Fragility of Reputation and Clustering in Risk-Taking

JOB MARKET PAPER

Guillermo L. Ordoñez*

February, 2008

Abstract

I study the interplay between reputation and risk-taking in a dynamic stochastic environment where it is ex-ante efficient for firms to engage in safe projects, but ex-post preferred to invest in risky ones, appropriating surplus from lenders. By introducing fundamentals, I interpret the model as a dynamic global game in which strategic complementarities arise endogenously from reputation updating, overcoming pervasive multiple equilibria. I find that even though reputation deters opportunistic behavior, it introduces fragile incentives which may lead to large changes in aggregate risk-taking in response to small changes in aggregate fundamentals, inducing financial crises and credit crunches.

*UCLA Economics Department. guilord@ucla.edu. I would like to specially thank Andy Atkeson and David K. Levine for their unwavering support and advice. For useful comments and discussions I also thank Fernando Alvarez, Gabriele Camera, V.V. Chari, Jernej Copic, Christian Hellwig, Johannes Horner, Hugo Hopenhayn, Narayana Kocherlakota, Bob Lucas, Lee Ohanian, Chris Phelan, Larry Samuelson, Rob Shimer, Nancy Stokey, Pierre-Olivier Weill, Mike Woodford and Bill Zame and seminar participants at UCLA, Minneapolis Fed, Washington University in St. Louis, Wharton, Iowa, Brown, NYU Stern, Chicago, Yale, LSE, CREI, Pompeu Fabra, Minnesota, Princeton, ASU and Columbia. First version: July 2007. The usual waiver of liability applies.

1 Introduction

Reputation concerns deter opportunistic behavior by creating a link between past actions and expectations about future actions. Consider, for example, an environment in which lenders provide funds to firms whose risk-taking decisions and profits are unobservable. Firms could take excessive risk, appropriating most of the benefits from large successes and imposing most of the losses from big failures on lenders. This inefficient risk-taking reduces lending and increases its cost. However, if firms generate signals correlated to decisions, lenders could use those signals to construct reputation and offer better lending conditions to firms with better reputation. Firms are then afraid of losing their reputation and are deterred from taking excessive risk. This role for reputation has been extensively discussed in the literature[1].

The point of this paper is to argue that these reputational incentives are fragile because they may suddenly collapse, inducing big changes in aggregate risk-taking in response to small changes in aggregate fundamentals. This sudden shift in behavior may have a large impact on economic outcomes such as corporate failures, credit conditions, interest rates, and returns to investors. Hence reputation may have been an unnoticed detonator of financial collapses and credit crunches characterized by confidence crises. In normal times, lenders have confidence in firms with good reputation and no confidence in firms with bad reputation. In bad times, lenders lose confidence in almost all firms and lending breaks down[2].

I construct a model where incentives to take risk monotonically vary with a stochastic aggregate fundamental. All firms can invest in risky projects and some of them (strategic firms) can also invest in safer projects, with a lower probability of default and a higher probability of generating good signals. A firm's reputation is defined as the probability that lenders assign to the firm being the strategic type. Reputation is Bayesian updated by lenders from observing the signals, and firm incentives are shaped in large part by the concern for their reputation. To protect their reputation, strategic firms engage in safe projects when otherwise they would have preferred to opportunistically take risky ones.

In the absence of any equilibrium selection device, it is not possible to draw firm conclusions about the interplay between reputation and risk-taking, since this model delivers multiple

[1] Among the most influential papers on reputation are Milgrom and Roberts (1982), Kreps and Wilson (1982), Fudenberg and Levine (1992), Holmstrom (1999) and Mailath and Samuelson (2001). Literature specifically relating reputation and risk-taking was pioneered by Diamond (1989)

[2] Recent examples of how a loss of confidence in ratings (a measure of reputation) can fuel crises were the financial problems experienced by many countries in August 2007. Since the implementation of Basel II regulations, under which banks holding AAA assets are allowed to keep less capital and lend more, banks around the world have been filling their vaults with AAA- rated structured products, specially CDOs. In August 2007 Central Banks were forced to inject large amounts of liquidity into the overnight money markets because banks were charging very high rates to lend to each other since they lost confidence on the meaning of AAA backed securities (The Economist. "The game is up" and "Surviving the markets", 08/16/07).

equilibria. For some range of fundamentals, if lenders believe that strategic firms will play safe, then these firms will indeed have incentives to play safe to increase the probability of good signals. The reason is that good signals will be in part attributed to the firm using a safe project and then the firm being strategic. Contrarily, if lenders believe that strategic firms will undergo risky projects, then firms will indeed have incentives to take risks. In this case, good signals will be just attributed to good luck in risky projects. This strong dependence of reputation formation on lenders beliefs about firms choices is at the heart of the reputation fragility. It is irrelevant whether or not a firm has a good reputation if lenders are convinced the firm will choose the risky project.

I use techniques from global games to select a unique equilibrium that is robust to information perturbations. I assume that after the lending contract has been negotiated, but before deciding the project, firms observe a noisy signal of the fundamental. This is a key technical part of the analysis since the model is a non-standard global game. Strategic complementarities arise endogenously from reputation formation and are affected by the dynamic structure of the game. Hence standard conditions for global games to work, such as uniform limit dominance, are not assumed but obtained endogenously. Uniqueness is characterized, for each reputation level, by a threshold in fundamentals below which all firms with that reputation level take risks. This result generates a well-defined probability of risk-taking (the probability fundamentals are below the threshold) to draw conclusions about the effectiveness of reputation to deter excessive risk.

The intuition behind the collapse of reputation relies on two types of incentives to choose safe projects. First, safe projects increase the probability that firms continue operating. These "continuation incentives" increase with reputation since firms with better reputations face lower borrowing rates in the future and hence have higher expected future profits. Second, safe projects increase the probability of generating good signals, which are used for reputation formation. Because of learning, these "reputation incentives" are high for intermediate and low for extreme reputation levels. The reason is that neither firms with very high nor very low reputation can change their reputation quickly, whereas intermediate firms can. The fragility of reputation arises from combining these two types of incentives. Consider the intermediate reputation level at which reputation incentives are maximized. For reputation levels below that point, as reputation increases, both continuation and reputation incentives to increase, generating a big reduction in the fundamental threshold below which risk-taking is preferred. For reputation levels above that point, as reputation increases, continuation incentives still increase but reputation incentives decrease, compensating each other and generating small changes in thresholds. Since thresholds are less sensitive to reputation when reputation is high, risk-taking becomes attractive for firms with an increasingly larger range of reputation levels as fundamentals decline.

Finally, we relate the predictions of the model with data. First, taking credit ratings of corporate bonds as a proxy for reputation and ratings transitions as a proxy for reputation formation, we show reputation evolves gradually and changes less in bad times than in good times. Second, we discuss recent empirical evidence suggesting that both risk-taking behavior (measured by idiosyncratic risk) and corporate defaults tend to cluster "excessively" in recessions (Campbell et al. (2001) and Das et al. (2007) respectively). These empirical findings seem consistent with the model implications for reputation evolution and risk-taking clustering.

This paper combines two separate strands of literature - reputation and global games. The model is closely related to Diamond (1989) and Mailath and Samuelson (2001) who study reputation incentives in state invariant contexts. As in Diamond (1989), firms behavior affects reputation through the probability of continuation in business. As in Mailath and Samuelson (2001), firms behavior affects reputation through the generation of signals correlated to actions. I introduce these two channels in a single framework, finding that the combination is more than the sum of parts since it leads to the result that reputation incentives are fragile. Their papers also have multiple equilibria. While Diamond (1989) deals with it analyzing extreme equilibria, Mailath and Samuelson (2001) focus on the most efficient one. Since this paper is interested in understanding the time variation and state dependence properties of reputation incentives, we have explicitly tackled the multiplicity issue.

The model is also related to the literature on dynamic global games. I follow Chassang (2007) and Toxvaerd (2007) to solve for uniqueness. However, my model presents additional complications since strategic complementarities are not hard-wired into payoffs but arise endogenously from reputation updating, and hence are tied to the dynamic structure of the game. This paper also contributes to the scarce literature of learning in global games. While most papers study cases in which players learn about a policy maker or a status quo (e.g., Angeletos, Hellwig, and Pavan (2006) and Angeletos, Hellwig, and Pavan (2007)), my model deals with the opposite case in which the market learns about players' types, generating coordination problems. To the best of my knowledge this is the first paper that exploits fundamental-driven incentives to create a reputation global game model and select a unique equilibrium.

In the next two sections I describe a full version of the model (also considering consumers) and discuss equilibrium multiplicity when fundamentals are perfectly observed. In sections 4 and 5, I show how to select a unique equilibrium using a dynamic global games approach when fundamentals are observed with noise. In section 6 I discuss the fragility of reputation, characterized by big changes in risk-taking in response to small changes in fundamentals. In section 7, I use numerical simulations to illustrate the role of reputation in magnifying crises. In section 8, I show the predictions of the model are consistent with data on reputation dynamics and clustering in risk-taking. In the last section, I make some final remarks.

2 The Model

2.1 Description

The economy is comprised of a continuum of long-lived, risk neutral firms (with mass 1) that produce a good or provide a service, an infinite number of risk neutral lenders who fund firms to produce and consumers who buy the good or service from firms.

2.1.1 Firms

Each firm runs a unique project. The activity of all firms faces an identical market and industry risk, hence differences in results across firms are only induced by their production decisions, generating a purely idiosyncratic risk component. There are two ways to produce. Safe technologies (s) that have been previously proven to deliver a high probability of success, and risky technologies (r) that may lead to the discovery of cheaper and better production alternatives but also reduce the probability of success.

There are two types of firms, defined by their access to these production technologies. Strategic firms $\mathcal{S}$ can decide whether to follow safe or risky technologies[3]. Risky firms $\mathcal{R}$ do not have access to safe technologies, so they do not have a choice but to follow risky technologies. Reputation is defined by $\phi = Pr(\mathcal{S})$, the probability of being a strategic firm. The introduction of these two types are based on my (maybe pessimistic) belief that all firms can play risky but not all of them can play safe. While all firms can perform trial-error procedures, not all of them have access to well-designed procedures[4].

Firms cannot accumulate assets. At the beginning of the period, the firm negotiates a loan (normalized to 1) to produce. Then, strategic firms should decide whether to use safe or risky technologies[5]. At the end of the period, after production has taken place, the firm may continue (c) or die (d), with probabilities depending on the technology used. If the firm continues, two possible signals (good g or bad b), correlated with the technology chosen, are generated[6].

Probabilities are,

$$Pr(c|s) = p_s > Pr(c|r) = p_r \tag{1}$$

[3] I will also use interchangeably play safe or take safe actions (s) and play risky or take risky actions (r)

[4] Another possible assumption is that non-strategic firms only have access to safe technologies. In this case the characterization of equilibrium is different but the main result of large changes in aggregate behavior in response to small changes in fundamentals is the same. However, I believe a better description of reality is that some firms are restricted to use superior technologies rather restricted to use inferior ones.

[5] Since in this section I focus on a given period t reputational problem, I am not using any subscript to refer to time. In the next section, when introducing dynamic considerations, I will explicitly denote periods by subscripts.

[6] These two signals can be interpreted as results from production. Good signals are the firm growth or high-quality production. Bad signals are the production of defective products or the provision of a low-quality service.

$$Pr(g|c, s) = \alpha_s > Pr(g|c, r) = \alpha_r \tag{2}$$

Hence, the unconditional probability of good signals is higher using safe technologies than using risky ones (i.e., $p_s\alpha_s > p_r\alpha_r$). Additionally, assume that the unconditional probability of bad signals is higher playing risky than playing safe (i.e., $p_r(1-\alpha_r) > p_s(1-\alpha_s)$)[7].

2.1.2 Lenders and Consumers

Lenders and consumers cannot observe the technology used by the firm nor its profits but can observe whether the firm continues or not and, in case of continuation, whether it generates good signals (g) or bad signals (b). To give room for reputation to introduce incentives, signals are observable but unverifiable on court, which means interest rates charged by lenders and prices paid by consumers cannot be conditioned on observed signals.

Lenders provide funds to firms. There is an infinite number of risk neutral lenders whose outside option is a risk free interest rate $\overline{R} > 1$[8]. Repayment is characterized by a costly state verification with a bankrupt procedure that destroys the value of the output. This is a natural way to introduce truth-telling by firms. When profits are greater than the value of interest rates, it is always optimal for firms to repay and get the positive differential rather than default and file for bankruptcy. I assume that, conditional on continuation, firms can always pay back their loans, hence default occurs only in case of firm's death.[9].

Consumers buy production from firms. Consumers' utility depends on whether the signals are good or bad. If the firm generates good signals (for example the production of high-quality products) the utility for consumers is $u(g) = 1$. If the firm generates bad signals, consumers' utility is $u(b) = 0$[10]. I assume consumers pay up front for the good or service and the market interaction is given by perfect price discrimination. Each consumer buys one unit of the good in each period and pays a price P, which is a function of their expectations about the probabilities of receiving good signals. This price does not depend on the firm's actions (which are not observable) or the signals (which are known only after the good is purchased). However, as will be shown later, P depends on the firm's reputation and on consumers' expectations about the probabilities the firm played risky.

[7] This assumption is not particularly relevant but introduces monotonicity in learning and spare us from dealing with awkward expressions and extra conditions.

[8] Since lenders are the long side of the market, there is no competition for funds. The introduction of such competition makes reputation effects more important and magnifies the results

[9] Nothing fundamental changes with this assumption but it simplifies the notation and eases the exposition. The analysis relaxing it, such that default also exists in case of continuation, reinforces results and is available upon request

[10] Without loss of generality we also assume consumers' utility if the firm dies is just 0, as in the presence of bad signals. A better assumption may be a negative utility in the case of firm death. However, the conclusions of the model are identical at the cost of complicating the exposition.

2.1.3 Cash Flows

If the firms dies, present and future cash flows are zero. If the firm continues, cash flows depend on the technology used. Expected instantaneous cash flows from playing safe are $\Pi_s(\theta) = \alpha_s \pi_{s,g}(\theta) + (1 - \alpha_s)\pi_{s,b}(\theta)$ and expected instantaneous cash flows from playing risky are $\Pi_r(\theta) = \alpha_r \pi_{r,g}(\theta) + (1 - \alpha_r)\pi_{r,b}(\theta)$. Cash flows also depend positively on a single-dimensional variable $\theta \in \mathbb{R}$, which represents the aggregate economic fundamentals that affect the profitability of the project[11]. Fundamentals θ are i.i.d. over time and distributed with a density $v(\theta)$, a mean $E(\theta)$, and a variance γ_θ.

To be more specific about the structure of these cash flows, $\pi_{a,j}(\theta) = A(\theta)[P - c_{a,j}(\theta)]$, for $a \in \{s, r\}$ and $j \in \{g, b\}$. $A(\theta)$ is the level of demand for the firm's product, which depends positively on the aggregate state of the economy (or fundamentals θ), P is the unit price and $c_{a,j}(\theta)$ is the average cost of production, which depends on the aggregate state of the economy, the technology used and the signal generated. I assume that for any fundamental θ, costs from playing risky are more volatile than costs from playing safe.

Assumption 1 $c_{r,g}(\theta) < c_{s,g}(\theta) < c_{s,b}(\theta) < c_{r,b}(\theta)$ *for all* θ

This assumption arises naturally from the definition of risky technology. For example, taking risks by cutting costs beyond safe procedures may be highly beneficial if the results obtained are good but can be very costly if the results are bad since it may be necessary to pay fines, face demands, cover guarantees, etc. Since the price P charged for the product does not depend on the technology used or the generated signals, this assumption immediately implies $\pi_{r,g}(\theta) > \pi_{s,g}(\theta) > \pi_{s,b}(\theta) > \pi_{r,b}(\theta)$ for all θ. Hence expected profits from risky actions are more volatile and centered around expected profits from safe actions.

With respect to the cost structure, I also assume both expected average costs from playing risky (i.e., $C_r(\theta) = \alpha_r c_{r,g}(\theta) + (1 - \alpha_r)c_{r,b}(\theta)$) and from playing safe (i.e., $C_s(\theta) = \alpha_s c_{s,g}(\theta) + (1 - \alpha_s)c_{s,b}(\theta)$) depend negatively on fundamentals. As will be discussed later P will decrease as fundamentals weaken, which means profits per unit of production decrease in bad times[12].

Hence, in bad times there is a reduction in total expected instantaneous cash flows from two channels. On the one hand, demand decreases from a reduction in $A(\theta)$. On the other, average profits also decrease from an increase in expected average costs. Finally, I assume changes in C_r are less sensitive to fundamentals than changes in C_s.

[11] For tractability reasons we assume fundamentals only affect the profitability of projects and not the probabilities of success. However, assuming for example that higher fundamentals increase probabilities of success from playing safe respect than from playing risky (i.e., p_s in relation to p_r) we would obtain the same results but with the complication that reputation updating varies in different states of the economy.

[12] Note this always happens with fixed costs of production when facing a demand reduction

Assumption 2 $\frac{\partial C_s}{\partial \theta} < \frac{\partial C_r}{\partial \theta} \leq 0$

What is really important about the assumption is not the direction of the inequality but rather the monotonic change in incentives as fundamentals vary. Unlike other reputation models, we allow incentives behind moral hazard and adverse selection to differ in different stages of the cycle. In this particular case, the assumption means that playing risky is more attractive in bad times than in good times. When fundamentals weaken, expected instantaneous cash flows decrease more when using safe technologies than risky ones, for example if in the latter case, firms adjust easily to a changing environment.

The particular direction of the assumption can be justified in two ways. First, under limited liability, there will be a maximum cost $\bar{c} = P$ above which it is not possible for the firm to cover the consequences of its actions (such as fines, demands, etc.). Under this situation, even if $\frac{\partial C_s}{\partial \theta} = \frac{\partial C_r}{\partial \theta}$, since $c_{r,b}$ is the highest possible cost, it binds first with $\bar{c}$. Hence, in expectation, average costs from safe actions effectively increase faster than average costs from risky actions in bad times. Since potential losses are bounded while potential gains are not, it is more attractive to take risk in bad times than in good times. In other words, the highest variance distribution gets truncated faster in its left tail. Second, in bad times experimentation in new production procedures is cheaper. This idea has been extensively discussed since Schumpeter, who believed recessions are good opportunities for firms to innovate and try new ways to produce.

2.2 Timing

The timing of the model is:

- Firms, lenders and consumers observe the reputation level ϕ of all firms. Firms acquire a loan of 1 from lenders. Lenders' outside option is a risk free interest rate $\overline{R} > 1$
- Fundamentals θ (that only affect profits) are realized by everybody in the economy[13].
- The firm decides between following safe (s) or risky (r) technologies.
- Production occurs and the firm either continues (c) or dies (d).
- If the firm dies, it defaults on the loan.

[13]The timing in which fundamentals are observed will be relevant later to select a unique equilibrium. An alternative, and possibly more realistic, assumption is that a subset of fundamentals is observed before the loan is negotiated while another subset is observed before production but after lending

- If the firm continues, it pays to lenders the negotiated interest rate $R > 1$ and sell the product to consumers at a price P.
- After the sale, the firm generates good (g) or bad (b) signals of its actions. Lenders and consumers observe those unverifiable signals and use them to update the reputation from ϕ to ϕ'

This is the timing in each period. Since reputation only makes sense in a dynamic context, the game will consist in many repeated periods with this sequence of interactions and decisions. We will discuss the results of the model in both finite and infinite horizon versions of the repeated game.

2.3 Definition of Equilibrium

Before formally defining the equilibrium, we discuss some preliminaries concerning the properties of reputation updating by lenders and consumers and the definition of the value function that firms maximize.

2.3.1 Reputation Updating

When updating a firm's reputation, lenders and consumers have a prior about the firm's reputation and have beliefs about whether the firm plays risky. These two ingredients deserve a detailed explanation.

The model assumes lenders and consumers receive a public signal g or b about the firm's actions[14]. Unfortunately, a model with common, public realizations has many equilibria where reputation does not have the asset characteristics we are focusing on and where an implausible degree of coordination between firm behavior and the market belief about the firm behavior is required[15]. I eliminate these equilibria by requiring behavior to be Markov. However, even when restricting attention to Markovian strategies, reputation formation still depends on beliefs about the probabilities the firm plays risky.

[14] The obvious and natural alternative is that each agent in the market receives an idiosyncratic signal. However the idiosyncrasy of signals present the same problems to analyze than model of private monitoring. Obstructing the ability to coordinate continuation play, it imposes serious constraints on the ability to construct equilibria. See a complete discussion in Mailath and Samuelson (2006), Ch. 18

[15] One of these equilibria can be, for example, to play safe for certain fundamentals until the first bad result happens and then play risky afterward. In this particular equilibrium reputation does not exist as we interpret it, and beliefs about firms' behavior requires implausible degrees of complexity and coordination. See discussion in Mailath and Samuelson (2001)

Since we focus on Markovian strategies, the sufficient statistic about the firm's type is the reputation level ϕ. Let $x(\phi, \theta)$ be the probability a strategic firm with reputation ϕ plays risky when the fundamental is θ. Additionally, let $\widehat{x}(\phi, \theta)$ be lenders and consumers' beliefs about the probability a strategic firm with reputation ϕ plays risky when the fundamental is θ. By Bayesian updating, after observing a continuing firm generating good signals,

$$Pr(\mathcal{S}|c, g) = \phi_g(\phi, \widehat{x}) = \frac{[p_r \alpha_r \widehat{x} + p_s \alpha_s (1 - \widehat{x})]\phi}{[p_r \alpha_r \widehat{x} + p_s \alpha_s (1 - \widehat{x})]\phi + p_r \alpha_r (1 - \phi)} \tag{3}$$

and, after observing a continuing firm generating bad signals,

$$Pr(\mathcal{S}|c, b) = \phi_b(\phi, \widehat{x}) = \frac{[p_r(1 - \alpha_r)\widehat{x} + p_s(1 - \alpha_s)(1 - \widehat{x})]\phi}{[p_r(1 - \alpha_r)\widehat{x} + p_s(1 - \alpha_s)(1 - \widehat{x})]\phi + p_r(1 - \alpha_r)(1 - \phi)} \tag{4}$$

where ϕ_g is the posterior after the observation the firm continued with good signals and ϕ_b the posterior after the observation the firm continued with bad signals, given a prior ϕ.

2.3.2 Profits

For the moment, we will focus on the static problem that firms have to solve just assuming a fixed stream of continuation values for different ϕ in the future. I impose three restrictions on continuation values $V(\phi)$. First, they are well-defined. That this is indeed the case will be shown in Section 5, where a fully fledged dynamic model is considered. Second, they are positive, which is clear since profits are bounded below by zero. Finally, they are monotonically increasing in the reputation level ϕ. Even when this seems a natural assumption because reputation is a valuable asset, it is also a useful assumption for expositional purposes. In Section 5, I discuss the conditions for this assumption to hold and why it is convenient to discuss the results but not critical to obtain them.

