Optimal Monetary and Fiscal Policies in a Search Theoretic Model of Monetary Exchange*

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Abstract

Search models of monetary exchange commonly assume that terms of trade in decentralized markets are determined via Nash bargaining. Bargaining frictions add to the classical intertemporal distortion present in most monetary models, whereby agents work today to obtain cash that can be used only in future transactions. More important, bargaining frictions may cause underproduction in decentralized markets and monetary equilibrium allocations to be inefficient. In this paper, we study the properties of optimal fiscal and monetary policy within the framework of Lagos and Wright (2005). We abstract from revenue-raising motives to focus on the welfare-enhancing properties of optimal policy. We show that subsidies in decentralized markets can be implemented to alleviate underproduction, while money is still essential. Deviations from the Friedman rule may be large, and the existence of fiscal and monetary policies results in considerable welfare gains. When lump sum monetary transfers are not available, a positive production subsidy may be inflationary and welfare reducing. However, sales taxes in the decentralized market and production taxes in the centralized market may increase welfare. The optimality of the Friedman rule in this case depends crucially on the bargaining power of the buyer, and equilibria are not first best.

JEL Codes: C70, E40.

Keywords: money, bargaining, search, inflation, fiscal policy

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*We would like to thank Narayana Kocherlakota, Ricardo Lagos, Neil Wallace, Chris Waller, Randy Wright, Steve Williamson as well as the participants of the Midwest Macro meetings held at the Federal Reserve Bank of Cleveland, the 2007 Annual SED meeting held in Prague, and the 2007 Money, Banking, Payments and Finance conference organized by the Federal Reserve Bank of Cleveland for their helpful comments.

†The views expressed are those of the individual authors and do not necessarily reflect official positions of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or the Board of Governors.

1 Introduction

A fundamental intertemporal friction present in most traditional monetary models is that monetary exchanges entail agents paying a cost today (production) to receive a future benefit: money that can be used to purchase goods in future trades. The properties of optimal fiscal and monetary policies in this class of models are well understood and usually involve deflating at the rate of time preference to counteract the aforementioned intertemporal friction. A new literature based on search and bargaining models, has emerged since the work of Kiyotaki and Wright (1991, 1993). An essential objective of this new literature is formalizing the role of money as a medium of exchange, thus providing microeconomic foundations for monetary economics. In this class of models the fundamental intertemporal friction of traditional monetary models is also present. However, it seems desirable to establish whether and if so, exactly how search and bargaining frictions influence the properties of optimal fiscal and monetary policy. The latter question motivates our work.

In particular, this paper studies the properties of optimal fiscal and monetary policies and the welfare costs of inflation in a version of the widely used search model of Lagos and Wright (2005). We focus on the possibility of improving the welfare properties of monetary equilibrium allocations. Hence, our study of optimal policy abstracts from all revenue-raising motives.

The key features of our economic framework are quasilinear preferences and the possibility of trade in both decentralized and centralized markets. These features keep the model analytically tractable and easy to quantify. It is also known that when lump sum monetary transfers are the only instrument available and if the buyer does not have all the bargaining power monetary equilibria are not efficient. Moreover, the welfare costs of inflation are substantially larger than those found in models in which money is introduced with ad hoc assumptions and the Friedman rule is the unique optimal policy.

Our analysis considers alternative fiscal instruments to study the following issues. First, can fiscal and monetary policy restore efficiency of equilibria when the buyer does not have all the bargaining power? Second, what is the magnitude of the welfare costs of inflation once fiscal policy is also available? Finally, is the Friedman rule an optimal policy when negative lump sum transfers are not available?

Two frictions hinder the efficiency of equilibrium in the Lagos and Wright (2005) framework. First, in monetary exchanges agents pay a cost today (production) to receive a future benefit (money that can be used to purchase goods in future trades).[1] The second friction is a direct consequence of the properties of Nash's solution to the bargaining problem.[2] In particular, unless the buyer has all the bargaining power, Nash's solution implies that the buyer's surplus from a given match is not monotone in monetary holdings. Thus, buyers hold too little cash

[1]Note that this feature is also present in applied models with *ad hoc* assumptions regarding fiat money.
[2]For more details see Aruoba, Rocheteau, and Waller (2007).

and underproduction exists in the decentralized market. When lump sum monetary transfers are the only instrument and the buyer does not have all the bargaining power, the Friedman rule eliminates the first friction and attenuates the impact of the second. However, equilibrium allocations are not efficient. In this paper, we propose different fiscal and monetary policies that can restore efficiency of monetary equilibrium.

Because two potential sources of friction are present in the economy, a complete tax system requires two instruments. More important, constructing appropriate fiscal policies in a micro-founded monetary model requires consideration of the anonymity of trading partners and the impossibility of record keeping. Thus, it is necessary to construct taxes on activities that occur in the decentralized market without violating information restrictions. Implementation of our fiscal and monetary policies requires that agents disclose their money holdings. Agents that increase their money holdings (producers) are given a monetary subsidy.[3] Incentives are well aligned for agents to truthfully report money holdings because a subsidy is received.

Production subsidies, paid in money, can be used to increase production in the decentralized market but they may be inflationary. If costless lump sum monetary transfers are available, these can be used to extract the money introduced through the subsidy and thus inflation can be easily contained. In this environment, we find multiple combinations of taxes, subsidies, and (sometimes strictly positive) inflation rates such that efficiency is attained. In addition, the Friedman rule is always an optimal policy regardless of the value of the bargaining power of the buyer. Moreover, since equilibrium under the optimal policy is efficient, the welfare costs of increasing inflation from the Friedman rule rate to 10% are up to 8% of lifetime consumption. The latter is 1.6% points higher than what is obtained without fiscal policy.

When lump sum monetary transfers are not available, the production subsidy is inflationary, which may magnify the distortions of the model. Hence, we consider a new set of fiscal instruments. We first consider a sales tax in the decentralized market because this allows the government to retire money from circulation, thus making the Friedman rule feasible. We also introduce a production tax in the centralized market, which alters the bargaining position of agents, and ultimately results in higher production in the decentralized market. In this type of environment, we find monetary equilibrium is never efficient. The Friedman rule is optimal only when the buyer has relatively low bargaining power. Finally, we find that the welfare gains of having fiscal and monetary policies in place are substantial.

The paper closest in spirit to ours is that of Aruoba and Chugh (2008), who study the dynamic Ramsey problem in the Lagos and Wright (2005) framework. However, these authors are interested in analyzing the business cycle frequency properties of optimal monetary and fiscal policy with positive government expenditures in a model that includes government bonds and

[3]Note that in order to implement these policies, the government does not need to know the identity of each buyer and seller, thus preserving the anonymity of trading partners and the necessity of having money as a medium of exchange.

capital assets. Fiscal and monetary instruments are restricted to (i) production and capital taxes in the centralized market and (ii) to open market operations. Equilibrium in their model is not efficient, the Friedman rule is typically not optimal and inflation is stable over time. These authors also find that because capital is under accumulated, the optimal policy includes a subsidy on capital income. In a different environment, in which the total number of trade matches is determined by a matching function and search intensities are optimally chosen by households, Ritter (2007) finds that an optimal policy may consist of both a positive tax rate and a positive nominal interest rate. Monetary, but not fiscal, policy alters the agent's bargaining position, leaving a special role for a deviation from the Friedman rule.

The findings of this paper and those of Aruoba and Chugh (2008) and Ritter (2007) confirm the observation by Kocherlakota (2005) and Wright (2005) that fiscal and monetary policies may have important interactions, particularly in frameworks with microeconomic foundations for the existence of fiat money, and should always be jointly considered in the design of optimal government policy.

