Equity Issuance and Dividend Policy under Commitment*
Work in Progress

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Abstract. This paper studies a model of corporate finance in which firms use stock issuance to finance investment. We assume that the firm is "rational" and therefore recognizes the relationship between future dividends and stock prices. Under this assumption, future variables enter in the constraints of the firm, so that the problem is not recursive in a standard sense and the Bellman equation does not hold. This implies that the model has to be solved with recursive contracts methods such as the ones used, for example, in models of optimal macroeconomic policy or in risk sharing models with participation constraints. In addition, financial policy may be time inconsistent. First, we characterize several cases where time consistency arises. Second, we compare numerically the full commitment (and potentially time inconsistent) solution of a "rational" firm to the one of a "naive" firm that ignores the relationship between current price and future dividends. Our results suggest that growing firms will pay lower dividends at the beginning and promise higher dividends in the future. This allows them to raise cheaper external funds through a higher value of stocks, accumulate more capital, and grow faster.

Keywords: Stock Issuance; time inconsistency; dividend policy
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1. Introduction

Although there is an enormous amount of work on consumers' portfolio choice in dynamic stochastic models, there is a lot less formal work on firms' choice of assets in order to finance investment. A big part of the literature on firm financing has focused on the debt/equity choice, while less attention is paid to issues such as the importance of dividends versus repurchases in the payout composition or the interplay between dividends, stock issuance and investment. This is understandable, since dividends have been the prominent form of payout during many years, but this is not so anymore. In particular, repurchases have grown considerably since the 80's, the proportion of dividend paying firms has declined, and most firms have increased their payout through repurchases. In this paper, we study a dynamic stochastic general equilibrium framework that can be used to analyze these issues.

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Our framework differs from the literature in several important aspects. First, we focus on the choice of dividends and stocks and therefore abstract from the debt/equity choice. In other words, we assume that firms can finance investment with internal funds and outside equity. The choice of the firm is, then, about how many stocks to issue (or repurchase), how much to pay out as dividends and how much to invest. Second, most dynamic analyses of financial policy share the common features of ruling out repurchases and obtaining the pecking order theory. In this setting, the pecking order theory would imply that firms will use outside equity only if internal funds are not enough to finance the optimal level of investment. As a consequence, firms will not increase dividends while they are issuing equity. Given the empirical implausibility of these results, we focus on cases under which the pecking order theory does not hold. Third, the literature typically breaks the Modigliani Miller irrelevance theorem by adding financing frictions. While we allow for general frictions, we also consider different objectives for the firm (or different managerial compensations) under which the manager is in conflict with the objective of market investors, the conflict arising from different degrees of risk aversion. One way to theoretically rationalize these objectives is to consider them as a reduced form agency issues (not modelled here).

Given the above departures from existing literature, we study the following issue. Under rational expectations and non-bubble stock prices, market investors will only hold the stocks issued by firms if the stock price is equal to the present discounted value of future dividends. *Rational* firms selling their stock in a competitive market should recognize this relationship between future dividends and current stock prices, which we label price-dividend mapping. In other words, they should realize that their own plans for future dividends influence the current stock price and, therefore, the amount of funds that can be raised by issuing stocks today. While the relationship between future dividends and current prices can be important for the determination of financial policy, the literature has avoided considering this link by assuming value maximization as a firm objective and/or very particular financing frictions. In such a setup the issues we discuss below do not arise.

The assumption of rational firms is both theoretically and empirically appealing. However, it poses several technical difficulties. If future variables (in this case, future dividends) enter the current firm's budget constraint, the problem of the firm might not be recursive in the standard sense and standard dynamic programming is not applicable. The problem of the firm is then of a similar nature to a problem of optimal macroeconomic policy, since it chooses today's dividend and stock issuance in a way consistent with promises about future dividends that induce shareholders to hold the stock. As in models of optimal policy, financial policy is generally time inconsistent. Using the techniques of recursive contracts developed in Marcet and Marimon (2008), we formulate the rational choice recursively by adding a co-state variable that captures past promises about dividends to be paid out today.

We argue that the lack of recursivity is a very general feature arising in most setups in which a firm uses stock issuance to finance its investment. Similarly, it would also arise if a firm uses stock repurchases to distribute profits to the stockholders. The key ingredient generating this result when markets are incomplete is the disagreement among different types of stockholders (or among these and the manager). As mentioned earlier, we consider an extreme form of market incompleteness by not allowing the firm to issue any asset except equity. However, the issues we analyze would continue to arise if other securities such as bonds were introduced as long as there is disagreement.

After discussing how to formulate the general problem recursively, we characterize several special cases that differ in the financing frictions and the compensation of the manager. According to the survey by Murphy (1999), the main components of US CEO compensation are a fixed salary, a bonus linked to performance, stock options and other forms of compensation, including restricted stock. We study three different forms of compensation that correspond to different combinations of these components. The first is linked to cash flows and it corresponds to value maximization if managers and shareholders have the same risk aversion, the second is linked to firm stock and the third is linked to cash flows and one period stock options. Further, we distinguish between symmetric and asymmetric frictions, with the difference being that the first imposes the friction on total dividends and on the total value of stocks, whereas the second imposes different frictions on the number of stocks and on the per share prices and dividends.

Our main results can be summarized as follows. First, if the manager's compensation is linked to cash flows, we find that financial policy is time consistent under agreement (value maximization), while it is time inconsistent under other forms of compensation. Second, when financing frictions are asymmetric, the rational solution might be different to the one in which the firm does not take into account the relationship between future dividends and current prices, even if there is agreement between the managers and shareholders. We denote this latter solution as *naive*, since it implies that firms ignore that future dividends influence today's price. This result is particularly important, since it implies that one cannot ignore the price-dividend mapping, even if the objective of the manager is value maximization.

Most of the literature has studied settings in which the naive and rational solutions coincide. One obvious case are the papers assuming either a fixed dividend rule or no equity issuance. Clearly, if no equity can be issued, there is no room for the stock price to play any role in influencing the firm's investment, since the stock price is irrelevant for firm financing. A less obvious case concerns models with equity issuance and dividend payments that follow the standard approach of value maximization with either symmetric financing frictions or frictions under which the solution follows the pecking order result (see e.g. Gomes (2000) and Gomes et al (2003)). In these cases, the naive solution coincides with the rational solution and the rational solution is time consistent due to the implicit agreement between stockholders and managers. These papers represent an enormous progress relative to the previous literature and they have shed light on a number of issues. However, these are very special cases that are not validated by observed firm behavior, since many firms pay dividends and issue stocks in the same period, while there is evidence of the absence of value maximization in the data. In order to go beyond these models we argue that the equilibrium concept we discuss is the natural one.

Our paper is also related to other strands of literature. The empirical literature on firm financing is large but does not use often explicit modelling, making it hard to formulate hypotheses to be tested. Explicit modelling has been used to address issues such as firm dynamics in a dynamic infinite horizon setting (see e.g. Hopenhayn (1992), Cooley and Quadrini (2001), Covas and Den-Haan (2007) and Quadrini and Jermann (2005)). Although the aforementioned papers vary greatly in their scope and objectives, their models share the common feature that firms maximize their market value subject to symmetric financing frictions. In contrast, we allow for conflicting objectives for the market investors and the managers and we abstract from issues such as firm heterogeneity and the size and age distribution of firms, which are central to those papers. Moreover, we focus on cases under which the pecking order result breaks down and explicitly model the dividend, stock and stock price interactions by allowing managers to anticipate the effects of dividend policy on the firm's stock price. Our paper is also related to a large strand of corporate finance literature. The possibility of time inconsistent financial policy is, in a way, implicitly recognized in the seminal work of Modigliani and Miller (1961), since their consideration that dividend policy is undetermined but gives today's value of the firm is taking into account the link we discuss. The difference is that, as we argue, this link influences choices every period because the stock price matters every period, while Modigliani and Miller only considered this in the price of the first period. As to the issue of time inconsistency, the only explicit mention in the literature of corporate finance we have found is in Miller and Rock (1985), who study a two period model with private information. In contrast, our paper assumes full information but disagreement between stockholders and managers and focuses on the interaction between future dividends and current stock issuance. Given this, we provide an alternative reason for the presence of time inconsistency.

There are a number of issues that this article does not address. Throughout the paper, we assume that firms have full commitment. That is, the manager announces all dividends that will be paid in the future under any contingency at period zero and (due to reputational issues or some other institutional arrangements that we do not model explicitly) this promise is fulfilled in the future. An alternative could have been to assume that firms follow time consistent policies and to use the solution techniques designed by Klein, Krusell and Ríos-Rull (2007) in a number of papers on optimal fiscal policy. Both alternatives make extreme assumptions on the level of commitment of managers and a more realistic approach should ultimately look at intermediate cases of partial commitment. One of the main reasons we focus on the full commitment equilibrium is that, given the current knowledge of stochastic dynamic solution techniques, recursive contracts can be used in much larger problems. Another interesting issue to study is what conditions and institutions should be embedded in a firm so as to restore time consistency. For this purpose, a number of equilibrium concepts on how to sustain the full commitment solution have been developed in the macroeconomics literature and they could also be exported to the dynamic corporate finance problem that we address. Here, we do not pursue these lines and simply assume full commitment.

Other important issues we do not address are why dividends are smooth or why dividends are paid at all. There is a very basic reason why firms pay dividends in our setup: under rational expectations and non-bubble prices zero dividends in all periods imply zero stock prices, so that the firm would be unable to finance itself with stock. As to dividend smoothing, some versions of our model simply impose that firm managers dislike dividend variability. This could be thought of as capturing an optimal contract in which the firm's board of directors has solved an agency problem determining that the payout to the manager should be according to dividend payments or the stock price of the firm. This has been justified in various ways by the literature of hidden information but we take the compensation as given and discuss other issues. Finally, a potentially interesting extension that we do not address involves studying the case in which the firm takes into account the effects of dividend policy on both stock prices and households' consumption.[1] These are all interesting issues that we leave for future research.

The paper is organized as follows. The model is presented in Section 2. Section 3

[1] Note that this case would be the closest one to the Ramsey optimal taxation literature. There is an important difference, however, since the Ramsey concept is typically associated with a benevolent government, while the firm is not benevolent in our framework.

presents the main theoretical results. In particular it shows how to formulate the problem recursively under full commitment and it discusses the issue of time inconsistency. Moreover, it characterizes the cases under which the solution is time consistent. Section 4 presents several examples that differ in the financing frictions and the compensation of the manager. One of the examples has an analytical solution and a proof of time inconsistency is provided. Finally, Section 5 summarizes and concludes.

2. The Model

Time is discrete and indexed by $t = 0, 1, 2...$The only source of uncertainty in the economy is an exogenous technology shock θ. The economy is populated by a continuum of identical investors and by a continuum of identical firms.

2.1. Market Investors. Households or market investors can trade in stocks issued by the firm and in one period risk free bonds that are assumed to be in zero net supply. They solve the following problem:[2]

$$E_0 \sum_{t=0}^{\infty} \delta^t u\left(c_t\right) \text{ s.t.} \tag{1}$$

$$c_t + p_t\left(s_t^h - s_{t-1}^h\right) + p_t^b b_t^h \leq d_t s_{t-1}^h + b_{t-1}^h$$

where s^h and b^h denote the household holdings of stocks and bonds respectively. Optimality implies that the price of the safe bond p_t^b is equal to:

$$p_t^b = \delta E_t \frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)} \tag{2}$$

In addition, the equilibrium stock price depends on the stream of dividends according to the following equation:

$$p_t = \delta E_t \frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)}\left[p_{t+1} + d_{t+1}\right] = E_t \sum_{j=1}^{\infty} \delta^j \frac{u'\left(c_{t+j}\right)}{u'\left(c_t\right)} d_{t+j} \tag{3}$$

where we have imposed a non-bubble solution requiring that $\lim_{j\to\infty} E_t \delta^j \frac{u'(c_{t+j})}{u'(c_t)} p_{t+j} = 0$.

Given the consumption process, equation (3) illustrates how the market maps a given dividend process $\{d_{t+j}\}_{j=1}^{\infty}$ into a process for the current stock price p_t. Throughout the paper, we call this relationship the price-dividend mapping. It is important to note that this mapping reflects that the stock market is perfectly competitive. As stated in the introduction, firms take as given that the market imposes this mapping and they take the mapping into account when deciding on its financial and investment policy. We assume that firms take the consumption process of the household-investors as given.[3]

[2] We implicitly assume that household-investors are subject to the natural borrowing limit.

[3] This is justified, for example, if there is a continuum of identical firms subject to the same shock or, more generally, if each firm has a minuscule impact on the consumption of the market stockholders.

2.2. Firms. We focus on a production economy in which the cash flow n of the firm is a function of both the technology shock and the aggregate capital stock k, which is determined endogenously. In particular, we assume that the firm owns and accumulates the capital stock that it uses for production each period. The cash flow is then equal to total production net of investment,

$$n_t = F\left(\theta_t, k_{t-1}\right) - k_t \tag{4}$$

where $F\left(\theta_t, k_{t-1}\right) = \theta_t f(k_{t-1}) + (1-\eta)k_{t-1}$ and η is the depreciation rate of capital.

Each period t, the firm can obtain external financing by issuing new stocks that are traded at price p_t. It distributes a dividend per stock of d_t to the stockholders and it faces financial frictions that are represented by the general function $\mathcal{C}_t \equiv \mathcal{C}(x_t, x_{t-1}, \theta_t)$, where $x_t \equiv (k_t, s_t, d_t, p_t)$. This general formulation encompasses capital adjustment costs, costs of equity issuance and costs of changing dividends amongst others. To simplify notation, we define the net cash flows net of financing costs as $n_t^c = n_t - \mathcal{C}_t$. If we let s_t be the quantity of stocks outstanding at time t, with $s_{-1} = 1$, the budget constraint of the firm is given by

$$d_t s_{t-1} + k_t \leq F(k_{t-1}) + p_t\left(s_t - s_{t-1}\right) - \mathcal{C}_t \tag{5}$$

Apart from $\mathcal{C}_t$, the firm also faces frictions of the form

$$\mathcal{B}_t \equiv \mathcal{B}(x_t, x_{t-1}, \theta_t) \leq 0 \tag{6}$$

representing for example limits on equity issuance/repurchases or lower bounds on dividend payments.

