Is There a Plausible Theory for Decision under Risk?

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Theories of decision under risk that model risk averse behavior with decreasing marginal utility of money have been critiqued with concavity calibration arguments. This paper introduces a convexity calibration that applies to decision theories that represent risk aversion with nonlinear transformation of probabilities. We explain the duality of calibrations that imply implausible large stakes risk aversion for theories that represent risk averse preferences with nonlinear transformations of (a) payoffs or (b) probabilities. The dual calibrations make clear why plausibility problems with theories of decision under risk are fundamental. Heretofore, calibration critiques have been based on thought experiments. This paper reports real experiments that provide data on the empirical relevance of the critiques.

Keywords: Decision Theory, Risk, Calibration, Experiments

1. INTRODUCTION

Rabin (2000) identified a simple pattern of small stakes risk aversion for which the expected utility of terminal wealth model implies implausible large stakes risk aversion. Neilson (2001) extended Rabin's concavity calibration critique to apply to rank dependent utility of terminal wealth. Safra and Segal (2008) showed that a "stochastic" version of Rabin's calibration pattern can produce anomalies for some additional non-expected utility models in which preferences are defined on terminal wealth. Cox and Sadiraj (2006) derived calibration results for models with preferences defined on income, such as cumulative prospect theory, the expected utility of income model, and the expected utility of initial wealth and income model. Rubinstein (2006) took the concavity calibration critique to time preferences under risk.

All these studies build on the same pattern of small stakes risk aversion, the one that first appeared in Rabin (2000). None of the papers identifies a simple pattern of small stakes risk aversion that applies to the dual theory of expected utility (Yaari, 1987) that has constant marginal utility of money for risk averse preferences and represents risk aversion with nonlinear transformation of probabilities.[2] Furthermore, previous calibrations challenge

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[1] *This is a revision and extension of our 2005 working paper titled "On the Empirical Plausibility of Theories of Risk Aversion." The present paper incorporates several additional experiments run in three countries over several years. We are grateful to three anonymous referees and Glenn W. Harrison, Peter P. Wakker, and Nathanial T. Wilcox for helpful comments and suggestions. Financial support was provided by the National Science Foundation (grant numbers DUE-0226344, DUE-0622534, and IIS-0630805).*

[2] *Safra and Segal (2008, pgs. 1145, 1152-1153) explain that their stochastic small stakes risk aversion pattern does not generally hold for the dual theory.*

cumulative prospect theory and rank dependent utility theory only through their concave utility (or "value") functions, not through their probability transformation functions. Finally, Wakker (2005) shows that none of the previous calibrations apply to third generation prospect theory with variable reference points. In these ways, previous literature leaves open the question of how fundamental is the calibration critique of theories of decision under risk. Proposition 1 and its corollary fill all three of these remaining gaps in the critique.

In section 2, we clarify questions about applicability of calibration patterns to Yaari's (1987) dual theory of expected utility and other decision theories with nonlinear probability transformations. We explain that the independence axiom of expected utility theory can be used to identify patterns of small stakes risk aversion that imply implausible large stakes risk aversion for dual theory, cumulative prospect theory (Tversky and Kahneman, 1992), and rank dependent utility theory (Quiggin, 1993) but *not* for expected utility theory. We also explain that (deterministic) patterns of small stakes risk aversion used by Rabin and all subsequent authors satisfy Yaari's *dual* independence axiom. We explain that these different patterns imply implausible large stakes risk aversion for expected utility theory, cumulative prospect theory, and rank dependent utility theory but *not* for dual theory.

Sections 3 and 4 present dual calibration propositions and corollaries. Proposition 1 identifies patterns of small stakes risk aversion that have implausible large stakes risk aversion implications for dual theory of expected utility. We explain why these patterns have no *calibration* implication for expected utility theory. Proposition 2 uses small stakes risk aversion patterns, appearing in previous literature, that have implausible large stakes risk aversion implications for expected utility theory. We explain why these patterns have no *calibration* implication for dual theory of expected utility. Each proposition has a corollary that extends the calibration to rank dependent utility theory and cumulative prospect theory. The dual corollaries show that theories that model risk aversion with transformations of both probabilities and payoffs are subject to both types of calibration critique.

Calibration exercises are theoretically interesting. However, their relevance to evaluating the plausibility of theories of decision under risk is ultimately an empirical question. Do real people make choices that reveal patterns of small stakes risk aversion that (a) conform to the *dual* independence axiom but (b) imply implausible large stakes risk aversion for expected utility theory? Do real people make choices that reveal other patterns of small stakes risk aversion that (c) conform to the independence axiom but (d) imply implausible large stakes risk aversion for the *dual* theory of expected utility theory? Do real people make choices that reveal patterns that have implausible implications for theories, such as cumulative prospect theory and rank dependent utility theory, that incorporate risk aversion with nonlinear transformations of both money payoffs and probabilities? Previous literature has not reported any empirical tests of postulated patterns of small stakes risk aversion that have calibration implications. The present paper reports seven experiments that provide data for such tests.

Researchers encounter especially difficult problems in conducting real experiments with empirical validity of suppositions in calibration propositions. We explain these problems in section 5. Our solutions to these problems were implemented in experiments conducted over several years in three countries with idiosyncratic opportunities for implementing a variety of experimental designs and protocols. These experiments are briefly explained in sections 6 and 7 (and more details are provided on a web page). Analyses of data from the experiments are also reported in sections 6 and 7.

2. INDEPENDENCE, DUAL INDEPENDENCE, AND CALIBRATION PATTERNS

We start with two examples that illustrate calibration dualities. Let L_p denote the lottery that pays \$40 with probability p or \$0 with probability $1-p$. Let the (1/10)-mixture of (lottery or certain payoff) S and lottery L_p be the compound lottery with probability 1/10 for S and probability 9/10 for L_p. Suppose that the certain payoff of \$10 is preferred to the lottery $L_{1/2}$ by a risk averse agent. By the independence axiom, an expected utility maximizing agent prefers any (1/10)-mixture of \$10 and L_p to the (1/10)-mixture of $L_{1/2}$ and L_p, for all $p \in (0,1)$. Do these small stakes risk preferences imply implausible large stakes risk aversion with expected utility theory? The answer is "no"; here is a counterexample (to "yes"). Expected utility theory with CRRA utility function $u(x) = x^{\frac{1}{3}}$ implies that the agent satisfies the supposed pattern of small stakes risk aversion but has plausible large stakes risk aversion in that she prefers the 50/50 lottery that pays \$0 or \$801 to \$100 for sure. What does dual theory tell us about this pattern of risk preferences? Dual theory predicts implausible large stakes risk aversion. For example, if the agent prefers the (1/10)-mixture of \$10 and L_p to the (1/10)-mixture of $L_{1/2}$ and L_p for all $p \in \{0/18, 1/18, \cdots, 18/18\}$ then dual theory of expected utility has the implausible implication that the agent also prefers \$100 for sure to the 50/50 lottery that pays \$0 or \$1.9 million by Proposition 1 below. Paradoxically, this pattern of small stakes risk aversion: (a) implies implausible large stakes risk aversion for dual theory of expected utility; but (b) has *no* implication of implausible large stakes risk aversion for expected utility theory and, furthermore, conforms to the independence axiom of that theory

Next, consider the type of small stakes risk aversion pattern used in previous literature. One of the examples in Rabin (2000) is based on the supposition that certain payoff in amount $\$w$ is preferred to a 50/50 lottery that pays $\$(w-100)$ or $\$(w+105)$, for all amounts of initial wealth w between \$100 and \$300,000. As reported by Rabin, the expected utility of terminal wealth model implies that an agent with these small stakes risk preferences will also prefer \$10,000 for sure to the 50/50 lottery that pays \$0 or \$5.5 million when her initial wealth is \$290,000. What does dual theory tell us about this pattern of risk preferences? By the dual independence axiom, if an agent prefers $\$x$ to the 50/50 lottery that pays $\$(x-100)$ or $\$(x+105)$ for *some* value of x then she does so for *all* values of x (see appendix A.1). A dual expected utility maximizing agent with probability transformation function $f(p) = p^3$, for example, prefers $\$x$ to the 50/50 lottery that pays $\$(x-100)$ or $\$(x+105)$ for *any* value of x; however, he also prefers the 50/50 lottery that pays \$0 or \$80,000 to \$10,000 for sure, which is plausible large stakes risk aversion. Paradoxically, this pattern of small stakes risk aversion: (a) implies implausible large stakes risk aversion for expected utility theory; but (b) has *no* implication of implausible large stakes risk aversion for dual theory of expected utility and, furthermore, conforms to the dual independence axiom of that theory.

3. CALIBRATIONS FOR PROBABILITY TRANSFORMATIONS

We introduce a calibration proposition for dual theory of expected utility and a corollary that applies to cumulative prospect theory and rank dependent utility theory. Designs of experiments reported in section 6 are based on this proposition and corollary.

3.1. Calibrations for Dual Theory of Expected Utility

Let $\{y_2, p; y_1\}$ denote a binary lottery that pays the larger amount of money y_2 with probability p and the smaller non-negative amount of money y_1 with probability $1-p$. Let $\{y_3, p_3; y_2, p_2; y_1\}$ denote a three-outcome lottery that pays non-negative amounts: y_3 with probability p_3; y_2 with probability p_2; and y_1 with probability $1-p_2-p_3$. We use the convention $y_j > y_{j-1}$ for all j. Consider the $2n-1$ pairs of lotteries $A_i = \{cx, i/2n; 0\}$ and $B_i = \{cx, (i-1)/2n; x, 1/n; 0\}$, $i = 1, 2, \cdots, 2n-1$. In each pair, lottery B_i is constructed from lottery A_i by transferring probability mass $1/2n$ from both the highest payoff cx and the lowest payoff 0 to the other payoff x. The highest payoff cx in a B_i lottery is assumed to be more than twice the other positive payoff x; that is, $c > 2$.