The reminder of our paper is organized as follows. The model is presented in Section 2 and contains the main results of the paper and derives the properties of optimal fiscal and monetary policy under different government instruments. Each subsection includes a set of numerical experiments deriving the quantitative implications of the theory, the welfare benefits of optimal fiscal and monetary policy, and the welfare costs of inflation. Section 3 summarizes our finding and conclusions.

2 The model

2.1 The economic environment

The economy has a continuum of agents that live forever. The representative agent of this economy derives utility from consumption and disutility from labor. Each period is divided into two subperiods labeled day and night. Consumption and production take place during both, day and night. Preferences over streams of consumption and labor during the day, denoted by x and h, respectively, and during the night, denoted by X and H, are represented by

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(x_t) - h_t + U(X_t) - H_t], \tag{1}$$

where the utility functions $u(.)$ and $U(.)$ are twice continuously differentiable, increasing, and strictly concave. Finally, we assume that $u(0) = 0$ and that all consumption goods are non-storable. Money is also available to agents in this economy, it is perfectly divisible, and storable in any quantity $m \geq 0$.

The day good comes in many varieties, and each individual agent produces a good that she does not consume. To simplify our analysis we assume that double coincidence of wants is impossible. Since no record-keeping is possible in the decentralized market, the only feasible trade during the day is the exchange of goods for money. Money is essential in sense that the welfare level achieved when money is available is higher than it would be possible without money. In other words, in an environment with anonymous trade and no-record keeping without money, it would be impossible for trade to take place in the decentralized market.

The probability of a meeting in the decentralized market is α. Moreover, given two agents, i and j, the probability that agent i consumes what agent j produces (a single coincidence) is $\sigma \leq \frac{1}{2}$. Symmetrically, the probability that j consumes what i produces is equal to σ. The probability that neither wants what the other produces is $1 - 2\sigma$.

During the night subperiod agents trade a general good that everyone can produce and wants to consume in a centralized Walrasian market. The only feasible trade during the night involves general goods and money.

The economy we have described until this point is exactly the same as that in Lagos and Wright (2005), who study the properties of optimal monetary policy with a single instrument: costless lump sum monetary transfers. In this section, we evaluate the efficiency properties of monetary equilibrium, and the welfare costs of inflation, once a monetary production subsidy, financed by money printing, is available. Of course, the informational requirements for the implementation of such policy must be considered carefully. For money to be essential it should not be possible to monitor transactions in decentralized markets. In our model, agents that request a subsidy must disclose changes in their money holdings between the last period's central market and the start of the current period's central market. A monetary subsidy at constant rate $s \geq 0$ is provided to agents that have increased their real money holdings (when they were producers in the day market).[4] The government must print money to pay for this subsidy. Notice that when a production subsidy is in place agents have a clear incentive to truthfully reveal changes in monetary holdings. In particular, truthful revelation results in a positive money transfer.

Knowledge of changes in money holdings by the government may still be considered as requiring too much information. It is important to note that our results can be extended to a framework in which the only required information is current money holdings before entering the centralized market. However, analysis of such a model is much more convoluted and obscures our point. Hence, we have chosen to simplify the analysis and proceed with the version of the model with stronger informational requirements.[5] Notice also that because no record keeping is

[4] All the redistribution of resources takes place in the Walrasian market where we assume there is perfect information, all actions are observable and that the government can collect and redistribute taxes.

[5] The economic intuition works as follows: consider an equilibrium sequence of money holdings, $\{m_t^e\}$, from the model that requires knowledge of changes in monetary holdings. Let this sequence be given. Define a subsidy

possible in the decentralized market, it would be very difficult for a coalition of agents to reach an agreement where money holdings are pooled with the purpose of obtaining a subsidy. Agents may be able to sustain coalitions in the centralized market, where information is complete, but the government also will be able to recognize such coallitions and design the subsidy so that only agents that actually produced can receive it.

The monetary production subsidy we consider can be mapped in real-life economies to policies that prescribe paying interest to *some* money holders, in our case sellers. This type of policy prescription has a long tradition in monetary economics and has been advocated by Tolley (1957) and Friedman (1959), among others. Regarding the feasibility and implementability of this policy, Feinman (1993) notes that the Federal Reserve has explicitly supported legislation authorizing the payment of interest on reserves since the 1970s.[6] Our paper then reexamines an established monetary policy prescription in the new search models of money.

The trades, timing, and redistributions of resources considered in this model are summarized in Figure 1.

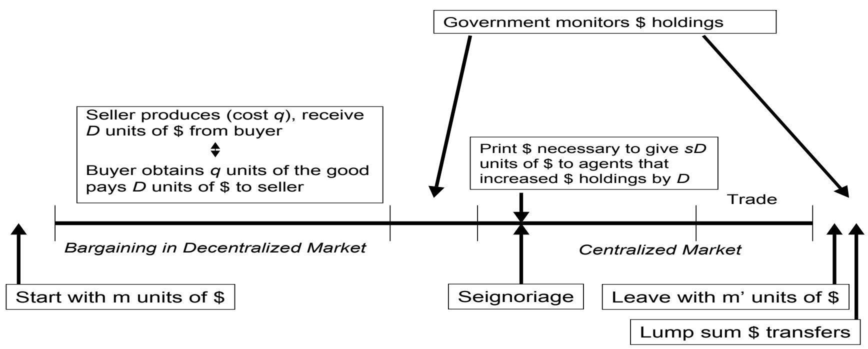

Figure 1: Timing.

The problem of the representative agent consists of maximizing expected utility while taking prices, subsidy rates, and the distribution of money holdings of other agents as given. During the decentralized market the agent knows that with probability α that she will trade, and that with probability σ she will be either a buyer or a seller. Notice that, to simplify our presentation, we abstract from double-coincidence meetings. Consumption (production), q, and money payments (receipts), D, in the decentralized market will be determined by bargaining. The representative agent knows the corresponding functional forms. We denote by m the money holdings of the rep-

rate function equal to zero when agents report money holdings lower than m_t^e, and equal to s if agents hold at least m_t^e. This function will not alter any of the important properties of equilibrium, in particular the fact that the distribution of money is degenerate. Furthermore, agents will find it optimal to choose money holdings equal to m_t^e. There are no incentives to deviate since reporting less money results in no subsidy. Conversely, the payoff of carrying more money is as good as it would be in the more restrictive model. The key features of stationary equilibria are the same as those of the more restricted model.

[6]We refer to Freeman and Haslag (1995) for more on this issue.

resentative agent, by $\tilde{m}$ the holdings of a partner in a given match, and by $F(\tilde{m})$ the distribution of money holdings. Then, the recursive formulation of the representative agent is characterized, first, by the value function associated to the day market

$$\begin{aligned} V_t(m) = \alpha\sigma \int & \{u\,(q(m,\tilde{m})) + W_t\,(m - D(m,\tilde{m}))\}\, dF_t(\tilde{m}) + \\ & + \alpha\sigma \int \{-q(\tilde{m},m) + W_t\,(m + (1+s)D(\tilde{m},m))\}\, dF_t(\tilde{m}) + \\ & + (1 - 2\alpha\sigma)W_t(m), \end{aligned} \tag{2}$$

which includes the value function $W(m)$ of trading in the centralized market (which is precisely defined below).

Terms of trade in the decentralized market are determined endogenously in a bargaining game. Following the literature, we consider the generalized Nash bargaining solution where the buyer has bargaining power $0<\theta\leq 1$, and threat points are given by no trade:

$$\begin{aligned} \max_{q,D} \{u(q) + W(m - D) - W\,(m)\}^{\theta} \{-q + W(\tilde{m} + (1+s)\,D0 - W\,(\tilde{m})\}^{1-\theta} \\ s.t.\ D \leq m. \end{aligned} \tag{3}$$

It is well understood, that the generalized Nash bargaining solution may not satisfy strong monotonicity. In a version of the model with no subsidies, Aruoba, Waller, and Rocheteau (2007) have shown that the lack of monotonicity of the buyer's surplus when $\theta<1$ causes monetary equilibrium to be inefficient. In particular, there will be underproduction in the decentralized market. As it is clear from equation (3), subsidies will affect the optimal D and q resulting from the Nash bargaining game. One of our main objectives is to understand how subsidies affect the buyer's surplus, and to determine whether the first best allocation can be reached.