In addition to stock issuance (external funds), the income of the firm is given by production (earnings or internal funds), which depends on past capital, today's productivity shock and the production function f. We also include a standard no-Ponzi game condition, requiring that total liabilities of the firm in the form of stocks, $(p_{t+1} + d_{t+1})s_t$, cannot grow faster than the interest rate. Formally, we require that in each period t,

$$\lim_{j\to\infty} E_t \delta^j \frac{u'\left(c_{t+j+1}\right)}{u'\left(c_t\right)} (p_{t+j+1} + d_{t+j+1}) s_{t+j} = 0. \tag{7}$$

Notice that the budget constraint in (5) implies that the firm has to decide on the investment level under incomplete financial markets, since it cannot insure against shocks by issuing state contingent debt. To see the importance of our incomplete markets assumption, the period by period budget constraint of the firm can be written as:

$$\left(d_t + p_t\right) s_{t-1} = E_t \sum_{j=0}^{\infty} \delta^j \frac{u\left(c_{t+j}\right)}{u\left(c_t\right)} n_{t+j}^c \tag{8}$$

where we have used the price dividend mapping and we have substituted forward for $(p_{t+j} + d_{t+j})s_{t+j-1}$ for $j \geq 1$. Using this relationship, Lemma 1 below establishes a useful result that we will use later on.[4]

Lemma 1. *For any sequence of capital investments, stocks and dividends, $\{k_t, s_t, d_t\}_{t=0}^{\infty}$, the period by period budget constraint of the firm in (5), the no-Ponzi game condition for*

[4] The proof of this lemma and of all the other results throughout the paper is provided in Appendix A.

the firm in (7) and the price dividend mapping in (3) are satisfied at all t if and only if the following constraints hold:

$$s_{-1}E_0\sum_{j=0}^{\infty}\delta^j\frac{u(c_j)}{u(c_0)}d_j = E_0\sum_{j=0}^{\infty}\delta^j\frac{u(c_j)}{u(c_0)}n_j^c \tag{9}$$

$$\frac{N_t}{D_t}\text{ is measurable with respect to information up to } t-1 \text{ for all } t>0 \tag{10}$$

where D_t and N_t represent the present value of dividends and cash flows net of financing costs respectively:

$$\begin{aligned} D_t &\equiv E_t\sum_{j=0}^{\infty}\delta^j\frac{u(c_{t+j})}{u(c_t)}d_{t+j} \\ N_t &\equiv E_t\sum_{j=0}^{\infty}\delta^j\frac{u(c_{t+j})}{u(c_t)}n_{t+j}^c \end{aligned}$$

In contrast to a framework in which markets are complete, Lemma 1 implies that the period by period budget constraint of the firm is not equivalent to the period zero consolidated budget constraint in (9). Under incomplete markets, the measurability conditions (10) also need to be satisfied.[5] In other words, while many dividend sequences satisfy (9), not all of them are feasible, since they have to adjust so that equation (8) is satisfied at each period t. To see this, assume, for example, that households are risk neutral, $\mathcal{B}_t \equiv 0$ and $\mathcal{C}_t \equiv 0$. Further, consider the constant stream of dividends $d_t = d = (1-\delta)\,E_0\sum_{j=0}^{\infty}\delta^j n_j$. This clearly satisfies equation (9) and the associated stock price is $p = d\frac{\delta}{1-\delta}$, while the budget constraint of the firm at $t \geq 1$ will only be satisfied if:

$$s_{t-1} = \frac{E_t\sum_{j=0}^{\infty}\delta^j n_{t+j}}{d+p}$$

But if cash flows are stochastic, the right hand side of this equation depends on information up to t, while the left side can only be chosen contingent on information up to $t-1$. Given this, a constant stream of dividends is not feasible if there is uncertainty and markets are incomplete. In contrast, they would be feasible if the firm was able to issue state contingent debt.

With respect to the objective of the firm, we assume that it chooses $\{d_t, s_t, k_t\}_{t=0}^{\infty}$ to maximize the following general function:

$$E_0\sum_{t=0}^{\infty}\delta^t V(x_t, x_{t-1}, \theta_t) \tag{11}$$

The previous function encompasses several objective functions that we will use in examples throughout the paper, including value maximization, which is the objective usually studied in the literature. In addition, we will also study other objectives that are consistent with the empirical evidence on US CEO compensation.

[5] The proof of this part follows closely the reasoning of Proposition 1 in Aiyagari, Marcet, Sargent and Seppala (2002).

2.3. Equilibrium. We now define an equilibrium, assuming that firms take the price dividend mapping into account when making their decisions. The problem of a rational firm is given by:

$$\max_{\{x_t\}} \sum_{t=0}^{\infty} \delta^t V(x_t, x_{t-1}, \theta_t) \text{ s.t.} \tag{12}$$

subject to

$$\begin{aligned} d_t s_{t-1} + k_t &= p_t (s_t - s_{t-1}) + F(\theta_t, k_{t-1}) - \mathcal{C}_t \\ \mathcal{B}_t &\leq 0 \end{aligned}$$

$$p_t = E_t \sum_{j=1}^{\infty} \delta^j \frac{u'(c_{t+j})}{u'(c_t)} d_{t+j} \tag{13}$$

Later on, we will compare the equilibrium allocations with rational firms with the ones arising if the firm is naive. Recall that a naive firm would not take into account the price dividend mapping (13), since it does not internalize the effects of dividends on stock prices.

Definition 1. *An equilibrium is a vector of household allocations* $x_h \equiv \{c_t, s_t^h, b_t^h\}_{t=0}^{\infty}$, *a vector of firm allocations* $x_f \equiv \{k_t, d_t, s_t\}_{t=0}^{\infty}$ *and a vector of prices* $p \equiv \{p_t^b, p_t\}_{t=0}^{\infty}$ *such that (i) given the firm policy* x_f *and price vectors* p, *the vector* x_h *solves the problem of the households in (1), (ii)* x_f *solves the problem of the firm in (34) and (iii) markets clear, namely,* $c_t = n_t^c$, $s_t^h = s_t$ *and* $b_t^h = 0$ *for all* t.

Before discussing the recursive formulation of the above problem, we want to emphasize that we feel that our assumption that firms understand the price-dividend mapping accordingly is a reasonable way to model the way firms decide on equity issued and dividends paid. Most firms see themselves as having to sell their equity in a competitive stock market and they have freedom to decide how much equity to issue and dividend to pay. Furthermore, we claim that knowledge of the price-dividend mapping corresponds to a standard definition of competitive behavior under incomplete markets. This needs some careful justification because, at first sight, it might seem that there is an element of monopolistic behavior since in the problem defined above firms choose stock prices.

To explain this point in detail let us build an analogy by considering two types of firms who face slightly different financing environment as the firm considered above. First consider a firm that has to finance investment under incomplete markets but the firm can only issue bonds of two different maturities. Say, the firm can only issue short bonds that mature in one period and long bonds that mature in N periods, for a given $N > 1$. Both are real riskless bonds that pay one unit of consumption at maturity. Assume, for simplicity, that the firm never buys back any of these bonds, so that the budget constraint of the firm is

$$b_{t-1}^1 + b_{t-N}^N \leq n_t^c + p_t^{b,1} b_t^1 + p_t^{b,N} b_t^N \tag{14}$$

where b_t^1, b_t^N are the amount of short and long bonds issued by the firm at time t and $p_t^{b,1}, p_t^{b,N}$ are the corresponding bond prices. It should be uncontroversial to claim that a standard definition of competitive equilibrium in this case would entail assuming that the firm chooses $\{b_t^1, b_t^N, n_t^c\}$ taking as given the price process $\left\{p_t^{b,1}, p_t^{b,N}\right\}$. The firm chooses the total cost of the portfolio of bonds issued $p_t^{b,1} b_t^1 + p_t^{b,N} b_t^N$, but it is obviously behaving competititvely since it takes prices as given.

Suppose now that we change this model very slightly. In particular, let's assume that the firm issues BP_t units of a portfolio of bonds. Investors can purchase units of this portfolio from the firm, but the short or long bonds can not be purchased separately. Let us denote the units of the short bond by sh_t so that $(1 - sh_t)$ is the share of the long bond in each portfolio. The firm can choose the share of long and short bonds sh_t, and it can choose the amount of bond portfolios issued BP_t each period. The firm sells each unit of portfolio of bonds for a price PP_t. Let us call this a *bond-portfolio-financing (*BPF) firm and let us assume again there is no buyback of previously issued bonds.

In this setupt the firm has to repay $sh_{t-1} PB_{t-1}$ short bonds plus $(1 - sh_{t-N})\, PB_{t-N}$ long bonds in period t. Therefore, the budget constraint of a BPF firm is

$$sh_{t-1} PB_{t-1} + (1 - sh_{t-N})\, PB_{t-N} \leq n_t^c + PB_t\ PP_t \tag{15}$$

A number of general equilibrium models would deliver the result that the following holds in equilibrium.

$$PP_t = sh_t\ p_t^{b,1} + (1 - sh_t)\, p_t^{b,N} \tag{16}$$

A natural definition of competitive behavior for a rational BPF firm would say that the firm takes the process $\left\{p_t^{b,1}, p_t^{b,N}\right\}$ and it takes as given that (16) holds. The firm can choose the share sh_t and change the price of the portfolio PP_t accordingly, but the firm behaves competitively in the bond market because it takes bond prices and the mapping (16) as given. In fact, the BPF firm is not doing anything different from the firm issuing only long and short bonds described above, it is just packaging the bonds differently. Equation (16) should then become a constraint in the BPF firm's problem and PP_t would become a choice variable in the firm's problem. In other words, a rational BPF firm is behaving competitively by choosing PP_t subject to (16).[6]

3. Recursive Formulation and Time Inconsistency

One of the difficulties of including the price dividend mapping (13) as a constraint in the problem of the firm is that this problem is not recursive, in the sense that the Bellman equation does not hold. This happens in many other models where future variables appear in the current choice set. The famous time inconsistency problem studied by Kydland and Prescott (1977) points out that this occurs in models of fiscal or monetary optimal policy.

The fact that the Bellman equation does not hold in the optimum means that there is no ground to state that the optimal choice is a time invariant function of the natural state variables. This means that we should not expect the optimal choice at time t, $x_t =$

[6] To make an even more basic analogy: consider a competitive firm that produces two goods jointly. For example, consider a winery that produces white and red wine. The winery sells bottles of red and white wine in neatly packaged wooden boxes, 6 bottles in each box. The firm chooses how many bottles of red or white wine go in each box. It would be natural to assume that a competitive firm should recognize that the price of the box depends on how many bottles of each kind are included. The firm can in a way choose the price of the 6-bottle box, by choosing how many bottles of each kind are included, but this is compatible with price-taking behavior since the firm takes as given the way that the market of wine drinkers values boxes with a different number of whites or reds.

(s_t, d_t, p_t, k_t) to be given by a time invariant function $F(x_{t-1}, \theta_t)$[7]. In addition, there are other complications arising from the fact that the Bellman equation does not hold. First, the value function is not a contraction mapping. Second, the optimal policy is likely to be time inconsistent, in the sense that the firm has incentives to promise a future path for dividends in period zero such that, if in a future period t' the firm is allowed to re-optimize (without any other institutional or reputational constraints binding the firm), it might change the decision about variables dated $t > t'$ relative to the optimum stated in period zero. In the present paper, we assume that the firm is fully committed to the promises it makes in period zero and that it simply does not consider the possibility of re-optimizing in the future.[8]

The macroeconomic literature has recently devoted a large amount of effort to solve models of this sort and to study the time inconsistency issue. We first concentrate on the discussion of how to compute an optimum and leave the discussion on time inconsistency for the end of the section.

3.1. Recursive Formulation. To write the problem of the rational firm recursively, one can follow the approach of Marcet and Marimon (2008) and write the Lagrangian as:

$$\begin{aligned} L \;=\; & E_0 \sum_{t=0}^{\infty} \delta^t [V(x_t, x_{t-1}, \theta_t) + \lambda_t \left(E_t \sum_{j=1}^{\infty} \delta^j u'(c_{t+j})\, d_{t+j} - u'(c_t)\, p_t \right) \\ & + \gamma_t u'(c_t) \left(F(\theta_t, k_{t-1}) + p_t(s_t - s_{t-1}) - k_t - d_t s_{t-1} - \mathcal{C}_t \right) - \xi_t u'(c_t) \mathcal{B}_t] \end{aligned}$$

where γ_t is the multiplier associated with the period t budget constraint and λ_t the multiplier on the price dividend mapping.[9]

The presence of expectations of future variables in the expression above implies that the problem is not recursive yet. Nevertheless, the Lagrangian can be rewritten in a recursive form by introducing a new state variable μ_t as follows:

$$\begin{aligned} L \;=\; & E_0 \sum_{t=0}^{\infty} \delta^t [V(x_t, x_{t-1}, \theta_t) - \lambda_t u'(c_t)\, p_t + \mu_{t-1} u'(c_t)\, d_t \\ & + \gamma_t u'(c_t) \left(F(\theta_t, k_{t-1}) + p_t(s_t - s_{t-1}) - k_t - d_t s_{t-1} - \mathcal{C}_t \right) - \xi_t u'(c_t) \mathcal{B}_t] \end{aligned}$$

where the co-state variable μ_t follows the law of motion:

$$\mu_t = \mu_{t-1} + \lambda_t \text{ with } \mu_{-1} = 0. \tag{17}$$

After rewriting the problem in this way it is clear that future variables do not enter today's objective function and that now past μ's appear in the objective. This suggests that

[7] Most commonly the state variables would not include p_{t-1} and d_{t-1}. Given the general objective and frictions discussed earlier, it could happen that those appear as natural state variables. See later for examples where this is the case.

[8] There is a recent literature studying time consistent equilibria in optimal policy in macroeconomics models where this equilibrium is different from the full commitment solution. See, for example, Klein, Krusell and Rios-Rull (2007). We will not consider these equilibria in this paper, but they would certainly be an interesting alternative to be studied.

[9] For convenience, we have multiplied the budget constraint and the price-dividend mapping by $u'(c_t)$, essentially renormalizing the multipliers γ_t and λ_t.

the optimal choice (with the additional assumption that θ is Markovian) can now be found by looking for a policy function such that

$$(x_t, \gamma_t, \lambda_t) = f(x_{t-1}, \theta_t, \mu_{t-1})$$

This policy rule, together with (17), determines the whole equilibrium path. Marcet and Marimon (2008) provide conditions guaranteeing that this is indeed the case and they show a saddle point functional equation that plays the role of the Bellman equation. In other words, the recursivity of the equilibrium is recovered by introducing the new state variable μ and two new decision variables γ and λ.