Total discounted profits for a given reputation level ϕ and observed fundamental θ, conditional on the probability of risk-taking x and on market's beliefs $\widehat{x}$ about that probability of risk-taking, are:

$$\begin{aligned}\widehat{V}(\phi, \theta | x, \widehat{x}) = \quad & x \quad [p_r[\Pi_r(\theta) - R(\phi)] + \beta p_r \left[\alpha_r \mathbf{V}(\phi_{g_{(\phi,\widehat{x})}}) + (1 - \alpha_r)\mathbf{V}(\phi_{b(\phi,\widehat{x})})\right]] \\ & + (1 - x)[p_s[\Pi_s(\theta) - R(\phi)] + \beta p_s \left[\alpha_s \mathbf{V}(\phi_{g_{(\phi,\widehat{x})}}) + (1 - \alpha_s)\mathbf{V}(\phi_{b(\phi,\widehat{x})})\right]]\end{aligned} \tag{5}$$

where

$$V(\phi, \theta | \widehat{x}) = \max_{x \in [0,1]} \widehat{V}(\phi, \theta | x, \widehat{x})$$

and $\mathbf{V}({\phi'}_{(\phi,\widehat{x})}) = \int_{-\infty}^{\infty} V(\phi', \theta'|\widehat{x}')v(\theta')d\theta'$ are elements of a given stream of continuation values $\Upsilon' = \{\mathbf{V}(\phi')\}_{\phi'=0}^{1}$

Note the value function depends on the reputation level (ϕ), fundamentals (θ), and beliefs the market assigns to the firm playing risky ($\widehat{x}$). Naturally, in equilibrium a strategy for firms uniquely determines the equilibrium updating rule the market must use if their beliefs are to be correct (i.e., $x = \widehat{x}$).

In what follows I focus on cutoff strategies in which the firm will decide to play risky if it observes a fundamental below a certain threshold and safe if it observes a fundamental above that threshold, such that

$$x(\phi, \theta) = \begin{cases} 0 & if \quad \theta > k^*(\phi) \\ 1 & if \quad \theta < k^*(\phi) \end{cases} \tag{6}$$

In Section 3 we show that even restricting attention to this type of strategies, we have a multiplicity of equilibria when fundamentals are common knowledge. In Section 4 we show that introducing noise in the observation of fundamentals, the unique equilibrium surviving iterated elimination of dominated strategies as the precision of signals goes to infinity, is a threshold strategy of this type. Intuitively this result arises from the monotonicity assumption of the relation between incentives to play safe and fundamentals.

2.3.3 Equilibrium

Definition 1 *A Markov perfect equilibrium in cutoff strategies is: cutoffs $k^*(\phi)$, interest rates $R(\phi)$ and posteriors ϕ_g and ϕ_b, such that*

- *Each firm with reputation ϕ observes θ and chooses $x^*(\phi, \theta)$ to maximize $\widehat{V}(\phi, \theta|x, \widehat{x})$ (given by equation 10) following a cutoff strategy such that*

$$x^*(\phi, \theta) = \begin{cases} 0 & if \quad \theta > k^*(\phi) \\ 1 & if \quad \theta < k^*(\phi) \end{cases}$$

- *Lenders charge $R(\phi)$ such that they obtain $\overline{R}$ in expectation.*
- *Posteriors ϕ_g and ϕ_b are updated using Bayes' Rule (equations 3 and 4).*
- *A strategy for ϕ firms uniquely determines the equilibrium interest rates and updating rule the market must use if their beliefs are to be correct (i.e., $\widehat{x}(\phi, \theta) = x^*(\phi, \theta)$).*

3 Multiplicity with Complete Information about Fundamentals

In this section we show there is a continuum of Markovian perfect equilibria in monotone cutoff strategies when firms perfectly observe fundamentals. This result arises from the impossibility of pinning down a unique belief for lenders and consumers to use in updating firms' reputation.

To achieve this result, we first discuss the dependence of the value and formation of reputation on lenders and consumers' beliefs about firms' actions. Then we discuss properties of the differential gains from playing safe rather than risky that are used to determine firms' optimal actions. Finally, we show equilibrium multiplicity in each period for a given stream of continuation values and discuss how this multiplicity problem becomes more serious as the horizon of the game grows.

3.1 Reputation and Beliefs

The next Proposition shows the role of reputation and beliefs as a source of multiplicity.

Proposition 1 *For a given reputation level ϕ and a fundamental θ, the reputation formation (measured by $\phi_g - \phi_b$) and the reputation value (measured by $R(\phi)$ and $P(\phi)$) decrease as lenders and consumers assign a greater probability the firm plays risky (i.e., greater $\widehat{x}(\phi, \theta)$).*

In the next subsections, we show this proposition by parts, first focusing on the formation of reputation and then in the value of reputation. Finally, we discuss the place of this result within the reputation literature.

Intuitively, when lenders and consumers assign a low probability of the firm playing risky, good signals are also signals that a firm had played safe with high probability and then it is more likely the firm is strategic. In this case, learning is easier and playing safe is a good way to increase probabilities of having good results and to increase reputation. On the contrary, when the market assigns a high probability of the firm playing risky, good signals are attributed to good luck rather than the use of safe procedures. In this case, since learning is difficult, firms do not have incentives to increase the probability of generating good signals by playing safe.

The value of reputation also depends on beliefs about risk-taking. If lenders believe it is very likely strategic firms play risky, they will charge high interest rates since it is less likely in expectation to recover the loan. If they believe firms play safe, they will charge low interest rates. Similarly, if consumers believe strategic firms played risky, the willingness to pay for the product is low because it is less likely to be a good product. However, if they believe strategic firms played safe, the willingness to pay for the product is higher for high reputation levels since it is more likely to enjoy good products.

Hence, reputation is a valuable asset, not because it represents an assumed intrinsic valuable characteristic, but because it increases instantaneous cash flows and reduces expected future interest rates by having access to safe actions. However, the magnitude of these effects in a given period depends heavily on the beliefs about the firm playing risky in that period. This property is the main source of multiplicity. If lenders and consumers believe the firm plays risky, not only do not update the reputation but also the reputation does not have any effect on increasing instantaneous cash flows or in reducing interest rates. This eliminates the deterring effects of reputation on risk-taking by making firms more prone to take risks. Contrarily, if lenders and consumers believe firms plays safe, both value and formation of reputation is important, preventing risk-taking by making firms more likely to play safe.

3.1.1 Reputation Formation

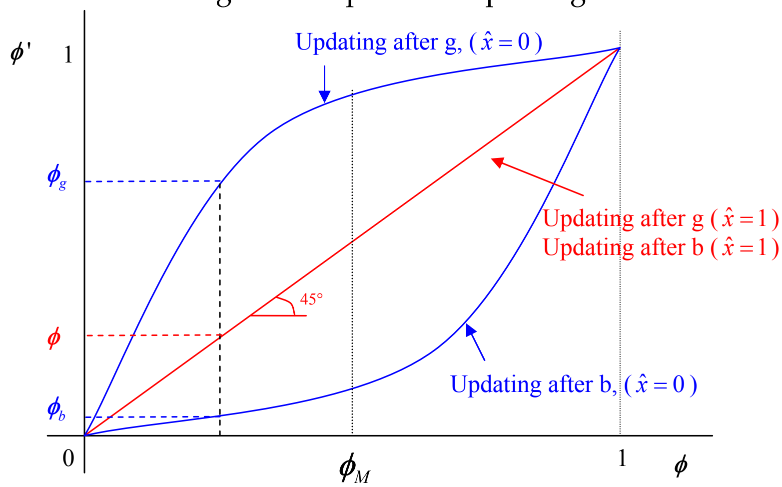
In this setting, the formation of reputation depends heavily on the beliefs of lenders and consumers about firm's actions. This is because reputation is not understood as the possession of an intrinsically valuable characteristic but the possession of a characteristic that only has value if it is really used.

Note from equations 3 and 4 that $\phi_g = \phi_b = \phi$ when $\widehat{x} = 1$ and $\phi_g \geq \phi \geq \phi_b$ when $\widehat{x} \leq 1$, with the gap $\phi_g - \phi_b$ increasing as $\widehat{x}$ goes to 0. Graphically, reputation evolves as in Figure 1. Reputation priors ϕ are represented in the horizontal axis and reputation posteriors ϕ' are represented in the vertical axis. Take, for example, the case in which lenders and consumers believe strategic firms play safe for sure (i.e., $\widehat{x} = 0$). In this case, given a current reputation level ϕ, the gain in terms of reputation of generating good signals rather than bad signals is determined by the gap $\phi_g - \phi_b$. Contrarily, when lenders and consumers believe strategic firms play risky for sure (i.e., $\widehat{x} = 1$), there is no gain in terms of reputation from generating good signals rather than bad ones. Recall also that when $\phi = 0$ or $\phi = 1$ there is no updating, no matter the signals nor the beliefs about the firm's actions. Contrarily, the maximum updating gap $(\phi_g - \phi_b)$ is obtained at an intermediate value ϕ_M for any value of $\widehat{x} < 1$.

3.1.2 Reputation Value

Now we will discuss the value of reputation in increasing expected profits by reducing interest rates and instantaneous cash flows and how this value decreases as beliefs of risk-taking increase.

Figure 1: Reputation Updating



First, interest rates decrease as reputation levels ϕ increase. Since I assume loans are negotiated before knowing fundamentals, interest rates are defined by the risk free interest rate $\overline{R}$ divided by the expected probability of continuation. Hence,

$$R(\phi) = \frac{\overline{R}}{Pr(c|\phi)} \tag{7}$$

where

$$Pr(c|\phi) = \int_{-\infty}^{\infty} [(p_r \widehat{x}(\phi, \theta) + p_s(1 - \widehat{x}(\phi, \theta)))\phi + p_r(1 - \phi)]v(\theta)d\theta$$

Note that $R(\phi) \in [\frac{\overline{R}}{p_s}, \frac{\overline{R}}{p_r}]$ and that $\frac{\partial R(\phi)}{\partial \phi} < 0$ for a fixed $\widehat{x}$. This is important because it is the first reason why firms would like to build and maintain reputation. For a given $\widehat{x}$, high reputation levels imply lenders charge lower interest rates to firms.

Since we are focusing on cutoff strategies, from equation 6 beliefs $\widehat{x}(\phi, \theta)$ are a function of the cutoff $\widehat{k}(\phi)$ that lenders and consumers believe a firm with reputation ϕ use. Hence, interest rates can be expressed as,

$$R(\phi|\widehat{k}) = \frac{\overline{R}}{Pr(c|\phi, \widehat{k})} \tag{8}$$

where

$$Pr(c|\phi, \widehat{k}) = (1 - \phi)p_r + \phi \left[p_r \mathcal{V}(\widehat{k}(\phi)) + p_s(1 - \mathcal{V}(\widehat{k}(\phi)))\right]$$

The sufficient condition for interest rates to decrease with reputation is that cutoff beliefs $\widehat{k}(\phi)$ are non-increasing in ϕ. As we will show, this is the case in equilibrium, in which beliefs are correct (i.e., $\widehat{k}(\phi) = k^*(\phi)$).

Second, instantaneous cash flows increase as reputation levels ϕ increase, since consumers are willing to pay a higher price P for the same product. This is common in most models in which firms care about having a reputation of producing high-quality products. Rather than just assuming this relation, we obtain it from our perfect price discrimination setup. Since consumers get a per period utility of 1 from the consumption of products under good signals and 0 from the consumption of products under bad signals, they are willing to pay for the product up to their reservation value.

$$P(\phi,\theta) = (\alpha_r \widehat{x}(\phi,\theta) + \alpha_s(1-\widehat{x}(\phi,\theta)))\phi + \alpha_r(1-\phi) \tag{9}$$

Note that $P(\phi,\theta) \in [\alpha_r, \alpha_s]$ and that $\frac{\partial P(\phi,\theta)}{\partial \phi} < 0$ for a fixed $\widehat{x}$. This is an additional reason why firms care about reputation. For a given $\widehat{x}(\phi,\theta)$, high reputation imply firms can charge higher price for their products. Again, since we are focusing on cutoff strategies, we can express prices also as,

$$P(\phi,\theta|\widehat{k}) = \begin{cases} (1-\phi)\alpha_r + \phi\alpha_s & if \quad \theta > \widehat{k}(\phi) \\ \alpha_r & if \quad \theta < \widehat{k}(\phi) \end{cases}$$

Similarly to the interest rates case, the sufficient condition for expected prices to increase with reputation is that cutoff beliefs $\widehat{k}(\phi)$ are non-increasing in ϕ. As we will show, this is the case in equilibrium, in which beliefs are correct (i.e., $\widehat{k}(\phi) = k^*(\phi)$).

Hence interest rates R and instantaneous cash flows Π_s and Π_r can be written fully explicitly as functions of fundamentals and reputation levels, $R(\phi,\overline{R})$, $\Pi_s(\phi,\theta)$ and $\Pi_r(\phi,\theta)$

3.1.3 Relation with the Literature

In this model, reputation is not intrinsically valuable, as would be the case of talents, quality or skills but it is defined by access to actions. To take advantage of a reputation of having access to a safe technology, lenders and consumers must also believe that the firm will in fact decide to play safe in that period. It is worthless to be seen as a firm that can choose if at the same time lenders and consumers believe the choice will be to play risky, the same action taken by firms that do not have a choice.

Since risk-taking is the product of a certain action rather than an intrinsic characteristic, reputation should be defined both using adverse selection (lenders and consumers do not know if the firm has the possibility to choose or not) and moral hazard (actions have value in themselves to lenders and consumers other than being just signals). A more general setting should allow reputation to be also a signal of the possession of an intrinsic value. We may think, for example, that a firm that knows how to play safe is also a firm that can produce better products simply because its managers are talented people. Assuming this extra effect would reinforce the monotonicity of the continuation values on ϕ, sustaining the main results. However, it also adds unnecessary elements to the exposition of the main conclusions[16].

This model differs importantly from other models relating reputation and risk-taking. Here we will discuss the main differences with the two most relevant related papers, Diamond (1989) and Mailath and Samuelson (2001)[17].

Diamond (1989) considers three types of firms - naturally risky, naturally safe, and strategic firms that can choose between risky or safe projects. He shows that strategic firms may choose safe projects and forego profits to enjoy lower interest rates in the future. A difference with our model is that firms signal their type just by continuing in business, hence reputation can only increase over time, being undistinguishable from age. Introducing a second set of signals after continuation, our model allows reputation to be constructed, destroyed and managed.

This last characteristic of our model is related to Mailath and Samuelson (2001) who in a different setting study a problem where reputation can vary depending on results and where strategic types try to separate from "bad" types. However, they do not consider firms can die as a result of their actions, not capturing continuation incentives on decisions.

Our model differ from these two paper in three important dimensions. First, it incorporates elements of continuation (as in Diamond (1989)) and elements of reputation based on results (as in Mailath and Samuelson (2001)). On the one hand, the combination of these two types of incentives in a single framework is critical to the main fragility result, hence being more than the sum of the parts. On the other hand it allows us to separate the interactions of lenders and consumers with the firm.

Second, our model let incentives for opportunistic behavior to vary with aggregate fundamentals. This is in stark contrast with both Diamond (1989) and Mailath and Samuelson

[16]Nevertheless, when relevant, we will show along the exposition how strengthening reputation also as an intrinsically valuable element reinforce the results

[17]Another relevant paper for us, even when not closely related, is Holmstrom (1999). He suggests that managers' incentives for risk-taking depend on their career concerns. When proposing projects, managers send to owners imperfect signals about their talent to determine which ones are good projects. The better the perceived talent the higher future wages. When wages are linearly related to talents and managers are risk neutral, they are indifferent about risk-taking decisions. However, if managers are risk averse, they prefer to propose that no investment should be taken, avoiding the risk of having a bad result. Since uncertainty is shared by the manager and the owners a "nicely behaved" pure strategy equilibrium always exists.

(2001) who analyze reputation in an invariant situation. This improvement sheds light on reputation effects over cycles.

Finally, as in these two papers, ours suffer a multiple equilibria problem. Diamond (1989) deals with multiplicity by focusing on the evolution of extreme equilibria (i.e., cases in which all strategic firms play risky or all of them play safe). Mailath and Samuelson (2001) only focuses on discussing the properties and conditions of the best equilibrium, the one that eliminates inefficiency completely. In our case, the introduction of fundamental-driven incentives naturally lead us to the use of a dynamic global games approach to select a unique equilibrium, which is robust to small perturbations in information about the state of the economy. This uniqueness is important to characterize risk-taking behavior by firms over economic cycles and to analyze the efficiency effects of reputation.

3.2 Differential gains from playing safe

Given cutoff strategies, we can redefine beliefs of risk taking at each fundamental θ as a function of cutoff beliefs. Following equation 6, $\widehat{x}(\phi,\theta)$ is a function of $\widehat{k}(\phi)$. Total discounted profits for a given reputation ϕ and fundamental θ, conditional on the probability of risk-taking x and on cutoff beliefs $\widehat{k}$, are:

$$\begin{aligned}\widehat{V}(\phi,\theta|x,\widehat{k}) = \quad x \quad & [p_r[\Pi_r(\theta) - R(\phi|\widehat{k})] + \beta p_r \left[\alpha_r \mathbf{V}(\phi_{g_{(\phi,\widehat{x}|\widehat{k})}}) + (1-\alpha_r)\mathbf{V}(\phi_{b_{(\phi,\widehat{x}|\widehat{k})}})\right]] \\ +(1-x) & [p_s[\Pi_s(\theta) - R(\phi|\widehat{k})] + \beta p_s \left[\alpha_s \mathbf{V}(\phi_{g_{(\phi,\widehat{x}|\widehat{k})}}) + (1-\alpha_s)\mathbf{V}(\phi_{b_{(\phi,\widehat{x}|\widehat{k})}})\right]]\end{aligned} \tag{10}$$

Since we are analyzing cutoff strategies we can define differential profits from playing safe rather than risky as $\Delta(\phi,\theta|\widehat{k}) = \widehat{V}(\phi,\theta|0,\widehat{k}) - \widehat{V}(\phi,\theta|1,\widehat{k})$.

$$\begin{aligned}\Delta(\phi,\theta|\widehat{k}) = p_s\Pi_s(\phi,\theta) - p_r\Pi_r(\phi,\theta) + \beta(p_s - p_r)\mathbf{V}(\phi) - (p_s - p_r)R(\phi|\widehat{k}) \\ +\beta[p_s\alpha_s - p_r\alpha_r][\mathbf{V}(\phi_{g_{(\phi,\widehat{x}|\widehat{k})}}) - \mathbf{V}(\phi)] \\ +\beta[p_r(1-\alpha_r) - p_s(1-\alpha_s)][\mathbf{V}(\phi) - \mathbf{V}(\phi_{b_{(\phi,\widehat{x}|\widehat{k})}})]\end{aligned} \tag{11}$$

These are the differential gains from playing safe for a firm ϕ that observes a fundamental θ, conditional on lenders and consumers having cutoff beliefs $\widehat{k}(\phi)$ and hence beliefs $\widehat{x}(\phi,\theta)$ of risk-taking in θ. The firm decides to play safe if $\Delta(\phi,\theta|\widehat{k}) > 0$ and risky if $\Delta(\phi,\theta|\widehat{k}) < 0$.

The next lemma shows how $\Delta(\phi,\theta|\widehat{k})$ depends on θ, $\widehat{k}$ and $\widehat{x}$

Lemma 1 $\Delta(\phi,\theta|\widehat{k})$ *is monotonically increasing in* θ*, monotonically decreasing in* $\widehat{x}$ *and monotonically non-increasing in* $\widehat{k}$

Proof We divide this proof in three steps.

• *Step 1:* $\frac{\partial \Delta(\phi,\theta|\widehat{k})}{\partial \widehat{k}} \leq 0$

Regardless of θ, it is straightforward to show, from equation 8, that $\frac{\partial R(\phi|\widehat{k})}{\partial \widehat{k}} \geq 0$

• *Step 2:* $\frac{\partial \Delta(\phi,\theta|\widehat{k})}{\partial \widehat{x}} < 0$

By decomposing the first component, we can write it explicitly as $(p_s\Pi_s(\phi,\theta) - p_r\Pi_r(\phi,\theta)) = (p_s - p_r)A(\theta)P(\phi,\theta|\widehat{k}) - [p_sC_s(\theta) - p_rC_r(\theta)]$. First, for a given θ, $\widehat{k}$ defines a $\widehat{x}$ and $(p_s\Pi_s - p_r\Pi_r)$ only depends on $\widehat{x}$ through prices. For a given ϕ and θ, as shown in Section 3.1.2, $\frac{\partial P(\phi,\theta|\widehat{k})}{\partial \widehat{x}} < 0$. Since $p_s > p_r$ and $A(\theta) > 0$, $\frac{\partial (p_s\Pi_s - p_r\Pi_r)}{\partial \widehat{x}} < 0$. Second, for a given ϕ and θ, as shown in equations (3) and (4), reputation gaps ($\phi_g - \phi$) and ($\phi - \phi_b$) decrease as $\widehat{x}$ increases. By assumption[18], $\mathbf{V}(\phi)$ is monotonically increasing in ϕ, hence $\mathbf{V}(\phi_g) - \mathbf{V}(\phi)$ and $\mathbf{V}(\phi) - \mathbf{V}(\phi_b)$ decrease as $\widehat{x}$ increases. The higher the beliefs assigned to the firm playing risky, the more difficult is the updating of reputation and the smaller the reputation gains from playing safe. Hence $\frac{\partial(\mathbf{V}(\phi_g) - \mathbf{V}(\phi))}{\partial \widehat{x}} < 0$ and $\frac{\partial(\mathbf{V}(\phi) - \mathbf{V}(\phi_b))}{\partial \widehat{x}} < 0$

• *Step 3:* $\frac{\partial \Delta(\phi,\theta|\widehat{k})}{\partial \theta} > 0$

As shown in *Step 1*, $(p_s\Pi_s(\phi,\theta) - p_r\Pi_r(\phi,\theta)) = (p_s - p_r)A(\theta)P(\phi,\theta|\widehat{k}) - [p_sC_s(\theta) - p_rC_r(\theta)]$. First total demand $A(\theta)$ increases with fundamentals. Second there is a reinforcement effect that comes through prices. As shown in the previous step, $\frac{\partial P(\phi,\theta|\widehat{k})}{\partial \widehat{x}} < 0$ and by cutoff strategies $\frac{\partial \widehat{x}}{\partial \theta} \leq 0$. Since $p_s > p_r$ and $P > 0$, then $\frac{\partial (p_s - p_r)A(\theta)P(\phi,\widehat{x})}{\partial \theta} > 0$. Finally, by assumption 2, $\frac{\partial [p_sC_s(\theta) - p_rC_r(\theta)]}{\partial \theta} < 0$ since $p_s > p_r$. Hence $\frac{\partial (p_s\Pi_s - p_r\Pi_r)}{\partial \theta} > 0$. Since the rest of components do not depend on θ, $\frac{\partial \Delta(\phi,\theta|\widehat{k})}{\partial \theta} > 0$.

Q.E.D.

Lemma 1 shows that differential gains from playing safe decrease as fundamentals go down (θ decreases) and as the beliefs of the firm playing risky go up, both in expectation ($\widehat{k}$ increases) and for a given θ ($\widehat{x}$ increases), which represents the source of multiplicity in this model. Intuitively, the reasons behind these relations are the following.

Differential gains from playing safe decrease as fundamentals weaken. When the state of the economy gets worse average costs in expectation increase less by experimenting than by following safe procedures, making more attractive to play risky in recessions.

Differential gains from playing safe decrease as beliefs of the firm playing risky increase. First, the price consumers are willing to pay for the good decreases because they assign a less probability of getting a good result. Since prices are lower, the gains from increasing the probability of remain alive by playing safe decrease. Second, in expectation default is

[18]This is an assumption for the moment since we will show this is the case in Section 5

more likely, interest rates are higher and firms are more prone to take risks since they become heavily indebted. Finally, a higher belief that the firm plays risky reduces the updating of beliefs, reduces the gain in terms of reputation from getting good results and makes less attractive to play safe.

Two important features, Uniform Limit Dominance and Single Crossing properties, can be obtained from analyzing the differential gain from playing safe.