The recursive formulation of the representative agent's problem is completed by the following definition of the value function associated with the centralized market:

$$\begin{aligned} W_t(m_t) = \max_{X,H,m_{t+1}} \{U(X) - H + \beta V_{t+1}(m_{t+1} + T)\} \\ s.t.\ X = H + \phi_t(m_t - m_{t+1}), \end{aligned} \tag{4}$$

where ϕ_t denotes the value of money balances at the centralized market. Finally, the monetary authority can provide lump sum monetary transfers, T, after trades have concluded in the centralized market.

To close the model, notice that the money supply is determined by the government. Money supply must always equal the money demand and thus

$$M_t = \int m dF_t(m) \forall t,$$

where M_t denotes the money supply. The government must print money in order to fund production subsidies. Recall that in the decentralized market only a fraction $\alpha\sigma$ of the population actually trades goods for money. Let τ_{m_1} be the growth rate in the money supply that results from paying subsidies. This yields the relation

$$\alpha\sigma s \int\int D(m, \tilde{m}) dF_t(m) dF(\tilde{m}) = \tau_{m_1,t} M_t. \tag{5}$$

Observe then that money subsidies have the potential of generating inflation. Let $\tau_{m_2,t}$ be the growth rate in the money supply by the end of the centralized market, when all lump sum monetary transfers have occurred. The total growth in the money supply from one period to the next is then $(1+\tau_{m_1})(1+\tau_{m_2})$. Since costless negative lump sum transfers are available, $\tau_{m_2} < 0$, it is possible to undo the aforementioned inflationary pressures.

Finally, we impose the market-clearing condition that total demand must equal the available supply, namely

$$H = X. \tag{6}$$

2.2 Equilibrium

An equilibrium for this economy consists of sequences of prices and money-holding distributions $\{\phi_t, F_t\}$, production and money payments in the decentralized market $\{q_t, D_t\}$, and production, consumption, and money carried for future purchases from the centralized market $\{H_t, X_t, m_{t+1}\}$ that meet the following conditions

1. $\{q_t, D_t, X_t, H_t, m_{t+1}\}$ solve the representative agent's problem taking the Nash bargaining functions, prices, subsidies, lump sum monetary transfers, and the distribution of money holdings as given.

2. The government funds subsidies by money printing, that is, equation (5) holds at all $t \geq 0$.

3. All markets clear, and all aggregate resource constraints are satisfied at all $t \geq 0$.

4. There is consistency between beliefs and the actual distribution of money.

2.3 Analysis of the model

The first important property of equilibrium in this model is that the value function of the centralized market during the night subperiod is linear in m, with slope ϕ. This result is easily derived by solving for H in the constraint of equation (4) and substituting its value into the objective function. The linearity of the value function associated with trading in the centralized market keeps the model tractable. In particular, it implies that all agents choose m_{t+1} independently of the money balances, m_t, with which they entered the market.

The linearity of $W(.)$ also simplifies the bargaining problem in the decentralized market as follows:

$$\max_{q,D} \{u(q) - D\}^{\theta} \{-q + \phi D (1+s)\}^{1-\theta} \tag{7}$$

$$s.t.\ D \leq m. \tag{8}$$

After multiplying both sides of constraint (8) by ϕ, it is clear that the above maximization problem depends only on real monetary balances, $z_t \overset{def}{=} \phi_t m_t$. If we let $d \overset{def}{=} \phi D$, the solution to the generalized Nash bargaining problem (7) is given by the following result.

Proposition 1 *Given a subsidy rate s, an interior solution to the generalized Nash bargaining problem is given by:*

$$d(z, \tilde{z}) = \begin{cases} z \ \textit{if}\ z < z^u \\ z^u \ \textit{if}\ z \geq z^u \end{cases}$$

$$q(z, \tilde{z}) = \begin{cases} \hat{q} \ \textit{if}\ z < z^u \\ q^u \ \textit{if}\ z \geq z^u \end{cases}$$

where q^u and z^u are the solutions to the first-order conditions of the maximization problem (7), ignoring the cash constraint of the buyer [equation (8)]:

$$\theta \{-q^u + z^u (1+s)\} u'(q^u) = (1-\theta) \{u(q^u) - z^u\} \tag{9}$$

$$\theta \{-q^u + z^u (1+s)\} = (1-\theta) \{u(q^u) - z^u\} (1+s). \tag{10}$$

When the cash constraint is binding, $d = z$, then $\hat{q}$ is given by the solution to the first-order condition of (7):

$$\theta \{-\hat{q} + z (1+s)\} u'(\hat{q}) = (1-\theta) \{u(\hat{q}) - z\}. \tag{11}$$

As Proposition 1 indicates, a key feature of the model is that the functions characterizing the bargaining game do not depend on the real money holdings of the seller. The latter is a key property of Lagos and Wright (2005) and carries over to our version of the model with subsidies. The new element here is that the function determining production in the decentralized market is positively related to the subsidy rate in the centralized market.

Corollary 2 *For each value of the real money holdings of the buyer, production in the decentralized market, $q(z)$, is increasing in s.*

Proof. To show that output is increasing in s, notice that equations (9) and (10) deliver

$$u'(q^u) = \frac{1}{1+s}.$$

Hence, the concavity of $u(.)$ yields that q^u is increasing in s. To establish that $\hat{q}$ is increasing in s, we apply the implicit function theorem to equation (11), which yields

$$\frac{\partial \hat{q}}{\partial s} = -\frac{\theta z u'(\hat{q})}{\theta\left(-u' + u''\left(-\hat{q} + z(1+s)\right)\right) - (1-\theta)\, u'}.$$

Thus, given the monotonicity and concavity of $u(.)$, the participation constraint $-\hat{q}+z(1+s) \geq 0$, the above derivative is positive; thus, $\hat{q}$ is increasing in s. ■

As can be seen from Corollary 2, the well-known result from public finance that subsidies to production tend to increase it, also holds here.

Observe that Proposition 1 and Corollary 2 take the money holdings of the agent as given. Obviously, the agent chooses its money holdings optimally. We proceed to analyze the determinants of this decision. The structure of the bargaining solution for this model simplifies greatly the problem of the representative agent. This is captured by Corollary 3 below. The resulting characterization of the representative agent's problem is used repeatedly throughout our analysis.

Corollary 3 *Under the conditions of Proposition 1, the problem of the representative consumer can be written as follows:*

$$\begin{aligned} W_t(m_t) = \max_{m_{t+1}, X} & [\{U(X) - X + \beta(\phi_{t+1} - \phi_t)m_{t+1}\} \\ & + \alpha\sigma\beta \left\{u(q(m_{t+1})) - \phi_{t+1} D(m_{t+1})\right\}] \\ & + \alpha\sigma\beta \int \left\{-q(\tilde{m}) + (1+s)\phi_{t+1} D(\tilde{m})\right\} dF_{t+1}(\tilde{m}). \end{aligned} \tag{12}$$

As is now well understood, additional regularity conditions on $u(.)$ can be imposed so that the solution to m_{t+1} to the above problem is unique.[7] The latter yields a degenerate distribution of money holdings, which keeps the model analytically tractable.