In the present setting, the multiplier μ_{t-1} captures the promises that have been made in the past about the dividend in period t, d_t. Since there are no past promises to be kept at the beginning of time, the optimal choice entails setting $\mu_{-1} = 0$. On the other hand, at $t = 1$, there is an inherited promise from period 0, $\mu_0 = \lambda_0 = \gamma_0(s_0 - s_{-1})$, which arises from the fact that p_0 depends on the choice for future dividends.[10] The firm (if it is fully committed to the optimal plan) will have to remember the promise made in all past periods about today's dividend payments. Similarly, as we consider dividends further away in the future (d_2, d_3 etc.), these are linked with promises made in past periods. As reflected by its law of motion, the co-state μ_{t-1} adds up all of these past promises and summarizes them in a single number.

More intuition can be obtained by considering the first order conditions arising from this problem. To do this, let $\mathcal{C}_t \equiv 0$, $\mathcal{B}_t \equiv 0$ and $V(x_t, x_{t-1}, \theta_t) \equiv v(d_t)$ in order to save on notation for the rest of this section.[11] This implies that the compensation of the manager is tied to the dividends per share. The first order conditions for this case are given by:

$$v'(d_t) = \gamma_t s_{t-1} u'(c_t) - u'(c_t)\, \mu_{t-1} \tag{18}$$

$$\gamma_t u'(c_t)\, p_t = E_t[\gamma_{t+1} u'(c_{t+1})\, (d_{t+1} + p_{t+1})] \tag{19}$$

$$\gamma_t u'(c_t) = \delta E_t\left[\gamma_{t+1} u'(c_{t+1})\, F'(\theta_{t+1}, k_t)\right] \tag{20}$$

$$\lambda_t = \gamma_t (s_t - s_{t-1}) \tag{21}$$

The second and third equations represent the stock Euler equation (19) and the capital Euler equation (20) respectively, which are fairly standard. The last condition is the first order condition for the stock price and it allows us to write the co-state as:

$$\mu_t = \mu_{t-1} + \gamma_t (s_t - s_{t-1}) \text{ with } \mu_{-1} = 0. \tag{22}$$

We focus on the condition describing the optimal dividend choice (18). A marginal increase in d_t yields a direct utility benefit of $v'(d_t)$ but it has a cost in terms of lost resources at t that is equal to $\gamma_t s_{t-1} u'(c_t)$. A naive firm that does not realize the relationship between its stock price and its dividend policy would only have to consider these two effects. In other words, a naive firm would have $\mu_t = 0$ in all periods t.

On the other hand, a rational firm has to take into account the fact that the dividend choice at time t will affect stock prices in all previous periods. In particular, a marginal

[10] See equation (22) below.

[11] The following intuitive arguments describing the meaning of μ can be readily used for the more general case. The additional terms that would appear would not affect the interpretation of μ.

increase in d_t also implies increases in the stock prices of all previous periods and this in turn affects the resources available in all these periods. If the firm has been issuing stocks ($\mu_{t-1} > 0$), this price effect is positive, since it implies more funds raised for the same level of stock issuance. Conversely, if the firm has been repurchasing stocks in the past ($\mu_{t-1} < 0$), a dividend increase has a negative effect on past resources.

The previous discussion implies that the multiplier μ_{t-1} summarizes the effect of a marginal change in d_t on all previous periods' resources and it can be positive or negative depending on the history of stock issuance and repurchase. Thus, despite having a maximization problem where the Bellman equation does not hold, by adding the co-state variable μ, we can make the solution recursive. This is due to the fact that, even though the whole past history is needed to make decisions at any point in time t, the recursive contracts formulation allows us to summarize all the relevant information in just one variable, μ_{t-1}. The nature of time inconsistency is that the firm will always be tempted to follow a policy where μ is re-set to zero and only the fact that the firm is fully committed will prevent this from happening.

Consider now a naive firm that does not exploit the interaction between dividends and stock prices. In this case, the equilibrium is characterized by equations (5), (2), (3), (19), (20) and by the following first order condition with respect to dividends:

$$v'(d_t) = \gamma_t u'(c_t)\, s_{t-1} \tag{23}$$

In principle, one would expect that the law of motion of the system above can be written as a time invariant function in the natural state variables $(\theta_t, k_{t-1}, s_{t-1})$.[12] In this case, for a given law of motion for prices, the Bellman equation applies and the solution is time consistent.

3.2. Time Inconsistency. In what follows, we discuss the issue of time inconsistency that can potentially arise when firms are rational. As shown above, it seems that a rational firm can in general improve its stance by credibly promising a certain path for dividends. In this way, the firm can achieve a certain price today for a certain number of stocks issued. However, when tomorrow comes and investors have already bought the firm's stock, it is likely that the manager has an incentive to deviate if he is not fully committed. This is due to the fact that adjusting the dividends will not affect the current stock price, since it depends only on future dividends. In other words, if the institutions allow him to do so, it seems that it is better for the manager to renege on past promises and the policy under rational firms is therefore likely to be time inconsistent.

We first describe a standard definition of time consistency. Let the full commitment solution be given by $x_0^* \equiv \{c_t^*, d_t^*, s_t^*, p_t^*, k_t^*\}_{t=0}^{\infty}$. Given a time period $\tilde{t}$, define the "time-$\tilde{t}$" continuation problem as:

[12] We say "one would expect" because in general one can not rule out equilibria with more state variables. The equilibrium here is given by a fixed point in the space of pricing functions and, even though $(\theta_t, k_{t-1}, s_{t-1})$ is a minimum set of state variables, there could be other equilibria with more state variables. In situations like this the papers on dynamic stochastic equilibria often simply *assume* that agents follow a Markov strategy with the above set of state variables. This is what we do in the computed examples. Since the naive equilibrium is not the focus of our paper, we do not discuss this issue any further.

$$
\begin{aligned}
& \max_{\left\{x_t^f\right\}_{t=\tilde{t}}^{\infty}} E_{\tilde{t}} \sum_{t=\widetilde{t}}^{\infty} \delta^{t-\widetilde{t}} V(x_t, x_{t-1}, \theta_t) \text{ s.t.} \\
d_t s_{t-1} &= p_t \left(s_t - s_{t-1}\right) + n_t^c \text{ for all } t \geq \widetilde{t} \\
p_t &= E_t \sum_{j=1}^{\infty} \delta^j d_{t+j} \text{ for all } t \geq \widetilde{t} \\
\mathcal{B}_t &\leq 0 \text{ for all } t \geq \widetilde{t} \\
s_{\widetilde{t}-1} &= s^*_{\widetilde{t}-1}, k_{\widetilde{t}-1} = k^*_{\widetilde{t}-1}, d_{\widetilde{t}-1} = d^*_{\widetilde{t}-1}, p_{\widetilde{t}-1} = p^*_{\widetilde{t}-1}
\end{aligned}
$$

Denote the solution to this problem by $x_{\tilde{t}}^{**} \equiv \{c_t^{**}, p_t^{**}, d_t^{**}, s_t^{**}, k_t^{**}\}_{t=\tilde{t}}^{\infty}$. Note that this is the solution that would arise if, having followed the full commitment solution up to time $\tilde{t}$, the manager decided to re-optimize and choose the best solution from then on, ignoring the plans that were involved in the solution x_0^* that was optimal from the standpoint of period zero.

Definition 2. *The problem is time consistent at time $\tilde{t}$ if*

$$
\begin{bmatrix} c_t^{**} \\ p_t^{**} \\ d_t^{**} \\ s_t^{**} \\ k_t^{**} \end{bmatrix} = \begin{bmatrix} c_t^{*} \\ p_t^{*} \\ d_t^{*} \\ s_t^{*} \\ k_t^{*} \end{bmatrix} \quad \text{for all } t \geq \tilde{t} \tag{24}
$$

The problem is time consistent if it is time consistent for all $\tilde{t} > 0$.

The fact that time inconsistency may arise in the present setup is reflected formally in the recursive formulation. The same (time-invariant) policy function F has to be used for all periods with μ_{t-1} as an argument. The co-state μ_{t-1} is determined endogenously every period from past actions and it captures promises that have been made about today's dividends. The fact that there are no past commitments in the first period is reflected in $\mu_{-1} = 0$. The temptation to re-optimize is reflected in the fact that, if the manager was allowed to do so in period $\tilde{t}$ without being restricted to honor past commitments, he would want to follow a policy that implies re-setting $\mu_{\tilde{t}-1} = 0$ and following the optimal policy F from then onwards. If the manager is fully committed to following the announced policy, however, he will plug in the actual $\mu_{\tilde{t}-1}$ in the policy function.

Although it is well known that time-inconsistency can arise in the presence of a constraint such as the price-dividend mapping, it is sometimes the case that the solution displays time consistency. This is discussed in the following section.

3.3. Time consistency and compensation linked to cash flows. In what follows, we characterize a setting where the problem of the manager is time consistent. In this case, the plans that the manager makes for future dividends and stock issuance will indeed be fulfilled in the future, even if he is offered the opportunity to re-optimize. The results are summarized by the following proposition.[13]

[13] The proof of this proposition and of all other propositions throughout the text is provided in the appendix.

Proposition 1: *Let* $V(x_t, x_{t-1}, \theta_t) \equiv V[d_t s_{t-1} - p_t(s_t - s_{t-1})]$. *The time-0 policy is time consistent in period* s *if either a)*$\frac{V'_{t+1}}{V'_t} = \frac{u'(c_{t+1})}{u'(c_t)}$ *for* $t \geq s$ *or b)* $\mu_{s-1} = 0$, *where* $V'_t = V'[d_t s_{t-1} - p_t(s_t - s_{t-1})]$

The proof of proposition 1 relies on showing that, when reoptimizing at period $t = s$, the same prices and allocations can be supported with a suitable renormalization of multipliers if conditions a) or b) are satisfied. Several remarks are worth noting. First, the proposition holds for a particular firm objective that depends on the cash flows net of financing costs $n_t^c = d_t s_{t-1} - p_t(s_t - s_{t-1})$. In contrast, the problem might be time inconsistent under any other compensation sheme, a conjecture that we investigate in the next section. Second, the proposition implies that time consistency arises under two different conditions.

On the one hand, part a) states that the problem is time consistent if there is agreement between the manager and the shareholders. An important implication of this result is that time consistency arises under the usual objective of value maximization, which can be expressed as:

$$E_0 \sum_{t=0}^{\infty} \delta^t \frac{u'(c_t)}{u'(c_0)} [d_t s_{t-1} - p_t(s_t - s_{t-1})] \tag{25}$$

To see that value maximization satisfies the requirements of the proposition, notice that maximizing the present value of $V[d_t s_{t-1} - p_t(s_t - s_{t-1})]$ is equivalent to maximizing

$$E_0 \sum_{t=0}^{\infty} \delta^t \frac{V'_t}{V'_0} [d_t s_{t-1} - p_t(s_t - s_{t-1})] = E_0 \sum_{t=0}^{\infty} \delta^t \frac{u'(c_t)}{u'(c_0)} [d_t s_{t-1} - p_t(s_t - s_{t-1})]$$

where the equality follows from condition a). Two important properties of value maximization are worth noting. First, the formulation in (25) uses the relationships defined earlier and, in particular, the price dividend mapping that arises from the optimization problem of the investors. In fact, the (cum-dividend) value of the firm at time $t = 0$ is given by $(p_0 + d_0)s_{-1}$. Using the price dividend mapping and the period by period budget constraint of the firm, we can then re-write the value of the firm as follows:

$$\begin{aligned}(p_0 + d_0)s_{-1} &= n_0^c + p_0 s_0 = n_0^c + \delta E_0 \frac{u'(c_1)}{u'(c_0)}(p_1 + d_1)s_0 \\ &= n_0^c + \delta E_0 \frac{u'(c_1)}{u'(c_0)} n_1^c + \delta E_0 p_1 s_1 ... = E_0 \sum_{t=0}^{\infty} \delta^t \frac{u'(c_t)}{u'(c_0)} n_t^c \end{aligned} \tag{26}$$

To make the point clearer, notice that a manager who ignored the price dividend mapping and who literally maximized the value of the firm taking stock prices as given would treat p_0 as outside of his control and decide that the optimum is to pay everything out as dividends today and close down the firm in one period. A manager could only go from $(p_0 + d_0)s_{-1}$ to (25) or (26) if he understood the link between future dividends and p_0 every period. Second, in the absence of financing frictions, the following Lemma uses Lemma 1 to establish that the level of investment is independent of the financial policy of the firm. In other words, financial policy is indeterminate under value maximization.

Lemma 2. *If* $\mathcal{C}_t \equiv 0$ and $\mathcal{B}_t \equiv 0$, *then given a capital sequence* $\{k_t\}_{t=0}^{\infty}$, *there are many feasible financial choices* $\{d_t, s_t\}_{t=0}^{\infty}$ *that are compatible with the firm budget constraint* *and the price dividend mapping. Moreover, under these different financial choices, the real allocations and firm value are unchanged.*

This Lemma establishes a Modigliani-Miller-like result under value maximization in the absence of financing frictions, in the sense that many financial choices are feasible once a capital sequence that is consistent with (20) has determined the cash flow process. Given this, we will always impose financing frictions if the objective of the firm is value maximization.

On the other hand, part b) of Proposition 1 shows that agreement between the manager and the shareholders is sufficient but not necessary for the solution to be time consistent. In other words, time consistency can also arise under disagreement. This case is discussed in Proposition 2 below.

Proposition 2: *If compensation is linked to cash flows and frictions are symmetric, in the sense that they are of the form* $\mathcal{C}_t = C\left(k_t, k_{t-1}, p_t\left(s_t - s_{t-1}\right), d_t s_{t-1}\right)$ and $\mathcal{B}_t = \mathcal{B}\left(k_t, k_{t-1}, p_t\left(s_t - s_{t-1}\right), d_t s_{t-1}\right)$*, then* $\mu_t = 0$ *for all* t.