Suppose that an agent prefers the three outcome lottery B_i to the two outcome lottery A_i for all $i = 1, 2, \cdots, 2n-1$. Define $K(t,n) = 1 + \sum_{j=1}^{n} (t-1)^j \Big/ \sum_{i=1}^{n} (t-1)^{1-i}$. Let $\succsim$ and $\succ$, respectively, indicate weak and strong preference. Let N denote the set of positive integers.

Proposition 1 (calibration for dual theory). *Let* $n \in N$ and $c > 2$ be given. Suppose that

P(1*) $\{cx, (i-1)/2n; x, 1/n; 0\} \succsim \{cx, i/2n; 0\}$, for all $i = 1, 2, \cdots, 2n-1$.

Then $z \succ \{zK(c,n), 0.5; 0\}$, for all $z > 0$.

Proof: see appendix A.2.

Note that, for $c > 2$, $K(c,n) \to \infty$ as $n \to \infty$. Therefore, the larger is the value of n, the more extreme are the implications from the calibration. This implies that for any K, as big as one chooses, there exists a large enough n such that weak preference for the three outcome lottery B_i over the two outcome lottery A_i, for all integers $i = 1, 2, \cdots, 2n-1$, implies a preference for z for sure over the risky lottery $\{zK, 0.5; 0\}$ for all $z > 0$.[3]

Some implications of Proposition 1 are reported in Table 1. For example, with c = 3.5 and n = 10, Proposition 1 tells us that for this pattern the dual theory predicts that the agent prefers 100 for sure to a lottery that pays 953,000 or 0 with even odds, as reported in the First DU Calibration column and third row of Table 1. The Second DU Calibration column of Table 1 reports calibrations for the small stakes risk aversion patterns with c = 4 (such as the one described in section 2). With c = 4 and n = 10, this pattern implies that 100 for sure is preferred to the 50/50 lottery that pays 0 or 5.9 million.

3.2. Calibrations for Rank Dependent Utility Theory and Cumulative Prospect Theory

The following corollary to Proposition 1 applies to rank dependent utility theory and cumulative prospect theory.[4] The proposition for dual theory, with functional that is linear in payoffs, incorporates the assumption that $c > 2$, which implies that the highest payoff cx in a B_i lottery is more than twice the amount of the middle payoff x in the lottery. In the

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[3] *Note that this proposition does not use an assumption that the probability transformation is everywhere convex.*

[4] *Cumulative prospect theory transforms both probabilities and payoffs differently for losses than for gains. However, for the specific lotteries considered in this section that involve only gains, cumulative prospect theory (with 0-reference point) does not differ from rank dependent utility theory.*

corollary, we restate the assumption for money payoff transformations $\upsilon(\cdot)$, that can be nonlinear, as $\upsilon(cx)/\upsilon(x) > 2$. Let $\upsilon^{-1}(\cdot)$ be the inverse function of $\upsilon(\cdot)$. One has:

Corollary 1 (calibration for cumulative prospect theory and rank dependent utility theory). Suppose that condition P(1*) is satisfied and that $\upsilon(cx)/\upsilon(x) > 2$. Then $z \succ \{\upsilon^{-1}(\upsilon(z)K(\upsilon(cx)/\upsilon(x), n)), 0.5; 0\}$, for all $z > 0$.

Proof: see appendix A.2.

Specification of a money transformation function $\upsilon(\cdot)$ implies pairs of values of c and x that satisfy the condition $\upsilon(cx)/\upsilon(x) > 2$ and can be used to illustrate implications of Corollary 1. For example, the fifth column in Table 1 reports calibrations for rank dependent utility theory (RD) and cumulative prospect theory (PT) using the money transformation function $\upsilon(y) = y^{0.88}$ (Tversky and Kahneman, 1992, p. 311). With this money transformation function, lottery payoffs $cx = 40$ and $x = 10$ satisfy the condition. As reported in the third (n = 50) row of the fifth column, if the agent rejects the lottery $\{40, p; 0\}$ in favor of the lottery $\{40, p-0.01; 10, 0.02; 0\}$ for all $p \in \{0.01, 0.02, \cdots, 0.99\}$ then calibration for cumulative prospect theory and rank dependent utility theory implies that 100 for sure is preferred to the even odds lottery that pays 0.78×10^{21} or 0.

3.3. No Calibration Implication for Expected Utility Theory

The patterns of risk aversion postulated in Proposition 1 have no calibration implication for expected utility theory, as is apparent from the following. A utility functional for expected utility theory, with the normalization that the utility of zero payoff equals zero, implies that the three outcome lottery $\{cx, (i-1)/2n; x, 1/n; 0\}$ is preferred to the two outcome lottery $\{cx, i/2n; 0\}$, *for all* i, if the utility of payoff cx is less than twice the utility of payoff x (that is, $u(cx) < 2u(x)$). There is no implication for the rate at which marginal utility of money decreases, hence nothing to calibrate for expected utility theory (see section 2 for an illustrative example).

4. CALIBRATIONS FOR PAYOFF TRANSFORMATIONS

We report a calibration proposition for expected utility theory and a corollary that applies to cumulative prospect theory and rank dependent utility theory. Designs of experiments reported in section 7 are based on this proposition and corollary.

4.1. Calibration of Patterns of Risk Aversion for Expected Utility Theory

We now examine the large stakes risk aversion implications of postulated patterns of small stakes risk aversion for expected utility theory. These implications hold for all three expected utility models discussed in Cox and Sadiraj (2006), the expected utility of terminal wealth model, the expected utility of income model, and the expected utility of initial wealth and income model. For bounded intervals of income, Proposition 2 states a concavity calibration result for expected utility theory with weakly concave utility of money payoff function $u(\cdot)$.[5] Consider sure payoffs in amounts x and lotteries that pay $x+g$ or $x-\ell$. Let $\langle x\rangle$ denote the largest integer smaller than x and define $r(t)=(1-t)\ell/tg$. One has:

Proposition 2 (calibration for expected utility theory on finite domains). Let $p\in(0,1)$ and $0<\ell<g$ be given such that $pg-(1-p)\ell>0$. Suppose that

(P.2*) $x \succsim \{x+g,\, p; x-\ell\}$, for all integers $x\in[m,M]$, $M>m\geq 0$.

Then for all $z\in[m+(g+\ell)\ln(q-qp)/\ln q, M]$, $z\succ\{G,p;m\}$ for all G such that

$$(*)\ \ G < M+(g+\ell)\frac{2q-1}{1-q}+Aq^{-\left\langle\frac{M-m}{g+\ell}\right\rangle},$$

where $q=r(p)$ and $A=\left(q(1-q^{\left\langle\frac{z-m}{g+\ell}\right\rangle})\dfrac{1-p}{p}-q^{\left\langle\frac{z-m}{g+\ell}\right\rangle}\right)\dfrac{g+\ell}{1-q}$.

Proof: See appendix A.3.

Note that for any given m and z, the third term on the right hand side of inequality $(*)$ increases geometrically in M because $q<1$ (which follows from $pg-(1-p)\ell>0$). This implies that for any amount of gain G, as big as one chooses, there exists a large enough interval in which preference for x over a risky lottery $\{x+g,0.5;x-\ell\}$, for all integers x from the interval $[m,M]$, implies a preference for z for sure to the risky lottery $\{G,0.5;m\}$. We use inequality $(*)$ in Proposition 2 to construct the illustrative examples in Table 2.

Suppose that an agent prefers the certain amount of income x to the lottery $\{x+110,0.5;x-100\}$, for all integers $x\in[1000,M]$, where values of M are given in the "Rejection Intervals" column of Table 2. In that case all three expected utility (of terminal wealth, income, and initial wealth and income) models predict that the agent prefers receiving the amount of income 3,000 for sure to a risky lottery $\{G,0.5;1000\}$, where the values of G

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[5] *See Rabin (2000) and Cox and Sadiraj (2006) for concavity calibrations on unbounded domains.*

are given in the "First EU Calibration" column of Table 2. For example, if $[m, M] = [1000, 50000]$ then $G = 0.1\times 10^{13}$ for all three expected utility models. According to the entry in the "Second EU Calibration" column and M = 30,000 row of Table 2, expected utility theory predicts that if an agent prefers certain payoff in amount x to lottery $\{x+90, 0.5; x-50\}$, for all integers x between 1,000 and 30,000, then such an agent will prefer 3,000 for sure to the 50/50 lottery with positive outcomes of 1,000 or 0.12×10^{56}.

4.2. Calibrations for Rank Dependent Utility Theory and Cumulative Prospect Theory

The following corollary to Proposition 2 applies to rank dependent utility theory and cumulative prospect theory with zero-income reference point (Kahneman and Tversky, 1992). The proposition for expected utility theory, with functional that is linear in probabilities, incorporates the assumption that $pg-(1-p)\ell > 0$. In the corollary, we restate the assumption for probability transformation functions $h(\cdot)$, that can be nonlinear, as $h(p)g-[1-h(p)]\ell > 0$.

