z(0,\delta) < 0$. If $b_l = b_h$, $Z(b_h, \delta) > 0$ as $b_l^w \leq b_l^* < b_h$ where the last inequality follows from Proposition 1. By continuity there exists a unique $b_l^w \in (0, b_h^*)$ such that $Z(b_l^w, \delta) = 0$.

We know from the implicit function theorem that $\frac{db_l^w}{d\delta} = -\frac{\frac{\partial Z}{\partial \delta}}{\frac{\partial Z}{\partial b_l}}$. I have shown above that $\frac{\partial Z}{\partial b_l} > 0$. As $\frac{\partial Z}{\partial \delta} = -\frac{1}{2}\Pr(r_{-i} < b_{-i}|l)W(l,\cdot) < 0$, this proves the second statement in the lemma.
■

Observe that b_l^w represents the optimal bid of a low-type player given that at time zero she anticipates that she will wait at time one (in case the other player also won her tract) and given that she will not drill in case $r_{-i} > b_{-i}$. As b_l^w depends on the discount factor and on drilling probabilities, in the following paragraphs I will sometimes write $b_l^w(\delta, \lambda(l), \lambda(h))$ instead of b_l^w. Observe also that my Δ-function (defined by equation 24) not only depends on s_i, $\lambda(l)$, and $\lambda(h)$, but also on b_l (in particular $\Pr(H|s_i, r_{-i} < b_{-i})$ is decreasing in b_l, and player i's posterior influences her drilling vs. waiting decision) and δ. To stress this dependency, in what follows I will often write $\Delta(s_i, \lambda(l), \lambda(h), b_l, \delta)$ instead of $\Delta(s_i, \lambda(l), \lambda(h))$.

Lemma 7 *Suppose player i is a low-type player. If the discount rate increases and if $b_{-i} = b_l^w$ if $s_{-i} = l$ and $b_{-i} = \frac{1}{2}(p-c)$ if $s_{-i} = h$, then player i has more incentives to wait. Formally,* $\frac{d\Delta(l,\lambda(l),\lambda(h),b_l^w,\delta)}{d\delta} < 0$.

Proof: The stated inequality is equivalent to $\frac{d\Delta(l,\cdot)}{d\delta} = \frac{\partial \Delta(l,\cdot)}{\partial \delta} + \frac{\partial \Delta(l,\cdot)}{\partial b_l^w}\frac{db_l^w}{d\delta} < 0$. Using 24, the inequality is equivalent to

$$-W(l,\cdot) + \left[\frac{\partial \Pr(H|l, r_{-i} < b_{-i})}{\partial b_l^w} - \delta\frac{\partial W(l,\cdot)}{\partial b_l^w}\right]\frac{db_l^w}{d\delta} < 0. \tag{27}$$

It follows from the proof of Lemma 6 that

$$\frac{db_l^w}{d\delta} = \frac{\frac{1}{2}\Pr(r_{-i} < b_{-i}|l)W(l,\lambda(l),\lambda(h))}{1 - \frac{1}{2}\delta\frac{\partial \Pr(r_{-i}<b_{-i}|l)W(l,\cdot)}{\partial b_l^w}}. \tag{28}$$

Replacing $\frac{db_l^w}{d\delta}$ in 27 by the right-hand side of 28, and rearranging, one has:

$$\frac{d\Delta(l,\cdot)}{d\delta} < 0 \Leftrightarrow \frac{1}{2}\Pr(r_{-i} < b_{-i}|l)\frac{\partial \Pr(H|l, r_{-i} < b_{-i})}{\partial b_l^w} < 1 - \frac{1}{2}\delta\frac{\partial \Pr(r_{-i} < b_{-i}|l)}{\partial b_l^w}W(l,\cdot).$$

Observe that $\frac{1}{2}\delta W(l,\cdot) < 1$ and that $\frac{\partial \Pr(r_{-i}<b_{-i}|l)}{\partial b_l^w} = \Pr(s_{-i}=l|s_i=l) < 1$. My last inequality is then satisfied as its left-hand side is negative, while its right-hand side is positive. ■

Lemma 8 *Suppose player i is a high-type player. Suppose $b_{-i} = b_l^w$ if $s_{-i} = l$ and $b_{-i} = \frac{1}{2}(p-c)$ if $s_{-i} = h$. Then:*

$$\begin{aligned}\frac{d\Delta(h,\cdot)}{d\delta} &= -W(h,\cdot) + \frac{\partial \Pr(l|h, r_{-i} < b_{-i})}{\partial b_l}\Bigg\{ -(1-\delta)[\Pr(H|h,h) - \Pr(H|h,l)] \\ &- \delta\Big\{[\Pr(H, drill|h,h) - \Pr(H, drill|h,l)]c + (1-\mathcal{I})[\Pr(H, wait|h,h) - \Pr(H, wait|h,l)] \\ &- \mathcal{I}[\lambda(h) - \lambda(l)]c\Big\}\Bigg\}\frac{db_l^w}{d\delta},\end{aligned} \tag{29}$$

where $\mathcal{I} = 1$ if $\Pr(H|h, r_{-i} < b_{-i}, a_{-i,1} = wait) \geq c$ and equals zero otherwise. Moreover, if either δ is sufficiently low, or p sufficiently high, $\frac{d\Delta(h,\cdot)}{d\delta} < 0$.