Hereafter, our analysis will be restricted to stationary monetary equilibrium. These equilibria have prices that grow at a constant rate and real money holdings are strictly positive. Finally, for ease of presentation we restrict our analysis to the utility functions:

$$\begin{aligned} u(c) &= \frac{(q+b)^{1-\eta} - b^{1-\eta}}{1-\eta}, \\ U(X) &= B\log(X), \quad \text{with } \; B, \; b > 0, \; \text{ and } 0 < \eta < 1, \end{aligned}$$

[7] See Wright (2008) for more on this issue.

which correspond to the preferences used by Lagos and Wright (2005) for their quantitative analysis.

The problem of the representative agent can be written as above, in which money holdings are chosen and q is determined in the bargaining game. Notice, however, that it is also possible to think of the problem of the representative agent as that of choosing q, with real money holdings given by $z(q)$. This alternative characterization of the representative agent's problem is useful later in establishing the welfare properties of equilibrium.

Proposition 4 *Consider any given subsidy and a sequence of prices that grows at rate* $(1+\pi)$, *then the representative agent's problem can be written as:*

$$(X^e, q^e) = arg \max_{X, q \in [0, \bar{q}]} U(X) - X + \alpha\sigma\left(\left(1 - \frac{1+\pi}{\beta}\right) z(q) + \alpha\sigma\left\{u(q) - z(q)\right\}\right) \tag{13}$$

Moreover, the solution to the above problem satisfies $q^e{<}q^u$ *and* $z{<}z^u$.

Proof. From Proposition 1 we know that if $z > z^u$ then $q = q^u$ for all z. For monetary equilibrium to exist we must satisfy $\left(1 - \frac{1+\pi}{\beta}\right) \leq 0$. If the latter term is equal to zero, then the objective of the representative household is constant for all $z{>}z^u$, and it is strictly decreasing if $\left(1 - \frac{1+\pi}{\beta}\right){<}0$. Thus, it suffices to consider the range $z{\leq}z^u$. From the first-order condition, equation (11), it is possible to define the output that solves the bargaining problem as a function of z. This function is invertible so that

$$z(q) \equiv \frac{(1-\theta)\, u(q) + \theta u'(q) q}{\theta(1+s)u'(q) + (1-\theta)}.$$

It is easy to verify that $-z'(q){<}0$ for all q. To evaluate the monotonicity properties of the buyer's surplus $\{u(q) - z(q)\}$, we follow Aruoba, Rocheteau, and Waller (2007). In particular, the first order condition of the bargaining problem with respect to q yields

$$\frac{\theta}{(1-\theta)}\left\{-q + z(1+s)\right\} u'(q) = \left\{u(q) - z\right\}.$$

Then, we substitute z into the left-hand side of the above equation to determine the following:

$$\frac{\theta\,(1-\theta)\, u'}{\theta(1+s)u' + (1-\theta)}\left[u(q)\,(1+s) - q\right] = \left\{u(q) - z\right\}. \tag{14}$$

Taking derivatives shows that the left hand side of equation (25) is non-monotone and that it is negative as $q \nearrow q^u$. The latter fact, paired with $-z'(q){<}\ 0$, implies that in any optimum $q^e{<}q^u$, so that $z{<}z^u$. ■

Proposition 4 explicitly shows the trade-off of holding real balances and consuming. In particular, if a seller could turn the proceeds from her production into immediate consumption, as in a static or frictionless model, then the seller would produce until marginal utility equals marginal cost. In a monetary exchange economy, however, the proceeds from production consist of cash that can only be spent in the future.

2.4 Optimal fiscal and monetary policy

This section considers the government's problem of choosing subsidies and monetary transfers to maximize social welfare with full commitment. Thus, we are contemplating an environment in which the government sets an inflationary and fiscal plan that will not change over time.

Definition 5 *An inflation rate and a production subsidy* (π^*, s^*) *are optimal if they solve the following problem:*

$$\max_{\tau_{m_2}, s} U(B) - B + \alpha\sigma(u(q^e) - q^e) \tag{15}$$

$$s.t. \;\; q^e \in \arg\max_q \left(\left(1 - \frac{1+\pi}{\beta}\right) z(q) + \alpha\sigma \left\{u(q) - z(q)\right\} \right) \tag{16}$$

$$\tau_{m_1} = s\alpha\sigma \tag{17}$$

$$(1+\pi) = (1+\tau_{m_2})(1+\tau_{m_1}) \tag{18}$$

$$\left(1 - \frac{1+\pi}{\beta}\right) \leq 0. \tag{19}$$

According to Definition 5, the government's problem consists of choosing inflation and subsidy rates that maximize social welfare subject to the constraint that production and consumption in both markets are stationary monetary equilibria.

Observe that the availability of lump sum monetary transfers at the end of the centralized market can neutralize any increase of the money supply from the payment of monetary subsidies at the centralized market (where we measure inflation).

Molico (2006), Bhattacharya, Haslag, and Martin (2005), and Deviatov and Wallace (2001), among others, have provided examples in which a policy that consists of increasing the money supply through lump sum transfers induces some redistribution across individuals. Our paper also emphasizes the importance of distributional effects when examining the Friedman rule. The source of our heterogeneity is the asymmetric fiscal treatment of buyers and sellers. Redistribution of resources between buyers and sellers is possible through production subsidies and lump sum injections/withdrawals of money in the decentralized market. To illustrate the importance of distributional effects consider an economy where inflation is higher than the Friedman rule. If the growth rate of the money supply is lowered there are two effects. All agents are better off because the monetary inefficiency is reduced. However, wealth is transferred from agents who hold little money to those who hold more. This effect may worsen the position of those with little money. If transfers are allowed, then society can undo the latter effect with the result of all agents being better off. Society cannot undo the latter effect without transfers, and the Friedman rule is not necessarily Pareto optimal.

The main result of this section establishes that fiscal and monetary policies can restore the efficiency of monetary equilibria in spite of the non-monotonicity of the buyer surplus implied by the Nash bargaining solution.

Proposition 6 *Consider any given value of the buyer's bargaining weight,* $0 < \theta \leq 1$, *and any given inflation rate* $\pi^* \geq \beta - 1$. *Then, as* $b \to 0$, *there exist values of* s^* *and* $\tau^*_{m_2}$ *that solve the optimal taxation and achieve first-best equilibrium allocations in both markets.*

Proof. Efficiency in the decentralized market requires $q^*= 1$. Since $q^u \geq q^e$ and $q^u = (1+s)^{\frac{1}{\eta}} - b$, a necessary condition for efficiency is $s^* \geq 0$. Moreover, it is then possible to solve for s^* in the first-order condition (16) at $q^e= 1$. Moreover, we show in the Appendix that the first-order condition characterizes the solution to the households problem q^e. Hence, efficiency in decentralized trades can be achieved. Notice that $X^*= B$ satisfies the first order condition of the household in the central market, and is also first best, establishing the desired result. ■

Proposition 6 suggests that the extra resources given to producers through production subsidies can provide extra incentives for sellers to produce up to the efficient level. Given that production subsidies must be monetized and the government has access to lump sum monetary taxes the government can always undo the inflation that may result from these subsidies. Finally, we note that the Friedman rule belongs to the set of optimal policies as long as the bargaining power of the buyer is strictly positive.

2.5 Quantitative analysis

We now study the quantitative implications of the theory for the efficiency of equilibrium, the welfare gains of optimal fiscal and monetary policy, and the welfare costs of inflation. Notice that the standard velocity equation

$$MV = PY,$$

where M is the money demand, P is the price level and Y is output, can be easily mapped into the variables of this model. First, the price level corresponds, in the model, to the prices of goods in a centralized market, $\frac{1}{\phi}$. Real output Y in units of the centralized market equals $B + \sigma\phi M$, and real money balances equal $\phi z(q)$, which equal ϕM in equilibrium. Hence,

$$V = \frac{B + \sigma z(q)}{z(q)}.$$

Finally, notice that equation (13) can be used to determine q, and therefore V, as a function of the nominal interest rate $\frac{1+\pi}{\beta} - 1$. The latter then can be used to derive the money demand implied by the model.