Corollary 2 (calibration for cumulative prospect theory and rank dependent utility theory). Let positive numbers g, ℓ, and $p \in (0,1)$ be given such that $h(p)g-[1-h(p)]\ell > 0$. Suppose that statement (P.2*) is satisfied. If $\upsilon(\cdot)$ is (weakly) concave then for $q = r(h(p))$ and for all $z \in [m+(g+\ell)\ln(q-qh(p))/\ln q, M]$, $z \succ \{G, p; m\}$ for all G that satisfy inequality $(*)$ in Proposition 2 with $A = \left(q(1-q^{\left\langle \frac{z-m}{g+\ell} \right\rangle})\dfrac{1-h(p)}{h(p)} - q^{\left\langle \frac{z-m}{g+\ell} \right\rangle} \right)\dfrac{g+\ell}{1-q}$.

Proof: See appendix A.3.

For cumulative prospect theory, Tversky and Kahneman's (1992, p. 300) probability weighting function for binary lotteries with non-negative payoffs is $w^{+}(0.5) = 0.42$. For rank dependent utility theory, Quiggin's (1993, p.52) probability transformation function, q(p) for binary lotteries has $q(0.5) = 0.58$. In our notation, $h(0.5) = w^{+}(0.5) = 1-q(0.5) = 0.42$. Preference for a certain payoff x over the lottery $\{x+90, 0.5; x-50\}$ satisfies the assumption in Corollary 2 because $h(0.5)90-[1-h(0.5)]50 > 0$. As shown in the First PT & RD Calibration column and M = 30,000 row of Table 2, rank dependent utility theory and cumulative prospect theory with $h(0.5) = 0.42$ imply that an agent will prefer 3,000 for sure to the 50/50 lottery with positive payoffs of 1,000 or 0.40×10^{27}.

As another illustrative example, suppose that an agent prefers certain payoff in amount x to the lottery $\{x+30, 0.5; x-20\}$, for all integers x between 1,000 and 6,000. This pattern satisfies the assumption in Corollary 2 because $h(0.5)30-[1-h(0.5)]20 > 0$. According to the entry in the Third EU Calibration column and $M = 6{,}000$ row of Table 2, expected utility theory predicts that an agent who rejects these lotteries will prefer 3,000 for sure to the lottery with 0.5 probabilities of gaining 1,000 or 0.4×10^{20}. As shown in the right-most column of Table 2, cumulative prospect theory and rank dependent utility theory imply than an agent who rejects these same lotteries will prefer 3,000 for sure to the lottery with 0.5 probabilities of gaining 1,000 or 2.9 million.

4.3. No Calibration Implication for Dual Theory of Expected Utility

The patterns of risk aversion postulated in Proposition 2 have no calibration implication for dual theory of expected utility, as is apparent from the following. Let $f(\cdot)$ be the transformation function for decumulative probabilities. The utility functional for dual theory is always linear in payoffs but is linear in probabilities if and only if the agent is risk neutral. The sure payoff x is preferred to the lottery $\{x+g, p; x-\ell\}$ if $x > f(0.5)\times[x+g]+[f(1)-f(0.5)]\times[x-\ell]$. In dual theory, $f(1)=1$; hence x is preferred to the lottery $\{x+g, p; x-\ell\}$, *for all* values of $x > \ell$, if $f(0.5) < \ell/(\ell+g)$. There is nothing to calibrate for dual theory (see section 2 for an illustrative example).

5. EXPERIMENTAL DESIGN ISSUES

The calibration propositions and corollaries demonstrate that prominent theories of decision under risk may have implausible implications. But such a calibration critique of decision theory has unknown empirical relevance in the absence of data that provide support for the “calibration patterns” of risk aversion that are postulated in the propositions and corollaries. We next discuss issues that arise in designing experiments with these calibration patterns.

5.1. Power vs. Credibility with Probability Calibration Experiments

Table 1 illustrates the relationship between the ratio c of high to middle payoff in the three-outcome lottery and the difference between probabilities in adjacent terms in the calibration (determined by the value of n in $\frac{i}{2n}-\frac{i-1}{2n}$). The design problem for probability transformation function calibration experiments is inherent in the need to have a fine enough partition of the [0,1] interval for the calibration in Proposition 1 to lead to the implication of implausible risk aversion in the large.

There are two problems with big values of the partition parameter n. First, a subject's decisions may involve trivial financial risk because the differences between all the moments of the distributions of payoffs for the three-outcome lottery $\{cx,(i-1)/2n;x,1/n,0\}$ and the two-outcome lottery $\{cx,i/2n;0\}$ become insignificant as n increases. For example, if c = 4, x = \$25, and $n=500$ then the lotteries are $\{\$100,(i-1)/1000;\$25,1/500;\$0\}$ and $\{\$100,i/1000;\$0\}$. In that case, the difference between expected values of the two-outcome and three-outcome lotteries is 5 cents (for all i). For the same n, c, and x values, the difference between standard deviations of payoffs for the two-outcome and three-outcome lotteries, at i = 500, is 4 cents. The second problem with large n is that adjacent probabilities differ by only $1/2n$ while the subject's decision task is to make $2n$ choices. For example, for $n=500$ adjacent probabilities differ by 0.001 and the subjects' decision task is to make 1,000 choices. In such a case, the subjects would not be sensitive to the probability differences and the payoffs would arguably not dominate decision costs because of the huge number of choices needing to be made. In contrast, if the length of each subinterval is $1/10$ (i.e. $n=5$) then the difference in expected payoffs between the two-outcome and three-outcome lotteries is \$5 for the above values $cx=\$100$ and $x=\$25$, and for i = 5 the difference in standard deviations is \$4.17; furthermore, the subjects' decision task is to make only 10 choices. The calibration implications of n = 5 are less spectacular than for n = 500, as shown in Table 1, but the resulting experimental design can credibly be implemented. In our experiments, we use relatively low values of the partition parameter n.

5.2. Affordability vs. Credibility with Payoff Calibration Experiments

Table 2 illustrates the relationship between the size of the interval $[m,M]$ in the left-most column, used in the supposition underlying a utility of money payoff calibration, and the size of the high gain G in the result reported in the other columns of the table. Payoff transformation function calibration experiments involve tradeoffs between what is affordable and what is credible, as we shall next explain.

As an example, consider an experiment in which subjects were asked to choose between $\$x$ for sure and the binary lottery $\{\$x+\$110,0.5;\$x-\$100\}$ for all x between m = \$1,000 and M = \$350,000. Suppose the subject always chooses the certain amount $\$x$ and that one of the subject's decisions is randomly selected for payoff. Then the expected payoff to a single subject would exceed \$175,000. With a sample size of 30 subjects, the expected payoff to subjects would exceed \$5 million, which would clearly be unaffordable. But why use payoffs denominated in U.S. dollars? Proposition 2 is dimension invariant. Thus, instead of interpreting the figures in Table 2 as dollars, they could be interpreted as dollars divided by 10,000; in that case the example experiment would cost about $500 for subject payments and clearly be affordable. So what is the source of the difficulty? The source of the difficult tradeoff for experimental design becomes clear from inspection of Proposition 2: the unit of measure for m and M is the same as that for the loss and gain amounts ℓ and g in the binary lotteries. If the unit of measure for m and M is $1/10,000 then the unit of measure for ℓ and g is the same (or else the calibration doesn't apply); in that case the certain payoff becomes $\$0.0001x$ and the binary lottery has payoffs in amounts $\$0.0001x$ + $0.011 or $\$0.0001x$ - $0.010, which involves trivial financial risk of 2.1 cents.

The design problem for concavity calibration experiments with money payoffs is inherent in the need to calibrate over an $[m, M]$ interval of sufficient length for the calibrations in Proposition 2 and Corollary 2 to lead to the implication of implausible risk aversion in the large. There is no way to avoid this problem; the design of any experiment on payoff transformation function calibration will reflect a tradeoff between affordability of the payoffs and credibility of the incentives. In our experiments, we address this problem in two ways by: (a) conducting some experiments in India, where we can afford to use $[m, M]$ intervals of rupee payoffs that are sufficiently wide for calibration to have bite; and (b) conducting an experiment in Germany, partly on the floor of a casino, which makes use of large contingent euro payoffs affordable.

6. EXPERIMENTS WITH PROBABILITY TRANSFORMATION THEORIES

We ran four experiments with calibration patterns for probability transformation theories identified in Proposition 1 and Corollary 1 in Germany, India, and the United States. We explain the common design features and idiosyncratic lotteries in these experiments and present a more detailed discussion of one experiment to provide a representative example. We begin with the example.

6.1. Experimental Design: An Example

Subjects in one experiment parameterization were asked to make choices for each of the nine pairs of lotteries shown in Table 3. The fractions in the rows of the table are the probabilities of receiving the prizes in the two outcome (option A) and three outcome (option B) lotteries. Each row of Table 3 shows a pair of lotteries included in the experiment. The subjects were *not* presented with a fixed order of lottery pairs, as in Table 3.[6] Instead, each lottery pair was shown on a separate (response form) page. Each subject picked up a set of response pages that were arranged in independently drawn random order. He or she could mark choices in any order desired. On each decision page, a subject was asked to choose among a two outcome lottery (option A in some row of Table 3), a three outcome lottery (option B in the same row of Table 3), and indifference ("option I").