Proof: Using an identical reasoning as the one present in Lemma 7,

$$\frac{d\Delta(h,\cdot)}{d\delta} = -W(h,\cdot) + \left[\frac{\partial \Pr(H|h, r_{-i} < b_{-i})}{\partial b_l} - \delta\frac{\partial W(h,\cdot)}{\partial b_l}\right]\frac{db_l^w}{d\delta}. \tag{30}$$

As $-W(h,\cdot) < 0$, as $\frac{db_l^w}{d\delta} > 0$, as $\frac{\partial \Pr(H|h,r_{-i}<b_{-i})}{\partial b_l} < 0$, and as $\frac{\partial W(h,\cdot)}{\partial b_l}$ is bounded, $\exists\tilde{\delta} \in (0,1]$ such that $\forall \delta < \tilde{\delta}$, $\frac{d\Delta(h,\cdot)}{d\delta} < 0$. Inserting 23 into 30, using the fact that $\frac{\partial \Pr(l|h,r_{-i}<b_{-i})}{\partial b_l} = -\frac{\partial \Pr(h|h,r_{-i}<b_{-i})}{\partial b_l}$, and rewriting, yields 29. The last claim in the lemma then follows from the fact that $\lim_{p\to 1}\frac{\partial \Pr(l|h,r_{-i}<b_{-i})}{\partial b_l} = 0$. ■

Lemma 9 *Suppose players bid $b_l^w(\delta, \lambda(l), \lambda(h))$ if they possess signal l and $\frac{1}{2}(p-c)$ if they possess signal h. Suppose also that signals are sufficiently precise. Then: $\frac{d\Delta(l,\lambda(l),\lambda(h),b_l^w,\delta)}{d\lambda(l)} < 0$, $\frac{d\Delta(l,\lambda(l),\lambda(h),b_l^w,\delta)}{d\lambda(h)} < 0$ and $\frac{d\Delta(h,\lambda(l),\lambda(h),b_l^w,\delta)}{d\lambda(h)} < 0$.*

Proof: Suppose $s_i = l$. One has: $\frac{d\Delta(l,\lambda(l),\lambda(h),b_l^w,\delta)}{d\lambda(s_{-i})} = \frac{\partial\Delta(l,\cdot)}{\partial\lambda(s_{-i})} + \frac{\partial\Delta(l,\cdot)}{\partial b_l^w}\frac{db_l^w}{d\lambda(s_{-i})}$. It follows from 24 that

$$\frac{\partial\Delta(l,\cdot)}{\partial\lambda(s_{-i})} = -\delta\frac{\partial W(l,\cdot)}{\partial\lambda(s_{-i})}, \text{ and that } \frac{\partial\Delta(l,\cdot)}{\partial b_l^w} = \frac{\partial \Pr(H|l, r_{-i} < b_{-i})}{\partial b_l^w} - \delta\frac{\partial W(l,\cdot)}{\partial b_l^w}. \tag{31}$$

It follows from the implicit function theorem that $\frac{db_l^w}{d\lambda(s_{-i})} = -\frac{\frac{\partial Z}{\partial\lambda(s_{-i})}}{\frac{\partial Z}{\partial b_l^w}}$, where the Z-function is defined by equation 26. One has

$$\frac{\partial Z}{\partial\lambda(s_{-i})} = -\frac{1}{2}\delta\Pr(r_{-i} < b_{-i}|l)\frac{\partial W(l,\cdot)}{\partial\lambda(s_{-i})}, \text{ and } \frac{\partial Z}{\partial b_l^w} = 1 - \frac{1}{2}\delta\frac{\partial \Pr(r_{-i} < b_{-i}|l)W(l,\cdot)}{\partial b_l^w}. \tag{32}$$

Remember that in the proof of Lemma 6, I have shown that $\frac{\partial Z}{\partial b_l^w} > 0$. On the basis 31 and 32, $\frac{d\Delta(l,\cdot)}{d\lambda(s_{-i})} < 0 \Leftrightarrow$

$$-\delta\frac{\partial W(l,\cdot)}{\partial\lambda(s_{-i})} + \left\{\frac{\partial \Pr(H|l, r_{-i} < b_{-i})}{\partial b_l^w} - \delta\frac{\partial W(l,\cdot)}{\partial b_l^w}\right\}\frac{\frac{1}{2}\delta\Pr(r_{-i} < b_{-i}|l)\frac{\partial W(l,\cdot)}{\partial\lambda(s_{-i})}}{1 - \frac{1}{2}\delta\frac{\partial \Pr(r_{-i}<b_{-i}|l)W(l,\cdot)}{\partial b_l^w}} < 0$$

$$\Leftrightarrow \frac{1}{2}\Pr(r_{-i} < b_{-i}|l)\frac{\partial \Pr(H|l, r_{-i} < b_{-i})}{\partial b_l^w} < 1 - \frac{1}{2}\delta \frac{\partial \Pr(r_{-i} < b_{-i}|l)}{\partial b_l^w} W(l, \cdot). \tag{33}$$

Observe that $\frac{\partial \Pr(H|l,r_{-i}<b_{-i})}{\partial b_l^w} < 0$, that $\frac{\partial \Pr(r_{-i}<b_{-i}|l)}{\partial b_l^w} = \Pr(l|l)$, and that $W(l, \cdot) < 1$. Hence, inequality 33 is satisfied as its left-hand side is negative, while its right-hand side is positive.