Uniform Limit Dominance determines extreme fundamentals $\underline{\theta}(\phi|\widehat{k})$ and $\overline{\theta}(\phi|\widehat{k})$ for each reputation value ϕ when cutoff beliefs are $\widehat{k}$ such that, for all $\theta < \underline{\theta}(\phi|\widehat{k})$ it is optimal to play risky and for all $\theta > \overline{\theta}(\phi|\widehat{k})$ it is optimal to play safe, no matter what lenders and consumers believe firms decide given that fundamental θ (i.e., no matter $\widehat{x}(\phi,\theta)$). While $\underline{\theta}(\phi|\widehat{k})$ is obtained for $\widehat{x}=0$ in which reputation is heavily updated, $\overline{\theta}(\phi|\widehat{k})$ is obtained for $\widehat{x}=1$ in which reputation does not change.

Lemma 2 *(Uniform Limit Dominance)*

- *For each ϕ and $\widehat{k}$, $\exists\underline{\theta}(\phi|\widehat{k})$ such that $\Delta(\phi,\underline{\theta}|\widehat{k},\widehat{x}=0)=0$*
- *For each ϕ and $\widehat{k}$, $\exists\overline{\theta}(\phi|\widehat{k})$ such that $\Delta(\phi,\overline{\theta}|\widehat{k},\widehat{x}=1)=0$*

Following this notation, we can define $\underline{\theta}(\phi|-\infty)$ the value of θ for which it is indifferent to play risky or safe if $\widehat{x}=0$ and the lowest possible interest rate for that ϕ is charged ($R(\phi|-\infty)=\frac{\overline{R}}{p_r(1-\phi)+p_s\phi}$). We can also define $\overline{\theta}(\phi|\infty)$ the value of θ for which it is indifferent to play risky or safe if $\widehat{x}=1$ and the highest possible interest rate for that ϕ is charged ($R(\phi|\infty)=\frac{\overline{R}}{p_r}$). Naturally, $\underline{\theta}(\phi|-\infty)\leq\underline{\theta}(\phi|\widehat{k})$ and $\overline{\theta}(\phi|\infty)\geq\overline{\theta}(\phi|\widehat{k})$ for all $\widehat{k}$.

The following two lemmas describe Single Crossing properties, which allows us to identify a unique cutoff in the set of fundamentals (θ) and on the set of beliefs ($\widehat{x}$) that make a particular firm indifferent between playing risky or safe, given a fixed $\widehat{k}$.

Lemma 3 *(State monotonicity) For every reputation level ϕ and cutoff belief $\widehat{k}$, fix a $\widehat{x}(\phi,\theta)$ for all θ and there exists a unique $\theta^*\in[\underline{\theta}(\phi|\widehat{k}),\overline{\theta}(\phi|\widehat{k})]$ such that $\Delta(\phi,\theta|\widehat{k},\widehat{x})<0$ for $\theta<\theta^*$, $\Delta(\phi,\theta|\widehat{k},\widehat{x})=0$ for $\theta=\theta^*$ and $\Delta(\phi,\theta|\widehat{k},\widehat{x})>0$ for $\theta>\theta^*$. Furthermore, θ^* is increasing in $\widehat{k}$ and $\widehat{x}$.*

Proof By Lemma 1 $\Delta(\phi,\theta|\widehat{k})$ is increasing in θ and by Assumption 2 there is a unique crossing on the space of fundamentals since, as they rise, the value of playing risky increases monotonically at a lower rate than the value of playing safe. Hence there is a unique θ^* such that $\Delta(\phi,\theta^*|\widehat{k},\widehat{x})=0$. Since $\widehat{x}\in[0,1]$ then, by definition, $\theta^*\in[\underline{\theta}(\phi|\widehat{k}),\overline{\theta}(\phi|\widehat{k})]$. By Lemma 1, $\Delta(\phi,\theta|\widehat{k})$ is decreasing in $\widehat{k}$ and $\widehat{x}$, then θ^* is increasing in $\widehat{k}$ and $\widehat{x}$. If the beliefs of the firm playing risky or the interest rate increase, the firm will strictly prefer to play risky at the previous θ^*, requiring an increase to recover the indifference. Q.E.D.

Lemma 4 *(Belief single crossing) For every reputation level ϕ and cutoff belief $\widehat{k}$, fix a $\theta \in [\underline{\theta}(\phi|\widehat{k}), \overline{\theta}(\phi|\widehat{k})]$ and there exists a unique $\widehat{x}^*$ such that $\Delta(\phi, \theta|\widehat{k}, \widehat{x}) > 0$ for $\widehat{x} < \widehat{x}^*$, $\Delta(\phi, \theta|\widehat{k}, \widehat{x}) = 0$ for $\widehat{x} = \widehat{x}^*$ and $\Delta(\phi, \theta|\widehat{k}, \widehat{x}) < 0$ for $\widehat{x} > \widehat{x}^*$. Furthermore, $\widehat{x}^*$ is increasing in θ.*

Proof By Lemma 1 $\Delta(\phi, \theta|\widehat{k})$ is monotonically decreasing in $\widehat{x}$. This ensures there is a unique crossing in beliefs $\widehat{x}$. Hence there is a unique $\widehat{x}^*$ such that $\Delta(\phi, \theta|\widehat{k}, \widehat{x}^*) = 0$, where $\widehat{x}^* \in [0, 1]$. Since, by Lemma 1, $\Delta(\phi, \theta|\widehat{k})$ is increasing in θ, so is $\widehat{x}^*$. If fundamentals improve the firm will strictly prefer to play safe at $\widehat{x}^*$, requiring an increase in the beliefs the firms plays risky $\widehat{x}^*$ to recover the indifference. Q.E.D.

3.3 Multiple Equilibria

The model exhibits multiple equilibria when firms perfectly observe fundamentals.

Proposition 2 *For all reputation levels $\phi \in (0, 1)$, there is a continuum of equilibrium strategy cutoffs $k^*(\phi) \in [\underline{\theta}(\phi|\underline{\theta}), \overline{\theta}(\phi|\overline{\theta})]$. For reputation $\phi = 1$ there is finite multiple equilibria when $\gamma_\theta \to 0$. For reputation $\phi = 0$, there is always a unique equilibrium cutoff $k^*(0)$.*

Proof The Proposition follows directly from lemmas 2 - 4. A cutoff $k^*(\phi)$ is an equilibrium strategy only if it's a best response for any realization of the fundamental θ. Take a cutoff $k^*(\phi)$ such that $k^*(\phi) \in [\underline{\theta}(\phi|k^*), \overline{\theta}(\phi|k^*)]$. The existence of such a case is guaranteed by Lemma 2. From the cutoff strategy, $x(\phi, \theta) = 0$ for all $\theta > k^*(\phi)$ and $x(\phi, \theta) = 1$ for all $\theta < k^*(\phi)$. From Lemma 4, at $\theta = k^*(\phi)$, indifference occurs at some $0 < x^*(\phi, k^*) < 1$. The cutoff $k^*(\phi)$ is an equilibrium because, for all $\theta > k^*(\phi)$, $\Delta(\phi, k^*|k^*) > 0$ and hence it is optimal for the firm to play safe (i.e., $x(\phi, \theta) = 0$). Similarly, for all $\theta < k^*(\phi)$, $\Delta(\phi, k^*|k^*) < 0$ and hence it is optimal for the firm to play risky (i.e., $x(\phi, \theta) = 1$). Now take an arbitrarily close cutoff $k^{**}(\phi) = k^*(\phi) + \varepsilon$ such that $0 < R(k^{**}(\phi)) - R(k^*(\phi)) < \delta$, where an arbitrarily small $\varepsilon > 0$ allows to define an arbitrarily small δ. By the discontinuity on beliefs (sudden jump from $x = 1$ to $x = 0$ at $\theta = k^*$)and the same reasoning described above, $k^{**}(\phi)$ is also an equilibrium cutoff strategy. Inductively it is possible to define a continuum of equilibrium strategy cutoffs.

The bounds of the equilibrium cutoffs $[\underline{\theta}(\phi|\underline{\theta}), \overline{\theta}(\phi|\overline{\theta})]$ are determined in the following way. $\underline{\theta}(\phi|\underline{\theta})$ is the value of the cutoff that determines an interest rate $R(\underline{\theta})$ and considers the gains from reputation ($\widehat{x} = 0$). Similarly, $\overline{\theta}(\phi|\overline{\theta})$ is the value of the cutoff that determines a higher interest rate $R(\overline{\theta})$ and does not consider the gains from reputation ($\widehat{x} = 1$). The condition for these bounds to be unique and all $\theta \in [\underline{\theta}(\phi|\underline{\theta}), \overline{\theta}(\phi|\overline{\theta})]$ to constitute an equilibrium is that $p_s \frac{\partial \Pi_s}{\partial \theta} - p_r \frac{\partial \Pi_r}{\partial \theta} \geq \frac{\partial R(\phi|k^*)}{\partial k^*}$. This condition basically requires interest rates do not jump suddenly with changes in cutoffs, or in other words, since $\mathcal{V}(k^*)$ determines $R(\phi|k^*)$, the distribution of fundamentals has a variance big enough.

For $\phi = 1$ the only source of possible multiplicity comes from different fixed points of beliefs $\widehat{k} = b(\widehat{k})$, where $b(\widehat{k})$ is the best response to cutoff beliefs $\widehat{k}$. In this case there is not continuum of equilibria since there is no discontinuity of differential payoffs generated by reputation (i.e., $\Delta(1, \theta|\widehat{k}, \widehat{x} = 0) = \Delta(1, \theta|\widehat{k}, \widehat{x} = 1)$). A unique equilibrium exists when there is no jumps of lending rates as cutoffs change $p_s \frac{\partial \Pi_s}{\partial \theta} - p_r \frac{\partial \Pi_r}{\partial \theta} \geq \frac{\partial R(\phi|k^*)}{\partial k^*}$.

For $\phi = 0$ there is a unique equilibrium because there is a unique possible interest rate given by $R(0) = \frac{\overline{R}}{p_r}$. Furthermore, reputation updating does not happen. Hence, $\Delta(0, \theta|\widehat{k}, \widehat{x} = 0) = \Delta(0, \theta|\widehat{k}, \widehat{x} = 1)$ for $R(0)$ and $[\underline{\theta}(0|\underline{\theta}), \overline{\theta}(0|\overline{\theta})]$ collapses into a singleton given by the unique equilibrium strategy cutoff $k^*(0)$ Q.E.D.

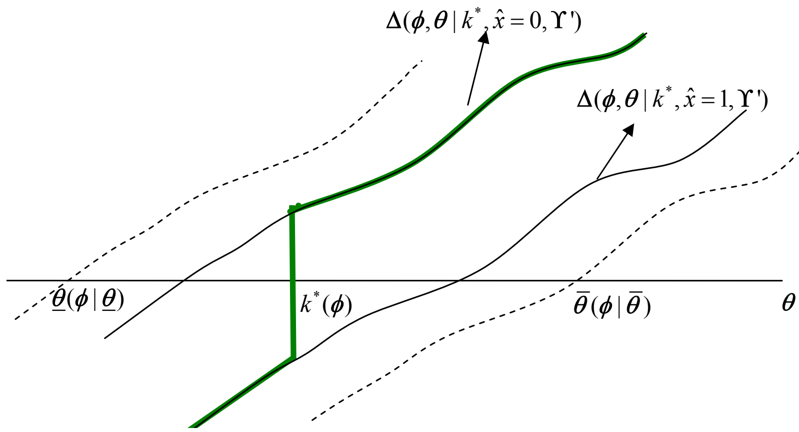
This multiplicity characterized by a continuum of equilibrium cutoffs for each ϕ is pervasive to draw conclusions about the effectiveness of reputation to deter excessive risk-taking. Since each cutoff represents a different ex-ante probability that ϕ firms take risks ($\mathcal{V}(k^*(\phi))$), equilibria are ranked in terms of firm's instantaneous cash flows, prices paid by consumers and interest rates charged by lenders. The higher the equilibrium cutoff, the more likely it is for firms to take risks, the lower the price consumers are willing to pay in expectation and the higher the interest rates charged by lenders.

An intuitive explanation of the multiplicity is provided through Figure 2. Take a proposed cutoff $k^*(\phi)$ for some reputation level $\phi \in (0, 1)$ such that $k^*(\phi) \in [\underline{\theta}(\phi)|k^*), \overline{\theta}(\phi)|k^*)]$. This is an equilibrium because it is optimal to play safe for all $\theta > k^*(\phi)$ (since $\Delta(\phi, \theta|k^*, \widehat{x} = 0) > 0$ for all $\theta > k^*(\phi)$) and it is optimal to play risky for all $\theta < k^*(\phi)$ (since $\Delta(\phi, \theta|k^*, \widehat{x} = 1) < 0$ for all $\theta < k^*(\phi)$). The function $\Delta(\phi, \theta|k^*)$ for different fundamentals is the bald function with a discontinuity at $k^*(\phi)$ in Figure 2. The cutoff $k^*(\phi)$ is an equilibrium strategy because it is a best response for any realization of the fundamental θ such that beliefs are correct.

An arbitrarily small increase in the cutoff generates an arbitrarily small increase in the interest rate. If interest rates do not change suddenly, they cannot overcome the discontinuity generated by the reputation effects that sudden changes in beliefs generate. Hence, it is possible to find equilibrium cutoffs arbitrarily close and hence a continuum of equilibria. As we move the cutoff to the right of $k^*(\phi)$, interest rates increase, reducing $\Delta(\phi, \theta|\widehat{k})$ for all θ[19]. Given the discontinuity introduced by reputation at the equilibrium cutoff strategy, the new cutoffs constitute equilibria until $\overline{\theta}(\phi|\overline{\theta})$ is reached. The same is true as we decrease cutoffs from k^* towards $\underline{\theta}(\phi|\underline{\theta})$. These extremes, determined by extreme beliefs and lending rates, constitute bounds to equilibrium cutoffs.

[19]Recall that, by assuming $p_s \frac{\partial \Pi_s}{\partial \theta} - p_r \frac{\partial \Pi_r}{\partial \theta} \geq \frac{\partial R(\phi|k^*)}{\partial k^*}$, I impose changes in payoffs to be greater than changes in interest rates, for a given change in fundamentals, making this process smooth.

Figure 2: Equilibria Multiplicity



This is the typical result of multiple equilibria in reputation settings in which "strategic" types try to separate from "bad" types rather than pooling with "good" types. The multiplicity relies heavily on the impossibility of pinning down beliefs to update reputation, as discussed in Mailath and Samuelson (2006), Mailath and Samuelson (2001) and Diamond (1989).

Up to this point we have highlighted the multiplicity that arises in a given period, for a given stream of value functions assigned to the future Υ'. Once we introduce dynamics, the multiplicity problem increases, making it very difficult to draw any conclusion about the effects of reputation on risk-taking behavior. The intuition of this result is straightforward. Since in each period multiplicity exists, multiple streams of continuation values for different ϕ, which are consistent with multiple equilibria in future periods, can be used to construct $\Delta(\phi,\theta)$. Introducing extreme continuation values determined by the highest ($\overline{\Upsilon}'$) and the lowest ($\underline{\Upsilon}'$) probability of risk-taking in all future periods for all reputation levels, it is possible to construct extreme bounds $\underline{\theta}(\phi|\underline{\theta},\overline{\Upsilon}') < \underline{\theta}(\phi|\underline{\theta},\Upsilon')$ and $\overline{\theta}(\phi|\overline{\theta},\underline{\Upsilon}') > \overline{\theta}(\phi|\overline{\theta},\Upsilon')$ such that the region of multiplicity in a given period expands when compared with the case of a unique stream of continuation values assumed so far.

It is important to distinguish the multiplicity determined just from the determination of interest rates and the multiplicity introduced by reputation. The multiplicity introduced by the determination of interest rates arises from the possibility of multiple fixed points in which the beliefs about the cutoff firms use are equal to the best response of firms to those beliefs (i.e., $\widehat{k}(\phi) = b(\widehat{k}(\phi))$). Generically this multiplicity will be finite and easy to eliminate with certain assumptions on the distribution of fundamentals[20]. The multiplicity introduced by reputa-

[20]For example, if we assume fundamentals are normally distributed $\theta \sim \mathcal{N}(E(\theta),\gamma_\theta)$, the sufficient condition for uniqueness is given by $p_s\frac{\partial \Pi_s}{\partial \theta} - p_r\frac{\partial \Pi_r}{\partial \theta} \geq \frac{\phi}{\gamma_\theta}\frac{\overline{R}(p_s-p_r)^2}{\sqrt{2\pi}[(1-\phi)p_r+\phi p_s-\phi(p_s-p_r)\mathcal{V}(\theta)]^2}$. As can be seen, when $\phi = 0$ the condition is always fulfilled (by assumption 5 the left-hand side term is positive), leading to the unique equilib-

tion incentives arises from the discontinuity of differential payoffs at the equilibrium cutoff $k^*(\phi)$. This allows for the determination of a continuum of indeterminate equilibria, which is impossible to eliminate just with assumptions on the distribution of fundamentals.

In this environment comparative statics and comparative dynamics analysis are not trivial since there is no explicit theory to guide the selection of equilibrium, leaving a big role to self-fulfilling beliefs and payoff irrelevant sunspots. However, as noted by Morris and Shin (2000), what really creates the multiplicity is the assumption of complete information and common knowledge of fundamentals that at the same time implies an implausible degree of coordination and capacity to predict rivals' behavior in equilibrium. In the following section we show that the introduction of few noise in the observation of fundamentals leads to the selection of a unique equilibrium.

4 Uniqueness with Incomplete Information about Fundamentals

In this section we slightly modify the assumption about complete information of fundamentals and the timing in which they are realized. We assume firms observe a noisy signal of the aggregate fundamental before deciding which technology to use. After production takes place, fundamentals are realized by firms, lenders, and consumers[21]. The signal observed by the firm is not observable by lenders and consumers, who can only infer it from observing the real fundamental. This modification allows us to select a unique market's belief about the probability a ϕ strategic firm takes risks. Having a unique belief, we can select a unique equilibrium in the reputation environment. More precisely, the assumptions about the information technology are

Assumption 3 *Each firm i observes a signal $z_i = \theta + \sigma\epsilon_i$ where $\epsilon_i \sim F$ identically and independently distributed across i*

Given θ the distribution of signals z is then given by $F(\frac{z-\theta}{\sigma})$.

Assumption 4 *(Monotone likelihood ratio property). For $a > b$, $\frac{f(a-\theta)}{f(b-\theta)}$ is increasing in θ*

rium discussed in Proposition 2. In general, without reputation concerns, uniqueness can be obtained when the variance γ_θ is big enough with respect to the reputation level.

[21] The assumption about the timing fundamentals are observed is important. Otherwise, if interest rates or prices reveal, through the aggregation of information by the market, the true fundamental before production, the whole point of introducing heterogeneity through signals to pin down a unique equilibrium disappears. See Atkeson (2001)

In words, this assumption means that a firm that receives a high signal, assigns a large probability that lenders and consumers believe the firm has in fact observed a high signal.

Introducing this assumption, the firm uses a cutoff strategy in the set of signals and not on the set of fundamentals, which are no longer observable. This means that for a history of fundamentals and a current signal about fundamentals z, a strategy for a firm with reputation ϕ picks a real number $z^*(\phi)$ with the interpretation that it uses safe technologies whenever $z > z^*(\phi)$ and risky ones whenever $z < z^*(\phi)$[22]. The next proposition estates that, assuming this information structure, there exists a unique Markovian perfect Bayesian equilibrium in monotone cutoff strategies for each reputation level ϕ, when signals are precise enough.

Proposition 3 *For a given ϕ, as $\sigma \to 0$, in equilibrium there exists a unique cutoff signal $z^*(\phi)$ such that $\Delta(\phi, z|z^*(\phi)) = 0$ for $z = z^*(\phi)$, $\Delta(\phi, z|z^*(\phi)) > 0$ for $z > z^*(\phi)$ and $\Delta(\phi, z|z^*(\phi)) < 0$ for $z < z^*(\phi)$, where $\Delta(\phi, z|z^*(\phi))$ are the expected differential gains from playing safe if a ϕ firm receives a signal z and lenders and consumers believe strategic firms ϕ use a cutoff $z^*(\phi)$.*

The proof is in the Appendix. Relaxing the assumption of common knowledge about fundamentals, when signals are very precise, allows us to use the approach provided by global games to select a unique belief concerning the probability that firms take risky actions. The intuitive proof is based on the iterated deletion of dominated strategies. Assume, for example, a strategic firm with reputation ϕ observes a signal $\underline{\theta}(\phi|\underline{\theta})$. In this case the firm would like to play risky even if the market uses a belief $\widehat{x}(\phi) = 0$. By receiving a low signal the firm also believes the fundamental is close to $\underline{\theta}(\phi|\underline{\theta})$. If fundamentals in fact happen to be $\underline{\theta}(\phi|\underline{\theta})$, lenders and consumers believe with some positive probability that the firm had observed a signal below $\underline{\theta}(\phi|\underline{\theta})$ hence having a belief $\widehat{x}(\phi) > 0$ (i.e., firm takes risks with some positive probability). However, with this belief, the firm would strictly prefer to play safe, not being an equilibrium a cutoff $\underline{\theta}(\phi|\underline{\theta})$. By continuity the same reasoning can be applied to signals above $\underline{\theta}(\phi|\underline{\theta})$. The same reasoning applies also to signals close to $\overline{\theta}(\phi|\overline{\theta})$.

We require $\sigma \to 0$ so the firm put more weight to its private signal than to the public signal given by the prior distribution of θ. The previous process of iterated deletion of dominated strategies results in a unique cutoff $z^*(\phi)$ such that the firm plays risky whenever $z < z^*(\phi)$ and safe whenever $z > z^*(\phi)$.

This uniqueness result remains once we consider the full-fledged dynamic model. In the next section we consider both the finite and infinite horizon game. In the finite horizon game it is always possible to define a unique sequence of equilibrium cutoffs as signals become very

[22]Recall each firm receives an idiosyncratic signal z_i. We get rid of the subindex for simplicity in notation. However, signals vary across firms and are not observed by lenders or consumers.

precise. In the infinite horizon game we can show there is a unique limit to the sequence of perfect Markovian equilibrium for the finite game.

5 Dynamics

In this section we show how to solve the model dynamically such that a unique sequence of cutoffs for each reputation level ϕ is obtained. We confirm that continuation values are well defined such that we can indeed use the propositions and proofs from previous sections, where a single period was considered. First I assume all firms live for a finite period of time T such that $\mathbf{V}_{T+1}(\phi) = 0$ for all ϕ. Afterwards I extend the results to an infinite horizon game as $T \to \infty$[23].

This extension is important for two reasons. First, reputation is an intrinsically dynamic process that must be studied dynamically to fully understand it. Second, since the previous sections were based on an assumed profile of continuation values, we must confirm they are always well defined and we must understand under what conditions they are monotonically increasing in reputation and how they may change results.

The following Lemma shows how continuation values for all reputation levels $\mathbf{V}_t(\phi)$ are indeed well defined at each period t based on the boundary condition $\mathbf{V}_{T+1}(\phi) = 0$ for all ϕ[24].