6.2. Experimental Design: Alternative Parameterizations and Protocols

We conducted four experiments on empirical validity of the small stakes risk aversion patterns postulated in Proposition 1 and Corollary 1. One experiment parameterization uses pairs of two outcome and three outcome lotteries $A_j = \{cx, j/10; 0\}$, and $B_j = \{cx, (j-1)/10; x, 2/10; 0\}$, for $j = 1, 2, \cdots 9$, in which $cx = 40$ and $x = 10$. We also ran experiments with the parameterizations $(cx, x) = (14, 4)$ and (400, 80).

The experiments were conducted in Magdeburg (Germany), Atlanta (U.S.A.) and Calcutta (India) with payoffs, respectively, in euros, U.S. dollars, and Indian rupees. The experiments used the following parameters:. Magdeburg 40/10: $cx =$ 40 euros, $x =$ 10 euros. Atlanta 40/10: $cx =$ 40 dollars, $x =$ 10 dollars. Atlanta 14/4: $cx =$ 14 dollars, $x =$ 4 dollars. Calcutta 400/80: $cx =$ 400 rupees, $x =$ 80 rupees.[7] Economic significance of the rupee payoffs is discussed in section 7.4. The payoff protocol used random selection of one decision for payoff, which is a standard procedure used in testing theories of decision under risk with or without the independence axiom. Experimental tests of random selection have generally reported consistency with the isolation effect of subjects focusing on individual decision tasks (Camerer, 1989; Starmer and Sugden, 1991; Beattie and Loomes, 1997; Cubitt, Starmer, and Sugden, 1998; Hey and Lee, 2005a, b; Laury, 2006; Lee, 2008). A home page appendix (http://excen.gsu.edu/jccox/subjects.html) reports subject instructions (in English), response forms (or pages), and detailed information on the protocol used in all of the experiments.

6.3. Implications of the Data for Dual Theory of Expected Utility

In testing for the presence of choice patterns that imply implausible risk aversion in the large, we aggregate choices of option B with choices of option I (indifference) because the

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[6] *An earlier version of the paper did report an experiment with the fixed order of choices shown in Table 3. Following referees' suggestions, we ran new experiments, reported here, without the fixed order.*

"if" statement in P(1*) in Proposition 1 involves weak preference for B over A. Aggregated choices of B and I are reported as B^I. Subjects' choice patterns are recorded as sequences of nine letters, ordered according to the probability of the high outcome. For example, the pattern [A, B^I, B^I, A, B^I, B^I, B^I, B^I, A] would indicate that a subject chose A (two outcome lottery) when the probability of the high outcome was $1/10, 4/10$ and $9/10$ - indexed as $j = 1$, 4, and 9- and chose B or I (indifference) for all other values of the index j. For the experiment with the parameterization as in Table 3, this choice pattern would mean the subject chose option A on (randomly ordered) pages with the lottery pairs in rows 1, 4, and 9 in the table and chose option B or option I on all other pages.

We use error-rate analysis for statistical inferences on proportion of subjects that made choices consistent with the supposition on small stakes patterns.[8] Choice probabilities are assumed to deviate from 1 or 0 by an error rate ε, as in Harless and Camerer (1994). Thus if B^I is preferred to A then Prob(choose B^I) = $1-\varepsilon$ and if B^I is *not* preferred to A then Prob(choose B^I) = ε, where $\varepsilon < 0.5$. The error rate model postulates that a subject with real preferences for B^I (respectively A) over A (respectively B^I) in all nine lottery pairs could nevertheless be observed to have chosen the other option in some rows. For example, according to this model a subject with underlying preferences [B^I, B^I, B^I, B^I, B^I, B^I, B^I, B^I, B^I] could, instead, have been observed to choose a different pattern such as [B^I, B^I, A, B^I, A, B^I, B^I, B^I, B^I], an event with probability $(1-\varepsilon)^7 \varepsilon^2$.

Stochastic choice Model I contains only the choice pattern with a sequence of nine B^I in the category "calibration pattern" and its mirror image with a sequence of nine A in the "other pattern." As shown in Table 1, the calibration pattern, all B^I in an experiment with n = 5 and $c = 4$ implies that 100 for sure is preferred to a 50/50 lottery that pays 24,400 or 0 according to the dual theory. According to Proposition 1, this calibration pattern also implies that 1,000 for sure is preferred to the 50/50 lottery that pays 244,000 or 0, which is again implausible risk aversion.

Model I is overly conservative in its specification of calibration patterns because other data patterns can be calibrated to imply implausible risk aversion. Stochastic choice Model II includes two patterns in the category "calibration patterns": the pattern with choice of B^I for

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[7] *We are grateful to the Centre for Experiments in Social and Behavioral Sciences, Department of Economics, Jadavpur University for use of their facilities to run experiments Calcutta 400/80 and Calcutta 90/-50 (reported in section 7).*

[8] *We are grateful to Nathaniel Wilcox for generous advice about this approach to data analysis and for supplying SAS code.*

index $j = 1, 2, \cdots, 8$ and the all B^I pattern (that is, $j = 1, 2, \cdots, 9$). The mirror images of these two patterns comprise the "other patterns" for Model II. Using a similar proof as in Proposition 1, it can be verified that these two calibration patterns of "no A except for for index $j = 9$" imply that 1,000 for sure is preferred to the 50/50 lottery that pays 81,000 or 0, which is implausible risk aversion. We also consider Model III which includes the patterns "no A except for indexes $j = 8$ and/or 9" in the category of calibration patterns. The mirror images of these four patterns comprise the other patterns for Model III. A prediction for these four calibration patterns in case of $n = 5$ and $c = 4$ is preference for 1,000 for sure to the 50/50 lottery that pays 27,000 or 0, which is arguably implausible risk aversion.

Table 4 reports results from maximum likelihood estimation of the proportion of subjects who exhibit the calibration patterns for Models I, II and III. Estimations are reported for a single error rate for all choices, for two different error rates (one error rate for choices with index j =1,..,4 and another one for choices with index j =5,..,9), and three different error rates (one error rate for choices with index $j = 1, 2, 3$, another error rate for choices with index $j = 4, 5, 6$, and another one for choices with index $j = 7, 8, 9$).

The first row of Table 4 shows results for Calcutta 400/80. For Model I with one error rate the estimated proportion of subjects who exhibited the calibration pattern is 0.72. The Wald 90 percent confidence interval is (0.58, 0.86). The 0.72 estimate is significant at one percent (as indicated by **). The other columns in the first row of Table 4 report the estimated proportions of subjects whose choice patterns in Calcutta 400/80 imply implausible risk aversion with the 1 error, 2 error, and 3 error rate versions of Models I, II, and III. These estimates vary only between 0.72 and 0.74, and they are all significant at one percent. We conclude that dual theory implies implausible large stakes risk aversion for 72 to 74 percent of the subjects in this experiment. The entries in bold font indicate the model that is selected by likelihood ratio tests; that is, with data from Calcutta 400/80, Model I with 1 error or 2 errors and Models II and III with 1 error, 2 errors, or 3 errors are all rejected in favor of Model I with 3 error rates.

The second through fourth rows of Table 4 show the estimated proportions of subjects whose choices are consistent with the calibration patterns for which dual theory implies implausible risk aversion in experiments Atlanta 40/10, Magdeburg 40/10, and Atlanta 14/4. Depending on the model and number of errors, the estimated proportion of subjects with data consistent with the calibration patterns in Atlanta 40/10 varies from 0.56 to 0.63, with all estimates significant at one percent. The estimates for data from Magdeburg 40/10 vary from 0.37 to 0.41, all significant at one percent. Estimates with data from experiment Atlanta 14/4 lie between 0.74 and 0.90 and all are significant at one percent. The entries in bold font indicate the model that is selected by likelihood ratio tests over all other models in that row.

6.4. Implications of the Data for Cumulative Prospect Theory and Rank Dependent Utility Theory

Corollary 1 applies to cumulative prospect theory and rank dependent utility theory. Using the notation in Corollary 1, the choice patterns included in "calibration patterns" have known calibration implications for these theories so long as the condition $\upsilon(cx)/\upsilon(x) > 2$ is satisfied. The value function in Tversky and Kahneman (1992, p. 311) satisfies this condition for our experiments with $(cx, x) = (14, 4)$, (40, 10), and (400, 80). Hence similar conclusions to those stated for dual theory of expected utility in section 6.3 apply here as well although the K-values will here depend on the ratio $\upsilon(cx)/\upsilon(x)$. For example, for the Calcutta 400/80 experiment, the prediction is preference of \$100 for sure over the 50/50 lottery with outcomes 29,700 or 0 in case of Model I and the value function with exponent 0.88 as in Tversky and Kahneman (1992).

6.5. Implications of the Data for Expected Utility Theory

As explained in section 3.3, expected utility theory implies that an agent's preference for option A or option B is the same for all lottery pairs in an experiment. There is no calibration implication for expected utility theory for an experiment with this design. However, expected utility theory can be tested with data from the experiment because the theory predicts that an agent will always choose the same option.