Suppose now that $s_{-i} = h$. Using a similar procedure as above and using Lemma 8, one has

$$\begin{aligned} \frac{d\Delta(h,\cdot)}{d\lambda(h)} &= -\delta \frac{\partial W(h,\cdot)}{\partial \lambda(h)} + \frac{\partial \Pr(l|h, r_{-i} < b_{-i})}{\partial b_l^w} \Bigg\{ -(1-\delta)[\Pr(H|h,h) - \Pr(H|h,l)] \\ &- \delta \Big\{ [\Pr(H, drill|h,h) - \Pr(H, drill|h,l)]c + (1-\mathcal{I})[\Pr(H, wait|h,h) - \Pr(H, wait|h,l)] \\ &- \mathcal{I}[\lambda(h) - \lambda(l)]c \Big\} \Bigg\} \frac{db_l^w}{d\lambda(h)}. \end{aligned}$$

It is easy to check that $\frac{\partial W(h,\cdot)}{\partial \lambda(h)} > 0$, and that $\frac{db_l^w}{d\lambda(h)}$ is bounded. The lemma then follows from the fact that $\lim_{p\to 1} \frac{\partial \Pr(l|h,r_{-i}<b_{-i})}{\partial b_l^w} = 0$ and from the fact that the term between curly brackets is bounded as well. ■

Define (b_l^d, δ^{c_1}) as a low-type player's bid and a discount factor such that

$$\Pr(H|l, r_{-i} < b_{-i}) - c > 0, \tag{34}$$

$$\Pr(H|l, r_{-i} > b_{-i}) < c, \tag{35}$$

$$\Pr(H|l, r_{-i} < b_{-i}) - c = \delta^{c_1} \Pr(H|l, r_{-i} < b_{-i})(1-c), \text{ and} \tag{36}$$

$$b_l^d = \frac{1}{2} E_{\frac{1}{2}}(U|l; \lambda(l) = \lambda(h) = 1). \tag{37}$$

δ^{c_1} represents the discount factor such that a low-type player is indifferent between drilling and waiting given that the other player won her tract, given that the other player will drill with probability one, and given that the other player optimally bid $\frac{1}{2}(p-c)$ if $s_{-i} = h$ and b_l^d if $s_{-i} = l$.

Lemma 10 *If $c < \frac{1}{2}$, there exists a unique (b_l^d, δ^{c_1}) which satisfies equations 34, 35, 36, and 37. Furthermore, $b_l^d > 0$ and $\delta^{c_1} \in (0, 1)$.*

Proof: It follows from 37 that b_l^d represents the optimal bid of a low-type player given that $\lambda(l) = \lambda(h) = 1$ and given that $\delta = \delta^{c_1}$. It then follows from Proposition 1 that $b_l^d < b_h^*$. Hence, $\Pr(H|l, r_{-i} > b_{-i}) < \Pr(H|l) = 1-p < c$. Hence, inequality 35 is satisfied. This result, combined with the fact that $\Pr(a_{-i,1} = wait|l, r_{-i} < b_{-i}; \lambda(h) = \lambda(l) = 1) = 0$, implies that the right-hand side of 36 represents $\delta^{c_1} W(l, 1, 1)$. This insight, combined with inequality 35 and equality 36, implies that equation 37 can be written as:

$$b_l^d = \frac{1}{2}\Pr(r_{-i} < b_{-i}|l)(\Pr(H|l, r_{-i} < b_{-i}) - c). \tag{38}$$

Call LHS (resp. RHS) the left-hand (resp. right-hand) side of the above equation. If $b_l = 0$, $LHS = 0 < \frac{1}{2}\Pr(s_{-i} = h|l)(\frac{1}{2} - c) = RHS$. If $b_l = b_h$, $LHS > 0 > \frac{1}{2} b_h(1 - p - c) = RHS$. This insight, combined with the fact that the RHS is decreasing in b_l (while its LHS is increasing), proves the existence of a unique $b_l^d > 0$ which satisfies the above equation. As $b_l^d > 0$, the right-hand side of 38 must be positive too, which implies that inequality 34 is satisfied. Plugging b_l^* (determined by equation 37) and $b_h^* = \frac{1}{2}(p - c)$ into 36 uniquely determines δ^{c_1}. Finally, it is straightforward to check that $\delta^{c_1} \in (0, 1)$. ■

Observe that b_l^d represents the optimal bid of a low-type player given that at time zero she anticipates that she will drill at time one (in case the other player also won her tract) and given that she will never drill in case $r_{-i} > b_{-i}$. Observe that b_l^d is independent of δ.

Lemma 11 *If $c < \frac{1}{2}$ and if $\delta < \delta_1^c$, there exists a unique monotone equilibrium in which $\lambda^*(l) = \lambda^*(h) = 1$, $b_h^* = \frac{1}{2}(p - c)$, and $b_l^* = b_l^d$.*

Proof: First, I show the existence of an equilibrium with the properties described in the lemma (Step 1). Next, I rule out other monotone symmetric equilibria (Step 2).

Step 1: By construction of (b_l^d, δ^{c_1}), $\Delta(l, 1, 1; b_l^d, \delta^{c_1}) = 0$. Hence, $\forall \delta < \delta^{c_1}$, $0 \leq \Delta(s_i, 1, 1, b_l^d, \delta^{c_1}) < \Delta(s_i, 1, 1, b_l^d, \delta)$, where the first and second inequalities respectively follow from Lemma 4 and equation 24. Hence, if $\delta < \delta^{c_1}$ and if player i expects player $-i$ to drill with probability one, it is a best reply for player i to drill with probability one. As $\Pr(H|l, r_{-i} > b_{-i}) < \Pr(H|l) < c$, and as $r \sim [0, 1]$, a low-type player cannot gain by bidding an amount different from b_l^d.