The previous proposition states that time consistency will arise if the objective of the manager depends on the cash flows net of financing frictions as long as the frictions are symmetric. We label a friction symmetric if it affects stocks and per share dividends equally or if they are imposed on the total value of stocks or the total value of dividends. Examples of such frictions are restrictions on repurchases, $p_t\left(s_t - s_{t-1}\right) \geq 0$, issuance costs $C\left(p_t\left(s_t - s_{t-1}\right)\right) = \left[p_t\left(s_t - s_{t-1}\right)\right]^n$ for $n \geq 1$ or minimum dividend payments, $d_t s_{t-1} \geq 0$. In contrast, examples of asymmetric frictions would be a limit on the number of stocks issued, $s_t - s_{t-1} \leq \Delta$ per share dividend targets $\tau_d\left(d_t - d\right)^n$ for $n \geq 1$ or costs in changing per share dividends, $\tau_d\left(d_t - d_{t-1}\right)^n$ for $n \geq 1$.

This result is particularly important, since the literature typically assumes symmetric frictions and the objective of value maximization. In fact, the literature has considered what we denote as DE-problem, given by

$$\begin{aligned} & \max_{\{k_t, e_t, D_t\}} E_0 \sum_{t=0}^{\infty} \beta^t V(k_t, k_{t-1}, D_t - e_t) \text{ s.t.} \\ D_t + k_t &= e_t + F(\theta_t, k_{t-1}) - \mathcal{C}(k_t, k_{t-1}, D_t, e_t) \\ \mathcal{B}\left(k_t, k_{t-1}, D_t, e_t\right) &\leq 0 \end{aligned} \tag{DE}$$

The proposition then shows that this problem is equivalent to the problem we consider in this paper, which we label the OP-problem:

$$\begin{aligned} & \max_{\{d_t, s_t, k_t, p_t\}} E_0 \sum_{t=0}^{\infty} \beta^t V(k_t, k_{t-1}, D_t - e_t) \text{ s.t.} \\ D_t + k_t &= e_t + F(\theta_t, k_{t-1}) - \mathcal{C}(k_t, k_{t-1}, D_t, e_t) \\ p_t &= E_t \sum_{j=1}^{\infty} \beta^j \frac{u'\left(c_{t+j}\right)}{u'\left(c_t\right)} d_{t+j} \\ \mathcal{B}\left(k_t, k_{t-1}, D_t, e_t\right) &\leq 0 \end{aligned} \tag{OP}$$

where $D_t = d_t s_{t-1}$ and $e_t = p_t\left(s_t - s_{t-1}\right)$. An important implication of this equivalence is that the issues we discuss do not arise, since the price dividend mapping is redundant. Note also that the DE problem is actually the model considered by Gomes (2000) and Gomes at all (2003), who assume that there is a cost of issuing equity, a no repurchase constraint and a lower bound on total dividends[14]. Proposition 2 shows then that these authors are justified in focusing on the naive case. On the other hand, the solution displays the well known pecking order result, implying that firms do not increase their dividend payments while they are issuing equity. This, and the fact that value maximization is not validated by the data leads us to study other firm objectives that are more in line with the empirical observations on manager compensation.

Finally, it is important to note that the result of Proposition 2 breaks with small departures from the standard setting, even under value maximization. In particular, Proposition 3 below states that one cannot ignore the price dividend mapping if $\mu_t \neq 0$.

Proposition 3. *If $\mu_t \neq 0$ for some t, then naive and fully rational policies are different.*

As already mentioned above, Proposition 3 implies that ignoring the p-d mapping could potentially lead to the wrong conclusion. In other words, the rational solution might be different from the naive solution, even if it is time consistent. In the next section, we show that this happens, for example, with asymmetric financing frictions.

4. Numerical Examples

In this section, we study specific examples that differ in the compensation of the manager and the financing frictions faced by firms. For simplicity, these and all the other examples throughout this section assume risk neutral households and no uncertainty. All the examples assume that the initial capital stock of the firm is lower than the steady state value, which is reached after a finite number of periods T. This implies that the firm is growing over time. Given this, the role of stock issuance is, precisely, to provide funding to invest in capital so that the firm can operate at the optimal level given by the modified golden rule. Since firms will achieve the optimal capital immediately (after one period) in the absence of frictions, we introduce such frictions.

As part of the analysis, the rational equilibrium will be compared to the case in which the firm is naive, in the sense that it does not take into account the effect of dividends on prices in the budget constraint. This comparison will be important for two reasons. First, if the rational and naive solutions coincide, we can conclude that the problem is time consistent, since the recursive multiplier μ is equal to zero every period. Second, if rational and naive solutions coincide, the price-dividend mapping can be ignored and the (easier) naive problem can be computed as a solution to the problem. On the other hand, we show that there exist situations (even in the absence of time inconsistency) where the solutions are different and thus ignoring the price-dividend mapping would lead to erroneous results.

According to the survey by Murphy (1999), the main components of CEO compensation in US are (i) a fixed part or base salary, (ii) a bonus mostly based on yearly performance, with the most common measure being accounting profits, (iii) stock options, which are typically non tradable and now constitute the largest component of compensation in US and (iv) other forms of compensation, including restricted stock. Murphy also notes that stock options have typically a strike price equal to the market value on date of grant and they reward only price appreciation (no dividends). Following the evidence, the objectives we consider are combinations of the different components in (i)-(iv). First, we analyze objectives

[14] Variations of this are analyzed in Cooley and Quadrini (2001), Covas and Den-Haan (2007), Quadrini and Jermann (2005) and Gomes et al (2003) amongst others.

linked to cash flows (component (ii)), a special case of which is value maximization. Second, we study a firm objective that links compensation to per share dividends, corresponding to component (iv). Third, we study a firm objective that links compensation to cash flows and stock options, corresponding to a combination of components (ii) and (iii).

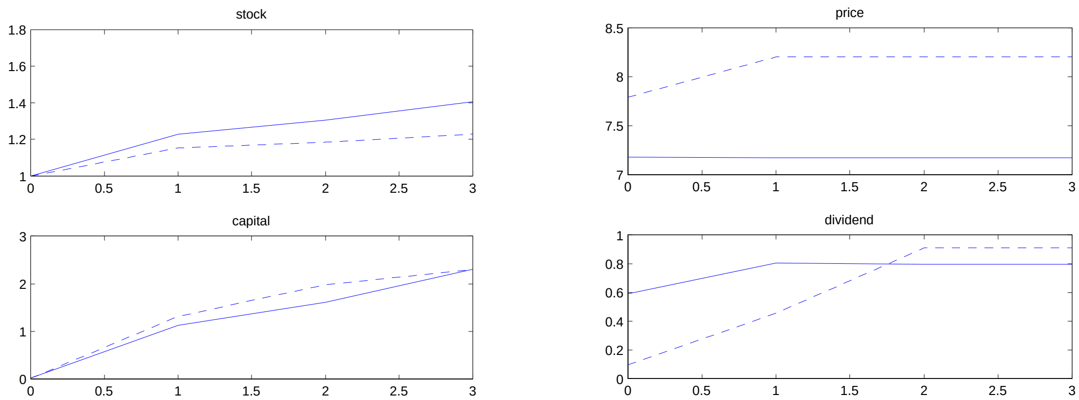
4.1. Value maximization with asymmetric frictions. In the first example, we assume value maximization as in Gomes(2000) and Gomes et all (2003) but introduce costs of changing per share dividends. This is intended to capture (in an admittedly crude fashion) the observation that per share dividends are very persistent, in the sense that they are very infrequently changed. This is an example in which the naive and rational solutions do not coincide in spite of the fact that the rational solution is time consistent. Formally, the rational firm solves:

$$\max_{\{d_t,k_t,s_t\}} \sum_{t=0}^{\infty} \delta^t \left[d_t s_{t-1} - p_t \left(s_t - s_{t-1}\right)\right] \text{ s.t.}$$

$$\begin{aligned}
d_t s_{t-1} &= F(k_t) - k_t + (1-\delta)k_{t-1} + p_t \left(s_t - s_{t-1}\right) \\
&\quad -\tau p_t^2 \left(s_t - s_{t-1}\right)^2 - \tau_d \left(d_t - d_{t-1}\right)^2 \\
d_t s_{t-1} &\geq 0,\ p_t \left(s_t - s_{t-1}\right) \geq 0 \\
p_t &= \sum_{j=1}^{\infty} \delta^j d_{t+j}
\end{aligned}$$

The time path for some of the endogenous variables in the model is displayed in the figure below. As reflected by the figure, the desire of firms to smooth dividends over time implies that both dividend payments and equity issuance occur simultaneously but the rational firm pays lower dividends at first and higher dividends in the future. By doing this, the firm can inflate the stock price and obtain cheaper external finance. As a result, the rational firm can grow faster.

Figure 1: Value maximization with costs in changing dividends



Stocks and Capital Stock

Stock Price and Dividends per Stock

4.2. Compensation linked to dividends. In what follows, we consider the case in which the firm maximizes the value of dividend payments according to an increasing and concave utility function v:

$$W_0 \equiv E_0 \sum_{t=0}^{\infty} \delta^t v\left(d_t\right) \tag{27}$$

This second objective, which we label as a risk averse firm in what follows, can be interpreted as a case in which the manager owns a fixed number of stocks in the firm and has no other sources of income. Formally, assume that managers hold a (fixed) number of stocks s^m and let the stocks held by investors be given by s_t^h, so that the total number of stocks in the economy is equal to $s_t = s_t^h + s^m$. Whereas the problem of the investors is the same as before, the income of the manager is equal to $c_{m,t} = d_t s_m$ and he maximizes the following objective:[15]

$$E_0 \sum_{t=0}^{\infty} \delta^t u_m(d_t)$$

where $u_m(d_t) \equiv v\left(d_t s^m\right)$. Upon a normalization of the number of stocks ($s^m = 1$), this alternative model would generate the same allocations as the objective in (27).

More generally, v can be justified as the contract that the manager has been offered to give him or her incentives to manage the firm properly. In a setting in which the optimal payout and investment are not observable, the manager is restricted from overinvesting or diverting funds by linking his compensation to the payout. In other words, there may be a signalling problem or hidden action mechanism in the background, that prompts the firm to offer a reward to the manager that is tied to the dividend. Indeed, many firms offer stocks or options as a form of payment to managers and managers are not allowed to sell these assets for a long time. We concentrate on the optimal stock issuance policy given v, but we can think of v as a reduced form of an incentive problem that we take as exogenous here but that, ideally, would be endogenized.

[15] Whereas this formulation assumes that the stocks of the manager are fixed, he can change his proportion in the firm by modifying the total number of stocks through issues and repurchases. Issues arising from the trade of shares between managers and shareholders are also discussed in Gorton and He (2006). Their focus is more on the interaction of agency issues and asset pricing and less on financial policy and investment.

Another interpretation of this utility function is that there are two types of stockholders: market stockholders and internal stockholders. Market stockholders would correspond to the investors (households) in this setting. Internal stockholders are somehow tied to this firm, either because they founded the firm, or because their human capital is particularly useful in this firm; they run the firm and they decide how much to invest and how many stocks to issue, while the utility $v(d_t)$ represents their direct preferences on the firm's performance.

An important implication of the fact that managers and shareholders have different preferences is that financial policy is fully determined even in the absence of financing frictions. This is due to the fact that the investment and financial decisions are linked through the presence of γ_t in the capital Euler condition (20). To provide a more intuitive explanation of why this is the case, we now show that firm risk aversion implies that firms care both about maximizing cash flows and smoothing dividend payments. This can be done by plugging the consolidated period 0 constraint in (9) into the objective function of the firm. The problem can then be rewritten as:

$$\max_{\{d,s,k\}} E_0 \sum_{t=0}^{\infty} \delta^t \left[v(d_t) - \xi_0 \frac{u'(c_t)}{u'(c_0)} d_t + \xi_0 \frac{u'(c_t)}{u'(c_0)} n_t^c) \right] \tag{28}$$

st. (10)

where ξ_0 is the Lagrange multiplier of (9). Expressed in this way, it is clear that the manager would like to maximize the expected, discounted *weighted* sum of two elements. The first element is $v(d_t) - \xi_0 \frac{u'(c_t)}{u'(c_0)} d_t$ and it depends only on dividends, whereas the second element is the cash flow weighted by $\xi_0 \frac{u'(c_t)}{u'(c_0)}$. This illustrates that investment and financing decisions are linked with risk averse firms, in the sense that a given financial policy will have to come up with a dividend policy that balances these two objectives. In this case, the presence of the part $v(d_t) - \xi_0 \frac{u'(c_t)}{u'(c_0)} d_t$ in the objective can be interpreted as the manager caring about minimizing the variability of dividends for a given cash flow. If the manager cared only about this part of the objective function, he would not choose capital efficiently (as under value maximization), since he would use it to smooth dividends. In fact, the optimal behavior of the manager has to balance the optimality of the capital choice that maximizes the cash flows and is best for the consumers with the desire to smooth dividends.

An analytical example. We now analyze an example for which we can obtain an analytical solution and which clearly illustrates the difference between naive and rational firms. In this example, the friction consists of a maximum amount of stock that can be issued in the first periods. This can be justified by the presence of transaction costs or due to the manager disliking that too many stocks are distributed, since this would cause a loss of his control in the firm. The manager solves:

$$\max_{\{d_t, s_t, k_t\}} \sum_{t=0}^{\infty} \delta^t v(d_t s_m) \quad \text{s.t.}$$

$$\begin{aligned} d_t s_{t-1} + k_t - (1-\eta) k_{t-1} &= p_t (s_t - s_{t-1}) + f(k_{t-1}) && (29) \\ s_t - s_{t-1} &\leq \Delta && (30) \\ k_{-1}, s_{-1} & \quad \text{given} && (31) \end{aligned}$$

where $s_t = s_t^h + s^m$ and $\Delta > 0$ is a fixed constant limiting the amount of stocks that can be issued. Under a rational firm, the manager also takes into account the following constraint:

$$p_t = \sum_{j=1}^{\infty} \delta^j d_{t+j} \tag{32}$$

As stated earlier, we assume that initial capital is much lower than the steady state capital. Formally, the steady state capital, which we denote by k^{GR} for 'golden rule', satisfies:

$$1 = \delta \left[f'(k^{GR}) + 1 - \eta \right] \tag{33}$$

and we assume that $k_{-1} < k^{GR}$.

No Bounds on Stock Issuance: $\Delta = \infty$. In the absence of uncertainty, the rational firm would be able to achieve the complete market solution if the constraint (30) was not present. That is, if $\Delta = \infty$, the rational manager would be able to issue a sufficiently large amount of stocks in the first period to finance the desired accumulation of capital at $t = 0$, achieving the first best capital in one step. In fact, the manager would be able to complete the markets with stock issuance so that $k_t = k^{GR}$ for all $t \geq 0$ and dividends are perfectly smoothed. In contrast, we now show that the naive firm would not be able to achieve this allocation. We do this analytically for the case with $v(.) = \log(.)$.