At the aggregate level, data show clear differences in fractions of observed B^I choices across lottery pairs. For example, the percentage of B^I choices in Calcutta 400/80 is 80% when the probability of the high outcome p is 0.1 in the binary lottery (and 0 in the three outcome lottery) but only 40% when p = 0.6. In Atlanta 14/4, Atlanta 40/10 and Magdeburg 40/10 the percentages vary, respectively, over the ranges 15 - 87%, 27 - 68% and 32 - 61%. These variations appear inconsistent with the "no-switching" hypothesis. Probit panel regressions of individual choices can be used to test whether the prediction "always choose the same option" is inconsistent with observed behavior. This prediction implies that the estimated coefficient for the right hand variable "lottery pair," indexed by $j = 1, 2, \cdots, 9$ (the same index used in section 6.3) should be insignificant. Equivalently, this prediction is that the estimated coefficient for the probability of getting the high (or low) payoff is insignificant. Probit panel regressions with random effects report coefficients on lottery pair indexes that are negative at one percent significance for data from three out of the four experiments.[9] The one exception is data from Magdeburg 40/10. A negative coefficient means the higher the probability of the high outcome, the lower the likelihood of choosing the three outcome lottery or indifference.

6.6. Implications of the Data for Expected Value Theory

The expected value of option A is always larger than the expected value of option B. The error rate model can be used to address the question whether the one choice pattern consistent with expected value theory is as consistent with the data as are choice patterns from alternative models. The one pattern ("all A") model is rejected at 1 percent significance by log likelihood tests for data from all six experiments reported in Table 4. The estimates from the probit panel regressions for expected utility theory imply rejection of the testable implication of expected value theory because the coefficients on the lottery pair index are significant.

7. EXPERIMENTS WITH PAYOFF TRANSFORMATION THEORIES

We ran three experiments with calibration patterns for payoff transformation theories identified in Proposition 2 and Corollary 2 in India and Germany. We explain the common features and idiosyncratic lotteries used in these experiments after presenting a detailed discussion of one experiment to provide a representative example.

7.1. Experimental Design: An Example

Subjects in one experiment parameterization were asked to make six choices between a certain amount of money x and a binary lottery $\{x+30, 0.5; x-20\}$ for values of x from the set {100, 1K, 2K, 4K, 5K, 6K}, where K = 1,000. Subjects were asked to choose among option A (the risky lottery), option B (the certain amount of money), and option I (indifference). The choice tasks given to the subjects for this parameterization are presented in Table 5. Each row of Table 5 shows a certain amount of money and paired lottery in a choice task included in the experiment. The subjects were *not* presented with a fixed order of decisions tasks, as in Table 5. Instead, each pair of sure payoff and lottery was shown on a separate (response form) page. Each subject picked up a set of response pages that were arranged in independently drawn random order. He or she could mark choices in any order desired.

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[9] *In addition to "lottery pair number," the panel regression includes subjects' answers to many questions about demographic characteristics and risk-taking attitudes. The questionnaire data are explained in an appendix on an author's home page (http://excen.gsu.edu/jccox/subjects.html).*

7.2. Experimental Design: Alternative Parameterizations and Protocols

We conducted three experiments on empirical validity of the small stakes risk aversion patterns postulated in Proposition 2 and Corollary 2. These experiments used the random decision selection payoff protocol. Calcutta $30/-20$: binary lotteries $\{x+30, 0.5; x-20\}$ and sure payoffs x from the set {100, 1K, 2K, 4K, 5K, 6K}, where K = 1,000; payoffs in rupees. Calcutta $90/-50$: binary lotteries $\{x+90, 0.5; x-50\}$ for values of x from the set {50, 800, 1.7K, 2.7K, 3.8K, 5K}, where K = 1,000; payoffs in rupees. Magdeburg $110/-100$: binary lotteries $\{x+110, 0.5; x-100\}$ for values of x from the set {3*K*, 9*K*, 50*K*, 70*K*, 90*K*, 110*K*}, where K = 1,000; payoffs in *contingent* euros.

An appendix on an author's home page (http://excen.gsu.edu/jccox/subjects.html) reports the subject instructions (in English), the response forms (or pages), and detailed information on the experiment protocol used in all of the experiments. Before presenting data, we discuss economic significance of the rupee payoffs in Calcutta experiments and the meaning of *contingent* euro payoffs in the Magdeburg experiment.

7.3. Economic Significance of the Rupee Payoffs

The exchange rates between the Indian rupee and the U.S. dollar during the years 2004 and 2008 in which the Calcutta experiments were run were, respectively about 42 to 1 and 47 to 1. These exchange rates can be used to convert the rupee payoffs discussed above into dollars. Doing that would not provide very relevant information for judging the economic significance to the subjects of the certain payoffs and risks involved in the Calcutta experiments because there are good reasons for predicting that none of the subjects would convert their rupee payoffs into dollars and spend them in U.S. markets. Better information on the economic significance of the payoffs to subjects is provided by comparing the rupee payoffs in the experiment to rupee-denominated monthly stipends of the student subjects and rupee-denominated prices of commodities available for purchase by students residing in Calcutta.

In 2004, student subjects' incomes were in the form of scholarships that paid stipends of 1,200-1,500 rupees per month in addition to the standard tuition waiver that each received. This means that the highest certain payoff used in the Calcutta $30/-20$ experiment (6,000 rupees) was equal to four or five months' stipend for the subjects. The daily rate of pay for the students was 40 to 50 rupees. Hence the amount at risk in the Calcutta $30/-20$ experiment lotteries (the difference between the high and low payoffs) was greater than or equal to a full day's pay. The amount at risk in the Calcutta $90/-50$ experiment (140 rupees) was almost three times as large.

A sample of commodity prices in Calcutta at the time of the 2004 experiment is reported in a table on an author's home page (http://excen.gsu.edu/jccox/subjects.html). Prices of food items were reported in number of rupees per kilogram. There are about 15 servings in a kilogram of these food items.[10] As reported in the home page table, for example, we observed prices for poultry of 45 – 50 rupees per kilogram. Hence, the size of the risk in the lotteries in Calcutta $30/-20$ (50 rupees) was equivalent to 15 servings of poultry. The price of a moderate quality restaurant meal was 15 – 35 rupees per person. Hence the 50 rupee risk in the experiment lotteries was the equivalent of about 1.5 – 3 moderate quality restaurant meals. The observed prices for local bus tickets were 3 – 4.5 rupees per ticket. This implies that the 50 rupee risk in the experiment lotteries was the equivalent of about 14 bus tickets. Again, the amount at risk in the Calcutta $90/-50$ experiment was about three times as large.

7.4. Contingent Euro Payoffs in Magdeburg

The Magdeburg $110/-100$ experiment included amounts x that were as large as 110K euros. We could credibly offer to pay such large amounts in *contingent* euros by using the following protocol. The experiment included two parts. In part 1 subjects made their choices between the sure amounts x and the lotteries in the MAX-Lab at the University of Magdeburg. They were told that whether their payoffs would be hypothetical or real depended on a condition which would be described later in part 2. After making their decisions the subjects were informed that real payoffs were conditional on winning gambles at the Magdeburg Casino. The payoff contingency was implemented in the following way. For each participant the experimenter placed €90 on the number 19 on one of the (four American) roulette wheels at the Magdeburg Casino. The probability that this bet wins is 1/38. If the bet wins, it pays 35 to 1. If the first bet won, then the experimenter would bet all of the winnings on the number 23. If both the first and second bet won, then the payoff would be €(35 × 35 × 90) = €110,250, which would provide enough money to make it feasible to pay any of the amounts involved in the part 1 decision tasks for that subject. The real payoff contingency was made credible to the subjects by randomly selecting three of them to accompany the

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[10] *There are 2.205 pounds per kilogram and 16 ounces in a pound, hence there are 35.28 ounces per kilogram. The U.S. Department of Agriculture's food pyramid guide defines a "serving" of meat, poultry, or fish as consisting of 2 – 3 ounces.*

experimenter to the casino and subsequently report to the others whether the experimenter had correctly placed the bets and recorded the outcomes.

7.5. Implications of the Data for Expected Utility Theory, Rank Dependent Utility Theory, and Original Cumulative Prospect Theory

The "if" statement in P(2*) in Proposition 2 involves weak preference for option B over option A. Therefore, in all tests we aggregate choices of option B with choices of option I (indifference) and denote the aggregated choice category as B^I. We report tests for incidence in the data of patterns of choices that, according to Proposition 2 and Corollary 2, imply implausible risk aversion in the large with expected utility theory and, for two of the experiments, with cumulative prospect theory and rank dependent utility theory.

We use error rate models to draw statistical conclusions from these data. Recall that this type of analysis takes into account that a subject with real preferences for option B^I rather than option A in all six rows could nevertheless be observed to have chosen B^I in five (or fewer) out of six rows. That is, the model assumes that a subject with real underlying preferences such as [B^I, B^I, B^I, B^I, B^I, B^I] could, instead, choose a different pattern, say [B^I, B^I, B^I, A, B^I, B^I], an event with probability $(1-\varepsilon)^5\varepsilon$, where ε is an error rate.

Models I, II, and III considered here are as follows. Model I includes only choices of all B^I (corresponding to M = 6,000 in Proposition 2 for the Calcutta 30/-20 experiment for example) as a calibration pattern and its mirror, all A's as the other pattern. Let the small stakes lotteries be {x+30, 0.5; x-20} for x from 100 to 6,000. According to Proposition 2, the choice pattern "all B^I" implies that 1,000 for sure is preferred to the lottery that pays $0.39\text{x}10^{23}$ or 0 with equal probabilities, which is implausible risk aversion. Model II (which corresponds to Proposition 2 with M = 5,000 for the Calcutta 30/-20 experiment) contains the Model I pair of (calibration and other) patterns, and one additional calibration pattern with A as the last entry (for x = 6,000) and its mirror image as an additional "other pattern." According to Proposition 2, the calibration patterns in Model II imply that getting 1,000 for sure is preferred to the 50/50 lottery that pays 0.12×10^{20} or 0, which is implausible risk aversion. Finally, Model III (which corresponds to Proposition 2 with M = 4,000 for the Calcutta 30/-20 experiment) contains patterns with four sequential B^I in the first four positions (for x = 100, 1000, 2000, and 4000) as calibration patterns and their mirror images as other patterns. With these calibration patterns, Proposition 2 implies that getting 1,000 for sure is preferred to the lottery that pays 0.36×10^{16} or 0 with equal probabilities, which is implausible risk aversion.