Step 2: It follows from Lemma 4 that I can without loss of generality restrict attention to candidate equilibria in which either (i) $\Delta(l, \lambda^*(l), \lambda^*(h)) < \Delta(h, \lambda^*(l), \lambda^*(h)) < 0$, or (ii) $\Delta(l, \lambda^*(l), \lambda^*(h)) < \Delta(h, \lambda^*(l), \lambda^*(h)) = 0$, or (iii) $\Delta(l, \lambda^*(l), \lambda^*(h)) \leq 0 < \Delta(h, \lambda^*(l), \lambda^*(h))$.

Suppose there exists an equilibrium in which $\Delta(l, \lambda^*(l), \lambda^*(h)) < \Delta(h, \lambda^*(l), \lambda^*(h)) < 0$, which implies that $\lambda^*(l) = \lambda^*(h) = 0$. Then, $\Delta(h, 0, 0) = (1 - \delta)[\Pr(H|h, r_{-i} < b_{-i}) - c]$, which is positive and thus contradicts our earlier assumption.

Suppose there exists an equilibrium in which $\Delta(l, \lambda^*(l), \lambda^*(h)) < \Delta(h, \lambda^*(l), \lambda^*(h)) = 0$, which implies that $\lambda^*(l) = 0$, $\lambda^*(h) \in [0, 1]$, and $b_l^* = b_l^w(\delta, 0, \lambda^*(h))$. We then, however, run into the following contradiction:

$$\begin{aligned} 0 &= \Delta(l, 1, 1, b_l^d, \delta^{c_1}) = \Delta(l, 1, 1, b_l^w(\delta^{c_1}, 1, 1), \delta^{c_1}) < \Delta(l, 0, \lambda^*(h), b_l^w(\delta^{c_1}, 0, \lambda^*(h)), \delta^{c_1}) \\ &< \Delta(l, 0, \lambda^*(h), b_l^w(\delta, 0, \lambda^*(h)), \delta) < \Delta(h, 0, \lambda^*(h), b_l^w(\delta, 0, \lambda^*(h)), \delta) = 0, \end{aligned}$$

where the second equality sign follows from the fact that if $(\lambda(l), \lambda(h)) = (1, 1)$ and if $\delta = \delta^{c_1}$, a low-type player is indifferent between drilling and waiting (which implies that $b_l^d = b_l^w(\delta^{c_1}, 1, 1)$), and where the first, second, and third inequalities respectively follow from Lemmas 9, 7, and 4.

Suppose there exists an equilibrium in which $\Delta(l, \lambda^*(l), \lambda^*(h)) \leq 0 < \Delta(h, \lambda^*(l), \lambda^*(h))$, which implies that $\lambda^*(l) \in [0,1]$, $\lambda^*(h) = 1$, and $b_l^* = b_l^w(\delta, \lambda^*(l), 1)$. We then run into the following contradiction:

$$\begin{aligned} 0 &= \Delta(l, 1, 1, b_l^w(\delta^{c_1}, 1, 1), \delta^{c_1}) \leq \Delta(l, \lambda^*(l), 1, b_l^w(\delta^{c_1}, \lambda^*(l), 1), \delta^{c_1}) \\ &< \Delta(l, \lambda^*(l), 1, b_l^w(\delta, \lambda^*(l), 1), \delta) \leq 0, \end{aligned}$$

where the first and second inequality respectively follow from Lemma 9 and Lemma 7. ■

Define (b_l^d, δ^{c_2}) as a low-type player's bid and a discount factor which satisfies 34, 35, and which also satisfies

$$\Pr(H|l, r_{-i} < b_{-i}) - c = \delta^{c_2} \Pr(H, h|l, r_{-i} < b_{-i})(1-c), \text{ and} \tag{39}$$

$$b_l^d = E_{\frac{1}{2}}(U|l; \lambda(l) = 0, \lambda(h) = 1). \tag{40}$$

δ^{c_2} represents the discount factor such that a low-type player is indifferent between drilling and waiting given that the other player won her tract, given that the other player waits (with probability one) if $s_{-i} = l$, given that the other player drills (with probability one) if $s_{-i} = h$, and given that the other player optimally bids $\frac{1}{2}(p-c)$ if $s_{-i} = h$ and b_l^d if $s_{-i} = l$. Observe also that, as a low-type player is indifferent between drilling and waiting, $b_l^d = b_l^w(\delta^{c_2}, 0, 1)$.

Lemma 12 *If $c < \frac{1}{2}$, there exists a unique (b_l^d, δ^{c_2}) which satisfies equations 34, 35, 39, and 40. Furthermore, $\delta^{c_2} \in (\delta^{c_1}, 1)$.*