Lemma 5 *For a given reputation ϕ and a period t, as $\sigma \to 0$, $x_t(\phi, z_t) = 0$ for all $z_t < z_t^*(\phi)$ and $x_t(\phi, z_t) = 1$ for all $z_t > z_t^*(\phi)$, where $z_t^*(\phi) = f(\overrightarrow{\boldsymbol{V}}_{t+1}(\phi))$ is the unique solution to the following equation (where $\overrightarrow{\boldsymbol{V}}_{t+1}(\phi) = [\boldsymbol{V}_{t+1}(\phi_b), \boldsymbol{V}_{t+1}(\phi_g)]$)*

$$\int_0^1 \Delta_t(\phi, z_t^*(\phi)|\widehat{x}_t) d\widehat{x}_t = 0 \tag{12}$$

$\boldsymbol{V}_t(\phi)$ is given recursively by the boundary condition $\boldsymbol{V}_{T+1}(\phi) = 0$ and by

$$\begin{aligned} \boldsymbol{V}_t(\phi) = f(\overrightarrow{\boldsymbol{V}}_{t+1}(\phi)) = &\int_{-\infty}^{f(\overrightarrow{\boldsymbol{V}}_{t+1}(\phi))} p_r[\Pi_r(\phi,\theta) - R_t(\phi) + \beta \boldsymbol{V}_{t+1}(\phi)] v(\theta) d\theta \\ &+ \int_{f(\overrightarrow{\boldsymbol{V}}_{t+1}(\phi))}^{\infty} p_s[\Pi_s(\phi,\theta) - R_t(\phi) + \beta E(\boldsymbol{V}_{t+1}(\phi'))] v(\theta) d\theta \end{aligned}$$

[23] While previous sections results were obtained for a given period t, in what follows we use the same arguments but denote explicitly each period by subscripts t.

[24] In order to solve this finite dynamic global game we follow Morris and Shin (2003), Toxvaerd (2007), Giannitsarou and Toxvaerd (2007) and Steiner (2006)

Proof To show the first part of the Proposition we must show we can solve the model as a series of static games that deliver a unique equilibrium (specifically, a unique cutoff for each ϕ) in each period t. At the last period T the cutoff $z^*_T(\phi)$ will be very high in general since $\mathbf{V}_{T+1}(\phi) = 0$ for all ϕ. Hence, when solving for risk-taking at the last period T, there will not be any future punishment from a possible death or from a potential loss of reputation.

At the last period T, $\Delta_T(\phi, \theta)$ is well defined for all ϕ and θ. A cutoff $z^*_T(\phi)$ can be obtained as shown in Proposition 3, from $\int_0^1 \Delta_T(\phi, z^*_T(\phi)|\widehat{x})d\widehat{x}$ where,

$$\Delta_T(\phi, z^*_T(\phi)|\widehat{x}) = p_s\Pi_s(z^*_T(\phi)) - p_r\Pi_r(z^*_T(\phi)) - (p_s - p_r)\frac{\overline{R}}{(1-\phi)p_r + \phi(p_r\widehat{x} + p_s(1-\widehat{x}))}$$

Once $z^*_T(\phi)$ is determined, it is possible to define the equilibrium interest rate

$$R(\phi|z^*_T(\phi)) = \frac{\overline{R}}{(1-\phi)p_r + \phi[p_r\mathcal{V}(z^*_T(\phi)) + p_s(1-\mathcal{V}(z^*_T(\phi)))]}$$

and the expected continuation value at T for each reputation level ϕ. For signals $z_T < z^*_T(\phi)$ firms play risky and for signals $z_T > z^*_T(\phi)$ they prefer to play safe. As $\sigma \to 0$, total discounted expected profits at T are closely approximated by,

$$\mathbf{V}_T(\phi) = \int_{-\infty}^{z^*_T(\phi)} p_r[\Pi_r(\phi,\theta) - R_T(\phi)]v(\theta)d\theta + \int_{z^*_T(\phi)}^{\infty} p_s[\Pi_s(\phi,\theta) - R_T(\phi)]v(\theta)d\theta \qquad (13)$$

Since equilibrium strategies are well defined and unique at period T (through $z^*_T(\phi)$), continuation values $\mathbf{V}_T(\phi)$ are well defined for all ϕ.

Now consider the decision of a firm ϕ at period $T-1$. The problem to be solved at $T-1$ is essentially a static one since $\mathbf{V}_T(\phi)$ are unique and well defined for all ϕ from equation 13. Then, $\Delta_{T-1}(\phi,\theta)$ is also well defined for all ϕ and θ. Using Proposition 3 there is a unique cutoff $z^*_{T-1}(\phi)$ and it is possible to obtain a unique and well defined $\mathbf{V}_{T-1}(\phi)$ for all ϕ.

By straightforward inductive reasoning, there will exist a unique sequence of cutoffs $\{z^*_t(\phi)\}_{t=0}^T$. Furthermore there will exist a unique sequence of expected total discounted profits for each reputation level ϕ at each period t. As $\sigma \to 0$,

$$\begin{aligned}\mathbf{V}_t(\phi) = &\int_{-\infty}^{z^*_t(\phi)} p_r[\Pi_r(\phi,\theta) - R_t(\phi) + \beta\mathbf{V}_{t+1}(\phi)]v(\theta)d\theta \\ &+ \int_{z^*_t(\phi)}^{\infty} p_s[\Pi_s(\phi,\theta) - R_t(\phi) + \beta E(\mathbf{V}_{t+1}(\phi'))]v(\theta)d\theta\end{aligned}$$

Q.E.D.

The next proposition shows that continuation values are always well defined and, given the boundary condition imposed by a final period T, the sequence of equilibrium cutoffs is unique.

Proposition 4 *In a finite game with final period T and a boundary condition $\mathbf{V}_{T+1}(\phi) = 0$ for all ϕ, as $\sigma \to 0$, continuation values $\{\mathbf{V}_t(\phi)\}_0^T$ are well defined and there is a unique equilibrium for the whole game given by a unique sequence of cutoffs $\{z_t^*(\phi)\}_{t=0}^T$ for each ϕ.*

The proof of the proposition is a direct application of Lemma 5 and is based on the idea that, if the dynamic global game has a finite final period, it can be solved as a sequence of static global games.

Before extending our conclusions to an infinite period game, we discuss how the backward determination of continuation values may lead to a convergence to a fixed point in continuation values for all reputation levels ϕ in periods t far enough from T. This is relevant because these fixed points represent bounded limits required to show that there is an infinite horizon equilibrium that is a unique limit of the finite horizon Markov perfect equilibrium[25].

First, recall that strategic complementarities that generate multiplicity in the reputational model arise endogenously from reputation formation, rather than being hard-wired into static payoffs, as is standard in the global game literature. This is important because reputation levels $\phi = 0$ and $\phi = 1$ do not show a multiplicity problem since lenders and consumers beliefs do not affect reputation updating. The fixed point of value functions for these two extreme reputation levels are given by parameters only and do not depend on value functions for other reputational levels. Hence, $\mathbf{V}(0)$ and $\mathbf{V}(1)$ can be used as anchors to determine value functions for the rest of reputation levels. Then, we can obtain the conditions for fixed points from analyzing these two extreme cases[26].

Define the fixed point continuation value when firms $\phi = 0$ play safe for sure as

$$\overline{\mathbf{V}}(0|s) = \frac{p_s \Pi_s(0) - \overline{R}}{1 - \beta p_s} \tag{14}$$

where $\Pi_s(\phi) = \int_{-\infty}^{\infty} \Pi_s(\theta) v(\theta) d\theta$

Define also the fixed point continuation value when firms $\phi = 1$ play risky for sure as

$$\overline{\mathbf{V}}(1|r) = \frac{p_r \Pi_r(1) - \overline{R}}{1 - \beta p_r} \tag{15}$$

[25] I haven't examined yet the broader issue of what other equilibria there might be in the infinite horizon game

[26] We will assume the distribution of fundamentals has a variance γ_θ large enough such that there is a unique equilibrium for $\phi = 1$

Let's introduce now a technical assumption to ensure a unique steady state[27]

Assumption 5 $\overline{\boldsymbol{V}}(1|r) > \overline{\boldsymbol{V}}(0|s)$

Using this assumption, the next Lemma shows the conditions for the continuation values to converge to a unique value as we iterate backwards from a large finite period T. Furthermore, we will characterize the type of behavior consistent to those conditions.

First, define the continuation value for which $z^*(0) = E(\theta)$ as

$$\widetilde{\mathbf{V}}(0) = \frac{p_r \Pi_r(0, E(\theta)) - p_s \Pi_s(0, E(\theta))}{\beta(p_s - p_r)} + \frac{\overline{R}}{\beta p_r} \tag{16}$$

and the continuation value for which $z^*(1) = E(\theta)$ as,

$$\widetilde{\mathbf{V}}(1) = \frac{p_r \Pi_r(1, E(\theta)) - p_s \Pi_s(1, E(\theta))}{\beta(p_s - p_r)} + \frac{\overline{R}}{\beta[p_r \mathcal{V}(E(\theta)) + p_s(1 - \mathcal{V}(E(\theta)))} \tag{17}$$

It is possible to show that $\widetilde{\mathbf{V}}(1) < \widetilde{\mathbf{V}}(0)$. The intuition is that, everything else constant, $z^*(1) < z^*(0)$ and then it is necessary that $\widetilde{\mathbf{V}}(1) < \widetilde{\mathbf{V}}(0)$ to compensate and to have the same cutoff $z^*(1) = z^*(0) = E(\theta)$

Lemma 6 *Convergence to a unique continuation value for each* ϕ

- *If* $\widetilde{\boldsymbol{V}}(0) < \overline{\boldsymbol{V}}(0|s)$, *(hence* $\widetilde{\boldsymbol{V}}(1) < \overline{\boldsymbol{V}}(1|r)$*), continuation values converge to a unique* $\overline{\mathbf{V}}(\phi)$ ***for each*** ϕ. *The probability of playing safe is close to 1 for all* ϕ.
- *If* $\widetilde{\boldsymbol{V}}(1) > \overline{\boldsymbol{V}}(1|r)$, *(hence* $\widetilde{\boldsymbol{V}}(0) > \overline{\boldsymbol{V}}(0|s)$*), continuation values converge to a unique* $\overline{\mathbf{V}}(\phi)$ ***for all*** ϕ. *The probability of playing safe is close to 0 for all* ϕ.
- *If* $\widetilde{\boldsymbol{V}}(0) > \overline{\boldsymbol{V}}(0|s)$ *and* $\widetilde{\boldsymbol{V}}(1) < \overline{\boldsymbol{V}}(1|r)$, *continuation values converge to a unique* $\overline{\mathbf{V}}(\phi)$ ***for each*** ϕ *only when* $v(z^*(0)) < 1$. *The probability of playing safe is strictly between 0 and 1 for all* ϕ.

This lemma is proved and graphical intuition is provided in the Appendix. The first two bullet points correspond to cases in which continuation values converge to a unique fixed point characterized by playing safe and risky almost surely, for all ϕ. The third case is more interesting since the fixed point is characterized by a positive probability of playing both safe and risky. However to achieve convergence to a fixed point it is also necessary that the

[27] This can be justified, for example, assuming disastrous cash flows from using safe procedures in very bad times.

variance of θ be big enough to avoid a cyclical pattern. For example, if θ is distributed as a normal, the condition is given by $\frac{1}{\gamma_\theta\sqrt{2\phi}} \exp - \frac{(z^*(0)-E(\theta))^2}{2\gamma_\theta^2} < 1$. If $\gamma_\theta \to 0$ this condition will not be fulfilled and a cyclical pattern will arise. If γ_θ is big enough, this condition will be fulfilled and convergence of continuation values will arise. This is in fact a similar condition to the one required to obtain a unique equilibrium in the static model without reputation.

Under the assumptions, when information becomes very precise $\sigma \to 0$ and continuation values converge to a fixed point in a finite period game, there is an infinite horizon equilibrium that is a unique limit of finite horizon Markov perfect equilibria, for all reputation values ϕ.

Proposition 5 *If $\boldsymbol{V}_T(\phi) \to \overline{\boldsymbol{V}}(\phi)$ for all ϕ as $T \to \infty$, given $\sigma \to 0$, there exists a sequence of cutoffs $\{z_t^*(\phi)\}_{t=0}^{\infty}$ for each ϕ that is a unique limit of the finite horizon Markov perfect equilibria described in Proposition 4.*

Proof Having shown uniqueness for an arbitrary finite horizon T, we must show the same reasoning is extended as $T \to \infty$. First note the value of taking safe actions and the value of taking risky actions are bounded and well behaved monotone functions of T when continuation values converges to a fixed point $\mathbf{V}_T(\phi) \to \overline{\mathbf{V}}(\phi)$ as $T \to \infty$. Second, note also $\Delta(\phi, z_t|z_t^*(\phi))$ represents the optimal trade off between the value from playing safe and the value of playing risky. Hence $\Delta(\phi, z_t|z_t^*(\phi))$ also converges to a unique limit as $T \to \infty$. Then $z_t^*(\phi)(T) \to z_t^*(\phi)(\infty)$ as $T \to \infty$, where $z_t^*(\phi)(T)$ is the equilibrium cutoff at t far enough from T and $z_t^*(\phi)(\infty)$ is the equilibrium cutoff at t in an infinite horizon game. Q.E.D.

When continuation values converge to a fixed point, which are the cases described by Lemma 6, there is an infinite horizon equilibrium that is a unique limit of the Markov perfect equilibria in the finite horizon version of the game. Rather than using the boundary condition we can use the steady state value $\overline{\mathbf{V}}(\phi)$ for each ϕ to solve backwards. When dynamics are characterized by cyclical behavior it is not possible to use a unique continuation value as a boundary condition. Hence, there would not exist a unique limit to the Markov perfect equilibrium as defined in Proposition 4.

In the full-fledged dynamic model, the ex-ante probability of risk-taking will be uniquely determined in each period by $Pr(z < z^*(\phi))$, or when $\sigma \to 0$, by $Pr(\theta < z^*(\phi))$. Since this unique belief is used in the reputation updating, a unique equilibrium exists in the reputation model. The unique equilibrium is based on payoff relevant fundamentals, rather than payoff irrelevant sunspots or self-fulfilling beliefs, which allows us to obtain conclusions about the determinants of the probability of risk-taking and about how this probability changes in response to parameters and fundamental variations. These considerations, which cannot be performed using models with multiple equilibria or models with a unique equilibrium based on sunspots or self fulfilling beliefs, are the subjects of the next section.

6 Reputation and Risk-Taking Behavior

In previous sections we obtained a unique equilibrium and showed how steady states continuation values are determined. In this section I use the results to analyze the determinants of risk-taking, the clustering behavior of risk-taking, and the fragility of reputation to deter inefficient risk-taking.

6.1 Determinants of Risk-Taking

Recall the ex-ante probability of risk-taking is given by $Pr(\theta < z^*(\phi)) = \mathcal{V}(z^*(\phi))$ when $\sigma \to 0$. Then, we have to analyze how the cutoffs $z^*(\phi)$ react to variables such as reputation levels ϕ, interest rates $R(\phi)$ and reputation concerns.

Proposition 6 *The ex-ante probability of risk-taking for a firm with reputation level ϕ decreases as reputation rewards ($\boldsymbol{V}(\phi_g) - \boldsymbol{V}(\phi)$) and reputation punishments ($\boldsymbol{V}(\phi) - \boldsymbol{V}(\phi_b)$) increase.*

Proof From equation (11), $\frac{\partial\Delta(\phi)}{\partial(\mathbf{V}(\phi_g)-\mathbf{V}(\phi))} = \beta(p_s\alpha_s - p_r\alpha_r) > 0$ and $\frac{\partial\Delta(\phi)}{\partial(\mathbf{V}(\phi)-\mathbf{V}(\phi_b))} = \beta(p_r(1 - \alpha_r) - p_s(1 - \alpha_s)) > 0$. Since the cutoff $z^*(\phi)$ is determined by equation (28), as the reputation rewards and punishments go up, $\Delta(\phi, \theta)$ also goes up and by Lemma 3 it is required a smaller signal as a cutoff $z^*(\phi)$ in order to maintain the indifference. Hence more reputation rewards and punishments imply reductions in the ex-ante probability of risk-taking. Q.E.D.

Reputation reduces excessive risk-taking behavior, a positive role of reputation widely discussed, informally in the press and casual discussions and formally by an extensive literature in reputation. This model also delivers this result, but explicitly solving the multiplicity that arises from different possible beliefs about the firm's actions.

Proposition 7 *The ex-ante probability of risk-taking increases with interest rates.*

Proof Since $\frac{\partial\Delta(\phi)}{\partial R(\phi)} = -(p_s - p_r) < 0$, by equation 28 it is straightforward to show $\frac{\partial z^*(\phi)}{\partial R(\phi)} \leq 0$ by using Lemma 3 Q.E.D.

When interest rates increase, firms are more indebted and moral hazard problems become more relevant. Incentives to follow risky procedures and hence the inefficiency of risk-taking behavior also increase. This result suggests, for example, that firms in underdeveloped countries with high $\overline{R}$, have greater incentives to take excessive risk and reputation concerns are less effective in deterring the resultant excessive risk-taking.

Proposition 8 *The ex-ante probability of risk-taking decreases with reputation in the range $\phi \in [0, \phi_M]$. Whether the probability of risk-taking increases or decreases in the range $\phi \in [\phi_M, 1]$ depends on the rate of increase $\frac{\partial[\beta \mathbf{V}(\phi) - R(\phi)]}{\partial \phi} > 0$ when compared with the rate of decrease $\frac{\partial[\mathbf{V}(\phi_g) - \mathbf{V}(\phi_b)]}{\partial \phi} < 0$. Furthermore, $\frac{\partial^2 z^*(\phi)}{\partial \phi^2} > 0$ for all $\phi \in [0, 1]$*

Proof As shown in section 3.1.1, $(\phi_g - \phi_b)$ achieves a maximum at ϕ_M. By assumption $\mathbf{V}(\phi)$ is monotonically increasing in ϕ (this will be discussed in detail in the next section), hence $(\mathbf{V}(\phi_g) - \mathbf{V}(\phi_b))$ also achieves a maximum at ϕ_M, being $\mathbf{V}(\phi_g) = \mathbf{V}(\phi_b)$ at $\phi = 0$ and $\phi = 1$. Hence, in the range $\phi \in [0, \phi_M]$, as ϕ increases, $P(\phi)$ increases, $\mathbf{V}(\phi)$ increases, $R(\phi)$ decreases and $\mathbf{V}(\phi_g) - \mathbf{V}(\phi_b)$ increases for all $\widehat{x} < 1$. From equation (11) it is clear that when ϕ increases in the range $[0, \phi_M]$, $\Delta(\phi, z)$ goes up and the cutoff $z^*(\phi)$ decreases, reducing the probability of risk-taking.

When ϕ is in the range $[\phi_M, 1]$ still $P(\phi)$ increases, $\mathbf{V}(\phi)$ increases and $R(\phi)$ decreases as ϕ increases, increasing $\Delta(\phi, z)$. We call this effect "continuation effect". However in this range, as ϕ goes up, $\mathbf{V}(\phi_g) - \mathbf{V}(\phi_b)$ goes down, decreasing $\Delta(\phi, z)$. We call this effect "reputation effect". Depending on which one of the effects is higher, $\Delta(\phi, z)$ can either increase or decrease, reducing or increasing the probability of risk-taking respectively.

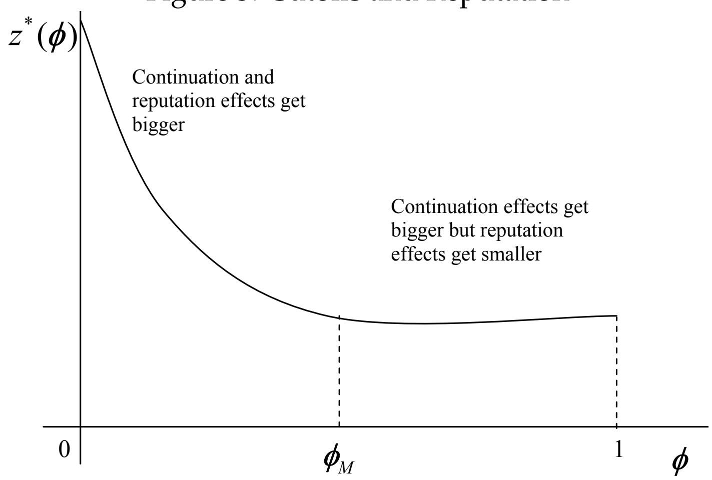
Recall that, using equation (28) $z^*(\phi)$ is obtained by considering all possible beliefs $\widehat{x} \in [0, 1]$, so even when comparing across different levels of reputation ϕ in average the shape of reputation updating is the one shown in Figure 1. If the impact of a higher reputation $\phi \in [\phi_M, 1]$ on current and continuation payoffs is greater than the impact in reducing reputation effects, the probability of risk-taking continues decreasing in this range of reputation levels as well. However, even when the direction in the probability of risk-taking is not clear and depends on parameters when $\phi \in [\phi_M, 1]$, it is possible to guarantee that the rate at which the probability of risk-taking decreases in the range $\phi \in [0, \phi_M]$ is higher than in the range $\phi \in [\phi_M, 1]$.

To show $\frac{\partial^2 z^*(\phi)}{\partial \phi^2} > 0$ it is enough to show $\frac{\partial^2 \Delta(\phi)}{\partial \phi^2} < 0$ for all $\phi \in [0, 1]$. Given $\frac{\partial z^*(\phi)}{\partial \phi}$ described above, $\frac{\partial^2 P(\phi)}{\partial \phi^2} < 0$, $\frac{\partial^2 R(\phi)}{\partial \phi^2} > 0$, $\frac{\partial^2 \mathbf{V}(\phi)}{\partial \phi^2} < 0$ and $\frac{\partial^2 \mathbf{V}(\phi_g) - \mathbf{V}(\phi_b)}{\partial \phi^2} < 0$. The intuition is that the combined decrease in $z^*(\phi)$ and ϕ creates a fast decrease in interest rates and increase in prices for low reputation levels, making $\mathbf{V}(\phi)$ concave on ϕ. The result is a convex schedule of cutoffs $z^*(\phi)$ Q.E.D.

Figure 3 is an example of the relation between the cutoff $z^*(\phi)$ and the reputation level ϕ. It shows the probability of risk-taking is higher for low reputation levels because continuation and reputation effects are not that important to deter these firms from taking risky actions. For low reputation levels, as ϕ increases, continuation values $\mathbf{V}(\phi)$, prices $P(\phi)$ and reputation punishments $\mathbf{V}(\phi_g) - \mathbf{V}(\phi_b)$ go up while $R(\phi)$ goes down, reducing $z^*(\phi)$. This is the case until

ϕ hits ϕ_M. For $\phi > \phi_M$, as ϕ increases, continuation effects still go up but reputation pressures go down. The cutoff will increase or decrease depending on which effect dominates.

Figure 3: Cutoffs and Reputation



Note that in case the probability of risk-taking is minimized at a value of $\phi_m < 1$ (i.e., the cutoff schedule goes down and then increases toward $\phi = 1$), prices would be maximized and lending rates minimized at ϕ_m, hence the value function would achieve the maximum at ϕ_m. In this case continuation values would not increase monotonically with ϕ. It would increase until ϕ_m and then decrease towards one.

However, it is possible to draw some conclusions with respect to ϕ_m. First, $\phi_m \in [\phi_M, 1]$ as discussed above. Second, ϕ_m is biased towards one (i.e., the value function is almost always monotonically increasing in ϕ) when the probability of playing safe in steady state is high for all ϕ. This makes the difference between $\overline{\mathbf{V}}(1)$ and $\overline{\mathbf{V}}(0)$ big enough such that incentives from continuation are large when compared with the incentives from reputation (this is the case described in the first bullet point of Lemma 6 or in Figure 17 at the Appendix). At the other extreme, when steady states are characterized by risk-taking as in 18, $\overline{\mathbf{V}}(1) = \overline{\mathbf{V}}(0)$, both incentives from continuation and from reputation would be nonexistent.