The top row in Table 6 shows estimated proportions of subjects whose choices satisfy the calibration patterns with the 1 error, 2 error, and 3 error rate versions of Models I, II, and III using data from Calcutta 90/-50. For this experiment, the estimated proportions for the 1 error rate version of Model I (M = 5,000) is 0.82, with Wald 90 percent confidence interval (0.70, 0.94). The estimated proportions for all models vary between 0.80 and 0.82; all are significant at one percent (indicated by **). We conclude that expected utility theory, rank dependent utility theory, and cumulative prospect theory imply implausible large stakes risk aversion for 80 to 82 percent of the subjects in the Calcutta 90/-50 experiment. The entries in bold font indicate the model that is selected by likelihood ratio tests.

The second row of Table 6 reports estimates for data from Calcutta 30/-20. The estimated proportions vary between 0.36 and 0.48, all significant at one percent. The estimations for Calcutta 30/-20 imply that 36 to 48 percent of the subjects in this experiment have implausible large stakes risk aversion with expected utility theory, rank dependent utility theory, and original cumulative prospect theory. Estimates in the third row for data from Magdeburg 110/-100 vary between 0.50 and 0.56; all are significant at one percent. The estimated percentage of 50 to 56 percent of subjects with implausible risk aversion for data from Magdeburg 110/-100 does not apply to rank dependent utility theory and cumulative prospect theory with a probability transformation function such that the transformed probability $h(0.5) < 0.476$ (for example, $h(0.5) = 0.42$, as in Kahneman and Tversky, 1992) since for such values the assumption $h(p)g - [1 - h(p)]\ell > 0$ of Corollary 2 is not satisfied by the lottery $\{x + 110, 0.5; x - 100\}$.

7.6. Implications of the Data for Dual Theory and Prospect Theory with Reference Point Editing

As explained in section 4.3, dual theory of expected utility implies that an agent's preference for option A or option B is the same for all lottery pairs in an experiment. Therefore, dual theory implies that an agent will reject the risky lottery $\{x + g, 0.5; x - \ell\}$ for all x if and only if he does so for one x.

In their development of cumulative prospect theory, Kahneman and Tversky (1992) dropped some of the elements of the original ("non-cumulative") version of prospect theory (Kahneman and Tversky, 1979). One element of the original version of prospect theory, known as "editing," can be described as follows. In comparing two prospects, an individual is said to look for common amounts in the payoffs, to disregard (or "edit") those common amounts, and then compare the remaining distinct payoff terms in order to construct a preference ordering of the prospects. Recent development and applications of "third generation" cumulative prospect theory (Kőszegi and Rabin, 2006, 2007; Schmidt, Starmer, and Sugden, 2008) have reintroduced editing in the form of reference point payoffs that differ from the zero-payoff reference point used by Tversky and Kahneman (1992). Non-zero reference points have implications for application of cumulative prospect theory to our experiments. For example, the concavity calibration in Corollary 2 is based on the supposition that an agent prefers the certain amount x to the lottery $\{x+g, p; x-\ell\}$ for all $x \in [m, M]$. But x is a common amount in the certain payoff and both possible payoffs in the lottery. If this common (or "reference point") amount x is edited, that is eliminated from all payoffs, then all comparisons are between the certain amount 0 and the single lottery $\{g, p; -\ell\}$ and there remains no interval $[m, M]$ over which to calibrate. In this way, editing of reference point payoffs can immunize prospect theory to critique by calibration of payoff transformation functions (Wakker, 2005).[11] But such editing does *not* immunize prospect theory from being tested with data from the experiment because it implies that a subject will reject the risky lottery $\{x+g, 0.5; x-\ell\}$ for one positive value of x if and only if he does so for all positive values of x. Therefore, prospect theory with editing has the same testable implication as dual theory with data from these experiments: a subject will consistently choose either the sure thing or the lottery.

At the aggregate level, data show clear differences in percentages of observed choices of the sure payoff across decisions. For example, the percentage of B^I choices is 78% when x = 4,000 but only 51% when x = 5,000 in the Calcutta 90/-50 experiment. In Calcutta 30/-20 and Magdeburg 110/-100 the percentages vary, respectively, over the ranges 25 – 53% and 43 – 55%. These variations appear inconsistent with the "no-switching" hypothesis. Probit panel regressions can be used to test the "no switching" prediction at the individual level. The prediction is that the estimated coefficient for "lottery pair" should not be significant. Probit panel regressions with individual-subject random effects yield parameter estimates for the decision pair variable that are significantly different from 0 at 5.2 percent significance level with data from two of the experiments but not with data from Calcutta 90/-50.

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[11] *Editing of reference point payoffs does not immunize prospect theory to critique by calibration of probability transformation functions, as in Corollary 1.*

7.7. Implications of the Data for Expected Value Theory

Every choice faced by a subject is between a sure payoff x and a lottery with expected value larger than x. Therefore, expected value theory predicts that the sure payoffs (option A) will never be chosen. The one pattern ("all A") model is rejected in favor of other models in Table 6 at 1 percent significance by likelihood ratio tests for data from all three experiments. The probit regression tests for dual theory and cumulative prospect theory with variable reference point also imply rejection of the testable implication of expected value theory because the coefficients on the lottery pair variable are significantly different from zero.

8. IS THERE A PLAUSIBLE DECISION THEORY FOR RISKY ENVIRONMENTS?

Prominent theories of decision making under risk model individuals' preferences over lotteries with nonlinear transformation of money payoffs and/or nonlinear transformation of probabilities. Previous calibration literature has focused on the possibly-implausible implications of modeling risk aversion with nonlinear transformation of money payoffs. This paper provides a dual critique that focuses on implications of nonlinear transformation of probabilities as well as nonlinear transformation of payoffs. The dual critique makes clear why plausibility problems with theories of decision under risk are fundamental. The dual critique produces a paradoxical insight into theories of risk aversion in that patterns of small stakes risk aversion that conform to the independence axiom (respectively, dual independence axiom) imply implausible large stakes risk aversion for the dual theory of expected utility (respectively, expected utility theory). Furthermore, theories that incorporate nonlinear transformations of both money payoffs and probabilities are shown by the dual corollaries to the propositions to be vulnerable to calibration problems coming from both of their nonlinear transformations.

Previous literature has offered no data supporting empirical relevance of supposed patterns of risk aversion that have calibration implications. This paper provides data from seven experiments. Further empirical testing may be needed. But the data now available provide support for empirical validity of risk aversion patterns underlying the calibrations. Accordingly, we conclude that the answer to the question about whether there exists a plausible theory for decision under risk may be "no."

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Table 1. Calibrations for Probability Transformations

$$100 \succ \{G, 0.5; 0\}$$

Rejection Intervals	First DU Calibration (c = 3.5)	PT & RD Calibration with υ(y) = $y^{0.88}$ (c = 3.5)	Second DU Calibration (c = 4.0)	PT & RD Calibration with υ(y) = $y^{0.88}$ (c = 4.0)	Third DU Calibration (c = 5.0)	PT & RD Calibration with υ(y) = $y^{0.88}$ (c = 5.0)
n	***G***	***G***	***G***	***G***	***G***	***G***
5	9,800	5,400	24,400	7,800	102,500	64,600
10	953,000	281,000	0.59×10^{7}	0.6×10^{6}	0.10×10^{9}	0.41×10^{8}
50	0.78×10^{22}	0.17×10^{20}	0.71×10^{26}	0.78×10^{21}	0.12×10^{33}	0.12×10^{31}
100	0.62×10^{42}	0.30×10^{37}	0.51×10^{50}	0.60×10^{40}	0.16×10^{63}	0.15×10^{59}
200	0.38×10^{82}	0.95×10^{71}	0.26×10^{98}	0.37×10^{78}	0.25×10^{123}	0.23×10^{115}
500	0.93×10^{201}	0.28×10^{175}	0.36×10^{241}	0.84×10^{191}	0.10×10^{304}	0.82×10^{283}

c= 3.5 Atlanta 14/4, c=4, Magdeburg 40/10, Atlanta 40/10; c=5, Calcutta 400/80

Table 2. Calibrations for Payoff Transformations

$$3,000 \succ \{G, 0.5; 1000\}$$

Rejection Intervals [1000, *M*]	First EU Calibration (*g*=110, ℓ=100)	Second EU Calibration (*g*=90, ℓ=50)	First PT & RD Calibration (*g*=90, ℓ=50)	Third EU Calibration (*g*=30, ℓ=20)	Second PT & RD Calibration (*g*=30, ℓ=20)
M	*G*	*G*	*G*	*G*	*G*
5,000	8,000	0.24×10^{10}	0.10×10^{7}	0.12×10^{17}	564,000
6,000	9,900	0.15×10^{12}	0.64×10^{7}	0.40×10^{20}	0.29×10^{7}
8,000	15,000	0.10×10^{16}	0.34×10^{9}	0.44×10^{27}	0.79×10^{8}
10,000	24,000	0.38×10^{19}	0.14×10^{11}	0.49×10^{34}	0.21×10^{10}
30,000	0.11×10^{9}	0.12×10^{56}	0.40×10^{27}	0.13×10^{105}	0.50×10^{24}
50,000	0.10×10^{13}	0.38×10^{92}	0.11×10^{44}	0.37×10^{175}	0.11×10^{39}