Result 1. *When $\Delta = \infty$, $v(.) = \log(.)$ and $k_{-1} < k_s$, the naive firm allocations are*

$$\begin{aligned}
k_t^N &= k^{GR} = \left[\frac{\frac{1}{\delta} - 1 + \eta}{\alpha} \right]^{\frac{1}{\alpha - 1}} \\
s_t^N &= \bar{s}^N \text{ for } t \geq 0 \\
d_t^N &= \bar{d}^N \text{ for } t \geq 1 \\
d_0^N &= \frac{n^{GR}}{s_{-1}} \\
p_t^N &= p^N = \frac{\delta}{1-\delta} \bar{d}^N
\end{aligned}$$

where k^{GR} denotes the 'golden rule' level of capital and n^{GR} is the corresponding cash flow. In contrast, the rational firm allocations are

$$\begin{aligned}
k_t^{FR} &= k^{GR} = \left[\frac{\frac{1}{\delta} - 1 + \eta}{\alpha} \right]^{\frac{1}{\alpha - 1}} \\
s_t^{FR} &= \bar{s}^{FR} \text{ for } t \geq 0 \\
d_t^{FR} &= \bar{d}^{FR} \text{ for } t \geq 0 \\
p_t^N &= p^{FR} = \frac{\delta}{1-\delta} \bar{d}^{FR}
\end{aligned}$$

Result 2. *The allocations of the naive and rational firms satisfy the following relationships:*

$$d_0^{FR} < d_0^N, \ \bar{d}^{FR} > \bar{d}^N, \ \bar{s}^{FR} < \bar{s}^N, \ p^{FR} > p^N$$

The proof of these results is provided in the appendix. The above allocations imply that both firms issue stocks in the first period and invest enough to jump to the optimal level of capital immediately. As a result, real allocations and the value of the firm are the same in both cases. However, financial policy and manager's welfare differ. Compared to the naive firm, the rational firm pays less dividends in the initial period and more in all future periods. This implies smoother dividends under rational firms with the added benefit that the stock price is always higher. This is irrelevant from period 1 onwards since there is no stock issuance, but it is important in period 0 in which both firms issue stocks.

This 'cheaper' external finance under rational firms that arises from credible promises about future dividends allows the rational firm to implement the complete markets allocation and achieve both smooth dividends and optimal capital accumulation[16]. In contrast, the naive firm has to sacrifice dividend smoothing to achieve the optimal investment. Even more interestingly, the naive firm needs to issue more stocks to achieve the optimal investment. This provides a hint that, in the presence of financial frictions (costly or limited external finance), the naive firm would be unable to invest as much as the rational firm. We explore this below by choosing a finite and relatively tight bound Δ.

Bounds on Stock Issuance: $\Delta < \infty$. Suppose there is a bound on stock issuance $\Delta < \infty$. For any $\Delta > 0$, there is a point in time after which the bound is not binding any longer. That is, capital has grown enough so that it is close to the steady state and one last period of stock issuance (that does not violate the issuance bound) is enough to reach the steady state. Suppose this happens after T periods. Then

$$s_t - s_{t-1} \leq \Delta \text{ for } 0 \leq t \leq T-1$$

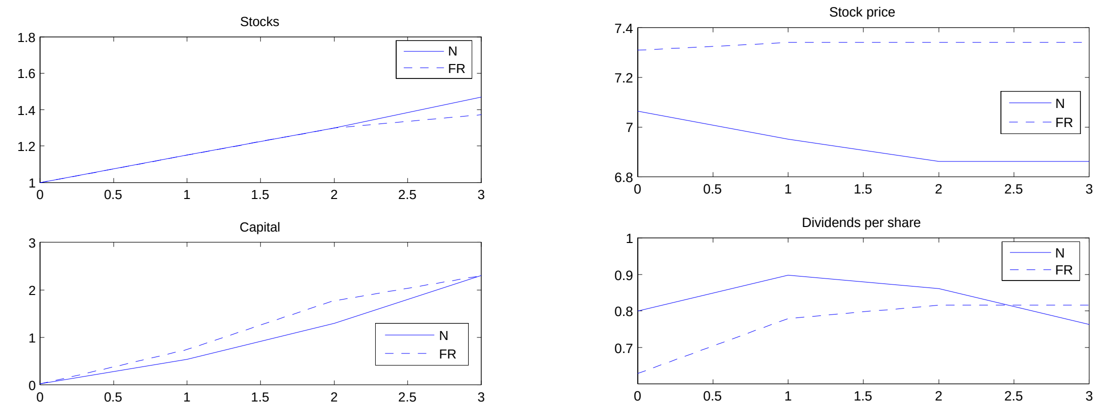
Starting at period $t = T$ and given s_{T-1} and k_{T-1}, the continuation problem is one where bounds on stock issuance are not binding any more and the solution given by the one in the previous section. We now analyze the naive and rational firm policies for this setup, where the policy for the rational firm turns out to be time inconsistent.

Since the bounds on stock issuance are binding for both the naive and rational firms, the levels of stock issuance are equal for both. However, the rational firm uses the price dividend mapping to obtain higher levels of external finance. In particular, by promising high dividends in the future, it ensures that the competitive price for its stock is higher and thus its external finance is higher for the same level of stock issuance. In turn, higher funds raised externally imply that the rational firm can grow faster and reach the optimal level of capital earlier.

We provide a numerical example in which we choose $\Delta = 0.35$ so that $T = 2$. This means that the limit on stock issuance binds for only two periods. The rest of the parameters are as follows: $\delta = 0.9$, $\eta = 0.1$, $\alpha = 0.36$, $\gamma = 1$ and $s_m = 1$. Initially, investor's stock holdings are one half of the total stocks, that is, $s_{-1} = s_{h,-1} + s_m = 2$. We consider a startup firm, that is, a firm that starts at a very low level of capital, $k_{-1} = 0.01k^{GR}$. The results are displayed in figures 1 and 2, where the solid lines depict the growth paths for the naive firm and the dashed lines depict those for the rational firm.

[16] As already shown in the previous section, the allocations of this example are time-consistent, since firms choose the value maximizing level of capital.

Figure 2: Risk averse firms and Issuance Bound

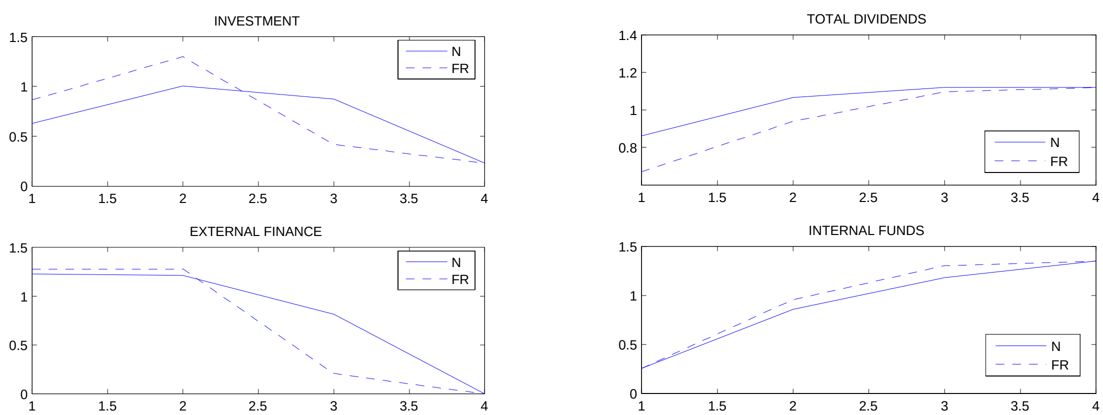


Stocks and Capital Stock

Stock Price and Dividends per Stock

The left panel of figure 2 depicts the stocks held by households starting at $s_{h,-1} = 1$. As we see, they grow at the same speed for both firms and for the first two periods. This illustrates that the limit on issuance binds for both firms, who are able to issue only Δ stocks in each of the two periods. Capital starts at a very low level and steadily grows towards the steady state. The steady state is reached in the first period in which the limits don't bind, which is period 2. But the rational firm manages to invest more initially and approach the steady state faster than the naive firm. This is for two reasons. First, the firm pays less dividends in the beginning, releasing more funds for investment. Second, the dividend policy includes a promise of high future dividends, which has the additional effect that stock prices are higher. As a result, a higher level of external funds is raised, allowing the firm to invest more initially and grow faster than a naive firm. Since the limit on stock issuance is not binding any more in the third period, both firms reach the golden rule level of capital and stop issuing any additional stocks.

Figure 3: Risk averse firms and Issuance Bound



The figures also reflect that the jump to the optimal capital requires much less stock issuance both because it is closer to the steady state and because its stock price is higher. The two firms remain at this level of capital and stocks from then on. Figure 3 depicts the levels of investment, external finance, total dividends and internal funds. As we see, the total dividend payout from then on is constant and equal for the two firms. However, having used less stock issuance to grow, the rational firm is in a position to pay higher dividends per stock from then on and maintain a higher price forever. Clearly, the rational manager achieves a higher welfare by using the additional knowledge from the price-dividend mapping. More interestingly, the value of the firm is also higher under rational managers, implying that investors are also better off.

Finally, Proposition 4 below shows that the equilibrium allocation under rational firms is time inconsistent for this example.

Proposition 4. *In a production economy with no uncertainty, bounds on stock issuance for the initial T periods and initial capital lower than the steady state, the problem is time inconsistent.*

The previous proposition shows that the example with the issuance bound and risk averse firms exhibits time inconsistency. This illustrates that a crucial factor generating time inconsistency in the model is that there is disagreement between the shareholders and the manager of the firm, generated by the fact that the firm is risk averse. However, there are still some cases under which the solution is time consistent with risk averse firms. These cases are described in detail in Appendix B but essentially include economies in which there are no financing frictions, or dividends are constant, or there is no uncertainty or the horizon is finite and there is full capital depreciation. A common implication of these cases is that the value maximizing level of capital stock is chosen.

4.3. Compensation linked to cash flows and stock options. We now consider a third alternative objective representing the case where managers are compensated through stock options and cash flows[17]. In particular, we assume that managers receive one period options every period at the fixed strike price p^s, which is chosen so that the options are exercised every period. We introduce costly equity issuance and a target for total dividend payout. Both frictions fall under the category of symmetric frictions. The manager solves[18]:

$$\max_{\{d_t,k_t,s_t\}} \sum_{t=0}^{\infty} \delta^t \left[d_t s_{t-1} - p_t \left(s_t - s_{t-1}\right) + \max\left(0, p_t - p^s\right)\right] \text{ s.t.}$$

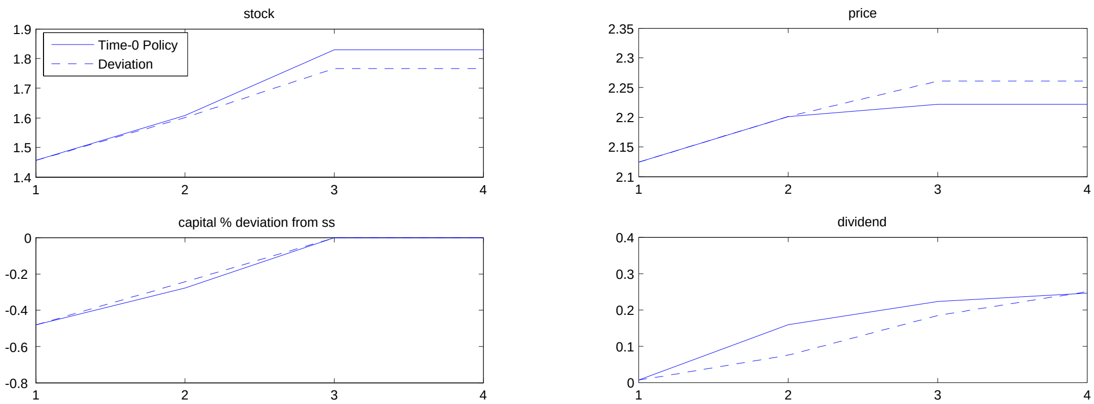
$$\begin{aligned}
d_t s_{t-1} &= F(k_t) - k_t + (1-\delta)k_{t-1} + p_t \left(s_t - s_{t-1}\right) \\
&\quad -\tau p_t^2 \left(s_t - s_{t-1}\right)^2 - \tau_d \left(d_t s_{t-1} - d^{ss} s^{ss}\right)^2 \\
d_t s_{t-1} &\geq 0,\ p_t \left(s_t - s_{t-1}\right) \geq 0,\ p_t = \sum_{j=1}^{\infty} \delta^j d_{t+j}
\end{aligned}$$

[17] This is the case that is most closely related to the empirical observation on CEO compensation. Recall that the three main components of CEO compensation are bonuses (linked to cash flows), stock opitons and base salary. We omit the base salary component since it makes no qualitative difference.

[18] The presence of the max operator in the objective would in general complicate the maximization problem considerably. We sidestep this issue by ensuring that the option is optimally exercised every period so that the max operator can be ignored.

The following figure displays the evolution of some of the key endogenous variables. Qualitatively, the story here is not any different than in the previous examples. The rational manager pays lower dividends at the beginning and higher dividends in the future, a strategy which allows him to obtain a higher external finance and grow faster. In fact, the naive manager here acts as if there was no stock option component in their compensation since they take prices as given. Naive managers would follow the value maximizing policy while rational manager exploit their decision making power in order to inflate the stock price and, hence, their own compensation.

Figure 4



Using this example, we also attempt to illustrate the nature of time inconsistency. We do this by considering the possibility of deviation in period $t = 2$: After the manager has chosen investment and financial policy for all the future under the assumption of full commitment, we consider what he would change if he were given the opportunity to re-optimize at $t = 2$. At that stage, past choices have already been realized and the manager inherits some levels of capital stock $k_1$and number of outstanding stocks s_1. He also inherits promises made about financial policy in the past (in the form of a positive μ_1), but is allowed to renege on those and set $\mu_1 = 0$.

As we see. The deviation in dividend policy is clearly aimed at raising stock prices p_2 and p_3. The way this is achieved is by lowering current dividends d_2 which do not affect these prices and promising higher dividends in the future (d^{ss}). By raising stock prices, the manager can raise external finance without too much dilution (s_3 is less under the deviating policy) and also grow faster (k_2 is higher under the deviating policy). As a result, he can deliver the promised higher dividends per share. Clearly the manager is better off by deviating which means the commitment policy is time inconsistent.