This implies that the nonlinear schedule in cutoffs remains, even in cases where continuation values are not monotonically increasing for all ϕ but decreases for values close to one. It is difficult to draw any further analytical conclusions since value functions and cutoffs for all ϕ in equilibrium should be obtained jointly (i.e., in equilibrium, value functions of a given ϕ have an impact in determining the value functions for all other reputation levels). We will show how to solve this schedule numerically and support these considerations in Section 7.

6.2 Risk-Taking Clustering

6.2.1 Sensitivity of risk-taking to fundamentals

It is important at this point to analyze what happens with risk-taking in the case of a reduction in fundamentals. If $\theta < z^*(\phi)$, for a given reputation value ϕ and a signal noise $\sigma \to 0$, then most firms with that reputation level receive in expectation a signal $z < z^*(\phi)$ and decide to take risks. Since lenders and consumers observe θ and also $z^*(\phi)$ in equilibrium, they can infer a ϕ firm has a high $x(\phi)$, prices will be low and reputation would not be heavily updated. Small changes in fundamentals around $z^*(\phi)$ induce sudden changes in risk-taking for firms with reputation ϕ.

The next proposition formalizes this idea.

Proposition 9 *For highly precise signals about fundamentals (i.e., $\sigma \to 0$), small changes in fundamentals θ around the optimal cutoff $z^*(\phi)$ may induce a sudden change in risk-taking behavior for firms with reputation level ϕ.*

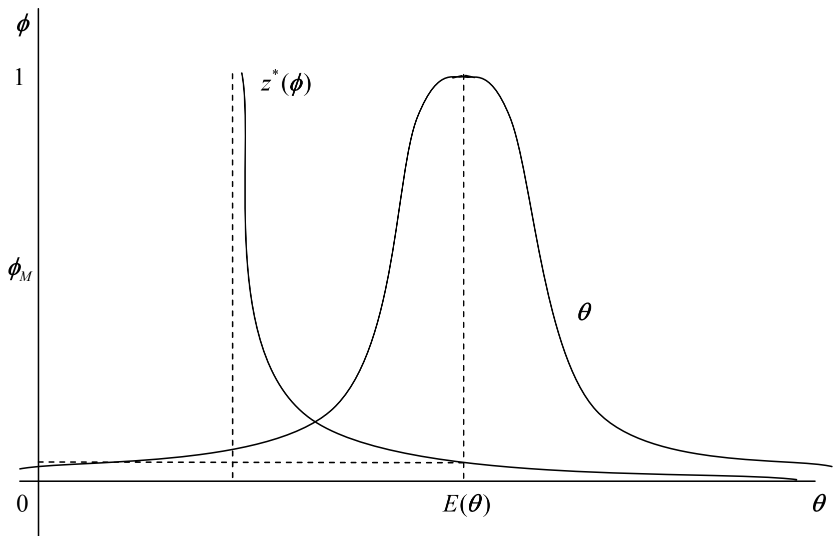
Proof Assume an equilibrium cutoff $z^*(\phi)$ for firms with reputation level ϕ. The market observes the fundamental value θ after firms have taken their actions from observing a signal $z = \theta + \sigma\epsilon$. Lenders and consumers know ϕ firms will decide to play risky when $z < z^*(\phi)$, hence the probability assigned a particular firm plays risky is given by $Pr(z < z^*(\phi)|\theta)$ or which is the same, $Pr(\epsilon < \frac{z^*(\phi)-\theta}{\sigma}|\theta)$. Since $\epsilon \sim F$, the probability a firm ϕ plays risky $x(\phi) = F(\frac{z^*(\phi)-\theta}{\sigma})$.

If σ is low and $\theta > z^*(\phi)$, then $x(\phi, \theta)$ is close to 1, prices are low and reputation formation is not important, reinforcing that firms want to play risky in that situation. Contrarily, if σ is low and $\theta > z^*(\phi)$, then $x(\phi, \theta)$ is close to 0, prices depend a lot on reputation and reputation is heavily updated, reinforcing that firms want to play safe in that situation. Q.E.D.

A lot of action can happen around the equilibrium cutoff to firms of the same reputation level when signals are highly precise, even when fundamentals do not show a significant change. However, the analysis so far has focused in the sudden change of behavior for firms with a particular ϕ value. What is the behavior in the aggregate assuming a particular distribution of reputation in the economy?

All firms with reputation ϕ will cluster when fundamentals go below the cutoff $z^*(\phi)$. As shown in Proposition 8 and Figure 3, cutoffs $z^*(\phi)$ are similar for values of reputation $\phi \in [\phi_M, 1]$. When the state of the economy is good, changes in fundamentals do not induce a change in risk-taking behavior for many reputation levels. Contrarily, in bad states of the economy, changes in fundamentals do induce a change in risk-taking behavior for many reputation levels. If the distribution of reputation levels is not heavily skewed towards low reputation firms, this will generate a big clustering in aggregate risk-taking when fundamentals weaken enough. This effect can be observed in Figure 4.

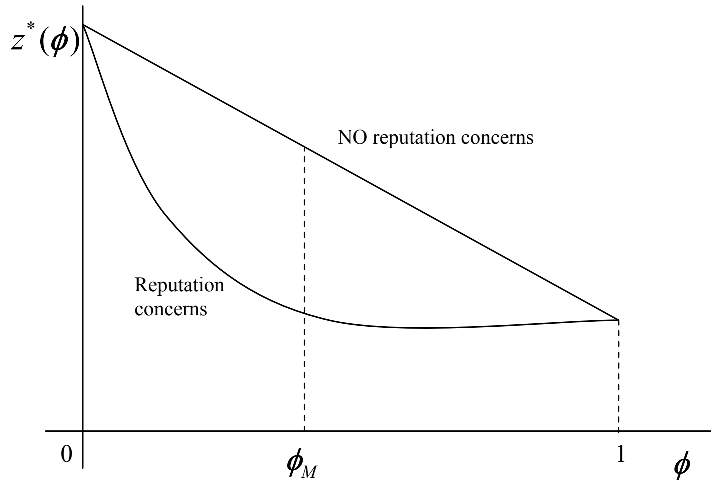
Figure 4: Cutoffs and fundamentals



This property of the model implies large spikes in risk-taking behavior with a short duration, where risk-taking increases even for firms with high reputation. In fact, this is exactly a feature we can observe in the data (as will be discussed in Section 8.2.1), generating sudden and big losses for investors in those particular events.

What is even more striking is that clustering occurs exactly because of the existence of reputation concerns. Assume momentarily that firms born with a prior about the probability of being strategic ϕ that they cannot modify by gaining or losing reputation. In this counter-natural exercise, which arises for example in the case of unavailability of information about signals, differential gains from playing safe are given by equation (11) but without the last two components that represent the incentives from reputation. In this case, when obtaining cutoffs for each reputation level without reputation concerns, only continuation effects are present in the computation, which eliminates the nonlinearity introduce by reputation through learning, as in Figure 1. Figure 5 shows in general the relation between cutoffs with and without reputation concerns. While cutoffs are the same at $\phi = 0$ and $\phi = 1$, for all other reputation levels the benefits from playing safe are higher with reputation concerns, reducing cutoffs. In particular, the difference between the cutoffs with and without reputation concerns reaches the maximum at ϕ_M, where the reputation gains from playing safe reach the maximum. Specific examples will be shown in Section 7.

Figure 5: Cutoffs with and without reputation concerns



6.2.2 Considerations about the distribution of reputation

The clustering result not only depends on the non-linear schedule of cutoffs but also on the distribution of reputation levels in the economy. In reality, the distribution of reputation seems to have a high mass among intermediate reputation levels (see Tables 12 and 13 in Section 8.1). In the theory, since cutoffs for each ϕ are independent of the distribution, it depends on specific assumptions about the birth of new firms in the economy. Denote $\omega_t(\phi)$ the fraction of firms with reputation ϕ in the economy at period t. Its expected evolution is

$$\begin{aligned}\omega_t(\phi) = b(\phi)D + \omega_{t-1}(\phi_0)\alpha_s(1-\mathcal{V}(z^*(\phi_0))) + \omega_{t-1}(\phi_1)(1-\alpha_s)(1-\mathcal{V}(z^*(\phi_1))) \\ -\omega_{t-1}(\phi)[\mathcal{V}(z^*(\phi)) + (1-p_r)(1-\mathcal{V}(z^*(\phi)))]\end{aligned} \tag{18}$$

where $b(\phi)$ is the birth rate of firms with reputation ϕ as a fraction of the total population of firms, $\omega_{t-1}(\phi_0)\alpha_s(1-\mathcal{V}(z^*(\phi_0)))$ is the fraction of firms with reputation ϕ_0 at $t-1$ that in expectation will be upgraded to ϕ in period t (where $\phi_0 = \frac{p_r\alpha_r\phi}{p_r\alpha_r\phi + p_s\alpha_s(1-\phi)}$). Similarly, ϕ_1 is the reputation level that is downgraded to ϕ. The negative expression represents the proportion of ϕ firms that die or change reputation out of ϕ.

Expressing birth rates for each ϕ as a function of a stationary expected distribution

$$b(\phi) = \omega(\phi)[2 - p_r(1-\mathcal{V}(z^*(\phi)))] - \omega(\phi_0)\alpha_s(1-\mathcal{V}(z^*(\phi_0))) - \omega(\phi_1)(1-\alpha_s)(1-\mathcal{V}(z^*(\phi_1))) \tag{19}$$

Recall from the definition of $b(\phi)$ I am not restricting the economy from shrinking or growing and I am not taking any stand on the shape of the distribution. Hence, it is possible to impose any stationary distribution of reputation assigning the correct birth rate from equation (19).

Given this degree of freedom it is relevant to discuss which are the assumptions on birth primitives required for a reputation distribution to overcome the non-linearity of cutoffs and to avoid clustering in risk-taking[28].

First, by learning properties, $\phi = 0$ and $\phi = 1$ are absorbing states. Assuming there is no renovation of firms in the economy, (i.e., $b(\phi) = 0$ for all ϕ), the economy will shrink with time and will converge to a distribution of firms with reputation 0 and 1, with a proportion given by the distribution of types in the economy. In this case the distribution of firms depends exclusively on the reputation dynamics of existing firms. Now assume all dying firms are replaced by newborns with a reputation $\phi = 0.5$[29], this renewal would fill intermediate reputation firms, making the distribution more evenly distributed or even with a greater mass at intermediate levels. Considering these effects, a distribution heavily skewed towards low reputation levels requires that most newborn firms are believed to have a very low initial reputation level, or which is the same, the proportion of strategic types is very small.

6.3 Efficiency considerations

In equation (11), the components of $\Delta(\phi, \theta)$ distinguish how effective is reputation to deal with inefficiencies that arise from adverse selection and moral hazard. Consider first the case of full information in which lenders and consumers know both firms' types and actions[30]. When strategic firms have a $\phi = 1$. The value in case they decide to play safe and risky respectively are $V(1, \theta|s)_{FI} = p_s\Pi_s(1, \theta) - p_s\frac{\overline{R}}{p_s} + \beta p_s\mathbf{V}(1)$ and $V(1, \theta|r)_{FI} = p_r\Pi_r(1, \theta) - p_r\frac{\overline{R}}{p_r} + \beta p_r\mathbf{V}(1)$.

Some specificities in the previous expressions are worth noting. First, since there are no reputation problems, the expected future value is always $\mathbf{V}(1)$. Second, interest rates reflect the real probability of default ($R_s(1) = \overline{R}/p_s$ and $R_r(1) = \overline{R}/p_r$), which means the cost of the capital for firms is always $\overline{R}$ in expectation. Finally, since consumers also observe actions, they pay a price reflecting the real probability of good results $P_s(1) = \alpha_s$ and $P_r(1) = \alpha_r$. In this sense $p_s\Pi_s(1, \theta) - p_r\Pi_r(1, \theta) = A(\theta)[(p_s\alpha_s - p_r\alpha_r) - (C_s(\theta) - C_r(\theta))]$

Hence, differential gains from playing safe under full information are,

$$\Delta(1, \theta)_{FI} = p_s\Pi_s(1, \theta) - p_r\Pi_r(1, \theta) + \beta(p_s - p_r)\mathbf{V}(1)$$

[28] Since we are dealing with simultaneous equations analytically intractable but easily solvable numerically, I will just make some general considerations based on numerical simulations available upon request

[29] This is the case, for example, if the prior is that 50% of firms are strategic and there is no further information about the firm at the time it arises

[30] In fact it is not interesting to have full information of actions and not types, since strategic firms can easily signal their type just by playing safe at least once

These are the first two components in equation (11) for the case of $\phi = 1$ when actions are observable. The value of the fundamental $\theta^*_{FI}(1)$ for which $\Delta(1, \theta^*_{FI}(1))_{FI} = 0$, determines the point below which it is efficient for strategic firms to play risky and above which it is efficient for strategic firms to play safe.

When lenders cannot observe firms' actions, they charge a unique interest rate $R(1)$, regardless of the technology used. This generates a moral hazard problem. Firms are more prone to take risks because they are not paying the premium for increasing default probabilities lenders would charge for taking risks. While firms appropriate all gains from good results, they impose to lenders the losses from bad results. Formally, in the case of non-observable actions, $V(1, \theta|s) = p_s\Pi_s(1, \theta) - p_s R(1) + \beta p_s \mathbf{V}(1)$ and $V(1, \theta|r) = p_r\Pi_r(1, \theta) - p_r R(1) + \beta p_r \mathbf{V}(1)$. The main difference with full information is a common interest rate $R(1) \in [\frac{\overline{R}}{p_s}, \frac{\overline{R}}{p_r}]$. Hence,

$$\Delta(1, \theta) = p_s\Pi_s(1, \theta) - p_r\Pi_r(1, \theta) + \beta(p_s - p_r)\mathbf{V}(1) - (p_s - p_r)R(1)$$

The last component in this equation represents the moral hazard incentives to play risky that come from the lending relationship. Since $\Delta(1, \theta) < \Delta(1, \theta)_{FI}$, it is easy to show $\theta^*_{FI}(1) < z^*(1)$. $R(1)$ wil be determined in equilibrium by $z^*(1)$. For fundamentals $\theta \in (\theta^*_{FI}(1), z^*(1))$, it is efficient that firms follow safe technologies but they prefer to take risks. So, moral hazard generates excessive and inefficient risk-taking.

So far, we have been discussing the inefficiency generated by moral hazard for a firm with reputation $\phi = 1$. However, the un-observability of actions opens the room for the problem of adverse selection. This is because actions produce a non-deterministic outcome that inhibit the market to fully learn about the type of the firm. Now, considering adverse selection together with moral hazard, take a reputation value $\phi < 1$. Since $P(\phi) < P(1)$, $R(\phi) > R(1)$ and $\mathbf{V}(\phi) < \mathbf{V}(1)$, then $\Delta(\phi, \theta) < \Delta(1, \theta)$. This means that $z^*(\phi) > z^*(1)$. Hence for the fundamentals $\theta \in (z^*(1), z^*(\phi))$, it is efficient that firms take safe actions but they prefer to take risks. So, adverse selection further increases inefficient risk-taking by firms.

Equation (11) shows the differential profits for any ϕ (given beliefs $\widehat{k}$) for the case with adverse selection, moral hazard and the possibility of reputation formation. The last two components increase $\Delta(\phi, \theta|\widehat{k})$, reverting the inefficiencies caused by moral hazard and adverse selection.

Reputation has a bright side. It reduces inefficient risk-taking. However, reputation also has a dark side. It reduces inefficient risk-taking in a way that generates sudden and isolated events of clustering of risk-taking, loss of confidence, big increases in default probabilities and huge losses by investors. Reputation concerns are effective in reducing excessive risk-taking, but there are states of the economy bad enough that reputation incentives break down and suddenly collapse.

7 Simulations

In this section we develop a numerical example to show how reputation concerns generate large changes in aggregate behavior as a response to small changes in fundamentals. We also discuss about reputation efficiency effects and the sizable negative effects of net returns to investors in those extreme situations.

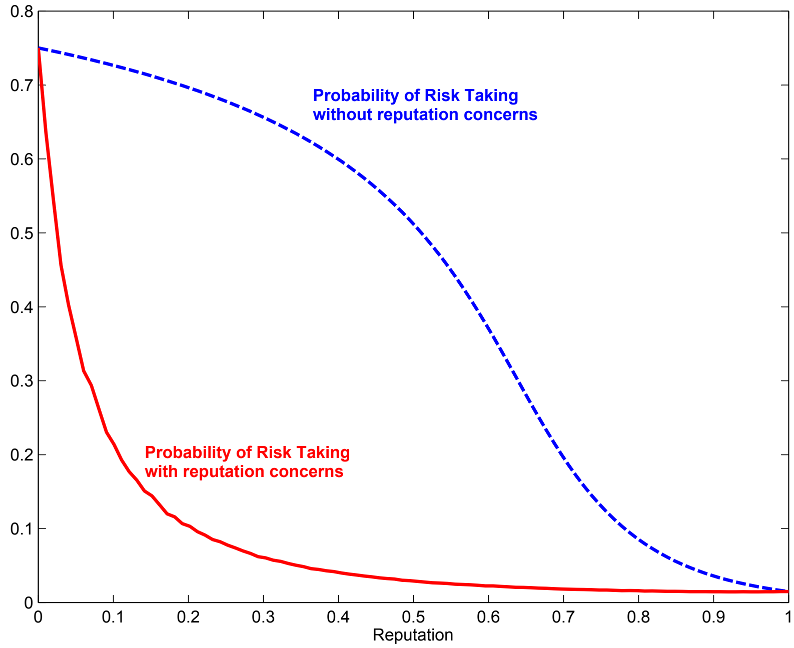
For simplicity, we don't introduce prices explicitly, so the market is composed only by lenders and all the results come only from interest rates. To be more specific, we assume Π_r is constant and $\Pi_s = \Pi_r + K + \psi\theta$, where $K < E(\theta)$ and $\psi > 0$, hence fulfilling assumptions 2 and 5 to ensure a unique steady state in continuation values. The computational procedure is described in the Appendix. Parameters in this particular exercise are $\beta = 0.95$, $\overline{R} = 1$, $\Pi_r = 1.6$, $K = -0.001$, $\psi = 0.4$, $p_s = 0.9$, $p_r = 0.7$, $\alpha_s = 0.8$, $\alpha_r = 0.4$ and $\theta \sim \mathcal{N}(0,1)$. These parameters have been chosen such that under full information risk-taking is efficient only for very low fundamental values, which arise with a probability 0.001%. This means risk-taking is almost never an efficient situation.

Figure 6 shows the ex-ante probability of risk-taking by firms with different reputation levels. The probability that intermediate firms take risks is much greater without reputation concerns than with reputation concerns. For example, the ex-ante probability a firm with a reputation level $\phi = 0.4$ takes risk is 60% without reputation concerns but only 4% with reputation concerns. Hence, the gap between the two curves shows the reduction in the ex-ante probability of inefficient risk-taking generated by reputation concerns. Even when reputation reduces inefficient risk-taking around intermediate reputation levels, it is not that successful for very low or very high reputation levels. Firms with very high reputation (ϕ around 1) and firms with very low reputation (ϕ around 0), have a probability of risk-taking around 3% and 75% respectively, whether or not they have reputation concerns.

Figure 7 shows the same intuition of risk-taking clustering as Figure 5. As fundamentals decline, reputation levels that enter into a phase of risk-taking gradually grow when reputation is not a concern but suddenly grow when reputation is a concern. Figure 8 shows expected value functions and lending rates for firms with different reputation levels ϕ. Firms with reputation concerns pay lower interest rates and have higher expected continuation value than firms without reputation concerns. This is because ex-ante probabilities of risk-taking are lower for all firms with reputation concerns, reducing current and future interest rates and increasing continuation values.

Figures 9 and 10 show the effects of sudden and isolated events of risk-taking in deep recessions, using 100 simulated periods. To do this exercise it is necessary to aggregate across firms, for which I assume a uniform reputational distribution. Since data seems to suggest

Figure 6: Ex-ante probability of risk-taking - with and without reputation



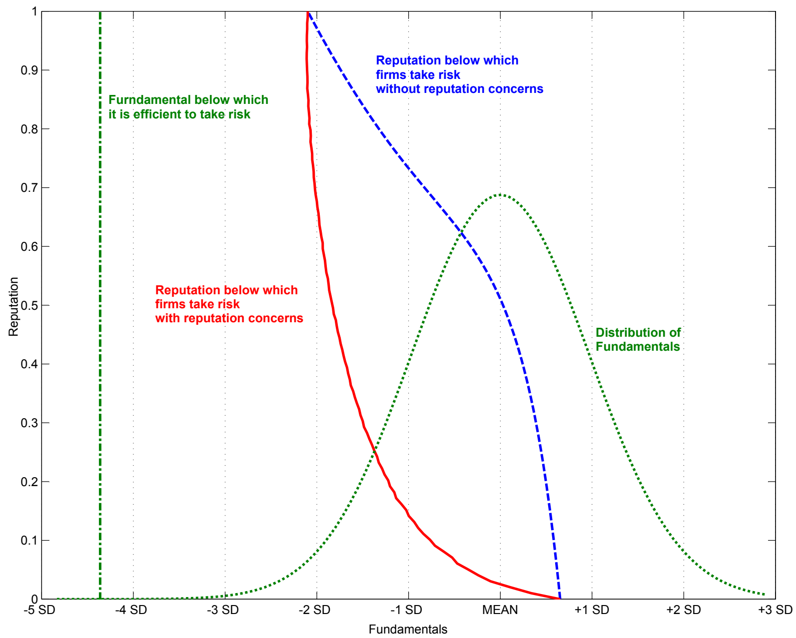
this distribution has a greater mass in intermediate reputation levels, this is a conservative assumption[31]. Without reputation concerns, risk-taking is more common and arises as a result of even small declines in fundamentals. With reputation concerns, inefficient risk-taking is greatly reduced in general, except in very deep recessions when reputation does not provide enough incentives to inhibit inefficient risk-taking, even for firms with high reputation.

Figure 9 shows aggregate probability of default. This number goes from 10% in the case no firm takes risk to 30% in the case all firms take risk. Figure 10 shows aggregate net returns. First we obtain individual net returns for each reputation level (computed by the lending rate charged to ϕ multiplied by the real probability of success by ϕ minus the risk free rate). Then we calculate the weighted sum of individual returns to obtain aggregate net returns, which will depend on fundamentals. When values of fundamentals decline enough, returns decline catastrophically since all firms, no matter their reputation, decide to take risks. Since lenders charge a low interest to high reputation firms, when those conditions arise, sudden losses are of high magnitude. With reputation concerns these rates are lower, then the losses are bigger.

We can also predict the returns of lending activities to firms with different reputation levels. Figure 11 shows simulated net returns of investors in firms with reputation levels $\phi = 0$, $\phi = 0.5$ and $\phi = 1$, for the cases with and without reputation concerns and for periods 55 to 75 in our simulation, when the two crises occur. In all cases, by the determination of lending

[31]see Tables 12 and 13 in Section 8.1

Figure 7: Cutoffs - with and without reputation



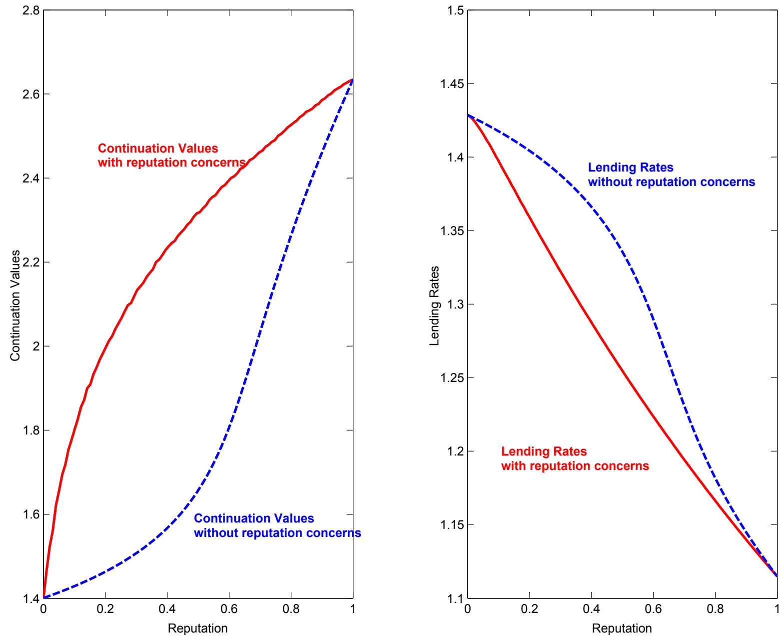
rates in equilibrium, expected net returns (before observing fundamentals) are zero.