Table 3. Choice Alternatives in Probability Transformation Experiment Magdeburg 40/10

Row	Option A		Option B		
	40 Euros	0 Euro	40 Euros	10 Euros	0 Euro
1	1/10	9/10	0/10	2/10	8/10
2	2/10	8/10	1/10	2/10	7/10
3	3/10	7/10	2/10	2/10	6/10
4	4/10	6/10	3/10	2/10	5/10
5	5/10	5/10	4/10	2/10	4/10
6	6/10	4/10	5/10	2/10	3/10
7	7/10	3/10	6/10	2/10	2/10
8	8/10	2/10	7/10	2/10	1/10
9	9/10	1/10	8/10	2/10	0/10

Table 4. Error Rate Models and Predictions for Probability Calibration Models

Experiment	Nr. Of Subjects	Model I			Model II			Model III		
		1 error	2 errors	3errors	1 error	2 errors	3 errors	1 error	2 errors	3 errors
Calcutta 400/80	40	0.72** (.58,.86)	0.74** (.60,.88)	**0.74** (.60,.87)**	0.72** (.58,.86)	0.73** (.58,.88)	0.74** (.59,.88)	0.73** (.59,.86)	0.72** (.57,.87)	0.73** (.58,.87)
		1000 ≻ {1 million, 0.5;0}			1000 ≻ {256000, 0.5;0}			1000 ≻ {64000, 0.5;0}		
Atlanta 40/10	22	0.56** (.37,.75)	0.62** (.42,.82)	0.62** (.34,.90)	0.59** (.39,.78)	**0.63** (.42,.83)**	0.63** (.41,.85)	0.60** (.39,.80)	0.61** (.38,.83)	0.60** (.35,.85)
		1000 ≻ {244000, 0.5;0}			1000 ≻ {81000, 0.5;0}			1000 ≻ {27000, 0.5;0}		
Magdeburg 40/10	31	0.38** (.20,.56)	**0.40** (.22,.59)**	0.41** (.22,.61)	0.37** (.19,.55)	0.39** (.21,.57)	0.40** (.21,.60)	0.40** (.21,.58)	0.40** (.22,.57)	0.41** (.22, .60)
		1000 ≻ {244000, 0.5;0}			1000 ≻ {81000, 0.5;0}			1000 ≻ {27000, 0.5;0}		
Atlanta 14/4	39	0.74** (.55,.93)	**0.90** (.81,.98)**	0.90** (.81,.99)	0.82** (.68,.96)	0.90** (.71,1.0)	0.90** (.77,1.0)	0.88** (.77,.99)	0.90** (.80,.99)	0.90** (.80,.99)
		1000 ≻ {98000, 0.5;0}			1000 ≻ {39000, 0.5;0}			1000 ≻ {15700, 0.5;0}		

Table 5. Choice Alternatives in Payoff
Calibration Experiment Calcutta 30/-20

Row	Option A	Option B
1	80 or 130	100
2	980 or 1,030	1,000
3	1,980 or 2,030	2,000
4	3,980 or 4,030	4,000
5	4,980 or 5,030	5,000
6	5,980 or 6,030	6,000

Table 6. Error Rate Models for Payoff Transformation Experiments

Experiment	**Nr. of subjects**	**Model I**			**Model II**			**Model III**		
		1 error	2 errors	3errors	1 error	2 errors	3 errors	1 error	2 errors	3 errors
Calcutta 90/-50 m = 50	40	**0.82** (.70,.94)**	0.81** (.69,.93)	0.81** .68,.94	0.81** (.69,.93)	0.80** (.67,.93)	0.80** (.66,.94)	0.82** (.69,.94)	0.80** (.68,.93)	0.81** (.67,.95)
		M=5,000: 1000 ≻ {0.13x10^{12}, 0.5; 0}			M=4000: 1,000 ≻ {0.69x10^{9}, 0.5; 0}			M=3,000: 1,000 ≻ {0.63x10^{7}, 0.5; 0}		
Calcutta 30/-20 m = 100	30	0.36* (.14,.59)	0.43** (.25,.62)	0.44** (.25,.64)	0.48** (.20,0.53)	0.43** (.17,.68)	0.46** (.27,.65)	0.48** (.30,.67)	0.37** (.20,.53)	**0.47** (.26,.68)**
		M=6,000: 1,000 ≻ {0.39x10^{23}, 0.5; 0}			M=5,000: 1,000 ≻ {0.12x10^{20}, 0.5; 0}			M=4,000: 1,000 ≻ {0.36 x10^{16}, 0.5; 0}		
Magdeburg 110/-100 m = 3000	42	0.54** (.39,.68)	0.55** (.41,.68)	**0.54** (.40,.68)**	0.54** (.39,.68)	0.56** (.43,.69)	0.50** (.41,.68)	0.50** (.32,.68)	0.52** (.38,.66)	0.50** (.36,.64)
		M = 110,000: 5,000 ≻ {0.26x10^{24}, 0.5;3,000}			M= 90,000: 5,000 ≻ {0.31x10^{20}, 0.5; 3,000}			M= 70,000: 5,000 ≻ {0.36x10^{16}, 0.5;3,000}		

APPENDIX

A.1. Concavity Calibration Pattern and the Dual Independence Axiom

Let Γ denote the set of all decumulative distribution functions. Let $\oplus$ denote the following operator: $\lambda G \oplus (1-\lambda)H = (\lambda G^{-1} + (1-\lambda)H^{-1})^{-1}$, $\forall G, H \in \Gamma$. The dual independence axiom as stated in Yaari 1987 (p. 99) is:

Axiom DI: If G, G', and H belong to Γ and α is a real number satisfying $0 \le \alpha \le 1$, then $G \succeq G'$ implies $\alpha G \oplus (1-\alpha)H \succeq \alpha G' \oplus (1-\alpha)H$.

Suppose that a dual expected utility agent rejects binary lottery $\{x+g, 0.5; x-\ell\}$ in favor of receiving x for sure for some $x > 0$. Then by axiom DI and continuity the agent rejects $\{y+g, 0.5; y-\ell\}$ in favor of getting y for sure for all positive y.

Let F_x denote the decumulative distribution function for the binary lottery and D_x the decumulative distribution for the degenerate lottery that pays x for sure. Then the agent's preference for the sure amount x over the binary lottery $\{x+g, 0.5; x-\ell\}$ is formally written as (∗) $D_x \succeq F_x$.

Let y be given. Without any loss of generality assume that $y > x$. Then take a sequence of $\alpha_n \in (0,1)$, $n \in N$ such that $\alpha_n \underset{n\to\infty}{\to} 1$. For each $\alpha_n, n \in N$ there exists a $z_n \ge y$ such that $y = \alpha_n x + (1-\alpha_n) z_n$. Let D_{z_α} denote the decumulative distribution for the degenerate lottery that pays z_n for sure. Statement (∗) and Axiom DI imply $\alpha D_x \oplus (1-\alpha) D_{z_\alpha} \succeq \alpha F_x \oplus (1-\alpha) D_{z_\alpha}$. Note that by definition of operator $\oplus$ and the construction of z_n, the expression on the left hand side is the degenerate lottery that pays y for sure whereas the one on the right is the binary lottery $\{y+\alpha_n g, 0.5; y-\alpha_n \ell\}$. So, the agent prefers getting y for sure to a 50/50 lottery with payoffs $y+\alpha_n g$ or $y-\alpha_n \ell$, for all α_n . By continuity our agent (weakly) prefers getting y for sure to the binary lottery $\{y+g, 0.5; y-\ell\}$.

A.2. Proof of Proposition 1 and Corollary 1

General result 1. Let a decision theory D represent preferences over finite discrete lotteries L with "utility functional"

(a.i) $$U(L) = \sum_{j=1}^{n} [f(\sum_{k=j}^{n} p_k) - f(\sum_{k=j+1}^{n} p_k)] v(y_j)$$

where $f(.)$ is the transformation of decumulative probabilities whereas $v(.)$ is the money transformation function. Suppose that

(a.ii) $\{cx,(i-1)/2n;x,1/n;0\} \succsim \{cx,i/2n;0\}$, for all $i=1,2,\cdots,2n-1$, and

(a.iii) $v(cx)/v(x)>2$

Using notation $C \equiv v(cx)/v(x)$ we show that getting z for sure is preferred to getting $v^{-1}(v(z)K(C,n))$ or zero with even odds, for $K(.,.)$ as defined in section 3.1.