4.4. Summary. Table 1 below displays the different examples we have considered.

Table 1

Objective	Frictions	N=R	TC
$\sum_t \delta^t n_t^c$	Symmetric	Yes	Yes
$\sum_t \delta^t n_t^c$	Asymmetric	No	Yes
$\sum_t \delta^t n_t^c + options$	Symmetric	No	No
$\sum_t \delta^t v\left(d_t\right)$	Asymmetric	No	No

As reflected by the table, time inconsistency only arises in the absence of value maximization, suggesting that it is a consequence of the disagreement between stockholders and managers. In fact, Proposition 1 shows exactly this: that if there is agreement about investment then there is no issue of time inconsistency. In addition, the table reflects that the naive solution is different to the rational solution in all cases in which frictions are asymmetric and the pecking order result is broken. As shown above, this implies that one can ignore the price dividend mapping in cases with value maximization and symmetric frictions or under which the solution has the pecking order property. However, even under value maximization, the pecking order result might not obtain for some type of financial frictions, in which case the naive solution might not be equal to the rational solution.

5. Conclusions

We have provided a way to formulate and solve a stochastic general equilibrium dynamic model of dividend and stock policy. The aim was to provide a framework within which a number of important issues can be addressed. The model proposed makes explicit the distinction between dividends and stock issuance or repurchases. It is thus well suited to analyze payout policy. In addition, the framework is also available for the analysis of questions regarding the interplay between payout policy and investment.

As a first implication of the theoretical analysis presented in the main section of this paper, we highlight the behavior of growing firms with regard to dividend payments. Typically, startup firms pay little or no dividends, while they funnel resources towards the available productive projects that lead to firm growth. One obvious theoretical explanation of this observation points at financial frictions that do not allow for unlimited funds being raised from external sources. Our framework provides another, complementary mechanism that can explain this observation. The idea is that young firms lack the burden of past promises about dividends and can therefore pay little now, while promising a lot of dividends for the future. This strategy allows them to raise external funds at more favorable prices by inflating the price of their stock. Using the cheaper external funds, they can also grow faster.

Our framework also provides a rationale for why a firm would prefer to use dividends as opposed to repurchases if the full commitment solution is taken as the benchmark case. As mentioned above, the reason is that dividend promises can be used to influence prices towards achieving cheaper external finance, while the same objective cannot be achieved through announcements in stock repurchases.

Finally, our work identifies a potential for time inconsistency in financial policy even in the absence of asymmetric information of the type considered by Miller and Rock (1985). We point out the complications arising from the need for commitment and we provide examples where the full commitment policy is time consistent and others where it is not. This raises the question of how the time consistent policy would look like, its efficiency properties and the arrangements that can be used to implement more efficient policies. We leave these questions for future research.

APPENDIX A: PROOFS

Proof of Lemma 1

To prove Lemma 1, we first show that the period-by period constraints in (5) and the price Euler equation from the consumers' problem in (3), together with the No-Ponzi scheme assumption, imply (9), (10). As we have already stated above, these imply (8). Since this holds for all $t \geq 0$, the equation evaluated at $t = 0$ implies (9). In addition, using the definitions of N_t and D_t, equation (8) implies $\frac{N_t}{D_t} = s_{t-1}$ so that (10) is satisfied.

To prove the converse, we show that given (9), (10) and (3), we can construct a sequence of stock holdings such that (5) is satisfied. First, define S_t as follows:

$$S_t \equiv \frac{N_t}{D_t}$$

so that S_t is measurable with respect to to information up to $t-1$. Then

$$\begin{aligned} D_t S_t &= n_t^c + E_t \sum_{j=1}^{\infty} \delta^j \frac{u\left(c_{t+j}\right)}{u\left(c_t\right)} n_{t+j}^c \\ &= n_t^c + \delta E_t \left[E_{t+1} \sum_{j=0}^{\infty} \delta^j \ \frac{u\left(c_{t+j}\right)}{u\left(c_t\right)} n_{t+j+1}^c \right] \\ &= n_t^c + \delta E_t \left[n_{t+1}^c + E_{t+1} \sum_{j=1}^{\infty} \delta^j \ \frac{u\left(c_{t+j}\right)}{u\left(c_t\right)} n_{t+j+1}^c \right] \\ &= n_t^c + \delta E_t \left[D_{t+1} S_{t+1} \right] \end{aligned}$$

But S_{t+1} is measurable with respect to information up to t, so that

$$D_t S_t = n_t^c + \delta S_{t+1} E_t \left[D_{t+1} \right]$$

Finally, noticing that $D_t = p_t + d_t$, we see that period-by-period budget constraint in (5) is satisfied for $s_{t-1} = S_t = \frac{N_t}{D_t}$.■

Proof of Proposition 1.

For simplicity, we provide the proof of Proposition 1 for $u\left(c\right) = c$ but it is straightforward to extend it to a concave function. If $u\left(c\right) = c$, the condition a) is replaced with $V\left(x\right) = x$. The Lagrangean of the firm's problem is:

$$\begin{aligned} L &= \max E_0 \sum_{t=0}^{\infty} \beta^t \left[V\left[d_t s_{t-1} - p_t \left(s_t - s_{t-1} \right) \right] - \lambda_t p_t + \mu_{t-1} d_t \right] + \\ &\quad + E_0 \sum_{t=0}^{\infty} \beta^t \gamma_t \left[F(\theta_t, k_t) - k_t + p_t \left(s_t - s_{t-1} \right) - \mathcal{C}_t \right] \\ &\quad - E_0 \sum_{t=0}^{\infty} \beta^t d_t s_{t-1} - E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \mathcal{B}_t \end{aligned}$$

while the law of motion for μ is given by:

$$\mu_t = \mu_{t-1} + \lambda_t$$

Suppose now that $\{c_t^*, k_t^*, s_t^*, p_t^*, d_t^*, \gamma_t^*, \xi_t^*, \mu_t^*\}_{t=0}^{\infty}$ solves the problem above given k_{-1}, s_{-1}, d_{-1}, p_{-1}, $\mu_{-1} = 0$ and consider a reoptimization at $t = s$ given k_{s-1}^*, $s_{s-1}^*, d_{-1}^*, p_{-1}^*$ and $\mu_{s-1}^{**} = 0$. We now show that if $\mu_{s-1} = 0$ or $V_{t+1}' = V_t'$, then the vector

$$\{c_t^{**}, k_t^{**}, s_t^{**}, p_t^{**}, d_t^{**}, \gamma_t^{**}, \xi_t^{**}, \mu_t^{**}\}_{t=s}^{\infty}$$

given by

$$\begin{aligned} c_t^{**} &= c_t^*, k_t^{**} = k_t^*, s_t^{**} = s_t^*, p_t^{**} = p_t^*, d_t^{**} = d_t^* \\ \gamma_t^{**} &= \frac{\gamma_t^*}{1+\mu_{s-1}^*}, \xi_t^{**} = \frac{\xi_t^*}{1+\mu_{s-1}^*}, \\ \mu_t^{**} &= \frac{\mu_t^* - \mu_{s-1}^* s_{t-1}}{1+\mu_{s-1}^*} \end{aligned}$$

for all $t \geq s$ satisfies the first order conditions of the reoptimization problem.

First, let $A_{d,t}^{**} = -\gamma_t^{**}\mathcal{C}_{d_t,t}^{**} - \xi_t^{**}\mathcal{B}_{d_t,t}^{**} - \beta E_t \gamma_{t+1}^{**}\mathcal{C}_{d_t,t+1}^{**} - \beta E_t \xi_{t+1}^{**}\mathcal{B}_{d_t,t+1}^{**}$. The first order condition for dividends is satisfied, since

$$\begin{aligned} 0 &= V_t^{**\prime} s_{t-1}^{**} - \gamma_t^{**} s_{t-1}^{**} + A_{d,t}^{**} + \mu_{t-1}^{**} \Leftrightarrow \\ 0 &= V_t^{*\prime} s_{t-1}^* - \frac{\gamma_t^* s_{t-1}^* + A_{d,t}^*}{1+\mu_{s-1}^*} + \frac{\mu_{t-1}^* - \mu_{s-1}^* s_{t-1}^*}{1+\mu_{s-1}^*} \Leftrightarrow \\ 0 &= \left(1+\mu_{s-1}^*\right) V_t^{*\prime} s_{t-1}^* - \gamma_t^* s_{t-1}^* + A_{d,t}^* + \mu_{t-1}^* - \mu_{s-1}^* s_{t-1}^* \end{aligned}$$

Further, using the dividend first order condition from the * problem,

$$0 = \mu_{s-1}^* V_t^{*\prime} s_{t-1}^* - \mu_{s-1}^* s_{t-1} \Leftrightarrow \mu_{s-1}^* = 0 \text{ or } V_t^{*\prime} = 1$$

Second, let $A_{k,t}^{**} = -\gamma_t^{**}\mathcal{C}_{k_t,t}^{**} - \xi_t^{**}\mathcal{B}_{k_t,t}^{**} - \beta E_t \gamma_{t+1}^{**}\mathcal{C}_{k_t,t+1}^{**} - \beta E_t \xi_{t+1}^{**}\mathcal{B}_{k_t,t+1}^{**}$. The capital first order condition is satisfied for the same allocations, since all multipliers are simply divided by a constant which can be cancelled:

$$\begin{aligned} \gamma_t^{**} - A_{k,t}^{**} &= \beta E_t \gamma_{t+1}^{**} F'(\theta_{t+1}, k_t^{**}) \Leftrightarrow \\ \frac{\gamma_t^* - A_{k,t}^*}{1+\mu_{s-1}^*} &= \beta E_t \left[\frac{\gamma_{t+1}^* F'(\theta_{t+1}, k_t^*)}{1+\mu_{s-1}^*}\right] \Leftrightarrow \\ \gamma_t^* - A_{k,t}^* &= \beta E_t \left[\gamma_{t+1}^* F'(\theta_{t+1}, k_t^*)\right] \end{aligned}$$

Third, plugging the above relationships in the stock Euler condition of the re-optimization problem gives:

$$\begin{aligned} &p_t^* \left(\gamma_t^* - \left(1+\mu_{s-1}^*\right) V_t^{*\prime}\right) + A_{s,t}^* = \beta E_t p_{t+1}^* \left(\gamma_{t+1}^* - \left(1+\mu_{s-1}^*\right) V_{t+1}^{*\prime}\right) \\ &+\beta E_t d_{t+1}^* \left(\gamma_{t+1}^* - \left(1+\mu_{s-1}^*\right) V_{t+1}^{*\prime}\right) \end{aligned}$$

Using the original stock Euler condition

$$\begin{aligned} p_t^* \mu_{s-1}^* V_t^{*\prime} &= \beta E_t p_{t+1}^* \mu_{s-1}^* V_{t+1}^{*\prime} + \beta E_t d_{t+1}^* \mu_{s-1}^* V_{t+1}^{*\prime} \Leftrightarrow \\ p_t^* V_t^{*\prime} &= \beta E_t \left(p_{t+1}^* + d_{t+1}^*\right) V_{t+1}^{*\prime} \text{ or } \mu_{1,s-1}^* = 0 \end{aligned}$$

Using the p-d mapping, this is clearly true if $V(x)=x$, implying that $V_t'=1$ for all t.

Last, we need to show also that the μ_t law of motion is satisfied.

$$\begin{aligned}\mu_t^{**} &= \mu_{t-1}^{**}+\left(s_t^{**}-s_{t-1}^{**}\right)\left(\gamma_t^{**}-V_t^{**\prime}\right)+A_{p,t}^{**}\Leftrightarrow\\ \mu_t^{*}-\mu_{t-1}^{*} &= \mu_{s-1}^{*}s_t^{*}-\mu_{s-1}^{*}s_{t-1}^{*}-\left(s_t^{*}-s_{t-1}^{*}\right)V_t^{*\prime}\mu_{s-1}^{*}\\ &\quad+\left(s_t^{*}-s_{t-1}^{*}\right)\left(\gamma_t^{*}-V_t^{*\prime}\right)+A_{p,t}^{*}\end{aligned}$$

Using the * problem law of motion, we cancel out most terms and are left with

$$0=\mu_{s-1}^{*}s_t^{*}-\mu_{s-1}^{*}s_{t-1}^{*}-s_t^{*}V_t^{*\prime}\mu_{s-1}^{*}+s_{t-1}^{*}V_t^{*\prime}\mu_{s-1}^{*}$$

This last condition is true if either $\mu_{1,s-1}^{*}=0$ or $V(x)=x$, implying that $V_t'=1$ for all t. ■

Proof of Lemma 2

Lemma 2 directly follows from Lemma 1. In particular, assume that financing frictions are zero and let the cash flow of the firm be given by $\{n_t\}_{t=0}^{\infty}$. Consider the equilibrium consumption process $\{c_t\}_{t=0}^{\infty}=\{n_t\}_{t=0}^{\infty}$. Consider any choice of stocks $\{\widetilde{s}_t\}_{t=0}^{\infty}$ such that $\widetilde{s}_t\neq 0$ almost surely and let $\left\{\widetilde{s}_t^h\right\}_{t=0}^{\infty}=\{\widetilde{s}_t\}_{t=0}^{\infty}$. Consistent with this choice of $\widetilde{s}$ we find the associated price to satisfy the following equation:

$$\widetilde{p}_t\widetilde{s}_t=E_t\left(\sum_{j=1}^{\infty}\delta^j\frac{u'\left(c_{t+j}\right)}{u'\left(c_t\right)}n_{t+j}\right)$$

and the divided process $\left\{\widetilde{d}\right\}$ to satisfy:

$$\widetilde{d}_t\widetilde{s}_{t-1}+\widetilde{p}_t\widetilde{s}_{t-1}=E_t\sum_{j=0}^{\infty}\delta^j\frac{u'\left(c_{t+j}\right)}{u'\left(c_t\right)}n_{t+j}$$

Now we have to show that such a stock, price and dividend processes satisfy budget constraints and pricing equations. First, notice that

$$\widetilde{p}_t\widetilde{s}_t=E_t\left(E_{t+1}\left(\sum_{j=1}^{\infty}\delta^j\frac{u'\left(c_{t+j}\right)}{u'\left(c_t\right)}n_{t+j}\right)\right)=E_t\left(\delta\frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)}E_{t+1}\sum_{j=0}^{\infty}\delta^j\frac{u'\left(c_{t+j}\right)}{u'\left(c_{t+1}\right)}n_{t+j+1}\right)$$

Using the definition of $\widetilde{d}_t\widetilde{s}_{t-1}+\widetilde{p}_t\widetilde{s}_{t-1}$ we also have that

$$\widetilde{p}_t\widetilde{s}_t=E_t\left(\delta\frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)}\left(\widetilde{d}_{t+1}\widetilde{s}_t+\widetilde{p}_{t+1}\widetilde{s}_t\right)\right)$$

so $\widetilde{s}_t$ cancels out and (3) holds. It is easy to see also that the above choices satisfy the budget constraint of the firm. We can find many other equilibria by changing $\{\widetilde{s}_t\}_{t=0}^{\infty}$.■

Proof of Proposition 2.