Lenders to firms with very low reputation ($\phi = 0$) charge the maximum possible interest rate. Since in equilibrium they are right, all firms are of type $\mathcal{R}$ and then, no matter the fundamentals, net returns are always zero. Lenders to firms with very high reputation values ($\phi = 1$) charge interest rates that assume there will be some fundamentals under which even these firms will take risks. When fundamentals are normal, investors make a small difference because risk-taking of these firms is infrequent. When risk-taking occurs, they lose a lot.

Since reputation does not make any difference for extreme values, these two lines are the same in both panels of Figure 11. When reputation is $\phi = 0.5$ and reputation concerns exist, ex-ante probability of risk-taking is small and the pattern is similar to the one observed for $\phi = 1$, with less gains in normal times and less, but more frequent, looses in very bad times. Without reputation concerns, intermediate reputation firms are more volatile since they enter more frequently in the phase of risk-taking. Hence it is necessary to pay more in good times to compensate for more frequent losses in bad times.

Even when results from this simulation are based on particular and arbitrary parameters, they are highly robust to changes in the numbers used as soon as they fulfill assumptions 2 and 5. In all cases, reputation concerns introduce incentives to deter inefficient risk-taking and generate a sudden wave of risk-taking, with big losses to investors, below a certain threshold.

Figure 8: Value functions and lending rates - with and without reputation



8 Some Testable Hypothesis

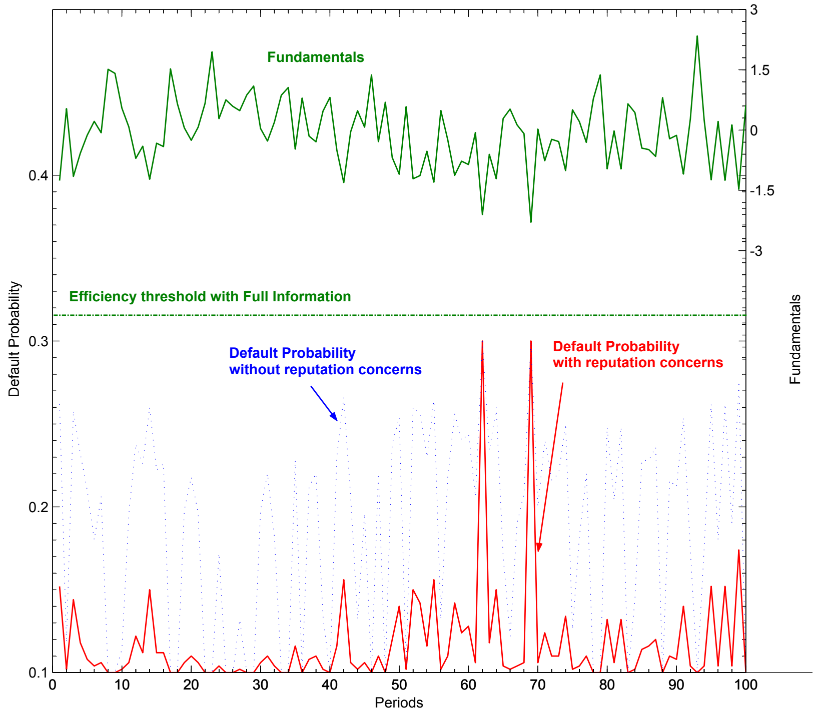
8.1 Reputation over the Cycle

According to the model, in economic troughs we should see two patterns of reputation formation. On the one hand, we should observe higher default and downgrading, both because these are times of weakening in fundamentals (even when we neutralized this effect in this model) and because firms decide to take more risks, being more prone to exit and to have bad results. On the other hand, we should observe fewer cases of reputation revision, since it is more difficult for the market to update beliefs.

It is however difficult to directly track the evolution of reputation in the market. Here I propose a novel approach. I analyze credit ratings to capture the idea of reputation and rating transitions to capture the idea of beliefs updating and learning.

As a first step, let's divide the reputation continuum from 0 to 1 in seven bins from Aaa to C. Aaa corresponds to the highest possible reputation (ϕ close to 1) while C corresponds to the lowest possible reputation (ϕ close to 0). With this interpretation, rating transitions deliver information on how reputation varies and how beliefs are updated in different phases of economic cycles. I use a detailed view of rating migration provided by Moody's yearly rating transition matrices. These matrices summarize the size and direction of rating movements,

Figure 9: Simulated probability of default - with and without reputation



including defaults[32], for the entire Moodys-rated universe, over specific time horizons.

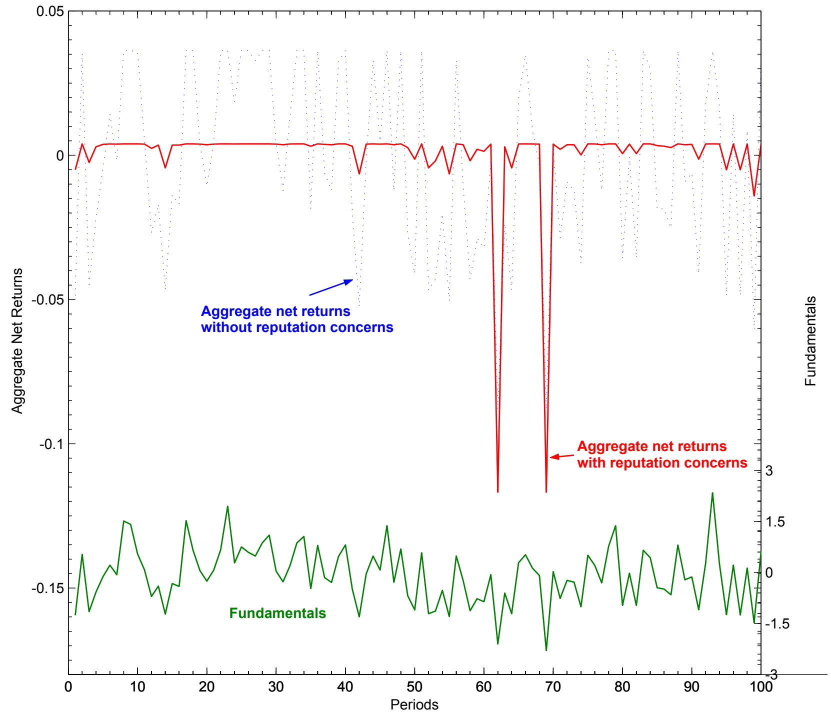
In Table 12 I compare rating transition and default rates in 2001 (the last recession year with high recorded idiosyncratic risk as a proxy of risk-taking behavior[33]) with averages for the 20-year period 1980-2000 for broad rating categories. This is a very rough way to look for clues about general features of reputation evolution and changes in reputation over the cycle.

First, focus on the average characteristics of rating transition matrices (first panel in Table 12). As can be observed in the concentration of transitions around the diagonal, upgrades or downgrades in reputation are given gradually rather than suddenly. In the model this is predicted by equations 3 and 4 and graphically shown in Figure 1. These patterns are consistent with our model, where reputation can be constructed, destroyed, and managed. As in Mailath and Samuelson (2001), this is in stark contrast with other more standard models

[32]These rates are calculated as fractions in which the numerator represents the number of issuers that defaulted on Moodys-rated debt in a particular time period and the denominator represents the number of issuers that could have defaulted on Moodys-rated debt in that time period. Moodys defines a bond default as *"any missed or delayed disbursement of interest and/or principal, bankruptcy, receivership, or distressed exchange where (i) the issuer offered bondholders a new security or package of securities that amount to a diminished financial obligation (such as preferred or common stock, or debt with a lower coupon or par amount) or (ii) the exchange had the apparent purpose of helping the borrower avoid default"*.

[33]According to Goyal and Santa-Clara (2003) and Davis et al. (2006), 200 and 2001 were years of particularly high idiosyncratic risk

Figure 10: Simulated aggregate net returns - with and without reputation

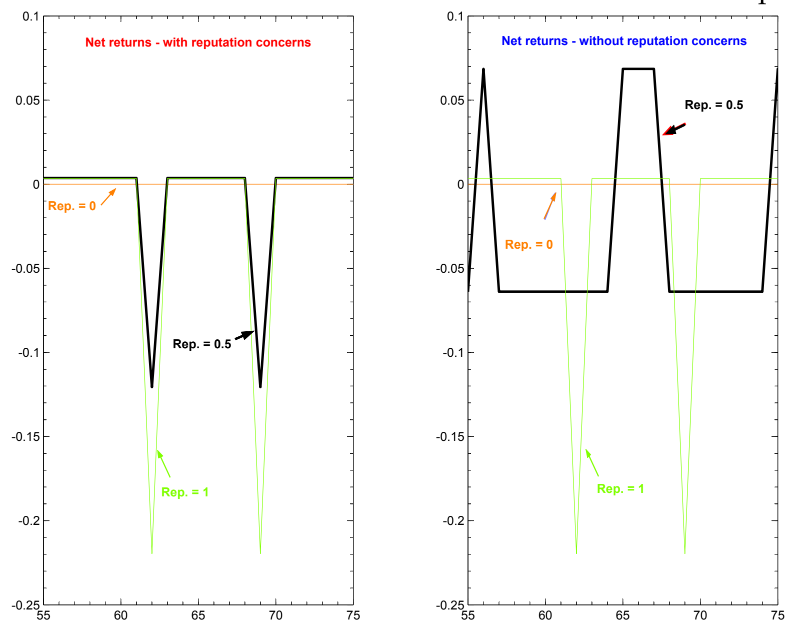


where reputation is intrinsically valuable (as Holmstrom (1999)), where "bad" types try to pool with "good" types and then reputation can be suddenly lost rather than managed over time (as Milgrom and Roberts (1982), Kreps and Wilson (1982) and Fudenberg and Levine (1992)) or where reputation can only increase over time or disappear (as Diamond (1989)).

Now we can focus on the comparison of rating transition across different states of the economy. We use the year 2001 both because it was the last recorded NBER recession date and because it was a year with a large degree of clustering in risk-taking behavior measured by firm-level volatility in returns. The shaded cells are those statistically different between the two transition matrices (at a 95% of confidence, one sided test). Comparing the two matrices, 2001 is characterized by a higher default rate (35% against 28% in average in the previous 20 years), mostly concentrated among low reputation firms.

Three facts consistent with our model predictions are worth noting. First, in recession firms take more risk, generating a higher probability of bad results (and downgrading) and a higher probability of default and exit. As can be seen, in 2001 there was more downgrading and less upgrading than on average since 1980. Second, by comparing the bolded diagonal elements of the two matrices, that indicate the frequency at which ratings have remained unchanged over respective periods, we can see that in 2001 the fraction of firms whose reputation was not updated is higher than its average since 1980. This denotes the difficulties in revising

Figure 11: Simulated individual net returns - with and without reputation



ratings in times with clustering in risk-taking. Furthermore, this difference between panels is more important for high reputation firms, which are the ones whose behavior changes the most in large recessions compared to normal times. Finally, transitions are more concentrated around the diagonal in recessions. The cells located far away from the diagonal are emptier in recessions than in normal times, showing the difficulties to update in recessions.

These patterns have also been noticed in other studies that try to document changes in ratings for other reasons. Here we highlight the finding from three sources, Bangia, Diebold, and Schuermann (2000), Moody's reports and Altman and Rijken (2006).

First, in Table 13 we repeat results from Bangia, Diebold, and Schuermann (2000), who use data from Standard & Poor's (rather than Moody's) from 1980-2000 and show US expansion quarters against US recession quarters, defined as periods above and below the trend respectively[34]. They also noticed that defaults and downgrades are more likely in recessions and that transitions are more concentrated around the diagonal in economic troughs. The fact that the numbers in the diagonal are smaller in some cases during depressions is more than compensated for the increase in downgrades in those particular periods. Furthermore,

[34]This comparison may hide our main point that under certain important weakening in fundamentals risk-taking behavior increases in a large degree as a result of a cluster behavior. Since recession dates as defined in Bangia, Diebold, and Schuermann (2000) also correspond to certain reductions in fundamentals that do not justify a sudden risk-taking behavior, the comparison of the tables may hide the main source of action. This is why in our original exercise in Table 12 we just used an extreme year characterized by a big weakening of fundamentals as defined by NBER

Figure 12: Reputation updating over the cycle

All-Corporate Average Rating Transition Matrix, 1980-2000 (percent)

Initial Rating	Terminal Rating: Aaa	Aa	A	Baa	Ba	B	Caa-C	Default	# of Firms
Aaa	**89.31**	10.15	0.50	---	0.03	---	---	---	2,585
Aa	0.96	**88.42**	10.04	0.38	0.16	0.02	---	0.04	8,085
A	0.08	2.34	**90.17**	6.37	0.81	0.22	---	0.02	15,210
Baa	0.09	0.39	6.42	**84.48**	6.92	1.39	0.12	0.20	10,066
Ba	0.03	0.09	0.50	4.41	**84.25**	8.65	0.52	1.54	8,816
B	0.01	0.04	0.17	0.58	6.37	**82.67**	2.98	7.17	7,437
Caa-C	---	---	---	1.10	3.06	5.89	**62.17**	27.77	1,025

All-Corporate Average Rating Transition Matrix, 2001 (percent)

Initial Rating	Terminal Rating: Aaa	Aa	A	Baa	Ba	B	Caa-C	Default	# of Firms
Aaa	**98.99**	1.01	---	---	---	---	---	---	121
Aa	0.31	**94.14**	5.24	0.16	---	0.16	---	---	714
A	0.26	2.38	**89.71**	6.80	0.34	0.17	0.17	0.17	1,312
Baa	0.20	0.30	3.72	**90.15**	4.32	0.80	0.20	0.30	1,081
Ba	---	---	1.23	6.94	**80.00**	9.38	1.23	1.23	540
B	---	---	0.12	1.01	5.95	**67.65**	15.50	9.78	1,055
Caa-C	---	---	---	---	---	2.67	**62.50**	34.82	266

by analyzing coefficients of variation, Bangia, Diebold, and Schuermann (2000) mention that *"results suggest that migration probabilities are more stable on contractions than on average"*

Figure 13: Reputation updating over the cycle (Bangia et al., 2000)

US Expansion Quarters (1981-1998) (percent)

Initial Rating	Terminal Rating: AAA	AA	A	BBB	BB	B	CCC	Default	# of Firms
AAA	**98.21**	1.66	0.11	0.02	0.02	---	---	---	6,581
AA	0.15	**98.08**	1.61	0.12	0.01	0.03	0.01	---	19,458
A	0.02	0.53	**98.06**	1.21	0.11	0.06	0.00	0.00	36,404
BBB	0.01	0.07	1.47	**96.94**	1.25	0.22	0.02	0.02	24,529
BB	0.01	0.03	0.19	1.93	**95.31**	2.25	0.16	0.12	18,161
B	---	0.02	0.07	0.10	1.70	**95.91**	1.31	0.88	20,002
CCC	0.05	---	0.19	0.23	0.47	3.57	**87.32**	8.17	2,129

US Recession Quarters (1981-1998) (percent)

Initial Rating	Terminal Rating: AAA	AA	A	BBB	BB	B	CCC	Default	# of Firms
AAA	**97.99**	1.76	0.25	---	---	---	---	---	795
AA	0.18	**96.89**	2.79	0.05	0.09	---	---	---	2,186
A	0.02	0.88	**96.44**	2.59	0.07	---	---	---	4,330
BBB	0.04	0.04	1.11	**96.31**	2.33	0.07	---	0.11	2,708
BB	---	0.06	0.06	1.39	**94.98**	2.72	0.42	0.36	1,655
B	---	0.06	0.06	0.11	0.72	**95.02**	2.27	1.77	1,806
CCC	---	---	---	---	---	1.20	**85.60**	13.20	250

Second, in several Moody's special reports, the same patterns are discussed. For example, the Historical Default Rates of Corporate Bond Issuers, 1920-1999 report states *"In spite of the higher default rates in 1999, overall rating volatility was lower than its average since 1980"*

Finally, Altman and Rijken (2006) try to rationalize the stability in rating transitions, especially on economic troughs. Their explanation is the "through-the-cycle" methodology that rating agencies use to construct and update their estimates. According to Moody's, ratings are stable because they intend to measure default risks in the long run and because modifications are made only when rating agencies are confident that observed changes in a company's risk profile are likely to be permanent. Altman and Rijken (2006) explanation is also based on a prudent behavior of agencies. When Moody's or Standard & Poors attribute ratings to bond obligors, they are engaged in a complex judgment. In particular, rating migrations are triggered when the difference between the actual agency rating and the one predicted by the model they use exceeds a certain threshold, modifying ratings only partially. Our explanation is not based on a prudent behavior by agencies but on difficulties in evaluating the policies of firms given the point of comparison of similar firms. In fact, agencies agree that recessions inhibit rating migrations since the elements used to consider whether a permanent change in overall risk status occurred or not are noisier than in normal times

As can be seen from our own analysis and from some evidence in the literature, differences in reputation formation over the cycle constitute evidence that in recessions, when risk-taking behavior clusters, reputation is not heavily updated. Hence, the deterring effect of reputation concerns over risk-taking behavior is seriously inhibited in times of weakening fundamentals.

8.2 Clustering in Risk-Taking Behavior

Here we discuss some indicative evidence that risk-taking behavior measured by idiosyncratic risk tends to cluster excessively in recessions. Furthermore, corporate default rates seem to follow a similar pattern which, in our model, is a direct consequence of risk-taking clustering.

Campbell et al. (2001) analyzes the trend and cyclical behavior of idiosyncratic risk measured as a firm level profit volatility. The construction of this indicator fixes market and industry risk, reflecting variations in volatility that happen exclusively as a result of changes occurring inside a firm, such as risk-taking by managers. They show not only that idiosyncratic risk more than doubles in recessions but also that the magnitude of this clustering cannot be explained only from a weakening in fundamentals.

Similarly, Das et al. (2007) do not only show that default rates cluster in recessions but also that the correlation in default cannot be explained merely by fundamentals. They go even further and suggest there seem to be a non-observed variable that is more active in bad times than in good times and may account for the non-explained degree of clustering. Considering the evidence from Campbell et al. (2001) and the findings about reputation formation over the cycle, our model suggests risk-taking behavior may be the non-observable variable that is highly active in bad times. Even more we propose as a potential explanation that, in recessions, reputation concerns do not work as an effective mechanism to deter risk-taking.

8.2.1 Idiosyncratic Risk as a Proxy for Risk-Taking

Campbell et al. (2001) show that idiosyncratic risk tends to cluster excessively in recessions. In fact market volatility and recessions help to predict firm-level volatility. However, even when recessions are highly correlated to firm-level volatility, they have a smaller effect on the predictable component of volatility. Even when idiosyncratic risk doubles in recessions, the predictable component only helps to explain an increase of about 1.5. Furthermore, they show idiosyncratic risks tend to have the most negative correlation with NBER recession dates. This represents an important unexplained clustering in firm-level volatility and potentially on risk-taking behavior over the cycle.

Figure 14 shows the idiosyncratic risk series from Campbell et al. (2001)[35]. Green bars show NBER-dated recessions. Hence, clustering occurs almost exclusively on economic downturns.

Figure 14: Aggregate idiosyncratic risk

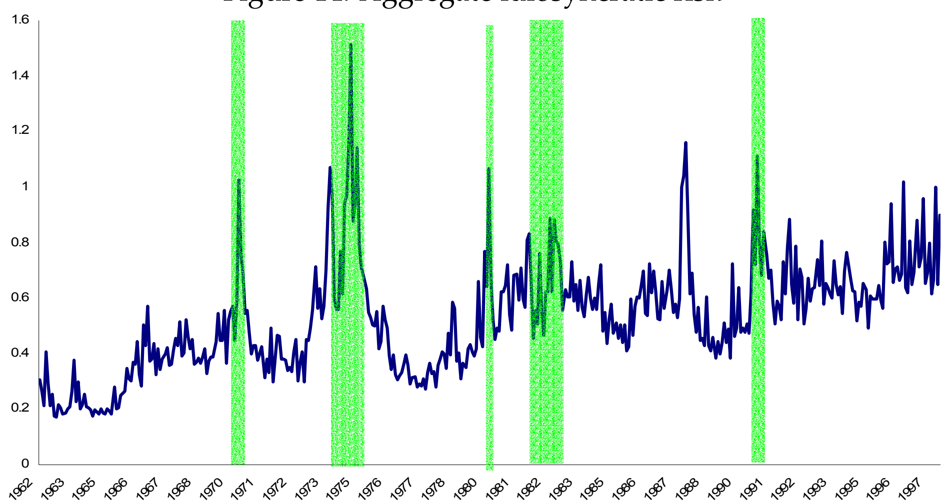
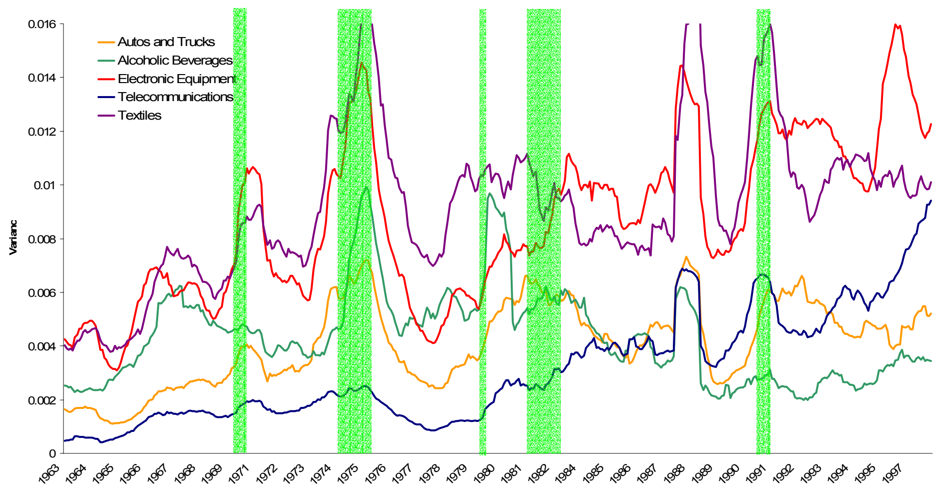


Figure 15 shows some representative examples of idiosyncratic risk in industries with important cycles. These cycles are not perfectly correlated. Campbell et al. (2001) show these are mostly driven by shock in industry-specific fundamentals. All the plotted industries' idiosyncratic risks cluster around recessions. Outside recession dates, idiosyncratic risks seem to

[35]Idiosyncratic risk is measured by the monthly volatility of daily firm-level returns from the CRSP data set, including firms traded on the NYSE, the AMEX, and the Nasdaq. Monthly volatility is adjusted by subtracting market and industry volatilities. Data is recorded for 49 industries following the classification from Fama and French (1997). Daily excess returns were calculated subtracting the 30-day T-bill return divided by the number of trading days in the month from daily returns.

follow their own cycle, with industries clustering at different times when industry-specific fundamentals weaken. See, for example differences in the evolution of idiosyncratic risk between electronic equipments and telecommunications since 1992 or the differences between textiles and alcoholic beverages between the crises of 1974 and 1982.