Proof: Inequality 35 is satisfied for the same reason as the one explained in the proof of Lemma 10. As $\lambda(l) = 0$ and $\lambda(h) = 1$, $\Pr(H|l, r_{-i} < b_{-i}, a_{-i,1} = wait) = \Pr(H|l, l) < c$. Both insights, combined with equality 39, imply that in this case equality 40 is also equivalent to 38. From the proof of Lemma 10, we know that there exists a unique $b_l^d > 0$ which satisfies equation 38 and inequality 34. Plugging b_l^d and $b_h^* = \frac{1}{2}(p-c)$ into 39 uniquely determines δ^{c_2}. Call LHS (RHS) the left-hand (respectively right-hand) side of equality 39. If $\delta^{c_2} = 1$, the inequality $LHS < RHS$ is equivalent to $(1-p)^2 b_l^*(1-c) < p(pb_l^* + (1-p)b_h^*)c$, which is satisfied. Hence, $\delta^{c_2} < 1$. Suppose $\delta^{c_2} \leq \delta^{c_1}$. On the basis of 36 and 39, we then run into the following contradiction:

$$\begin{aligned} \Pr(H|l, r_{-i} < b_{-i}) - c &= \delta^{c_2} \Pr(H, h|l, r_{-i} < b_{-i})(1-c) \\ &< \delta^{c_1} \Pr(H|l, r_{-i} < b_{-i})(1-c) = \Pr(H|l, r_{-i} < b_{-i}) - c. \end{aligned}$$

■

Lemma 13 *If $c < \frac{1}{2}$ and if $\delta \in (\delta^{c_1}, \delta^{c_2})$, there exists a monotone equilibrium in which $\lambda^*(l) \in (0,1)$, $\lambda^*(h) = 1$, $b_h^* = \frac{1}{2}(p-c)$, and $b_l^* = b_l^d$. Moreover, if signals are sufficiently precise, there exist no other monotone equilibria.*

Proof: The proof is divided in the same two steps as the ones in the proof of Lemma 11.

Step 1: As $\delta^{c_1} < \delta$, and as b_l^d does not depend on the discount factor, $\Delta(l,1,1,b_l^d,\delta) < \Delta(l,1,1,b_l^d,\delta^{c_1}) = 0$, where the inequality follows from equation 24. Similarly, $0 = \Delta(l,0,1,b_l^d,\delta^{c_2}) < \Delta(l,0,1,b_l^d,\delta)$. Hence, $\forall \delta \in (\delta^{c_1},\delta^{c_2})$, $\Delta(l,1,1,b_l^d,\delta) < 0 < \Delta(l,0,1,b_l^d,\delta)$. It then follows from Lemma 3 that there exists a unique $\lambda^*(l) \in (0,1)$ such that $\Delta(l,\lambda^*(l),1,b_l^d,\delta) = 0$. Lemma 4 then implies that $0 < \Delta(h,\lambda^*(l),1,b_l^d,\delta)$. Hence, it is a best reply for an high-type player to drill at time one with probability one. Given that a low-type player is indifferent between drilling and waiting, she cannot gain by setting $b_l \neq b_l^d$.

Step 2: It follows from Lemma 4 that I can, without loss of generality, restrict attention to candidate equilibria in which either (i) $\Delta(l,\lambda^*(l),\lambda^*(h)) < \Delta(h,\lambda^*(l),\lambda^*(h)) < 0$, or (ii) $\Delta(l,\lambda^*(l),\lambda^*(h)) < \Delta(h,\lambda^*(l),\lambda^*(h)) = 0$, or (iii) $\Delta(l,\lambda^*(l),\lambda^*(h)) < 0 < \Delta(h,\lambda^*(l),\lambda^*(h))$, or (iv) $0 < \Delta(l,\lambda^*(l),\lambda^*(h)) < \Delta(h,\lambda^*(l),\lambda^*(h))$.

Case (i) can be ruled out on the basis of the same argument as the one which appears in the proof of lemma 11.

Suppose there exists an equilibrium in which $\Delta(l,\lambda^*(l),\lambda^*(h)) < \Delta(h,\lambda^*(l),\lambda^*(h)) = 0$, which implies that $\lambda^*(l) = 0$, $\lambda^*(h) \in [0,1]$, and $b_l^* = b_l^w(\delta,0,\lambda^*(h))$. If p is sufficiently high, however, we run into the following contradiction:

$$\begin{aligned} 0 &= \Delta(l,0,1,b_l^w(\delta^{c_2},0,1),\delta^{c_2}) \leq \Delta(l,0,\lambda^*(h),b_l^w(\delta^{c_2},0,\lambda^*(h)),\delta^{c_2}) \\ &< \Delta(h,0,\lambda^*(h),b_l^w(\delta^{c_2},0,\lambda^*(h)),\delta^{c_2}) < \Delta(h,0,\lambda^*(h),b_l^w(\delta,0,\lambda^*(h)),\delta) = 0, \end{aligned}$$

where the first inequality follows from Lemma 9, the second inequality follows from Lemma 4, and where the third inequality sign follows from Lemma 8.

Suppose there exists an equilibrium in which $\Delta(l,\lambda^*(l),\lambda^*(h)) < 0 < \Delta(h,\lambda^*(l),\lambda^*(h))$, which implies that $\lambda^*(l) = 0$, $\lambda^*(h) = 1$, and $b_l^* = b_l^w(\delta,0,1)$. However, we then run into the following contradiction:

$$0 = \Delta(l,0,1,b_l^w(\delta^{c_2},0,1),\delta^{c_2}) < \Delta(l,0,1,b_l^w(\delta,0,1),\delta) < 0,$$

where the inequality follows from Lemma 7.