To prove proposition 2, we first establish the equivalence between the DE and the OP problems. For simplicity, let's assume that $\mathcal{B}_t\equiv 0$ and $u(c)=c$. Recall that the OP-problem is given by:

$$\max_{\{k_t,s_t,d_t,p_t\}}E_0\sum_{t=0}^{\infty}\beta^tV(k_t,k_{t-1}D_t-e_t)\text{ s.t.}\tag{34}$$

$$d_t s_{t-1} + k_t = p_t \left(s_t - s_{t-1} \right) + F(\theta_t, k_{t-1}) - \mathcal{C}\left(k_t, k_{t-1}, D_t, e_t \right) \tag{35}$$

$$p_t = E_t \sum_{j=1}^{\infty} \delta^j d_{t+j} \tag{36}$$

where total dividends D_t and new equity e_t are defined as

$$\begin{aligned} D_t &\equiv d_t s_{t-1} \\ e_t &\equiv p_t (s_t - s_{t-1}) \end{aligned} \tag{37}$$

Notice that given constraint (35) $D_t - e_t = n_t - \mathcal{C}_t$ are cash flows. We now prove the following properties of the solution of the OP-problem $\{k_t^*, s_t^*, d_t^*\}$.

1. $\{k_t^*, s_t^*, d_t^*\}$ is recursive in the natural state variables $(\theta_t, k_{t-1}, s_{t-1})$. In particular, it has the following recursive structure:

$$\begin{aligned} k_t^* &= F^k(k_{t-1}^*, \theta_t) \\ \begin{bmatrix} s_t^* \\ d_t^* \end{bmatrix} &= F^{sd}(k_{t-1}^*, \theta_t, s_{t-1}^*) \end{aligned}$$

   for time-invariant functions $F^k : R^2 \to R$ and $F^{sd} : R^3 \to R^2$

2. $\{k_t^*, s_t^*, d_t^*\}$ is time consistent

3. $\{k_t^*, s_t^*, d_t^*\}$ coincides with the solution to the problem of the naive manager

The proof of this result is based on the fact that the OP-problem is equivalent to the following DE-problem:

$$\max_{\{k_t, e_t, D_t\}} E_0 \sum_{t=0}^{\infty} \delta^t \; V(k_t, k_{t-1}, D_t - e_t) \text{ s.t.} \tag{38}$$

$$D_t + k_t - (1-\eta) k_{t-1} = e_t + \theta_t f(k_{t-1}) - \mathcal{C}(k_t, k_{t-1}, D_t, e_t) \tag{39}$$

We first prove the following.

a Given a sequence $\{k_t, s_t, d_t\}$ that is feasible in the original problem we can find $\{e_t, D_t\}$ that satisfies (37) and that is feasible in the DE problem for the same k series.

b Conversely, given $\{k_t, e_t, D_t\}$ that is feasible in the DE problem we can find a $\{s_t, d_t\}$ that satisfies (37), and that is feasible in the original problem for the same k series.

Part a) follows immediately from choosing $\{e_t, D_t\}$ that satisfies (37), plugging the results in (35) and observing it satisfies the only constraint in DE problem. For part b), given $\{k_t, e_t, D_t\}$ we build a process $\{s_t, d_t\}$ in the following way: first build the series of cash flows:

$$n_t \equiv F(\theta_t, k_{t-1}) - \mathcal{C}(k_t, k_{t-1}, D_t, e_t) - k_t$$

Then build $\{s_t, d_t\}$ recursively as follows. At any period $t \geq 0$, given s_{t-1} and the process $\{k_t, e_t, D_t\}$ find (s_t, d_t, p_t) for a given realization as follows:

$$
\begin{aligned}
d_t &= D_t / s_{t-1} && (40) \\
p_t &= \left(E_t \sum_{j=0}^{\infty} \delta^j \; n_{t+j} - D_t \right) \frac{1}{s_{t-1}} && (41) \\
s_t &= \frac{e_t}{p_t} + s_{t-1} && (42)
\end{aligned}
$$

With this solution we get s_t and can construct $(s_{t+1}, d_{t+1}, p_{t+1})$ and so on. It is clear that in this manner one can build a whole process $\{s_t, d_t, p_t\}$. Now we have

$$
\begin{aligned}
(p_t + d_t) s_{t-1} &= E_t \left(\sum_{j=0}^{\infty} \delta^j \; n_{t+j} \right) && (43) \\
&= n_t + \delta E_t \left(E_{t+1} \sum_{j=0}^{\infty} \delta^j \; \left[e_{t+1+j} + n_{t+1+j} \right] \right) \\
&= n_t + \delta E_t \left(p_{t+1} + d_{t+1} \right) s_t && (44)
\end{aligned}
$$

where the first equality follows from the fact that the process so constructed satisfies (40) and (41), the second equality uses the law of iterated expectations and simple algebra and the third equality uses (43) for period $t+1$ inside the expectation. On the other hand we have

$$(p_t + d_t) s_{t-1} = D_t - e_t + p_t s_t = n_t + p_t s_t$$

where the first equality follows from (37) and the second from the definition of cash flows. This together with (44) implies that

$$p_t = \delta E_t \left(p_{t+1} + d_{t+1} \right) = E_t \left(\sum_{j=1}^{\infty} \delta^j \; d_{t+j} \right)$$

so that (36) is also satisfied. In sum, all the constraints of the OP-problem are satisfied for the series that satisfies (40) to (42) and this proves part b).

Now note that given a sequence $\{k_t, s_t, d_t\}$, for the feasible sequence $\{e_t, D_t\}$ that is alluded to in part a), we have

$$E_0 \sum_{t=0}^{\infty} \delta^t V(k_t, k_{t-1}, D_t - e_t) = E_0 \sum_{t=0}^{\infty} \delta^t V(k_t, k_{t-1}, d_t s_{t-1} - p_t (s_t - s_{t-1}))$$

The same holds for any feasible sequence $\{k_t, e_t, D_t\}$ and the corresponding sequence $\{s_t, d_t\}$ that is mentioned in part b). This is because, in both cases, $D_t = d_t s_{t-1}$ and $e_t = p_t (s_t - s_{t-1})$. Therefore the maximum value of the original problem coincides with the maximum value of the DE problem. Formally, letting $\{k_t^{**}, e_t^{**}, D_t^{**}\}$ denote the solution of the original problem and let $\{k_t^*, s_t^*, d_t^*\}$ denote the solution to the original problem. Then,

$$E_0 \sum_{t=0}^{\infty} \delta^t V(k_t^{**}, k_{t-1}^{**}, D_t^{**} - e_t^{**}) = E_0 \sum_{t=0}^{\infty} \delta^t V(k_t^*, k_{t-1}^*, d_t^* s_{t-1}^* - p_t^* (s_t^* - s_{t-1}^*))$$

Now it is clear that the solution to the DE problem is recursive in the standard dynamic programming sense, since only past values of k (in addition the shock θ) constrain the feasible set for current e, D, k, therefore the optimal solution for the DE problem has the form

$$(k_t^{**}, e_t^{**}, D_t^{**}) = F^{DE}(k_{t-1}^{**}, \theta_t) \text{ for all } t, \text{ a.s.} \tag{45}$$

for some time-invariant policy function $F^{DE} : R^2 \to R^3$.

This means that the sequence $\{k_t, s_t, d_t\}$ corresponding to $\{e_t^{**}, D_t^{**}, k_t^{**}\}$ according to part b) of the results mentioned above achieves the maximum in the original problem. Since this corresponding $\{s_t, d_t\}$ sequence satisfes (40) to (42) then it is clear that for any t the variables (s_t, d_t) are a function of $(e_t^{**}, D_t^{**}, k_t^{**})$ and also of s_{t-1}^*, therefore, combining (40) to (42) with (45) we have

$$(s_t^*, d_t^*) = F(k_{t-1}^{**}, \theta_t, s_{t-1}^*) \tag{46}$$

for a time invariant function F. This proves part 1 of the proposition.

For part 2, consider the case where the manager reoptimizes at time $\bar{t}$ taking as given the "initial" state variables $(k_{\bar{t}-1}^{**}, \theta_{\bar{t}}, s_{\bar{t}-1}^*)$. We simply state that by a similar argument as above the reoptimized *original* problem is equivalent with the reoptimized *DE* problem. Since the DE problem satisfies a standard Bellman equation this problem is time consistent and the reoptimized series for k, e, D coincides with the original optimum announced at time zero for the DE problem $\{k_t^{**}, e_t^{**}, D_t^{**}\}_{t=\bar{t}}^{\infty}$. It is clear that the corresponding d, s series would also coincide with the preannounced one, so there is time consistency.

For part 3 of the proposition, note that the naive problem simply does not take into account (36) as a constraint. This means that the optimum of the DE problem is consistent with a series k, p, d, s that satisfies all constraints in the naive problem, since this problem simply has one fewer constraint than the original problem, namely (36). Therefore the maximum of the OP-problem is also the maximum of the naive problem.

Proof of Proposition 3.

We now show that the naive solution (N) is equal to the rational solution (R) iff $\mu_t = 0$ for all t. We do this for the case with $u(c) = c$. First, if $\mu_t^* = 0$ for all t, then the N and R foc are the same. Second, suppose that $\{k_t, s_t, p_t, d_t, \gamma_t, \mu_t\}_{t=0}^{\infty}$ solve the N problem so that

$$\begin{aligned}
\gamma_t \left(1 + \mathcal{C}_{k_t,t}\right) &= \beta \left[\gamma_{t+1} \left(F'(k_t) - \mathcal{C}_{k_t,t+1}\right)\right] \\
0 &= V_t' s_{t-1} - \gamma_t s_{t-1} - \gamma_t \mathcal{C}_{d_t,t} - \beta \gamma_{t+1} \mathcal{C}_{d_t,t+1} \\
p_t \left(\gamma_t - V_t'\right) - \gamma_t \mathcal{C}_{s_t,t} &= \beta p_{t+1} \left(\gamma_{t+1} - V_{t+1}'\right) + \beta \gamma_{t+1} \mathcal{C}_{s_t,t+1} \\
&\quad + \beta d_{t+1} \left(\gamma_{t+1} - V_{t+1}'\right)
\end{aligned}$$

Suppose that the same $\{k_t, s_t, p_t, d_t\}_{t=0}^{\infty}$ solve the rational problem, that is

$$\begin{aligned}
\gamma_t^{FR} \left(1 + \mathcal{C}_{k_t,t}\right) &= \beta \left[\gamma_{t+1}^{FR} \left(F'(k_t) - \mathcal{C}_{k_t,t+1}\right)\right] \\
p_t \left(\gamma_t^{FR} - V_t'\right) - \gamma_t^{FR} \mathcal{C}_{s_t,t} &= \beta p_{t+1} \left(\gamma_{t+1}^{FR} - V_{t+1}'\right) \\
&\quad + \beta \gamma_{t+1}^{FR} \mathcal{C}_{s_t,t+1} + \beta d_{t+1} \left(\gamma_{t+1}^{FR} - V_{t+1}'\right)
\end{aligned}$$

$$0 = V_t' s_{t-1} - \gamma_t^{FR} s_{t-1} - \gamma_t^{FR} \mathcal{C}_{d_t,t} - \beta \gamma_{t+1}^{FR} \mathcal{C}_{d_t,t+1} + \mu_{1,t-1}$$

$$\mu_{1,t} = \mu_{1,t-1} + (s_t - s_{t-1}) \left(\gamma_t^{FR} - V_t'\right) - \gamma_t^{FR} \mathcal{C}_{p_t,t} - \beta \gamma_{t+1}^{FR} \mathcal{C}_{p_t,t+1}$$

Clearly it has to be the case that $\frac{\gamma_{t+1}^{FR}}{\gamma_t^{FR}} = \frac{\gamma_{t+1}}{\gamma_t}$ for all t. Using the two dividend foc and subtracting one from the other

$$\left(1 - \frac{\gamma_t^{FR}}{\gamma_t}\right)\left[V_t' s_{t-1} + \mu_{1,t-1}\right] + \mu_{1,t-1} = 0$$

At $t = 0$, this implies $\left(1 - \frac{\gamma_0^{FR}}{\gamma_0}\right) V_0' s_{-1} = 0$. Assuming $V_0' \neq 0$, this implies that $\gamma_0^{FR} = \gamma_0$. By the previous condition, this also implies $\gamma_t^{FR} = \gamma_t$ for all t. As a result, from the R dividend first order condition for any t, it must be that $\mu_t = 0$.■

Proof of Results 1 and 2.