Figure 15: Idiosyncratic risk across some industries



Even when industry-specific fundamentals are important, the aggregate effect of depressions for all industries is easy to observe when comparing correlation of idiosyncratic risks across industries. The weighted average correlation of idiosyncratic risk[36] of the 49 industries considering NBER recession dates is 0.48. Taking recession dates and the October 1987 market crash out of the sample, the correlation is just half of that number, 0.24. Considering NBER recession dates and the 1987 market crash represent just 50 months out of 414 months in the sample (from July 1962 to December 1997), it is clear the aggregate effect of depressions in all industries when compared to cycles outside economic troughs.

Our model captures exactly this feature, firm-level volatility in industries tend to cluster at the same time when a large weakening of aggregate fundamentals occurs and it tends to cluster at different times when aggregate fundamentals do not show any relevant change but industry fundamentals do.

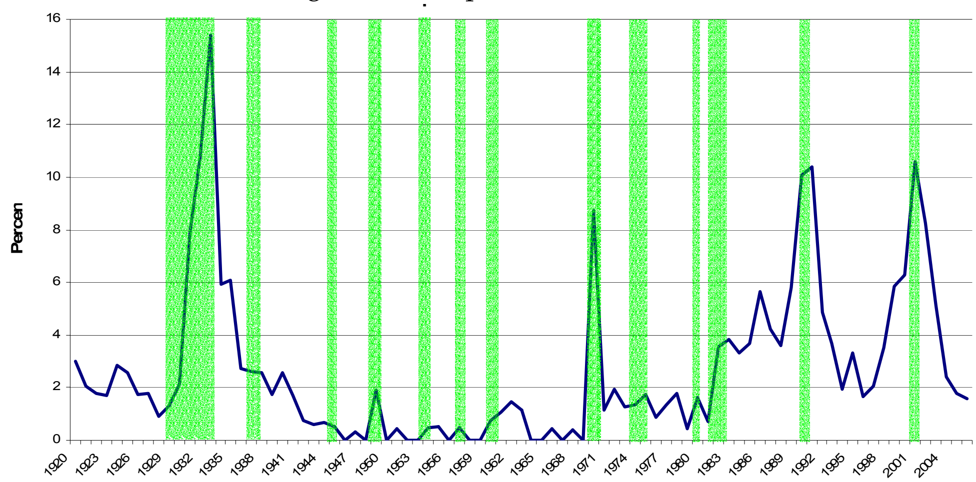
8.2.2 Corporate Default Rates as a Consequence of Risk-Taking

Another possible application of our results is the explanation of a seemingly puzzling new result in the literature. Corporate default rates cluster in recessions with a magnitude that

[36] Weights are based on market values using average market capitalization

cannot be easily explained merely by a weakening of fundamentals. Figure 16 shows corporate default rates of speculative grade bonds collected by Moody's from 1920 to 2006.

Figure 16: Corporate default rates



The relation between default probabilities and macro fundamentals has been widely documented in the literature. Koopman and Lucas (2005), Bangia, Diebold, and Schuermann (2000) and Nickell, Perraudin, and Varotto (2000) present wide evidence of the co-cyclicality between default rates in all industries and macro fundamentals. Stock and Watson (1989) indeed highlight the important role of default rates in the construction of leading indicators.

However, the question is whether the large degree of clustering experimented in recessions can be merely explained by weakening in fundamentals. Das et al. (2007) recently make an original effort to test how well fundamentals can explain clustering in corporate default rates. They test a standard doubly stochastic model of default under which, *"conditional on the paths of risk factors that determine all firm's default intensities, firm defaults are independent Poisson arrivals with these conditionally deterministic intensity paths."*. They find in the data evidence of the existence of default clustering beyond that predicted by the doubly stochastic model. Furthermore, introducing additional variables related to fundamentals (such as GDP growth), which may be missing covariates in the model, they find a degree of clustering that is not captured by the extended model either[37]. Calibrated estimations of Gaussian copula correlation, which is a measure of the degree of correlation in default times that is not captured by co-movement in default intensities, range between 1% to 4% in Das et al. (2007) to 20% in Akhavein, Kocagil, and Neugebauer (2005).

Das et al. (2007) propose that there seems to be a non-observed variable that is more active in

[37]Koopman et al. (2006) is an additional recent paper that shows cross-firm default correlation associated with observable factors cannot account for the large degree of time clustering in defaults found in the data.

bad times than in good times. In our model, this unobserved variable is risk-taking behavior, which clusters in bad times. Furthermore, we propose risk-taking is more active in bad times than in good times because the deterring effects of reputation are seriously hindered in economic depressions due to the existence of strategic complementarities in learning.

With a different methodology, Koopman et al. (2006) found that fundamentals (measured by the level of economic activity, bank-lending conditions, and financial markets variables) seem to be all important determinants of default rates. However, simple models seem to be significantly dynamically misspecified. Once they introduce in the model latent variables, macro fundamentals seem unable to explain the large degree of default that occurs in recessions. The question is open as to which missing latent variables capture the intensity of default. Contagion and frailty have been suggested as possible explanations[38].

Our model suggests to look at sudden changes in risk-taking behavior over the cycle as a potential factor behind a sudden jump in default rates. In the model default rates are a direct consequence of risk-taking behavior, captured by the exit state[39]. The probability of observing an exit (and hence a default) jumps from a number close to $(1 - p_s)$ to a higher one close to $(1-p_r)$ in big depressions. We have shown some empirical evidence that risk-taking behavior clusters in recessions, at a higher magnitude than fundamentals can possibly explain. This suggests that risk-taking may be a good avenue to explore sudden jumps in default rates.

Finally, recall the similarities between our numerical simulations in Section 7 and the data, especially the similarities between Figure 9 and Figure 16. Reputation reduces risk-taking (and hence default rates) most of the time, generating sudden and isolated events of clustering in risk-taking, financial crises, big losses to investors and, eventually, credit crunches. If reputation formation is somehow inhibited, countries would experience more volatility in default rates, higher interest rates and more frequent but also less dramatic crises.

9 Conclusions

Reputation concerns deter excessive risk-taking behavior. This is a widely accepted property of reputation, both on formal and informal grounds. This paper studies these deterring effects when incentives for inefficient risk-taking vary with the state of the economy. The main finding is that reputation effects may suddenly collapse, leading to large changes in aggregate risk-taking as a response to small changes in fundamentals.

[38] Even when these answers cover part of the story they have some problems. Schonbucher (2003) shows that if ”contagion” fully explained the large degree of clustering it should not be the case that in default times both partner and competitor firms have a higher default probability. Yu (2005) has found some inconclusive evidence of ”frailty” and learning after default as a correlation device.

[39] Recall in the model we assumed there is no default from a reduction of cash flows

In the model reputation is the probability of being a firm with access to a safe technology. Since all firms have access to risky technologies (experimentation, for example), firms' unobservable types are defined by the unobservable actions they can take. Firms that can choose between safe and risky actions want to distinguish themselves from firms that do not have a choice. A higher reputation allows firms to pay lower interest rates and to charge higher prices. However, since types are defined by action availability, reputation does not have an intrinsic value. The reputation of being able to choose between safe and risky actions does not really matter if lenders and consumers also believe the choice will be to take risks. In this sense reputation is fragile because its value is a combination of having a certain type and behaving in a certain way. None of these two conditions is important without the other.

In the model reputation can be constructed, destroyed and managed. However, this desirable property comes with a cost in terms of equilibria multiplicity. To overcome this problem I interpret the reputation model extended with fundamentals as a non-standard dynamic global game in which strategic complementarities arise endogenously from reputation formation, and hence depend on the dynamic structure of the game, rather than being hard-wired into static payoffs as is common in the global games literature. This allows us to select a unique equilibrium robust to information perturbations.

I provide empirical support for my theory using data on corporate credit-rating transitions over the business cycle. I show that credit ratings evolve more slowly in bad times than in good times, which supports my prediction that reputation formation is more gradual and more difficult in bad times. I also discuss recent literature that shows that risk taking and defaults cluster in time, especially around recessions, at a large degree that cannot be explained just from a weakening in fundamentals. These empirical results are also consistent with the large degree of clustering predicted by the model.

Finally, the model suggests several new policy implications. First, under periods of "clustering in risk taking" credit ratings are likely to be uninformative about default probabilities. Even firms with AAA bond ratings, for example, may have incentives in bad times to undertake risky projects which greatly increase the probability of default. This introduces a warning sign against relying on ratings as a basis to determine the right capital that banks should hold, as is the case with the Basel II regulations. My paper suggests that banking regulations which rely on official credit ratings may spread the effects of losses of confidence in borrowers more widely through the financial system, opening the doors to broader financial crises. Second, policies that promote credit bureaus facilitate learning and increase reputation incentives in domestic financial systems. My model implies that these policies have the potential to deter excessive risk taking but at the same time may exacerbate credit crises.

References

Akhavein, Jalal D., Ahmet E. Kocagil, and Matthias Neugebauer. 2005. "A Comparative Empirical Study of Asset Correlations." Working Paper, Fitch Ratings, New York.

Altman, Edward, and Herbert Rijken. 2006. "A Point-in-Time Perspective on Through-the-Cycle Ratings." *Financial Analysts Journal*.

Angeletos, George-Marios, Christian Hellwig, and Alessandro Pavan. 2006. "Signaling in a Global Game: Coordination and Policy Traps." *Journal of Political Economy* 114 (3): 452–484.

———. 2007. "Dynamic Global Games of Regime Change: Learning, Multiplicity, and the Timing of Attacks." *Econometrica* 75 (3): 711–756.

Athey, Susan. 2002. "Monotone Comparative Statics Under Uncertainty." *Quaterly Journal of Economics* 117 (1): 187–223.

Atkeson, Andrew G. 2001. "Discussion on Morris and Shin." *NBER Macroeconomics Annual 2000*, ed. by B.S. Bernanke and K.S. Rogoff. Cambridge,MA: MIT Press, 161–170.

Bangia, Anil, Francis Diebold, and Til Schuermann. 2000. "Ratings Migration and the Business Cycle, with and Application to Credit Portfolio Stress Testing." Mimeo, New York University.

Campbell, John, Martin Lettau, Burton Malkiel, and Yexiao Xu. 2001. "Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk." *The Journal of Finance* 56 (1): 1–43.

Chassang, Sylvain. 2007. "Fear of miscoordination and the robustness of cooperation in dynamic global games with exit." Mimeo, MIT.

Das, Sanjiv, Darrell Duffie, Nikunj Kapadia, and Leandro Saita. 2007. "Common Failings: How Corporate Defaults Are Correlated." *The Journal of Finance* 62 (1): 93–118.

Davis, Steven, John Haltiwanger, Ron Jarmin, and Javier Miranda. 2006. "Volatlity and Dispersion in Business Growth Rates: Publicly Traded versus Privately Held Firms." NBER Working Paper 12354.

Diamond, Douglas. 1989. "Reputation Acquisition in Debt Markets." *The Journal of Political Economy* 97 (4): 828–862.

Fama, Eugene, and Kenneth French. 1997. "Industry Costs of Equity." *Journal of Financial Economics* 93:153–194.

Fudenberg, Drew, and David K. Levine. 1992. "Maintaining a Reputation when Strategies are Imperfectly Observed." *Review of Economic Studies* 59:561–579.

Giannitsarou, Chryssi, and Flavio Toxvaerd. 2007, April. "Recursive Global Games." Mimeo, University of Cambridge.

Goyal, Amit, and Pedro Santa-Clara. 2003. "Idiosyncratic Risk Matters!" *Journal of Finance* 58 (3): 975–1008.

Holmstrom, Bengt. 1999. "Managerial Incentive Problems: A Dynamic Perspective." *Review of Economic Studies* 66 (1): 169–182 (January).

Koopman, Siem, Roman Krauss, Andre Lucas, and Andre Monteiro. 2006. "Credit Cycles and Macro Fundamentals." Tinbergen Institute Discussion Paper 023/2.

Koopman, Siem, and Andre Lucas. 2005. "Business and Default Cycles for Credit Risk." *Journal of Applied Econometrics* 20:311–323.

Kreps, David, and Robert Wilson. 1982. "Reputation and Imperfect Information." *Journal of Economic Theory* 27:253–279.

Mailath, George, and Larry Samuelson. 2001. "Who Wants a Good Reputation?" *Review of Economic Studies* 68:415–441.

———. 2006. *Repeated Games and Reputations*. Oxford University Press.

Milgrom, Paul, and John Roberts. 1982. "Predation, Reputation and Entry Deterrence." *Journal of Economic Theory* 27:280–312.

Morris, Stephen, and Hyun Song Shin. 2000. "Rethinking Multiple Equilibria in Macroeconomic Modelling." *NBER Macroeconomics Annual*.

———. 2003. Chapter Global Games: Theory and Applications of *Advances in Economics and Econometrics*, Dewatripont, M. and Hansen, L. and Turnovsky, S. Cambridge University Press.

Nickell, Pamela, William Perraudin, and Simone Varotto. 2000. "Stability of Rating Transitions." *Journal of Banking and Finance* 24:203–227.

Schonbucher, Philipp. 2003. "Information Driven Default Correlation." Mimeo, ETH Zurich.

Steiner, Jakub. 2006, October. "Coordination Cycles." CERGE-EI Working Paper 274.

Stock, JH, and M Watson. 1989. "New Indexes of Coincident and Leading Indicators." *NBER Macroeconomics Annual* 4:351–394.

Toxvaerd, Flavio. 2007. "Strategic Merger Waves: A Theory of Musical Chairs." Journal of Economic Theory, Forthcoming.

Yu, Fan. 2005. "Accounting Transparency and the Term Structure of Credit Spreads." *Journal of Financial Economics* 75:53–84.

A Appendix

A.1 Proof of Proposition 3

Proof To prove this proposition I proceed in four steps. First I derive the posterior density and distribution of θ given a signal z. Second, I prove there is a unique signal $z^*(\phi)$ that makes a strategic firm ϕ indifferent between taking risk or not, such that $z^*(\phi)$ is determined using Laplacian beliefs. Third I show that using $z^*(\phi)$ is a best response when the prior about θ follows a uniform distribution on the real line and both lenders and consumers believe $z^*(\phi)$ is the equilibrium cutoff. Finally we show that, as $\sigma \to 0$, the game with any prior distribution of θ uniformly converges to the unique solution proved in the previous step.

• **Step 1: Distributions of fundamentals conditional on signals**

Lemma 7 *The posterior density $f_{\theta|z}$ and distribution $F_{\theta|z}$ of θ given a signal z are given by,*

$$f_{\theta|z}(\eta|z) = \frac{v(\eta)f(\frac{z-\eta}{\sigma})}{\int_{-\infty}^{\infty} v(\theta)f(\frac{z-\theta}{\sigma})d\theta} \tag{20}$$

$$F_{\theta|z}(\eta|z) = \frac{\int_{-\infty}^{\eta} v(\theta)f(\frac{z-\theta}{\sigma})d\theta}{\int_{-\infty}^{\infty} v(\theta)f(\frac{z-\theta}{\sigma})d\theta} = \frac{\int_{\frac{z-\eta}{\sigma}}^{\infty} v(z-\sigma u)f(u)du}{\int_{-\infty}^{\infty} v(z-\sigma u)f(u)du} \tag{21}$$

Proof By Bayes' rule,

$$f_{\theta|z}(\theta|z) = \frac{v(\theta)f_{z|\theta}(z|\theta)}{f_z(z)} \tag{22}$$

where f_z and $f_{z|\theta}$ are the densities of z and $z|\theta$ respectively. Since z is the sum of θ and $\sigma\epsilon$, its density is given by the convolution of their densities, i.e., v and $f_{\sigma\epsilon}$. Considering that $F_{\sigma\epsilon}(\eta) = F(\eta/\sigma)$, $f_{\sigma\epsilon}(\eta) = \frac{f(\eta/\sigma)}{\sigma}$, then f_z can be defined as,

$$f_z(z) = \sigma^{-1}\int_{-\infty}^{\infty} v(\theta)f\left(\frac{z-\theta}{\sigma}\right)d\theta \tag{23}$$

We can obtain the distribution of the observed signal z after observing a fundamental θ.

$$F_{z|\theta}(\eta|\theta) = Pr(z \le \eta|\theta) = F\left(\frac{\eta-\theta}{\sigma}\right)$$

$$f_{z|\theta}(\eta|\theta) = \frac{dF_{z|\theta}(\eta|\theta)}{dz} = \sigma^{-1}f\left(\frac{\eta-\theta}{\sigma}\right) \tag{24}$$

Plugging equations 24 and 23 in 22, we obtain equation 20. The posterior distribution is obtained integrating over the density,

$$F_{\theta|z}(\eta|z) = \int_{-\infty}^{\eta} f_{\theta|z}(\theta|z)d\theta = \frac{\int_{-\infty}^{\eta} v(\theta)f(\frac{z-\theta}{\sigma})d\theta}{\int_{-\infty}^{\infty} v(\theta)f(\frac{z-\theta}{\sigma})d\theta}$$

and the expression in equation 21 follows from variable transformation $u = \frac{z-\theta}{\sigma}$ Q.E.D.

• **Step 2: Unique equilibrium cutoff** $z^*(\phi)$ **(using Laplacian beliefs)**.

Lemma 8 *There is a unique cutoff signal for each reputation ϕ such that $\Delta(\phi, z^*|z^*) = 0$, $\Delta(\phi, z|z^*) > 0$ for $z > z^*$ and $\Delta(\phi, z|z^*) < 0$ for $z < z^*$, where $\Delta(\phi, z|z^*)$ are the expected differential gains from playing safe for a firm with reputation ϕ that observes a signal z when lenders believe the cutoff the firm follows is $z^*(\phi)$.*

The cutoff $z^*(\phi)$ is obtained using Laplacian beliefs, where $\widehat{x} = F\left(\frac{z^*-\theta}{\sigma}\right)$ is the probability the firm plays risky when the fundamental is θ

$$\int_0^1 \Delta(\phi, z^*|\widehat{x})d\widehat{x} = 0 \tag{25}$$

Proof When fundamentals θ are not observed directly, the firm observes a signal z and lenders believe firms use a cutoff $z^*(\phi)$, the expected gains from playing safe are

$$\Delta(\phi, z, z^*) = E[\Delta(\phi, \theta|\widehat{x})|z] \tag{26}$$

where $\widehat{x}$ is just a function of the cutoff z^*.

$$\widehat{x} = F\left(\frac{z^* - \theta}{\sigma}\right) \tag{27}$$

Developing the expectation

$$\Delta(\phi, z, z^*) = \int_{-\infty}^{\infty} \Delta(\phi, \theta|\widehat{x})dF_{\theta|z}(\theta|z)$$

Note that $\theta = z^* - \sigma F^{-1}(\widehat{x})$. From equation 21, define

$$\Psi(\widehat{x}; z, z^*) = F_{\theta|z}(z^* - \sigma F^{-1}(\widehat{x})|z) = \frac{\int_{\frac{z-z^*}{\sigma}+F^{-1}(\widehat{x})}^{\infty} v(z-\sigma u)f(u)du}{\int_{-\infty}^{\infty} v(z-\sigma u)f(u)du}$$

Changing variables, from θ to $\widehat{x}$

$$\Delta(\phi, z, z^*) = \int_{-\infty}^{\infty} \Delta(\phi, z^* - \sigma F^{-1}(\widehat{x})|\widehat{x})d\Psi(\widehat{x}; z, z^*)$$

Laplacian beliefs arise from

$$\Psi(\widehat{x}; z, z^*) = Pr(\theta < z^* - \sigma F^{-1}(\widehat{x})|z) = F[\frac{z-z^*}{\sigma} + F^{-1}(\widehat{x})]$$

For $z = z^*$, $\Psi(\widehat{x}; z^*, z^*) = \widehat{x}$. Then, as $\sigma \to 0$

$$\int_0^1 \Delta(\phi, z^*|\widehat{x})d\widehat{x} = 0$$

By Lemmas 3 and 4, we know there is a unique solution $z^*(\phi)$ to this equation. Q.E.D.

The intuition behind the use of a uniform distribution of beliefs $\widehat{x}$ to obtain the solution is straightforward. Adapting the discussion in Morris and Shin (2003), the key to understanding this feature is to consider the answer to the following question asked by a firm. "My signal has a realization z. What are the chances that lenders assign a probability smaller than η to me having a signal smaller than z?". If the true state is θ, the probability the firm observe a signal below z is given by $F\left(\frac{z-\theta}{\sigma}\right)$. This probability is smaller than η if $\frac{z-\theta}{\sigma} < F^{-1}(\eta)$, or when

$$\theta > z - \sigma F^{-1}(\eta)$$

The probability of this event, conditional on z

$$Pr(\theta > z - \sigma F^{-1}(\eta)|z) = Pr(z - \sigma\epsilon > z - \sigma F^{-1}(\eta)) = F(F^{-1}(\eta)) = \eta$$

Considering in particular the cutoff $z^*(\phi)$, we can define $\widehat{x} = F\left(\frac{z^*(\phi)-\theta}{\sigma}\right)$, $Pr(\widehat{x} < \eta) = \eta$, hence the cumulative distribution of $\widehat{x}$ is the identity function, implying the density of $\widehat{x}$ is uniform over the unit interval. In words, if $z^*(\phi)$ happens to be the switching point of an equilibrium strategy, then playing safe or risky should be indifferent for the firm given lenders and consumers beliefs $\widehat{x}$ are uniformly distributed in [0,1].

• **Step 3: Best response with uniform priors over fundamentals.**

Now we need to verify that there exists indeed an equilibrium in which a firm with reputation ϕ plays risky whenever $z < z^*(\phi)$ and plays safe whenever $z > z^*(\phi)$. Signals z allow firms to have an idea not only about the fundamental but also about the signal lenders and consumers believe the firm has observed. Following Toxvaerd (2007), I first assume θ is drawn from a uniform distribution on the real line, hence an improper distribution with infinite probability mass. It is possible to normalize the prior distribution assuming $v(\theta) = 1$, simplifying the density to $f_{\theta|z}(\theta|z) = \sigma^{-1}f(\frac{z-\theta}{\sigma})$ and the distribution to $F_{\theta|z}(\theta|z) = F(\frac{z-\theta}{\sigma})$

First, we will denote $\widetilde{\Delta}(\phi, z|z^*)$ the case with a uniform prior distribution of fundamentals. We can redefine $\widetilde{\Delta}(\phi, z|\widehat{x})$ as $\widetilde{\Delta}(\phi, z|\widehat{z}(\phi))$ by writing $\widehat{x} = F\left(\frac{\widehat{z}(\phi)-\theta}{\sigma}\right)$, where $\widehat{z}(\phi)$ is the cutoff that the market believes strategic players ϕ use and z is the signal received by the firm. In words, since the market believes ϕ strategic firms follow the cutoff $\widehat{z}(\phi)$, when updating reputation and knowing the real fundamental, the market assigns a probability $\widehat{x} = F\left(\frac{\widehat{z}(\phi)-\theta}{\sigma}\right)$ the firm observed a signal z below the cutoff and played risk.