Suppose there exists an equilibrium in which $0 < \Delta(l,\lambda^*(l),\lambda^*(h)) < \Delta(h,\lambda^*(l),\lambda^*(h))$, which implies that $\lambda^*(l) = \lambda^*(h) = 1$, and $b_l^* = b_l^d$. We then, however, run into the following contradiction: $0 = \Delta(l,1,1,b_l^d,\delta^{c_1}) > \Delta(l,1,1,b_l^d,\delta) > 0$. ■

Define $(b_l^w(\delta^{c_3},0,1),\delta^{c_3})$ as a low-type player's bid and a discount factor which satisfies 35 and which also satisfies

$$\begin{aligned} \Pr(H|h,r_{-i}<b_{-i}) - c &= \delta^{c_3}\Pr(H,h|h,r_{-i}<b_{-i})(1-c) \\ &+ \delta^{c_3}\Pr(s_{-i}=l|h,r_{-i}<b_{-i})\max\{0,\Pr(H|h,l)-c\} \end{aligned} \tag{41}$$

$$b_l^w(\delta^{c_3},0,1) = \frac{1}{2}\delta^{c_3}\Pr(r_{-i} < b_{-i}|l)W(l,0,1), \text{ and} \tag{42}$$

$$\Pr(H|l, r_{-i} < b_{-i}) - c < \delta^{c_3}W(l,0,1). \tag{43}$$

δ^{c_3} represents the discount factor such that an high-type player is indifferent between drilling and waiting given that the other player won her tract, given that the other player waits (with probability one) if $s_{-i} = l$, given that the other player drills (with probability one) if $s_{-i} = h$, and given that the other player bid $\frac{1}{2}(p-c)$ if $s_{-i} = h$ and $b_l^w(\delta^{c_3},0,1)$ if $s_{-i} = l$. Henceforth δ^{c_2} is computed out of equations 34, 35, 39, and 40 when $c < \frac{1}{2}$. If $c \geq \frac{1}{2}$, $\delta^{c_2} = 0$.

Lemma 14 *There exists a unique $(b_l^w(\delta^{c_3},0,1),\delta^{c_3})$ which satisfies equations 35, 41, 42, and 43. Moreover, $\delta^{c_3} \in (\delta^{c_2},1)$.*

Proof: One can check that equation 41 is equivalent to

$$\delta^{c_3} = 1 - \frac{2p(1-p)(1-\mathcal{I})(c-\frac{1}{2})b_l^w + (1-p)^2cb_h^*}{p^2(1-c)b_h^* + 2p(1-p)\mathcal{I}(\frac{1}{2}-c)b_l^w} \equiv \delta_A^{c_3}(b_l^w),$$

where $\mathcal{I} = 1$ if $c < \frac{1}{2}$ and is equal to zero otherwise. If $\mathcal{I} = 1$, $\delta_A^{c_3}$ is an increasing and *concave* function in b_l^w. If $\mathcal{I} = 0$, $\delta_A^{c_3}$ is a decreasing function in b_l^w. It can also be checked that equation 42 is equivalent to

$$\delta^{c_3} = \frac{2}{p(1-p)(1-c)b_h^*}b_l^w \equiv \delta_B^{c_3}(b_l^w),$$

which is increasing and *linear* in b_l^w. One can also check that $\delta_B^{c_3}(0) < \delta_A^{c_3}(0)$ and that $\delta_B^{c_3}(b_h^*) > \delta_A^{c_3}(b_h^*)$. By continuity, there exists a unique $(b_l^w(\delta^{c_3}),\delta^{c_3})$ which satisfies equations 41 and 42. Furthermore, $0 = \Delta(h,0,1,b_l^w(\delta^{c_3},0,1),\delta^{c_3}) > \Delta(l,0,1,b_l^w(\delta^{c_3},0,1),\delta^{c_3})$, where the inequality follows from Lemma 4. Hence, inequality 43 is satisfied. Inequality 35 is satisfied for the same reason as the one explained in Lemma 10.

Suppose $\delta^{c_3} \leq \delta^{c_2}$ and that $c < \frac{1}{2}$. We then run into the following contradiction:

$$0 = \Delta(l,0,1,b_l^w(\delta^{c_2},0,1),\delta^{c_2}) \leq \Delta(l,0,1,b_l^w(\delta^{c_3},0,1),\delta^{c_3}) < \Delta(h,0,1,b_l^w(\delta^{c_3},0,1),\delta^{c_3}) = 0,$$

where the first and second inequalities respectively follow from Lemmas 7 and 4. Suppose $\delta^{c_3} = 0$ and that $c > \frac{1}{2}$. Then $b_l^w(0,0,1) = 0$ and we run into the following contradiction

$$0 < \Pr(H|h,h) - c = \Delta(h,0,1,b_l^w(0,0,1),0) = 0.$$

If $\delta^{c_3} = 1$, the right-hand side of 41 can be rewritten as

$$\mathcal{I}[\Pr(H|h,r_{-i} < b_{-i}) - c] + \mathcal{I}\Pr(L,h|h,r_{-i} < b_{-i})c + (1-\mathcal{I})\Pr(H,h|h,r_{-i} < b_{-i})(1-c),$$

which is greater than its left-hand side. Hence, $\delta^{c_3} \in (\delta^{c_2},1)$. ■

Lemma 15 *If $\delta \in (\delta^{c_2}, \delta^{c_3})$ and if signals are sufficiently precise, there exists a unique monotone equilibrium in which $\lambda^*(l) = 0$, $\lambda^*(h) = 1$, $b_h^* = \frac{1}{2}(p-c)$, and $b_l^* = b_l^w(\delta, 0, 1)$.*

Proof: The proof is divided in the same two steps as the ones in the proof of Lemma 11.