For ease of comparison with existing analysis, we used one of the parameterizations derived by Lagos and Wright (2005). In this parameterization the model with taxes and subsidies set at zero provides the best fit of the model to the annual "money demand" data of the United States. In this calibration a period is interpreted as one year (over which the day and the night markets occur). The interest rate data employed in this exercise are the annual commercial paper rate while M is measured by $M1$. The sample period was 1900 through 2000.

The annual rate of time preference is set at $r = 0.04$. Moreover, we normalize α to 1 and $\sigma = 0.5$, which means that every agent always has an opportunity to either buy or sell in each meeting of the decentralized market. Lagos and Wright (2005) show that parameter θ is difficult to identify and report results for three different values of this parameter. We pick the set of parameters that yield the largest welfare costs of inflation, in particular, we let $\theta = 0.343$, $\eta = 0.39, b \approx 0$, $\epsilon = 1$ and $B = 1.78$. Hence, our quantitative analysis of the welfare costs of inflation can be taken as a measure of the maximum gains that can be obtained by having active monetary and fiscal policy. Some results for alternative parameterizations are also reported.

2.5.1 Quantitative implications of optimal fiscal and monetary policies

We start by clarifying the mechanisms behind our key theoretical results. Figure 2 depicts the representative agent's objective when inflation follows the Friedman rule.

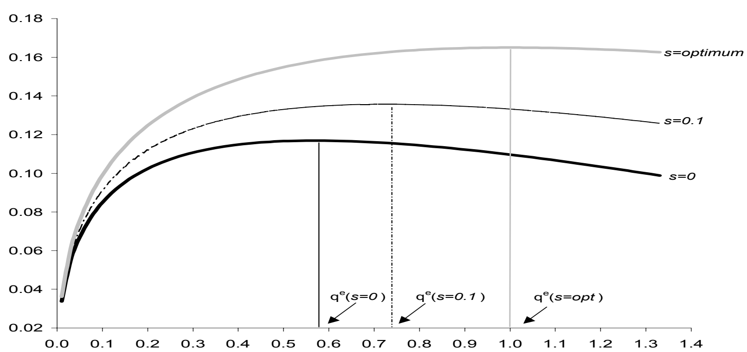

Figure 2: Objective of the representative agent as function of the subsidy rate and q. [Note: $q^u(s = 0) = 1$, $q^u(s = 0.1) = 1.28$, $q^u(s = opt) = 1.77$.]

Since we use the generalized Nash solution for the bargaining game in the decentralized market, the buyer's surplus is non-monotone in q^u, and the maximum is attained to the left of q^u. Moreover, Corollary 2 shows that q^u increases as the subsidy, s, increases. Hence, higher subsidies push both q^u and the representative household's optimal output, q^e, to the right. These are the economic forces behind Proposition 6, where it is shown that a level of subsidies can be chosen to restore efficiency of monetary equilibrium.

Second, we illustrate the properties of equilibria with and without fiscal policy. In particular, Table 1 presents different cases of fiscal and monetary policies that solve the government's problem with the resulting inflation, optimal output level for decentralized and centralized market, and the optimal subsidy rate. Table 1 also shows in the second row the Lagos and Wright (2005) experiment where the Friedman rule is the only optimal policy.

Table 1: Welfare Maximizing Policies

π	q	X	s
-3.8%	**0.56**	**1.78**	**-**
-3.8%	1.0	1.78	25%
0%	1.0	1.78	61%

As seen, the output costs of abstracting from optimal fiscal policy are significant: Output during the pairwise trade period falls by 40% relative to first best, which can be attained with a combination of monetary and fiscal policies. Notice that money printing required to pay for the subsidy would imply a total growth rate in the money supply of at least 12%. However, the inflation rates we report are low. Hence, an optimal policy requires undoing most of the money printing by extracting money after the night market is closed by using lump sum (negative) transfers. Finally, notice that the production subsidy is not a redundant tax since without such subsidy efficiency cannot be attained. Section 2.6 below shows that costless lump sum monetary transfers are not redundant either.

A more detailed analysis of the properties of the optimal subsidy rate is shown in Figure 3. The graphs illustrate the different monetary subsidy rates required to restore efficiency of monetary equilibrium under different inflation rates, and for values of the buyer's bargaining weight $\theta \in [0.2, 1]$.

The behavior of the optimal subsidy rate is quite intuitive. In particular, the subsidy rate decreases monotonically in the bargaining weight of the buyer and increases in the inflation rate. A higher bargaining power for the buyer means that less of the surplus associated with holding money will be taken away. As a result, current producers require a lower compensation to achieve the socially optimal level of production in the decentralized market. When the buyer has all bargaining power ($\theta \simeq 1$) then the Friedman rule suffices to make equilibrium allocations Pareto optimal. Under these circumstances no hold-up problem exists. With inflation above the Friedman rule, a hold-up problem is created and a positive subsidy rate is required to achieve optimality, even when the buyer has full bargaining power. Naturally, the size of the optimal subsidy increases as the bargaining power of the buyer decreases for any given inflation rate.

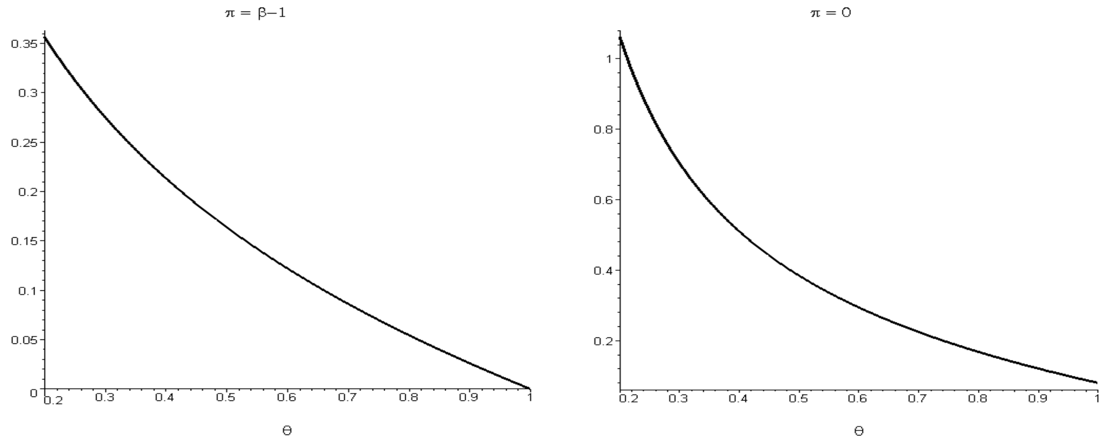

Figure 3: Optimal subsidy rates for different inflation rates and buyer's bargaining weights.

In summary, when lump sum monetary transfers are possible, multiple subsidy rates and (sometimes strictly positive) inflation rates exist that can yield the efficient allocation. In this environment, the Friedman rule is one of the possible policy options that is available to the government that yields efficiency. Moreover, a production subsidy is not a redundant tax. Finally, the Friedman rule belongs to the set of optimal policies regardless of the value of the bargaining power of the buyer.

2.5.2 The Welfare costs of inflation

Assessing the welfare costs of inflation requires a sound understanding of the benefits of monetary exchange. We now recast the classical analysis of the welfare costs of inflation in a setting where the existence and need for money is based on micro foundations and fiscal policies are considered.