Expected payoff gains from playing safe rather than risky, given signal z when the market believes strategic firms ϕ use cutoffs $\widehat{z}(\phi)$ are given by

$$\widetilde{\Delta}(\phi, z|\widehat{z}(\phi)) = \int_{-\infty}^{\infty} \widetilde{\Delta}\left(\phi, z|F\left(\frac{\widehat{z}(\phi)-\theta}{\sigma}\right)\right)\sigma^{-1}f\left(\frac{z-\theta}{\sigma}\right)d\theta$$

Changing variables introducing $m = \frac{\theta-\widehat{z}(\phi)}{\sigma}$

$$\widetilde{\Delta}(\phi, z|\widehat{z}(\phi)) = \int_{-\infty}^{\infty} \widetilde{\Delta}\left(\phi, z|F(-m)\right)\sigma^{-1}f\left(\frac{z-\widehat{z}(\phi)}{\sigma} - m\right)d\theta$$

We can rewrite it in a simpler way as

$$\widetilde{\Delta}(\phi, z|\widehat{z}(\phi)) = \widehat{\Delta}(\phi, z|\widehat{z}(\phi), z') = \int_{-\infty}^{\infty} B(z', m) D(z, m) dm$$

where $B(z', m) = \widetilde{\Delta}\left(\phi, z'|F(-m)\right)$ and $D(z, m) = \sigma^{-1} f\left(\frac{z-\widehat{z}(\phi)}{\sigma} - m\right)$. As shown in Athey (2002), because of the monotone likelihood property, $\widehat{\Delta}(\phi, z|\widehat{z}(\phi), z')$ inherits the single crossing property of $\widetilde{\Delta}(\phi, \theta|\widehat{x})$. This means it exists a $z^*(\phi, \widehat{z}(\phi), z')$ such that $\widehat{\Delta}(\phi, z|\widehat{z}(\phi), z') > 0$ if $z > z^*(\phi, \widehat{z}(\phi), z')$ and $\widehat{\Delta}(\phi, z|\widehat{z}(\phi), z') < 0$ if $z < z^*(\phi, \widehat{z}(\phi), z')$.

Assuming $z > z'$ and $\widehat{\Delta}(\phi, z|\widehat{z}(\phi), z) = 0$, we can show

$$\widehat{\Delta}(\phi, z'|\widehat{z}(\phi), z') \geq \widehat{\Delta}(\phi, z|\widehat{z}(\phi), z') \geq \widehat{\Delta}(\phi, z|\widehat{z}(\phi), z) = 0$$

The first inequality coming from the state monotonicity and the second from the action single crossing property. A symmetric argument holds for $z > z^*(\phi)$. Hence, there exists a best response $\chi : \mathbb{R} \to \mathbb{R}$ such that

$$\begin{array}{lll} \widetilde{\Delta}(\phi, z|\widehat{z}(\phi)) > 0 & if & z > \chi(\widehat{z}(\phi)) \\ \widetilde{\Delta}(\phi, z|\widehat{z}(\phi)) = 0 & if & z = \chi(\widehat{z}(\phi)) \\ \widetilde{\Delta}(\phi, z|\widehat{z}(\phi)) < 0 & if & z < \chi(\widehat{z}(\phi)) \end{array}$$

Since there exists a unique $z^*(\phi)$ that solves

$$\widetilde{\Delta}(\phi, z^*(\phi)|z^*(\phi)) = \int_0^1 \widetilde{\Delta}(\phi, z^*(\phi)|\widehat{x}) d\widehat{x} = 0 \tag{28}$$

Hence, $\chi(z^*(\phi)) = z^*(\phi)$, showing that there is a unique equilibrium in cutoff strategies for each ϕ such that

$$x^*(\phi, z) = \begin{cases} 0 & if \quad z > z^*(\phi) \\ 1 & if \quad z < z^*(\phi) \end{cases} \tag{29}$$

Q.E.D.

• **Step 4: Best response with general priors over fundamentals.**

Lemma 9 *As $\sigma \to 0$, $\Delta(z, z - \sigma\xi) \to \widetilde{\Delta}(z, z - \sigma\xi)$ uniformly.*

Proof First, $\Delta(z, z - \sigma\xi) \to \widetilde{\Delta}(z, z - \sigma\xi)$ continuously as $\sigma \to 0$

$$\Psi(\widehat{x}; z, z - \sigma\xi) = \frac{\int_{\xi + F^{-1}(\widehat{x})}^{\infty} v(z - \sigma u) f(u) du}{\int_{-\infty}^{\infty} v(z - \sigma u) f(u) du} \to 1 - F\left(\xi + F^{-1}(\widehat{x})\right) = \widetilde{\Psi}(\widehat{x}; z, z - \sigma\xi)$$

As in Toxvaerd (2007), we show convergence with respect to the uniform convergence norm, which implies uniform convergence. Uniformity ensures that the equivalence between the games with the two different assumptions about the prior distributions, is not a result of a discontinuity on $\sigma = 0$.

From the existence of dominance regions for each reputation level ϕ, $(-\infty, \underline{\theta}(\phi|\underline{\theta}))$ and $(\overline{\theta}(\phi|\overline{\theta}), \infty)$. Pick $\underline{z}(\phi) < \underline{\theta}(\phi|\underline{\theta})$ and $\overline{z}(\phi) > \overline{\theta}(\phi|\overline{\theta})$ and restrict attention to the compact sets $Z \equiv [\underline{z}(\phi), \overline{z}(\phi)]$ and $Z_\sigma \equiv [\underline{z}(\phi) - \sigma\xi, \overline{z}(\phi) - \sigma\xi]$. Hence $\Delta(\phi, z, z^*)$ maps into a compact set.

Define the sup-norm

$$\| \Delta(\phi) \| \equiv sup_{z,z^*}\{|\Delta(\phi, z, z^*)|\}$$

With respect to the Euclidean metric,

$$\begin{aligned}
&\forall \epsilon_1 > 0, \exists \delta_1 \, |z - z'| < \delta_1 \Rightarrow |\Delta(\phi, z, z^*) - \widetilde{\Delta}(\phi, z', z^*)| < \epsilon_1, \forall z^* \\
&\forall \epsilon_2 > 0, \exists \delta_2 \, |z^* - z^{*'}| < \delta_2 \Rightarrow |\Delta(\phi, z, z^*) - \widetilde{\Delta}(\phi, z, z^{*'})| < \epsilon_2, \forall z
\end{aligned}$$

This implies

$$\sqrt{(z - z')^2 + (z^* - z^{*\prime})^2} < \sqrt{\delta_1^2 + \delta_2^2}$$

By triangle inequality

$$\begin{aligned}
|\Delta(\phi, z, z^*) - \Delta(\phi, z', z^{*'})| &= |\Delta(\phi, z, z^*) - \Delta(\phi, z', z^*) + \Delta(\phi, z', z^*) - \Delta(\phi, z', z^{*'})| \\
&\leq |\Delta(\phi, z, z^*) - \Delta(\phi, z', z^{*'})| + |\Delta(\phi, z', z^*) - \Delta(\phi, z', z^{*'})| \\
&\leq \epsilon_1 + \epsilon_2
\end{aligned}$$

As $\sigma \to 0$

$$\| \Delta(\phi) - \widetilde{\Delta}(\phi) \| = sup_{z,z^*}\{\Delta(\phi, z, z^*) - \widetilde{\Delta}(\phi, z, z^*)\} \to 0$$

with respect to the sup-norm Q.E.D.

A.2 Proof and Intuition Lemma 6

Let's start with the case in which playing safe with a very high probability is a fixed point among continuation values (first bullet point in Lemma 6). If an important weakening of fundamentals occurs, firms will cluster in risk-taking behavior for a short period of time. This case is the simplest one and helps to develop a clear understanding of the intuition behind the next graphs.

Figure 17 shows $\mathbf{V}_t(\phi)$ as a function of the expected continuation value in $t+1$ ($\mathbf{V}_{t+1}(\phi)$). The lines labeled *"Always Risky"* and *"Always Safe"* represent cases in which strategic firms with a reputation level ϕ are believed to play always risky or always safe (as defined in equation 10 when $x = 1$ and $x = 0$) respectively. By Assumption 5, the intercept in the case in which firms are believed to play risky is higher than the intercept of the value when strategic firms are believed to play safe. Furthermore, the slope of the value when firms are believed to play risky (βp_r) is smaller than the slope of the value when strategic firms are believed to play safe (βp_s).

In Figure 17 we show the two extreme cases where $\phi = 0$ and $\phi = 1$. Take for example $\phi = 0$. In this case, regardless of the beliefs of lenders and consumers, $E(\mathbf{V}_{t+1}(\phi')) = \mathbf{V}_{t+1}(\phi)$ because reputation never improves or decays. A similar argument is true for $\phi = 1$.

However, these are the expected values in case lenders and consumers never change beliefs about risk-taking behavior, regardless of fundamentals or future payoffs. As shown in Lemma 5, the expected continuation value of a particular firm depends on the probabilities the fundamental θ lies below the optimal cutoff $z^*(\phi)$. Hence, expected values $\mathbf{V}_t(0)$ and $\mathbf{V}_t(1)$ are represented by the solid red lines.

For $\phi = 0$, $\Delta_t(0, z_t) = p_s\Pi_s(0) - p_r\Pi_r(0) + (p_s - p_r)[\beta\mathbf{V}_{t+1}(0) - R_t(0)]$. This differential determines $z_t^*(0)$, which is negatively related to $\mathbf{V}_{t+1}(0)$. It is possible to find a $\widetilde{\mathbf{V}}_{t+1}(0)$ for which $z_t^*(0) = E(\theta)$. For a small γ_θ, if $\widetilde{\mathbf{V}}_{t+1}(0)$ is to the left of the point $SS0$ in Figure 17, then the continuation value converges backward to $\overline{\mathbf{V}}(0)$ for a reputation level $\phi = 0$. Given a continuation value $\overline{\mathbf{V}}(0)$, strategic firms ϕ play safe with a very high probability.

A similar analysis is true at the other extreme, for $\phi = 1$. In this case $\Delta_t(1, z_t) = p_s\Pi_s(1) - p_r\Pi_r(1) + (p_s - p_r)[\beta\mathbf{V}_{t+1}(1) - R_t(1)]$. Note $\Delta_t(1, z_t) > \Delta_t(0, z_t)$ (for a given continuation value at $t+1$) from two effects. First, prices for $\phi = 1$ are greater than prices for $\phi = 0$ (in the case the market assigns some positive belief for the firm to play safe). Hence, $p_s\Pi_s(1) - p_r\Pi_r(1) > p_s\Pi_s(0) - p_r\Pi_r(0)$. Second, interest rates charged to firms with high reputation $\phi = 1$ are lower than those charged to $\phi = 0$, hence $R_t(1) < R_t(0)$.

It is important to note that even if lenders and consumers strangely believe that $\phi = 0$ is unlikely to play risky and $\phi = 1$ is unlikely to play safe, $R_t(1) < R_t(0)$ because they also believe the $\phi = 0$ firm cannot make a choice while the $\phi = 1$ firm is at least strategic for sure and sometimes will play risky.

Since $\Delta_t(1, z_t) > \Delta_t(0, z_t)$, $\widetilde{\mathbf{V}}_{t+1}(1) < \widetilde{\mathbf{V}}_{t+1}(0)$ and hence $\widetilde{\mathbf{V}}_{t+1}(1)$ is to the left of $SS1$. The intercept is higher for $\phi = 1$ than for $\phi = 0$ while slopes are the same. Hence, continuation values converge to a higher level for $\phi = 1$ than for $\phi = 0$, $\overline{\mathbf{V}}(1) > \overline{\mathbf{V}}(0)$.

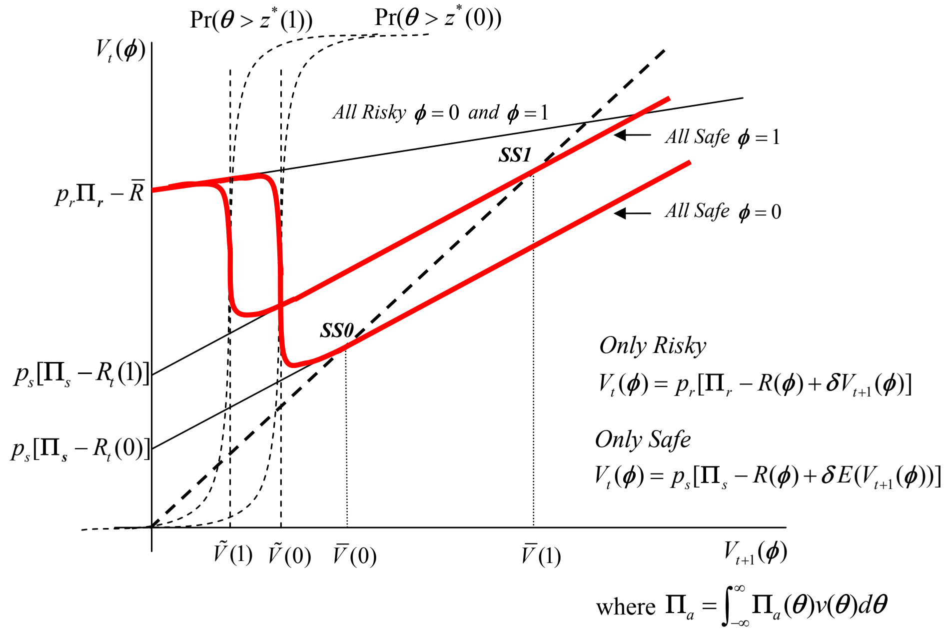
The lowest possible price and highest possible lending rate correspond to $\phi = 0$ because regardless of what a strategic firms would decide, lenders and consumers just believe that a firm is not strategic. For all other reputation levels, the potential behavior of strategic firms matters because lenders and consumers assign a probability that the firm is, in fact, strategic.

Hence, whenever $\widetilde{\mathbf{V}}_{t+1}(0)$ is to the left of the point $SS0$ in Figure 17, the continuation values for all reputation levels ϕ will converge to a unique value $\overline{\mathbf{V}}(\phi)$

The case depicted in Figure 17 not only shows that continuation values are well defined but also that they converge to a fixed point such that all firms will be playing safe with a very high probability (close to 1).

A similar analysis leads to convergence to continuation values dominated by risk-taking. This is the case depicted in Figure 18. The condition for this to be the case is that $\widetilde{\mathbf{V}}_{t+1}(1)$ exceeds

Figure 17: Unique "Safe" Continuation Value



the value at which $\mathbf{V}_t(1)$ crosses the 45-degree line. In this case all firms, regardless of their reputation, decide to take risk with a very high probability (close to 1). In this extreme case continuation values are well defined and converge to the same value for all reputation levels ϕ. Reputation does not play any role in introducing incentives in this particular situation where all firms almost always decide to play risky.

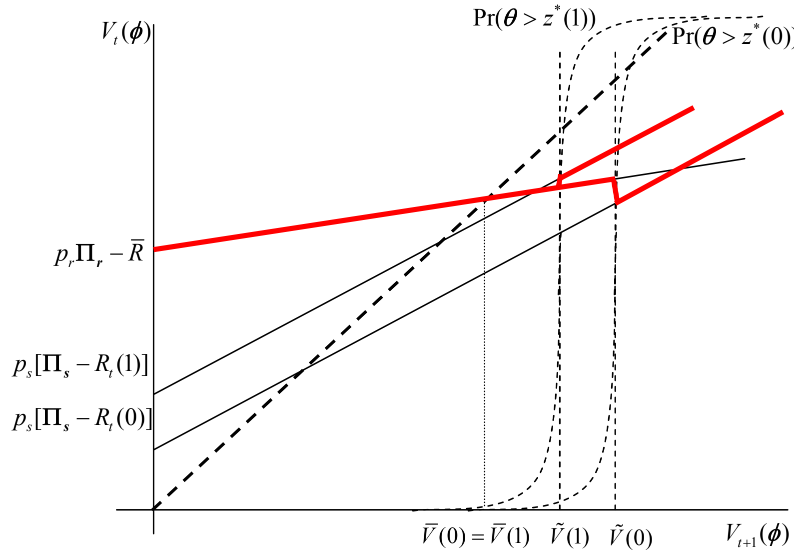
Steady states in which the market believes that firms will play safe or risky with a very high probability (close to 1) are not particularly interesting (even when the first case is able to explain situations of sudden and scarce risk-taking clustering, as shown in the data).

In Figure 19 we show the third possible situation in which, for $\phi = 0$ and $\phi = 1$, $\widetilde{\mathbf{V}}(0)$ and $\widetilde{\mathbf{V}}(1)$ lie between the points $SS0$ and $SS1$. To obtain convergence to a single continuation value (rather than cyclical movements) we must make the extra assumption that γ_θ is high enough.

The probability of risk-taking is higher for low reputation values than for high reputation levels. This implies monotonicity of continuation values on reputation levels ϕ

In the case of cyclic or even chaotic patterns, a full characterization of these cycles is always possible because continuation values are well defined, as shown in Proposition 4. However, it is possible for continuation values to behave counter-intuitively. Recall $\mathbf{V}_t(\phi)$ depends on $\mathbf{V}_{t+1}(\phi_g)$ and $\mathbf{V}_{t+1}(\phi_b)$, but these will depend on the specific part of the cycle that ϕ_g and ϕ_b will be playing at $t+1$. For example, it is possible that $\mathbf{V}_{t+1}(\phi_b) > \mathbf{V}_{t+1}(\phi_g)$ if ϕ_b will be playing risky next period while ϕ_g will be playing safe. In this case, reputation is reversed and firms may try to have a bad reputation to take advantage of the future risk-taking behavior of low reputation level firms.

Figure 18: Unique "Risky" Continuation Value

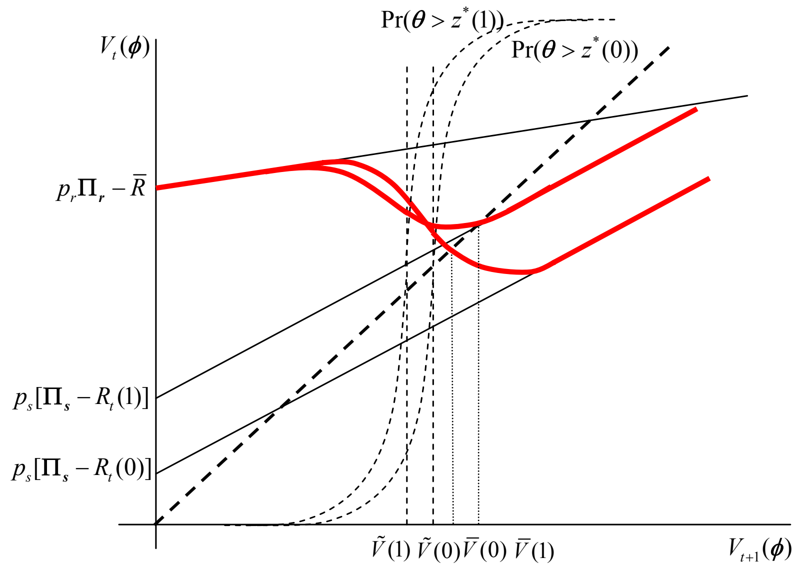


In our model, reputation is not treated as an intrinsic asset that makes the market blindly assign more value to firms with higher reputation. Because of this, it may be that firms do not try to achieve a higher reputation per se, but the reputation of firms more likely to play risky in order to pool with them and get more expected profits.

These pervasive effects of cycles come from the fact that reputation is not an asset per se but a signal of how well firms commit. Introducing reputation as an asset would eliminate this particular feature of the model, preserving, nevertheless, the existence of cyclical behavior.

In any case, as all models with univariate dynamics, this is not a compelling setting to analyze cycles since it heavily depends on the definition of the period length. Making the period shorter enough, for example, generates cycles at a higher frequency. Since the period length does not have any economic meaning but it changes the results of the model, this model is not effective to discuss cyclical properties.

Figure 19: Unique Continuation Value in Probabilities



A.3 Computational Procedure

We solve the model following the next procedure.

- Set a large grid of $\phi \in [0,1]$
- Solve for the full information case (efficiency)
  - Guess a $\mathbf{V}_{FI}(1)_0 = 0$.
  - Obtain θ_0^* by making $\Delta(1,\theta)_{FI} = p_s\Pi_s(1,\theta) - p_r\Pi_r(1,\theta) + \beta(p_s - p_r)\mathbf{V}_{FI}(1)_0 = 0$
  - Obtain $\mathbf{V}_{FI}(1)_1 = \left[\mathcal{V}(\theta_0^*)[p_r\Pi_r - \overline{R}] + \int_{\theta_0^*}^{\infty}[p_s\Pi_s(\theta) - \overline{R}]v(\theta)d\theta\right] / \left[1 - \beta(p_r + \mathcal{V}(\theta_0^*)(p_s - p_r))\right]$
  - Use $\mathbf{V}_{FI}(1)_1$ as the new guess and iterate until $\mathbf{V}_{FI}(1)_I - \mathbf{V}_{FI}(1)_{I-1} < \varepsilon$.
- Solve for the reputation case.
  - Guess a $\mathbf{V}(\phi)_0 = 0$ for all ϕ.
  - Using $\mathbf{V}(\phi)_0$ obtain $\Delta(\phi, z|\widehat{x})_0$ for large N_x beliefs $\widehat{x} \in [0,1]$.
  - Solve for $z^*(\phi)_0$ that makes $\frac{\sum \Delta(\phi,z|\widehat{x})_0}{N_x} = 0$
  - For all $\theta < (>) z^*(\phi)_0$, $x(\phi,\theta)_0 = 1 (= 0)$.
    * $R(\phi|z_0^*)_0$ follows from $z^*(\phi)_0$
    * $\phi_g(\phi,\theta)_0$ and $\phi_b(\phi,\theta)_0$ follow from $x(\phi,\theta)_0$.

- Obtain $\mathbf{V}(\phi)_1$ as

$$\mathbf{V}(\phi)_1 = \int_{-\infty}^{z^*(\phi)_0} p_r[\Pi_r - R(\phi|z_0^*)_0 + \beta\mathbf{V}(\phi)_0]v(\theta)d\theta$$
$$+ \int_{z^*(\phi)_0}^{\infty} p_s[\Pi_s(\theta) - R(\phi|z_0^*)_0 + \beta E(\mathbf{V}_{t+1}(\phi|0)_0)]v(\theta)d\theta$$

- Use $\mathbf{V}(\phi)_1$ as the new guess.
- Solve for $z^*(\phi)_1$ that makes $\frac{\sum \Delta(\phi,z|\widehat{x})_1}{N_x} = 0$.
- Iterate until $\mathbf{V}(\phi)_I - \mathbf{V}(\phi)_{I-1} < \varepsilon$ for all ϕ.

- Solve for the non reputation concerns case
  - Guess a $\mathbf{V}(\phi)_0 = 0$ for all ϕ.
  - Obtain $\theta^*_{MH}(\phi)_0$ by making $\Delta(\phi,\theta)_{MH} = p_s\Pi_s(\phi,\theta) - p_r\Pi_r(\phi,\theta) + \beta(p_s - p_r)[\mathbf{V}_{FI}(\phi)_0 - R(\phi|\theta^*_{FI})] = 0$
  - Obtain $\mathbf{V}_{MH}(\phi)_1 = \frac{\mathcal{V}(\theta^*_{MH}(\phi)_0)[p_r[\Pi_r - R(\phi)] + \int_{\theta^*_{MH}(\phi)_0}^{\infty}[p_s[\Pi_s(\theta) - R(\phi)]]v(\theta)d\theta}{\left[1-\beta(p_r + \mathcal{V}(\theta^*_{MH}(\phi)_0)(p_s - p_r))\right]}$ for each ϕ.
  - Use $\mathbf{V}_{MH}(\phi)_1$ as the new guess and iterate until $\mathbf{V}_{MH}(\phi)_I - \mathbf{V}_{MH}(\phi)_{I-1} < \varepsilon$.