Step 1: By construction of $(b_l^w(\delta^{c_2}, 0, 1), \delta^{c_2})$ and $(b_l^w(\delta^{c_3}, 0, 1), \delta^{c_3})$, $0 \geq \Delta(l, 0, 1, b_l^w(\delta^{c_2}, 0, 1), \delta^{c_2})$ and $0 = \Delta(h, 0, 1, b_l^w(\delta^{c_3}, 0, 1), \delta^{c_3})$. It then follows from Lemma 7 that $\forall \delta > \delta^{c_2}$, $\Delta(l, 0, 1, b_l^w(\delta, 0, 1), \delta) < 0$. Hence, it is a best reply for a low-type player to wait and thus to bid $b_l^w(\delta, 0, 1)$. Furthermore, it follows from Lemma 8 that if signals are sufficiently precise, $\forall \delta < \delta^{c_3}$, $\Delta(h, 0, 1, b_l^w(\delta, 0, 1), \delta) > 0$. Hence, it is a best reply for an high-type player to drill (with probability one).

Step 2: From Lemma 4, I can without loss of generality restrict attention to candidate equilibria in which either (i) $\Delta(l, \lambda^*(l), \lambda^*(h)) < \Delta(h, \lambda^*(l), \lambda^*(h)) < 0$, or (ii) $\Delta(l, \lambda^*(l), \lambda^*(h)) < \Delta(h, \lambda^*(l), \lambda^*(h)) = 0$, or (iii) $0 = \Delta(l, \lambda^*(l), \lambda^*(h)) < \Delta(h, \lambda^*(l), \lambda^*(h))$, or (iv) $0 < \Delta(l, \lambda^*(l), \lambda^*(h)) < \Delta(h, \lambda^*(l), \lambda^*(h))$.

Case (i) can be ruled out on the basis of the same argument as the one which appears in the proof of lemma 11.

Suppose there exists an equilibrium in which $\Delta(l, \lambda^*(l), \lambda^*(h)) < \Delta(h, \lambda^*(l), \lambda^*(h)) = 0$, which implies that $\lambda^*(l) = 0$, $\lambda^*(h) \in [0, 1]$, and $b_l^* = b_l^w(\delta, 0, \lambda^*(h))$. As signals are sufficiently precise and as $\delta < \delta^{c_3}$, we run into the following contradiction:

$$0 = \Delta(h, 0, 1, b_l^w(\delta^{c_3}, 0, 1), \delta^{c_3}) < \Delta(h, 0, 1, b_l^w(\delta, 0, 1), \delta) \leq \Delta(h, 0, \lambda^*(h), b_l^w(\delta, 0, \lambda^*(h)), \delta) = 0,$$

where the first and second inequalities respectively follow from Lemmas 8 and 9.

Suppose there exists an equilibrium in which $0 = \Delta(l, \lambda^*(l), \lambda^*(h)) < \Delta(h, \lambda^*(l), \lambda^*(h))$, which implies that $\lambda^*(l) \in [0, 1]$, $\lambda^*(h) = 1$, and $b_l^* = b_l^d$. We then run into the following contradiction:

$$0 = \Delta(l, \lambda^*(l), 1, b_l^d, \delta) \leq \Delta(l, 0, 1, b_l^d, \delta) < \Delta(l, 0, 1, b_l^d, \delta^{c_2}) = 0,$$

where the first and second inequality respectively follow from Lemma 3 and equation 24.

Suppose there exists an equilibrium in which $0 < \Delta(l, \lambda^*(l), \lambda^*(h)) < \Delta(h, \lambda^*(l), \lambda^*(h))$, which implies that $\lambda^*(l) = \lambda^*(h) = 1$, and $b_l^* = b_l^d$. We then run into the following contradiction:

$$0 < \Delta(l, 1, 1, b_l^d, \delta) < \Delta(l, 1, 1, b_l^d, \delta^{c_2}) < \Delta(l, 0, 1, b_l^d, \delta^{c_2}) \leq 0,$$

where the second and third inequalities respectively follow from equation 24 and Lemma 3. ■

Lemma 16 *If $\delta \in (\delta^{c_3}, 1)$ and if signals are sufficiently precise, there exists a unique monotone equilibrium in which $\lambda^*(l) = 0$, $\lambda^*(h) \in (0, 1)$, $b_l^* = b_l^w(\delta, 0, \lambda^*(h))$ and $b_h^* = \frac{1}{2}(p-c)$.*

Proof: The proof is divided in the same two steps as the ones in the proof of Lemma 11.

Step 1: By construction of $(b_l^w(\delta^{c_3}0,1),\delta^{c_3})$, $0=\Delta(h,0,1,b_l^w(\delta^{c_3},0,1),\delta^{c_3})$. As signals are sufficiently precise and as $\delta^{c_3}<\delta$, we know from Lemma 8 that $\Delta(h,0,1,b_l^w(\delta,0,1),\delta)<0$. It can also be checked that $\Delta(h,0,0,b_l^w(\delta,0,0),\delta)>0$. It then follows from lemma 9 that there exists a unique $\lambda^*(h)$ such that $\Delta(h,0,\lambda^*(h),b_l^w(\delta,0,\lambda^*(h)),\delta)=0$. This result, combined with Lemma 4, implies that $\Delta(l,0,\lambda^*(h),b_l^w(\delta,0,\lambda^*(h)),\delta)<0$. Hence, it is a best reply for a low-type player to wait (and thus to bid $b_l^w(\delta,0,\lambda^*(h))$).