It is important to note that, according to Proposition 6, if fiscal and monetary policy adjust simultaneously, then for any given inflation rate there is a subsidy that makes monetary equilibria efficient. As a result, there are no welfare costs of inflation. We still consider it interesting, nevertheless, to perform an exercise similar to that of Lucas (2000). We thus compute the percentage of consumption that an agent, living in an economy with optimal fiscal and monetary policies, would be willing to give up in avoid being in an economy where the inflation rate varies, while fiscal policies are held fixed. Our results are reported in Table 2 below. The second and third columns of this table consider the case $\theta = 0.343$, which is the calibration in Lagos and Wright that yields the largest inflation welfare loss. To compare with the results of the previous literature, the second column of Table 2 reproduces the welfare costs of inflation in the case where lump sum monetary transfers are the only available tool. Finally, the last column of Table 2 reports the case $\theta \simeq 1$, where the costs of inflation are minimized.

Table 2 shows that the costs of inflation can be considerably larger than what Lagos and

Table 2: Welfare Costs of Inflation Starting from $\pi = -3.8\%$

π	$\theta = 0.343$ $s = 25\%$	$\theta = 0.343$ No Fiscal Policy	$\theta \simeq 1$ $s = 0$
-2%	0.5%	0.6%	0.03%
0%	1.6%	1.9%	0.2%
4%	3.9%	4.8%	0.5%
6%	4.9%	6.2%	0.7%
8%	7.4%	6.2%	1.0%
10%	8.4%	6.8%	1.3%

Wright originally found. The welfare costs of inflation of our benchmark economy, where the optimal policies are able to achieve efficiency, can be as large as 8% of consumption. Finally, the welfare costs of inflation increase as the bargaining power of the buyer decreases.

In this environment, the inflation tax introduces a wedge in the decision to invest in real balances. The extent of this distortion depends crucially on the assumed pricing mechanism. The basic intuition behind this large welfare cost of inflation is the notion of a hold-up problem as we move away from the Friedman rule. In other words, an agent that carries a dollar into the next period is making an investment with cost equal to the value of money. When the agent spends money, she reaps all of the returns to her investment if and only if $\theta \simeq 1$. Otherwise, the seller "steals" part of the surplus. Thus, whenever $\theta < 1$ there is a reduction in the incentive to invest, lowering the demand for money and hence production in the decentralized market. This phenomenon becomes more important once fiscal instruments are in place because more production is possible under the optimal subsidy rate.

Our previous findings then suggest that ignoring active fiscal policies can be quite costly. Thus knowing the empirical "money demand" curve is not enough; what really needs to be understood in order to correctly estimate the welfare cost of inflation are the micro economic foundations of the money demand, and especially how the terms of trade are determined and affected by policy actions.

2.5.3 The Welfare value of fiscal policy

We now measure the welfare value of optimal fiscal policy. Our analysis is symmetric to that in the previous section. In particular, we compute the lifetime consumption value of living in a world in which optimal policies are implemented, relative to living in a world where fiscal policy deviates from the optimum. Inflation is held constant throughout alternative experiments.

Our results are reported in Table 3, in which two initial optimal policies, denoted by π^* and s^*, are taken as departing points. The alternative, suboptimal, subsidy rates considered are reported in the first and third columns of the table, whereas the implied welfare costs are reported in the second and fourth columns.

Table 3: Welfare Costs of Fiscal Policy Starting from Optimality

$\pi^* = \beta - 1$, s* = 25%		$\pi^* = 0$, s* = 60%	
s	Welfare Cost	s	Welfare Cost
20%	0.04%	48%	0.17%
15%	0.16%	36%	0.62%
10%	0.37%	24%	1.33%
5%	0.65%	12%	2.25%
0%	1.01%	0%	3.35%

The first important observation is that the welfare costs of changing the subsidy rate from its optimal level to zero are substantial (up to three percent of lifetime consumption). More important, the welfare costs of deviating from the optimal fiscal policy are increasing in the departing inflation rate. The intuition behind the latter result is straightforward. Using the Friedman rule implies that the only distortion in the model is the lack of monotonicity of the buyer's surplus that results from Nash's bargaining solution. However, any deviation from the Friedman rule brings introduces an additional distortion: the hold-up problem. Clearly, the welfare costs of not providing a production subsidy when these two sources of frictions are active will be higher than when monetary policy follows the Friedman rule.

2.6 Alternative operating procedures for monetary policy

Our previous results crucially depend on the availability of costless lump sum monetary transfers in the centralized market. Hence, it seems important to study how the properties of monetary equilibrium change once lump sum transfers are not available. This is the purpose of this section.

As mentioned previously, two frictions inherent in this model render equilibrium inefficient whenever the buyer does not have all the bargaining power. Hence, if we remove lump sum taxes, the government only has one instrument, the subsidy rate, and it is very unlikely that optimality can be restored. Moreover, without negative lump sum monetary transfers inflation is directly proportional to the subsidy rate since the following condition applies:

$$(1 + \pi) = (1 + s\alpha\sigma) .$$

Hence, a production subsidy has an ambiguous effect on output at the decentralized market. A positive subsidy induces producers to increase output at any moment in time. However, the resulting higher inflation creates an intertemporal distortion, the hold-up problem, which lowers the incentives to produce. Indeed, the sign of $\frac{\partial q^e}{\partial s}$ can be either zero, positive, or negative, depending on the underlying parameterization of the model. For all calibrations considered so far the value of this derivative is negative. In fact, we were only able to find that a positive subsidy is optimal for extremely low discount factors ($\beta < 0.7$). In light of these observations, it seems natural to search for alternative instruments that may improve the efficiency of monetary equilibrium allocations whenever lump sum transfers are not available.

One potential mechanism to retire money from circulation is to transform the subsidy into a tax, that is $s < 0$. This strategy is effective in retiring money from circulation but it reduces production in the decentralized market, as suggested by Corollary 2. We consider instead a sales tax on decentralized market transactions. Such a tax is possible given that the government can monitor changes in money holdings before entering and leaving the central market. In particular, if an agent lowers its money holdings by a D amount (the agent was a buyer in the decentralized market), then the government collects an additional $\tau_b D$ units of money from the agent before entering the centralized market. Notice that with an appropriate value of the sales tax, the Friedman rule is feasible. Finally, this new fiscal tool also changes the bargaining problem of buyers and sellers.

Since there are two frictions in the model, a complete taxing system requires an additional instrument. We thus consider a production tax in the centralized market. This tax increases the cost of consuming in the centralized market, giving incentives for agents to increase their consumption in the decentralized market. In particular, by changing the production tax the government is effectively changing the cost of consuming in the centralized market, and thus the outside option of agents in the decentralized market. Moreover, the government can use the goods that it collects from the production tax and sell them in exchange for money. Then these money holdings can be retired from circulation, which is a form of open market operations based on taxes and goods.

The remainder of this section formalizes the economic mechanisms just described. The value function associated with trades in the centralized market is now determined by:

$$W_t(m_t) = \max_{X,H,m_{t+1}} \{U(X) - H + \beta V_{t+1}(m_{t+1})\} \tag{20}$$
$$s.t.\ X = H\,(1-\tau_N) + \phi_t(m_t - m_{t+1});$$

where τ_N is the tax rate on the production of the centralized market good.

After solving for X in the constraint and substituting into the objective function, this value function is still linear in m, with slope $\frac{\phi}{1-\tau_N}$. The bargaining problem of the representative household is now given by:

$$\max_{q,D} \left\{u(q) - \frac{\phi D}{1-\tau_N}\right\}^{\theta} \left\{-q + \frac{\phi D}{1-\tau_N}\right\}^{1-\theta} \tag{21}$$
$$s.t.\ \frac{D}{(1+\tau_b)} \leq m; \tag{22}$$

where τ_b is the sales tax rate in the decentralized market.

An important new feature of this bargaining problem is that, given m, production in the decentralized market is decreasing in τ_b and increasing in τ_N. The latter situation is established by the following proposition.