Naive firms. We start with the problem of a naive firm. This is given by:

$$\begin{aligned} & \max \sum_{t=0}^{\infty} \delta^t \log\left(d_t s_m\right) \\ \text{s.t. } d_t s_{t-1} &= p_t\left(s_t - s_{t-1}\right) + k_{t-1}^{\alpha} + (1-\eta)k_{t-1} - k_t \\ & s_{-1}, k_{-1} \text{ given} \end{aligned}$$

and the first order conditions are given by:

$$\begin{aligned} \frac{1}{d_t} &= \gamma_t s_{t-1} \\ \gamma_t p_t &= \delta\left[\gamma_{t+1}\left(d_{t+1} + p_{t+1}\right)\right] \\ \gamma_t &= \delta\gamma_{t+1}\left(1 - \eta + \alpha k_t^{\alpha-1}\right) \\ p_t &= \delta\left(d_{t+1} + p_{t+1}\right) \\ d_t s_{t-1} &= p_t\left(s_t - s_{t-1}\right) + k_{t-1}^{\alpha} + (1-\eta)k_{t-1} - k_t \end{aligned}$$

The stock Euler equation together with the price equation imply:

$$\gamma_t = \gamma_{t+1}$$

The fact that the multipliers γ_t are constant has two implications. First, the capital Euler equation implies

$$k_t = k^{GR} = \left[\frac{\frac{1}{\delta} - 1 + \eta}{\alpha}\right]^{\frac{1}{\alpha-1}}$$

Second, the dividend first order conditions give

$$d_t s_{t-1} = d_{t+1} s_t$$

We now show that the following dividends and stocks satisfy the above equilibrium conditions

$$\begin{aligned} s_t &= \bar{s} \text{ for } t \geq 0 \\ d_t &= \bar{d} \text{ for } t \geq 1 \\ d_0 &= \frac{n^{GR}}{s_{-1}} \end{aligned}$$

These allocations satisfy the price-dividend mapping as long as

$$p = \frac{\delta}{1-\delta}\bar{d}$$

and the dividend first order conditions as long as

$$d_0 s_{-1} = \bar{d}\bar{s}$$

Thus, all that remains is to find $\bar{d}$, d_0 and $\bar{s}$ so that the budget constraints are also satisfied. From the budget constraint from $t = 1$ onwards we can find

$$d_t s_{t-1} = \bar{d}\bar{s} = n^{GR}$$

where

$$n^{GR} = \left(k^{GR}\right)^{\alpha} - \eta k^{GR}$$

This also means that

$$d_0 = \frac{n^{GR}}{s_{-1}}$$

will ensure the dividend first order conditions and the budget constraints from period $t = 1$ onwards are satisfied. The period 0 budget constraint is:

$$\begin{aligned} d_0 s_{-1} &= p_0(s_0 - s_{-1}) + k_{-1}^{\alpha} + (1-\eta)k_{-1} - k_0 \\ n^{GR} &= \frac{\delta}{1-\delta}\bar{d}(\bar{s} - s_{-1}) + k_{-1}^{\alpha} + (1-\eta)k_{-1} - k_0 \end{aligned}$$

Using $\bar{d}\bar{s} = n^{GR}$, we can find $\bar{d}$ consistent with this budget constraint to be

$$\bar{d} = \frac{(1-\delta)\left(k_{-1}^{\alpha} + (1-\eta)k_{-1} - k^{GR}\right) + (2\delta - 1)\, n^{GR}}{\delta s_{-1}}$$

and therefore

$$\bar{s} = \frac{\delta n^{GR}}{(1-\delta)\left(k_{-1}^{\alpha} + (1-\eta)k_{-1} - k^{GR}\right) + (2\delta - 1)\, n^{GR}} s_{-1}$$

Note that if $k_{-1} = k^{GR}$, then $\bar{s} = s_{-1}$. But if $k_{-1} < k^{GR}$ then $\bar{s} > s_{-1}$ and $d_0 > \bar{d}$.

Rational firms. We now look at the problem of a rational firm. In this case, we don't need logarithmic utility to prove the result. The problem of the rational firm is:

$$\begin{aligned} & \max \sum_{t=0}^{\infty} \delta^t v\left(d_t s_m\right) \\ \text{s.t. } d_t s_{t-1} &= p_t\left(s_t - s_{t-1}\right) + k_{t-1}^{\alpha} + (1-\eta)k_{t-1} - k_t \\ p_t &= \delta\left(d_{t+1} + p_{t+1}\right) \\ & s_{-1}, k_{-1} \text{ given} \end{aligned}$$

The recursive Lagrangian is

$$L = \sum_{t=0}^{\infty} \delta^t \left[v\left(d_t s_m\right) + \mu_{t-1} d_t + \gamma_t \left(k_{t-1}^{\alpha} + (1-\eta)k_{t-1} - k_t - d_t s_{t-1}\right)\right]$$

and the equilibrium conditions are now

$$\begin{aligned}
s_m v'(d_t s_m) &= \gamma_t s_{t-1} - \mu_{t-1} \\
\gamma_t p_t &= \delta \left[\gamma_{t+1} \left(d_{t+1} + p_{t+1}\right)\right] \\
\gamma_t &= \delta \gamma_{t+1} \left(1 - \eta + \alpha k_t^{\alpha-1}\right) \\
p_t &= \delta \left(d_{t+1} + p_{t+1}\right) \\
d_t s_{t-1} &= p_t \left(s_t - s_{t-1}\right) + k_{t-1}^{\alpha} + (1-\eta) k_{t-1} - k_t \\
\mu_t &= \mu_{t-1} + \gamma_t (s_t - s_{t-1})
\end{aligned}$$

We provide an analytical solution to these conditions. The stock Euler together with the price equation imply

$$\gamma_t = \gamma_{t+1}$$

so the stock Euler implies

$$k_t = k^{GR} = \left[\frac{\frac{1}{\delta} - 1 + \eta}{\alpha}\right]^{\frac{1}{\alpha-1}}$$

just like under naive firms. Using the fact that $\gamma_t = \gamma_{t-1}$ for all $t \geq 1$ and the dividend first order conditions we have

$$\begin{aligned}
s_m v'(d_t s_m) - s_m v'(d_{t-1} s_m) &= \gamma_t s_{t-1} - \mu_{t-1} - \gamma_{t-1} s_{t-2} + \mu_{t-2} \\
&= (\gamma_t - \gamma_{t-1}) s_{t-1} = 0
\end{aligned}$$

so $d_t = d_{t-1}$ for all $t \geq 1$. The constant dividend level is found from the time 0 budget constraint

$$d_t = \bar{d} = \frac{(1-\delta)\left(k_{-1}^{\alpha} + (1-\eta) k_{-1} - k^{GR}\right) + \delta n^{GR}}{s_{-1}} \text{ for } t \geq 0$$

Given that, we can use the period 0 dividend first order condition to find γ_t :

$$\gamma_t = \gamma_0 = \frac{s_m v'(\bar{d} s_m)}{s_{-1}}$$

and the price is also constant and equal to

$$p_t = p = \frac{\delta}{1-\delta} \bar{d}$$

We can now compute the stocks from the intertemporal budget constraints for $t \geq 1$

$$\begin{aligned}
(\bar{d} + p) s_{t-1} &= \sum_{j=t}^{\infty} \delta^{j-t} n^{GR} = \frac{n^{GR}}{1-\delta} \Rightarrow \\
s_{t-1} &= \bar{s} = \frac{n^{GR}}{\bar{d}} \text{for } t \geq 1
\end{aligned}$$

It is straightforward to see that

$$\bar{s} = \frac{n^{GR}}{(1-\delta)\left(k_{-1}^{\alpha} + (1-\eta) k_{-1} - k^{GR}\right) + \delta n^{GR}} s_{-1} > s_{-1}$$

as long as $k_{-1} < k^{GR}$. Finally, the multipliers μ_t are constant after period 0 and equal to μ_0

$$\mu_t = \gamma_0(\bar{s} - s_{-1}) > 0 \text{ for } t \geq 0$$

Comparison. In what follows, we compare the allocations under naive and rational firms. We assume throughout that $k_{-1} < k^{GR}$. If the opposite holds, that is, if $k_{-1} > k^{GR}$, all the relationships are reversed. If $k_{-1} = k^{GR}$ allocations are trivially the same in the two setups.

Dividends are lower for the rational firm initially:

$$d_0^{FR} = \frac{(1-\delta)\left(k_{-1}^{\alpha} + (1-\eta)k_{-1} - k^{GR}\right) + \delta n^{GR}}{s_{-1}} < \frac{n^{GR}}{s_{-1}} = d_0^N$$

but higher from then onwards:

$$\begin{aligned} d_0^{FR} + \frac{\delta}{1-\delta}\bar{d}^{FR} &= \frac{\sum_{t=0}^{\infty}\delta^t n_t}{s_{-1}} = d_0^N + \frac{\delta}{1-\delta}\bar{d}^N \Rightarrow \\ \bar{d}^{FR} - \bar{d}^N &= \frac{1-\delta}{\delta}\left(d_0^N - d_0^{FR}\right) > 0 \end{aligned}$$

This last result implies that the stock price is always higher under rational firms

$$p_t^{FR} = \frac{\delta}{1-\delta}\bar{d}^{FR} > \frac{\delta}{1-\delta}\bar{d}^N > p_t^N$$

In turn, this implies that the naive firm needs to issue more stocks to achieve the optimal investment. This can be seen using the expressions for s_0 in the two cases. Letting $n_0 = k_{-1}^{\alpha} + (1-\eta)k_{-1} - k^{GR}$

$$\begin{aligned} s_0^N &= \bar{s}^N = \frac{\delta n^{GR}}{(1-\delta)\, n_0 + (2\delta - 1)\, n^{GR}} s_{-1} \\ s_0^{FR} &= \bar{s}^{FR} = \frac{n^{GR}}{(1-\delta)\, n_0 + \delta n^{GR}} s_{-1} \end{aligned}$$

$$\begin{aligned} s_0^N &> s_0^{FR} \Leftrightarrow \\ \delta\left[(1-\delta)\, n_0 + \delta n^{GR}\right] &> (1-\delta)\, n_0 + (2\delta-1)\, n^{GR} \Leftrightarrow \\ (\delta - 1)\left[(1-\delta)\, n_0\right] + \left(\delta^2 - 2\delta + 1\right) n^{GR} &> 0 \Leftrightarrow \\ (1-\delta)^2\left(n^{GR} - \left[(1-\delta)\, n_0\right]\right) &> 0 \end{aligned}$$

which is true for $k_{-1} < k^{GR}$.■

Proof of Proposition 4.

The first order conditions for the time 0 problem are given by:

$$\mu_t = \mu_{t-1} + \gamma_t(s_t - s_{t-1}) \text{ with } \mu_{-1} = 0$$

$$s_m v'(s_m d_t) = \gamma_t s_{t-1} - \mu_{t-1}$$

along with

$$\begin{aligned}\gamma_t &= \gamma_{t+1}\delta(f'(k_t)+1-\eta)\\ s_t &= s_{-1}+(t+1)\Delta, \quad \text{for } 0\le t\le T-1\end{aligned}$$

We now consider whether a re-optimization in future periods would lead the firm to deviate from the dividend plans announced in period zero. We use the superscript R to denote the solution if the firm re-optimizes in period $t=1$. The conditions for capital and the stock are the same as before. On the other hand, we have

$$\begin{aligned}\mu_t^R &= \mu_{t-1}^R+\gamma_t^R(s_t^R-s_{t-1}^R) \text{ for } t\ge 1\\ \mu_0^R &= 0\end{aligned}$$

$$s_m v'(s_m d_t^R)=\gamma_t^R s_{t-1}^R-\mu_{t-1}^R$$

This implies that the following equation holds for $t>1$:

$$v'(d_t^R)=v'(d_{t-1}^R)+(\gamma_t^R-\gamma_{t-1}^R)s_{t-1}^R$$

In addition, since the firm re-optimizes at $t=1$, we have

$$v'(d_1^R)=\gamma_1^R s_0$$

Suppose that the re-optimization choices are the same as the original ones, i.e. $d_t^R=d_t$, $s_t^R=s_t$ and $k_t^R=k_t$ for $t\ge 1$. We now show that this leads to a contradiction. If the re-optimized choices are the same as originally, the following must hold

$$v'(d_1) = \gamma_1^R s_0 \tag{47}$$

$$\gamma_2-\gamma_1 = \frac{v'(d_2)-v'(d_1)}{s_1}=\gamma_2^R-\gamma_1^R \tag{48}$$

In addition, for these choices of γ to be compatible with the same choice for capital in period 1, the following equation must also be satisfied:

$$\gamma_2^R\delta(f'(k_1)+1-\eta)=\gamma_1^R$$

but this cannot happen. In fact, if (48) holds, we have $\gamma_2^R=\gamma_1^R-\gamma_1+\gamma_2$ so that we need the following to be true

$$\begin{aligned}\gamma_1^R &= \gamma_2^R\delta(f'(k_1)+1-\eta)=(\gamma_1^R-\gamma_1+\gamma_2)\delta(f'(k_1)+1-\eta)\\ &= (\gamma_1^R-\gamma_1)\delta(f'(k_1)+1-\eta)+\gamma_1\end{aligned}$$

The last expression can only be equal to γ_1^R if either $\delta(f'(k_1)+1-\eta)=1$ or $\gamma_1^R=\gamma_1$. The first condition arises when capital is optimal, a case which gives rise to time consistency as shown in Proposition 3 below, but which we have excluded above by the choice of a low initial capital and an upper bound on issuance Δ that is binding for at least two periods (period 0 and 1). The second case can be excluded by the formulae for γ_1^R in (47) and for γ_1 in the original problem, since $\mu_0\neq 0$. Therefore the re-optimized solution cannot be the same as the original one and the time zero policy is time inconsistent in this example.■

References

[1] Aiyagari, R., A. Marcet, T. Sargent and J, Seppala (2002), "Optimal Taxation without State-Contingent Debt", *Journal of Political Economy,* 110 (6), pp. 1220-1254.

[2] Bhattacharya, S. (1979), "Imperfect Information, Dividend Policy and the "Bird in the Hand" Fallacy", *The Bell Journal of Economics*, 10 (1), pp. 259-270.

[3] Cooley, T. F. and V. Quadrini (2001), "Financial Markets and firm Dynamics", *American Economic Review*, 91(5), 1286-1310.

[4] Covas, F. and W. DenHaan (2007), "The Role of Debt and Equity Finance over the Business Cycle", manuscript

[5] Gorton, G. and P. He (2006), "Agency-Based Asset Pricing", NBER Working Paper 12084.

[6] Hopenhayn, H. (1992), "Entry, Exit and Firm Dynamics in Long Run Equilibrium", *Econometrica*, 60 (5), 1127-50.

[7] Klein, P., P. Krusell and J.V. Ríos-Rull (2007) "Time Consistent Public Policy" working paper, University of Pennsylvania.

[8] Kydland, F. and E. C. Prescott (1977). "Rules Rather than Discretion: The Inconsistency of Optimal Plans", *Journal of Political Economy*, 473-492.

[9] Marcet A, and R. Marimon (1999), "Recursive Contracts", Eco. No. 98/37, European University Institute.

[10] Miller, M. H. and K. Rock (1985), "Dividend Policy under Asymmetric Information", *The Journal of Finance,* Vol. 40, No. 4, pp. 1031-1051.

[11] Quadrini, V. and U. Jermann, (2005), "Financial Development and Macroeconomic Stability, manuscript.