Step 2: From Lemma 4, I can without loss of generality restrict attention to candidate equilibria in which either (i) $\Delta(l,\lambda^*(l),\lambda^*(h))<\Delta(h,\lambda^*(l),\lambda^*(h))<0$, or (ii) $\Delta(l,\lambda^*(l),\lambda^*(h))<0$ $<\Delta(h,\lambda^*(l),\lambda^*(h))$, or (iii) $0=\Delta(l,\lambda^*(l),\lambda^*(h))<\Delta(h,\lambda^*(l),\lambda^*(h))$, or (iv) $0<\Delta(l,\lambda^*(l),$ $\lambda^*(h))<\Delta(h,\lambda^*(l),\lambda^*(h))$.

Case (i) can be ruled out on the basis of the same argument as the one which appears in the proof of lemma 11.

Suppose there exists an equilibrium in which $\Delta(l,\lambda^*(l),\lambda^*(h))<0<\Delta(h,\lambda^*(l),\lambda^*(h))$, which implies that $\lambda^*(l)=0$, $\lambda^*(h)=1$ and $b_l^*=b_l^w(\delta,0,1)$. As signals are sufficiently precise, we then run into the following contradiction:

$$0<\Delta(h,0,1,b_l^w(\delta,0,1),\delta)<\Delta(h,0,\lambda^*(h),b_l^w(\delta,0,\lambda^*(h)),\delta)=0,$$

where the equality sign follows from Step 1 and where the second inequality follows from Lemma 9.

Suppose there exists an equilibrium in which $0=\Delta(l,\lambda^*(l),\lambda^*(h))<\Delta(h,\lambda^*(l),\lambda^*(h))$, which implies that $\lambda^*(h)=1$ and that $b_l^*=b_l^d=b_l^w(\delta,\lambda^*(l),1)$. As signals are sufficiently precise, we run into the following contradiction:

$$\begin{aligned} 0 &= \Delta(l,\lambda^*(l),1,b_l^w(\delta,\lambda^*(l),1),\delta)\leq\Delta(l,0,1,b_l^w(\delta,0,1),\delta)<\Delta(h,0,1,b_l^w(\delta,0,1),\delta) \\ &< \Delta(h,0,1,b_l^w(\delta^{c_3},0,1),\delta^{c_3})=0, \end{aligned}$$

where the first, second and third inequalities respectively follow from Lemmas 9, 4, and 8.

Suppose there exists an equilibrium in which $0<\Delta(l,\lambda^*(l),\lambda^*(h))<\Delta(h,\lambda^*(l),\lambda^*(h))$, which implies that $\lambda^*(l)=\lambda^*(h)=1$ and that $b_l^*=b_l^d$. We then run into the following contradiction:

$$0<\Delta(l,1,1,b_l^d,\delta)<\Delta(l,0,1,b_l^d,\delta)<\Delta(l,0,1,b_l^d,\delta^{c_2})\leq 0,$$

where the second and third inequalities respectively follow from Lemma 3 and equation 24 and from our finding (proven in Lemma 14) that $\delta^{c_2}<\delta^{c_3}<\delta$. ■

Proof of Proposition 5

Observe that if $c > \Pr(H|h,l)$ and if $\delta \in (\delta^{c_2}, 1)$

$$\begin{aligned} E_{\frac{1}{2}}(U|l,\mathcal{D}) = 0 &< E_{\frac{1}{2}}(U|l,\mathcal{ND}), \text{ and} \\ E_{\frac{1}{2}}(U|h,\mathcal{D}) &> E_{\frac{1}{2}}(U|h,\mathcal{ND}), \end{aligned} \tag{44}$$

where the second inequality follows from the paragraph which precedes Proposition 3. The auctioneer's choice of auction then depends on her prior beliefs about i's type. In the paragraph which precedes Proposition 5, however, I argued that if $c = \Pr(H|h,l)$ and if $\delta \in (\delta^{c_2}, 1)$, the high type is indifferent between both auction designs. As $E_{\frac{1}{2}}(U|s_i,k)$ $(k \in \{\mathcal{D}, \mathcal{ND}\})$ is continuous in c, there exists a $\bar{c}(\delta) > \Pr(H|h,l)$ such that $\forall\ c \in (\Pr(H|h,l), \bar{c}(\delta))$, $W(\mathcal{D}) < W(\mathcal{ND})$.

Suppose now that $c < \frac{1}{2}$ and that $\delta \in (\delta^{c_2}, 1)$. As a high-type player then always (i.e $\forall\ x^{\mathcal{D}}$ and $\forall\ x_2^{\mathcal{ND}}$) faces a non-negative gain of drilling,

$$E_{\frac{1}{2}}(U|h,\mathcal{D}; c < \Pr(H|h,l)) = E_{\frac{1}{2}}(U|h,\mathcal{ND}; c < \Pr(H|h,l)). \tag{45}$$

As is clear from 16, if $c < \frac{1}{2}$ the comparison between $E_{\frac{1}{2}}(U|l,\mathcal{D})$ and $E_{\frac{1}{2}}(U|l,\mathcal{ND})$ is not clear cut. We know, however, that if $c = \Pr(H|h,l)$, the low-type player gains more (and, thus, bids more) if bids are not disclosed. As $E_{\frac{1}{2}}(U|l,\mathcal{D})$ and $E_{\frac{1}{2}}(U|l,\mathcal{ND})$ are continuous in c, for c sufficiently close to $\Pr(H|h,l)$ a low-type player still prefers the auction without bid disclosure. This insight, proves the proposition. ■

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