Proposition 7 *For each value of the real money holdings of the buyer we have the following results: (i) given τ_N, $q(z)$ is decreasing in τ_b; and (ii) given τ_b, $q(z)$ is increasing in τ_N.*

The previous result holds for a given value of real money holdings and for any given value of the buyer's bargaining weight, $0 < \theta \leq 1$. However, real money holdings depend on the rate of return of money, which is denoted as

$$(1 + \pi) = (1 - \tau_b \alpha \sigma) \, .$$

The characterization of monetary equilibria and the definition of the government's problem are analogous to those described in Sections 2.3-2.4. Now the government chooses the sales and production tax rate that maximizes welfare (and our appendix provides the details). The next subsection illustrates the properties of optimal fiscal and monetary policies in this new environment.

2.6.1 Quantitative implications of optimal fiscal and monetary policies

For a given parameterization, the government's problem can be easily solved with standard numerical methods. We study how optimal policy responds when no lump sum transfers are available while using the calibration of Lagos and Wright (2005). Notice that a sales taxes make the Friedman rule a feasible strategy even without negative lump sum monetary transfers. In particular, it is possible to set $\tau_b > 0$ such that $\tau_{m_1} = \beta - 1$. However, this implementation of the Friedman rule cannot achieve the efficient outcome since the unconstrained output solution to the Nash bargaining problem (21) satisfies:

$$q^u = \left(\frac{1}{1 + \tau_b} \right)^{1/\eta} < 1,$$

Moreover, in the appendix we establish $q^e < q^u$ so that efficiency, which requires q^e=1, is not possible.

Hence, if lump sum money extraction is not available, the resulting equilibrium will not be first best and the government must choose the optimal trade-off between the different instruments at hand. Consider the strategy of lowering inflation (maybe up to the Friedman rule) by setting $\tau_b >$ 0. Lower inflation may increase money that agents take to the decentralized market. However, since q is decreasing in τ_b, lower inflation tends to lower equilibrium output in the bargaining stage. Similarly, increasing production taxes in the centralized market, τ_N, increases production in the decentralized market. Production taxes, however, distort output in the centralized market. The government faces a trade-off.

The first row of results in Table 4 reports a version of the model where no taxes nor subsidies are available; i.e., the Lagos and Wright case where inflation is constrained to be zero. Optimal policies for the benchmark experiment are reported in the second row of results in Table 4. Finally, the last row of Table 4 considers a new experiment where all parameter values are held constant but θ is changed to 0.95. This last experiment illustrates how the trade-offs in the optimal taxation problem may change as the bargaining power of the buyer increases.

Table 4: Welfare Maximizing Policies

θ	π	τ_b	τ_N	q^e	X
0.343	**0%**	-	-	**0.29**	**1.78**
0.343	-3.84%	8.42%	5.32%	0.81	1.7
0.950	-0.66%	1.24%	3.79%	0.87	1.71

As can be seen in Table 4, production and sales taxes have important quantitative implications for output and welfare. Relative to the constrained Lagos and Wright (2005) case, with all taxes set at zero, output in the decentralized market increases by a factor of 2.7 under the optimal policy. When fiscal instruments are possible, the optimal policy involves the use of sales taxes to implement the Friedman rule. The economic intuition behind the positive production tax displayed in Table 4 is derived from Proposition 7. Production in the decentralized market is increasing in τ_N, and the government faces a trade-off since higher τ_N lowers welfare in the centralized market. The optimal production tax equates the marginal welfare gains from the decentralized market with the marginal welfare losses of the centralized market.

Another interesting result in Table 4 is that the optimality of the Friedman rule depends on the bargaining power of the buyer. Recall that a higher bargaining power for the buyer means that less of the surplus associated with holding money will be taken away. As a result, it is necessary to give a lower compensation to current producers in order for them to achieve the socially optimal decentralized production. Moreover, reducing inflation, by setting $\tau_b > 0$, lowers production in the decentralized market. On the other hand, whenever the buyer is able to capture more of the full benefit from the match, it increases her incentives to hold money, causing q to increase. Thus, whenever the bargaining power of the buyer is low enough, the Friedman rule is optimal. As the bargaining power of the buyer increases we find that it is optimal to have a positive net nominal interest rate.

Until this point the government has been able to observe changes in the money holding of agents. Knowledge of changes in money holdings by the government may be considered as requiring too much information and too restrictive. With sales taxes in place, agents have a clear incentive to not truthfully reveal changes in monetary holdings. Hence, truthful revelation will only occur if the government can restrict from participating in the centralized market. Given logarithmic preferences for consumption at the centralized market agents will always truthfully reveal their money holdings.

3 Conclusions

The objective of this paper is to provide a better understanding of the interactions between monetary and fiscal policies in an economic environment with microeconomic foundations for fiat money. In particular, this paper derives the optimal monetary and fiscal policies in a standard search theoretic model of monetary exchange where production subsidies, sales taxes, and the possibility of injecting fiat money at different times of the day are possible.

One of our main results is to show that, even when terms of trade in the decentralized market are given by Nash's bargaining solution, some of the inefficiencies in the Lagos and Wright framework can be restored with appropriate fiscal policies. In particular, when lump sum monetary transfers are possible, a production subsidy financed by money printing can increase output in the decentralized market. In this environment we can interpret the monetary production subsidy as a policy that pays interest on money holdings to some agents. This type of policy prescription has a long tradition in monetary economics and has been advocated by Tolley (1957) and Friedman (1959).

We also showed there exist multiple subsidies and (sometimes strictly positive) inflation rates that yield the efficient allocation. The Friedman rule is one of the possible policy options that yields efficiency. Finally, the Friedman rule is always an optimal policy regardless of the bargaining power of the buyer.

When operating procedures for monetary policy prevent lump sum transfers, introducing sales taxes in the decentralized market and production taxes in the centralized increase welfare. The availability of a sales tax makes the Friedman rule a feasible policy. Moreover, introducing a production tax in the centralized market increases production in the decentralized market. In this new environment, the optimality of the Friedman rule depends largely on the bargaining power of the buyer. In particular, for sufficiently low values of the bargaining power of the buyer the Friedman rule is the unique optimal policy. In contrast, when the bargaining power is high enough then deviations from the Friedman rule may occur. Irrespective of this bargaining power, the efficient allocation cannot be achieved in a monetary equilibrium.

Finally, under any of the operating procedures for monetary and fiscal policy considered in this paper, with or without lump sum taxes, large welfare gains are achieved by having fiscal and monetary policies in place. Thus, ignoring active fiscal policies can be quite costly.

The findings of this paper confirm the conjecture of Kocherlakota (2005) and Wright (2005) that fiscal and monetary policies have important interactions in frameworks with micro foundations for the existence of fiat money; thus, they should always be jointly considered in the design of optimal government policy.