Proof. To simplify notation, let $\delta = 1/2n$. First note that, according to theory D, if $\{cx,(i-1)\delta;x,2\delta;0\} \succsim \{cx,i\delta;0\}$, for all $i=1,2,\cdots,2n-1$ then

(a.1) $$v(x)f((1+i)\delta)+\left[v(cx)-v(x)\right]f((i-1)\delta) \geq v(cx)f(i\delta),\ i=1,\ldots,2n-1$$

which is equivalent to

(a.2) $$f((1+i)\delta)-f(i\delta) \geq (C-1)\left[f(i\delta)-f((i-1)\delta)\right],\ i=1,\ldots,2n-1$$

Writing inequality (a.2) for $i+k(=1,\ldots,2n)$ and applying it $k-1$ other times one has

$$\begin{aligned} f((i+k)\delta)-f((i+k-1)\delta) &\geq (C-1)\left[f((i+k-1)\delta)-f((i+k-2)\delta)\right] \geq \ldots \\ &\geq (C-1)^k\left[f(i\delta)-f((i-1)\delta)\right] \end{aligned}$$

which generalizes as

(a.3) $$f(j\delta)-f((j-1)\delta) \geq (C-1)^{j-i}\left[f(i\delta)-f((i-1)\delta)\right],\ j=i,\ldots,2n$$

Next, if we show that

(a.4) $$f(0.5) \leq \left[f(0.5)-f(0.5-\delta)\right]\sum_{i=1}^{n}\left(\frac{1}{C-1}\right)^{i-1} \text{ and}$$

(a.5) $$1-f(0.5) \geq \left[f(0.5)-f(0.5-\delta)\right]\sum_{j=1}^{n}(C-1)^j$$

then we are done since inequalities (a.4) and (a.5) imply

$$\frac{1-f(0.5)}{\sum_{j=1}^{n}(C-1)^j} \geq f(0.5)-f(0.5-\delta) \geq \frac{f(0.5)}{\sum_{i=1}^{n}(C-1)^{1-i}},$$

and therefore $1 \geq f(0.5)\left[1+\sum_{j=1}^{n}(C-1)^j \Big/ \sum_{i=1}^{n}(C-1)^{1-i}\right]$. For any given z multiply both sides with $v(z)$ and note that the last inequality implies that $v(z) \geq f(0.5)v(z)K(C,n)$. That is, getting z for sure is preferred to getting $v^{-1}(v(z)K(C,n))$ or zero with even odds.

To show inequality (a.4) recall that 0.5 = $n\delta$, and write f(0.5) as below,

$$f(0.5)=\sum_{i=1}^{n}\left[f(i\delta)-f((i-1)\delta)\right]\leq\left[f(n\delta)-f((n-1)\delta)\right]\sum_{i=1}^{n}\left(\frac{1}{C-1}\right)^{i-1}$$
$$=\left[f(0.5)-f(0.5-\delta)\right]\sum_{i=1}^{n}\left(\frac{1}{C-1}\right)^{i-1}$$

where the inequality follows from inequality (a.3). Similarly, inequality (a.5) follows from

$$1-f(0.5)=\sum_{j=n+1}^{2n}\left[f(j\delta)-f((j-1)\delta)\right]\geq\left[f((n+1)\delta)-f(n\delta)\right]\sum_{j=1}^{n}(C-1)^{j-1}$$
$$\geq\left[f(0.5)-f(0.5-\delta)\right]\sum_{j=1}^{n}(C-1)^{j}$$

Proof of Proposition 1 (dual theory of expected utility).

In dual expected utility theory $v(z)=z$. If c > 2 then $v(cz)/v(z)=c>2$ and therefore the general result 1 applies for this particular $v(z)=z$; hence z for sure is preferred to getting $zK(c,n)$ or zero with even odds.

Proof of Corollary 1 (cumulative prospect theory and rank dependent utility theory).

It is a straightforward application of the general result 1 for $v(z)=\upsilon(z)$.

A.3. Proof of Proposition 2 and Corollary 2

General result 2. Let a decision theory D with "utility functional" U in statement (a.i) be given. Let $a=\ell$ and $b=g+\ell$. We assume here that v is (weakly) concave and differentiable (the proof extends straightforwardly to non-differentiable weakly concave functions; see also Rabin, 2000.). Suppose that

(a.iv) $x+a\succsim\{x+b,p;x\}$ for all integers $x\in(m,M)$, $m>0$, and

(a.v) $bf(p)>a$.

We show that for all $z\in\left(m+b+b\ln(1-f(p))/\ln q,M\right)$, $z\succ\{G,p;m\}$ for all G that satisfy inequality (*) in Proposition 2 with $q=r(f(p))$.

Proof. Let N be the largest integer smaller than (M-m)/b. Condition (a.iv) and the definition of N imply

(a.6) $$v(x+a)\geq(1-f(p))v(x)+f(p)v(x+b),\text{ for all integers } x\in(m,m+Nb).$$

First we show that (a.7) and concavity of v imply that for all $y \in (m, m+Nb)$

(a.7) $\quad v'(y+jb) \le q^j v'(y)$, for all $j \in \Psi_y$,

where $\Psi_y = \{ j \in \mathbb{N} \mid y+(j-1)b \in (m, m+Nb)\}$ and $q = (1/f(p)-1)/(b/a-1)$

Next, for any given z (as stated in Proposition 2) we show that

(a.8) $\quad v(m+Kb) \ge f(p)v(m+(K+J)b)+(1-f(p))v(m).$

where K is the largest integer smaller than $(z-m)/b$, and J be the smallest integer larger than $(\overline{G}-m)/b-K$ where $\overline{G}$ is the expression on the right hand side of inequality (*) in the statement of Proposition 2.

This completes the proof since all G that satisfy inequality (*) also satisfy $G < m+(K+J)b$, which together with (a.8) and the definition of K imply $v(z) > f(p)v(G)+(1-f(p))v(m)$.

To derive (a.7), first write $v(x+a) = f(p)v(x+a)+(1-f(p))v(x+a)$, next rewrite (a.6) with $x = y$, and finally group together terms with factors $f(p)$ and $1-f(p)$ on opposite sides of the inequality (a.6) to get

(a.9) $\quad (1-f(p))\left[v(y+a)-v(y)\right] \ge f(p)\left[v(y+b)-v(y+a)\right],\ \forall y \in (m, m+Nb)$.

Inequalities $\left[v(y+b)-v(y+a)\right]/(b-a) \ge v'(y+b)$ and $\left[v(y+a)-v(y)\right]/a \le v'(y)$, (both following from the weak concavity of v), inequality (a.9) and notation q imply

(a.10) $$v'(y+b) \le \left(\frac{1-f(p)}{f(p)}\frac{a}{b-a}\right)v'(y) = qv'(y),\ \forall y \in (m, m+Nb).$$

Iteration of inequality (a.10) j times, for $j \in \Psi_y$, gives inequalities that together imply statement (a.7):

$$v'(y+jb) \le qv'(y+(j-1)b) \le \ldots \le q^j v'(y).$$

To show statement (a.8), let y denote $m+Kb$ and note that if $J+K > N$ then

(a.11)
$$\begin{aligned} v(y+Jb)-v(y) &= \sum_{j=0}^{J-1}\left[v(y+(j+1)b)-v(y+jb)\right] \\ &\le b\left[(J-N+K)v'\big(y+(N-K)b\big)+\sum_{j=0}^{N-K-1} v'(y+jb)\right] \\ &\le bv'(y)\left[q^{N-K}(J-N+K)+\sum_{j=0}^{N-K-1} q^j\right] \end{aligned}$$

(In (a.11) the first inequality follows from (weak) concavity of φ and $J+K > N$ whereas the second one follows from statement (a.7).) If however $J+K \le N$ then one has (a.11′) $$v(z+Jb)-v(y)\le b\sum_{j=0}^{J-1}v'(y+jb)\le bv'(y)\frac{1-q^J}{1-q}$$

Similarly, one can show that

(a.12) $$v(y)-v(y-bK)\ge bv'(y)\sum_{k=0}^{K-1}\frac{1}{q^k}$$

Hence, in case of $J+K>N$, (a.11) and (a.12) imply that a sufficient condition for (a.8) is

(a.13) $$(1-f(p))\sum_{k=0}^{K-1}\frac{1}{q^k}\ge f(p)\left[q^{N-K}(J-N+K)+\sum_{j=0}^{N-K-1}q^j\right]$$

Substitute $\sum_{j=0}^{N-K-1}q^j=\frac{1-q^{N-K}}{1-q}$, and $\sum_{k=0}^{K-1}\frac{1}{q^k}=\frac{q^{1-K}-q}{1-q}$ in (a.13) to get

(a.14) $$J\le N-K+\frac{1}{q^{N-K}}\left(\frac{1-f(p)}{f(p)}\frac{q^{-K}-1}{1-q}q-\frac{1-q^{N-K}}{1-q}\right)=N-K+\frac{1}{1-q}+\frac{A}{b}q^{-N}$$

The last inequality is true since

$$\begin{aligned}J&\le(\overline{G}-m)/b-K+1=\left(M+b(2q-1)/(1-q)+Aq^{-N}-m\right)/b-K+1\\&\le\left(m+bN+bq/(1-q)+Aq^{-N}-m\right)/b-K+1\\&=N-K+1/(1-q)+q^{-N}A/b\end{aligned}$$

Finally, if $J+K\le N$, (a.11’) and (a.12) imply that a sufficient condition for (a.8) is

(a.15) $$\left(q^{-K}-1\right)q>f(p)/(1-f(p)) \; .$$

Note that definition of K and $z\in\left(m+b+b\ln(1-f(p))/\ln q, M\right)$ imply $q^{K-1}<1-f(p)$, hence (a.15) is satisfied.

Proof of Proposition 2 (expected utility theory).

It is a straightforward application of the general result 2 for $f(p)=p$ and $v(z)=u(z)$.

Corollary 2 (cumulative prospect theory and rank dependent utility theory).

It is a straightforward application of the general result 2 for $f(p)=h(p)$ and $v(z)=\upsilon(z)$