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Appendix

Proof of Proposition 7

Recall that the objective of the consumer, relative to q, can be written as follows:

$$\left(\left(1-\frac{1+\pi}{\beta}\right)z(q)+\alpha\sigma\left\{u(q)-z(q)\right\}\right).$$

It is easy to show that $z(q)$ is monotone in q (for a given set of taxes and subsidies). Thus the first term is monotone decreasing in q. The only relevant term to evaluate is the buyer's surplus. From the proof of Proposition 4, however, we know the buyer's surplus is, in fact, given by

$$\frac{\theta\left(1-\theta\right)u'}{\theta(1+s)u'+(1-\theta)}\left[u(q)\left(1+s\right)-q\right]$$

Given our assumption on the functional form of u, as $b\rightarrow 0$, the derivative of the above is given by:

$$\begin{aligned}
&\frac{-\eta\theta\left(1-\theta\right)q^{1-\eta}\left[\frac{q^{1-\eta}}{1-\eta}\left(1+s\right)-q\right]}{q^2\left[\theta\left(1+s\right)q^{-\eta}+\left(1-\theta\right)\right]}\\
&-\frac{\theta\left(1-\theta\right)q^{1-\eta}\left(1-\eta\right)\left[\frac{q^{1-\eta}}{1-\eta}\left(1+s\right)-q\right]\left(-\eta\frac{\theta(1+s)q^{1-\eta}}{q^2}\right)}{q\left[\theta\left(1+s\right)q^{-\eta}+\left(1-\theta\right)\right]^2}\\
&+\frac{\theta\left(1-\theta\right)q^{1-\eta}\left(1-\eta\right)\left[\frac{q^{-\eta}}{1-\eta}\left(1+s\right)-1\right]}{q\left[\theta\left(1+s\right)q^{-\eta}+\left(1-\theta\right)\right]}
\end{aligned}$$

After eliminating $\theta(1-\theta)$, and a little algebra, the sign of the above expression is given by:

$$\begin{aligned}
&\frac{-\eta\left[\frac{q^{1-\eta}}{1-\eta}\left(1+s\right)-q\right]}{q^2\left[\theta\left(1+s\right)q^{-\eta}+\left(1-\theta\right)\right]}\\
&+\frac{\left[q^{1-\eta}\left(1+s\right)-q\right]}{q\left[\theta\left(1+s\right)q^{-\eta}+\left(1-\theta\right)\right]}\\
&-\frac{\left[\frac{q^{1-\eta}}{1-\eta}\left(1+s\right)-q\right]\left[\theta\left(1+s\right)q^{1-\eta}\left(1-\eta\right)-\theta\left(1+s\right)q^{1-\eta}\right]}{q^2q\left[\theta\left(1+s\right)q^{-\eta}+\left(1-\theta\right)\right]^2}
\end{aligned}$$

Given we care only about the sign, we can multiply by a positive term, $\left[\theta\left(1+s\right)q^{-\eta}+\left(1-\theta\right)\right]^2$, throughout. We also divide, each of the terms in brackets by q. Hence, the sign of the derivative of the buyer's surplus is given by:

$$
\begin{aligned}
&-\eta\left(1-\theta\right)\left[\frac{q^{-\eta}}{1-\eta}\left(1+s\right)-1\right]+\\
&\left[q^{-\eta}\left(1+s\right)-1\right]\left[\theta q^{-\eta}\left(1+s\right)+\left(1-\theta\right)\right],
\end{aligned}
$$

which after further simplifications can be written as follows:

$$
\begin{aligned}
&\left(1+s\right)q^{-\eta}\left[\left(1-\theta\right)\left[1-\frac{\eta}{1-\eta}\right]-\theta\right]+\left(1-\theta\right)\left[\eta-1\right]+\\
&+q^{-2\eta}\left(1+s\right)^{2}\theta.
\end{aligned}
\tag{23}
$$

Notice that the second term in (23) is negative. The third term is positive and decreasing in q at rate 2η. The first term may be positive or negative. Nevertheless, it is decreasing in q at rate η. It is also easy to show that at q^*, defined by $q^{*-\eta}=\frac{1}{1+s}$, the above expression is negative. Notice that (23) goes to $+\infty$ as q goes to zero, and thus, by continuity, there is a $\hat{q}$ such that (23) equals zero. Finally, given the rates of decrease in q of the positive, and of the possibly negative term, it follows that (23) is negative for all $q>\hat{q}$. We conclude that a maximum q^e exists, that it is interior, and that it satisfies $q^e<q^*$ for any given s,π.

Proof of Proposition 8

To show output is increasing in $\frac{1}{1+\tau_b}$, notice that corresponding equations (9) and (10) of our new problem deliver the following:

$$u'(q^u)=1+\tau_b.$$

Hence, the concavity of $u(.)$ yields that q^u is increasing in $\frac{1}{1+\tau_b}$. Using the implicit function theorem we can also derive the following:

$$\frac{\partial\hat{q}}{\partial\left(1+\tau_b\right)}=-\frac{\theta z u'(\hat{q})}{\theta\left(-u'+u''\left(-\hat{q}+\frac{z}{(1+\tau_b)(1-\tau_N)}\right)\right)-\left(1-\theta\right)u'}.$$

Thus, given the monotonicity of $u(.)$, the participation constraint $-\hat{q}+\frac{z}{1+\tau_b}\geq 0$, and the concavity of $u(.)$, $\hat{q}$ is increasing in $\frac{1}{1+\tau_b}$. The monotonicity of q^* and $\hat{q}$ in $\frac{1}{1+\tau_b}$ establish (i). Similarly,

$$\frac{\partial\hat{q}}{\partial\left(1-\tau_N\right)}=\frac{\left[\frac{\theta z}{1+\tau_b}u'(q)+\left(1-\theta\right)z\right]\left(1-\tau_N\right)^{-2}}{\theta\left(-u'+u''\left(-\hat{q}+\frac{z}{(1+\tau_b)(1-\tau_N)}\right)\right)-\left(1-\theta\right)u'},$$

together with the fact that q^u is independent of $\left(1-\tau_N\right)$, delivers (ii).

Proof that $q^e<q^u$ and $\frac{z}{1-\tau_b}<z^u$

First of all note that given any sales and production tax rates (τ_b,τ_N) and a sequence of prices that grows at rate $(1+\pi)$, then the representative agent's problem can be written as:

$$(X^e, q^e) = arg\max_{[0,\bar{q}]} U(X) - \frac{X}{1-\tau_N} + \alpha\sigma\left(\left(1-\frac{1+\pi}{\beta}\right)\frac{z(q)}{1-\tau_N} + \alpha\sigma\left\{u(q) - \frac{z(q)}{1-\tau_N}\right\}\right) \quad (24)$$

In the same spirit as in Proposition 1, we know that if $\frac{z}{1-\tau_b}{>}z^u$ then $q = q^u$ for all z. Moreover, D is also fixed at z^u and for monetary equilibrium to exist we must satisfy $\left(1-\frac{1+\pi}{\beta}\right)\leq 0$. If the latter term is equal to zero then the objective of the representative household is constant for all $\frac{z}{1+\tau_b}{>}z^u$, and is strictly decreasing if $\left(1-\frac{1+\pi}{\beta}\right){<}0$. Thus, it suffices to consider the range $\frac{z}{1+\tau_b}\leq z^u$ and to study problem (13). From the first order condition, equation (11), it is possible to define the output that solves the bargaining problem as a function of z. This function is invertible so we have that:

$$z(q) \equiv \frac{(1-\theta)\,u(q) + \theta u'(q)q}{\theta\frac{1}{(1-\tau_N)(1+\tau_b)}u'(q) + \frac{(1-\theta)}{1-\tau_N}}.$$

It is easy to verify that $-z'(q){<}0$ for all q. To evaluate the monotonicity properties of the buyer's surplus $\left\{u(q) - \frac{z(q)}{1-\tau_N}\right\}$, we follow Aruoba, Rocheteau and Waller (2007). In particular, notice that the first order condition of the bargaining problem with respect to q yields:

$$\frac{\theta}{(1-\theta)}\left\{-q + \frac{z)}{(1-\tau_N)\,(1+\tau_b)}\right\}u'(q) = \left\{u(q) - \frac{z}{1-\tau_N}\right\}.$$

Then, we substitute z into the left hand side of the above equation to get

$$\frac{\theta\,(1-\theta)\,u'}{\theta u' + (1-\theta)\,(1+\tau_b)}\left[u(q) - q\,(1+\tau_b)\right] = \left\{u(q) - \frac{z}{1-\tau_N}\right\}. \quad (25)$$

Taking derivatives, one finds that the left hand side of equation (25) is non-monotone and that it is negative as $q \nearrow q^u$. The latter fact paired with $-z'(q){<}\ 0$ implies that in any optimum $q^e{<}q^*$, so that $\frac{z}{1+\tau_b}{<}z^*$.