Political Institutions and the Dynamics of Public Investment[1]

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Abstract

We present a theoretical model of the provision of a durable public good over an infinite horizion. In each period, there is a societal endowment of which each of n districts owns a share. This endowment can either be invested in the public good or consumed. We characterize the planner's optimal solution and time path of investment and consumption. We then consider alternative political mechanisms for deciding on the time path, and analyze the Markov perfect equilibrium of these mechanisms. One class of these mechanisms involves a legislature where representatives of each district bargain with each other to decide how to divide the current period's societal endowment between investment in the public good and transfers to each district. The second class of mechanisms involves the districts making independent decisions for how to divide their own share of the endowment between consumption and investment. We conduct an experiment to assess the performance of these mechanisms, and compare the observed allocations to the Markov perfect equilibrium.
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1 Introduction

Most public goods provided by governments are durable, and hence dynamic in nature. It takes time to accumulate them, and they depreciate slowly, projecting their benefits for many years. Prominent examples are national defense, environmental protection and public infrastructure. Although a large literature has studied public good provision in static models both theoretically and empirically, much less is known about dynamic environments. First of all: how serious is free riding in these cases? Laboratory experiments have shown that theoretical predictions tend to overestimate the seriousness of free riding in static environments.[1] No empirical evidence, however, has been collected for the more realistic dynamic environments discussed above, either from field data or from laboratory experiments Secondly, to what extent does the free rider problem depend on the institutions that govern public decision making? Again, here little work has been done except for static environments. The institutional environment does not only determine the extent to which the policy will reflect the welfare of the citizens when the policy is chosen, but it also determines the extent to which political actors will internalize the benefits that will accrue in future periods; in a word, how "shortsighted" the policy is. The dynamic nature of public goods and the institutional setting, therefore cannot be studied separately: on the one hand, the institutional setting will determine the shortsightedness of the policy and the nature of the free rider problem; on the other hand, the dynamic free rider problem should be an important factor in evaluating the institutional setting.

In this work we make a first attempt to answer these questions by proposing a theoretical model of dynamic public good provision under alternative institutional settings and testing it in a laboratory experiment. Experimental analysis is particularly important when studying a highly structured dynamic environment that cannot be easily replaced by field data; this is because strategic behavior can be observed only if there is a precise measurement of the "state variable" and the actions space available to the players. To our knowledge, this is the first experimental study of the dynamic accumulation process of a durable public good.

The economy we study has a continuum of infinitely lived citizens who live in n equal-sized districts. A durable public good can be accumulated and depreciates at rate $d < 1$. We consider two institutional mechanisms by which public good investment decisions can be taken. The first is a purely decentralized mechanism, which we call *Autarky*, whereby each district retains full property rights over its share of the societal endowment and in each period chooses on its own how to allocate it between investment in the public good (which

[1] See Ledyard [1995] for a survey. The failure of theoretical predictions seems more serious in cases where the equilibrium level of investment is zero. In experiments where the equilibrium level of investment is positive, the results are mixed, and sometimes very close to equilibrium or even underprovision. See, for example, Palfrey and Prisbrey [1996,1997], Palfrey and Rosenthal [1991], Holt and Laury [2008].

accrues benefits to by all districts) and private consumption, taking as given the current and future strategies of the other districts. The total economy-wide investment in the public good in each period is given by the sum of the district investments. In the second mechanism, the policy is taken by a centralized body, *the Legislature*, composed of a single representative from each district. The legislature is endowed with the power to tax and allocate revenues between public good investment and targeted transfers. Representatives bargain in the legislature over the allocation of resources. For both mechanisms, we characterize the trajectory of public policies that would result from a symmetric Markov perfect equilibrium, and compare them with the optimal policy of a benevolent social planner.

The equilibrium generates predictions about how the dynamics of investment are affected by the political mechanism used to make these decisions. The model implies that the legislative mechanism will generate a higher level of investment and a higher steady state of the public good than the autarky model. For both mechanisms, investment should continue until a steady state is reached. For both mechanisms, both the investment level along the transition path and the steady state are predicted to be substantially lower than in the benevolent planner's solution. These key predictions of the theory are confirmed by the experimental data. We do, however, observe some differences between the finer details of the theoretical predictions and the data. The clearest such deviation is a statistically significant overinvestment that characterizes the legislative model and, to a lesser extent, the autarchy model. This phenomenon is similar to the finding in experiments on static public good provision, but is more complex in our dynamic setting: we observe a large initial overinvestment in the early rounds, followed by a significant disinvestment approaching the equilibrium steady state.

Consistent with the equilibrium of the mechanisms, players are clearly forward looking, and the expected continuation value function are significant variables explaining voting behavior. However, we detect some evidence of non stationary behavior in proposals. For example, voters tend to punish past proposers who made proposals in a non-egalitarian way or "greedily" (by proposing policies that favor themselves too much). The observed punishments apparently are not sufficiently powerful to enforce an efficient outcome; indeed more than being part of a "trigger strategy" equilibrium, they seem to follow from a myopic behavioral response, perhaps motivated by an aversion to non-egalitarian proposals. We conclude that observed behavior lies somewhat in between the prediction of a purely forward looking Markov equilibrium, and an equilibrium in which agents look back in a limited way at the past to punish "unfair" proposals, resulting in only slight increases in investment above equilibrium levels.

This work contributes primarily to a growing literature on dynamic political economy. Two papers are related to the institutional settings that we study. The first is Battaglini and Coate [2006], who have studied public good accumulation in a legislative model. As in Battaglini and Coate, in our model legislative bargaining is dynamic in the sense that the policy choice at t will affect utilities and choices in the following periods through a change of the state variable, the level of accumulated public good. Although the solution of our model maintains some features of the solution in Battaglini and Coate, the equilibrium is different because the model of the underlying economy and the bargaining protocol are different. The second paper is Fershtman and Nitzan [1991] which studies a model similar to our autarky mechanism, except in the assumptions of continuous time and quadratic payoffs, and identifies the dynamic free riding effect.[2] Neither of these two papers, however, provides a comparative study of different institutional settings; and, given the differences in the models, do not present results that can be directly compared, even from a qualitative point of view. These two papers, moreover, do not provide any empirical evidence about behavior, under the mechanisms.

The findings of this paper contribute to the literature on dynamic political economy at a more general level. Previous work in the dynamic political economy literature typically explores a single specific political mechanism in a dynamic environment,[3] rather than conducting a comparative analysis of alternative institutions as in this paper. More importantly, this paper takes a first step in providing experimental evidence about the empirical validity of the equilibrium predictions.[4] This evidence is particularly useful for assessing the comparative static predictions of the model, as well as various assumptions common in the literature, e.g., the assumptions of symmetry and stationarity of the equilibrium. Much of the literature has focused attention on Markov equilibria in which strategies depend only on a restricted number of state variables;[5] some previous work has also focused on Pareto efficient equilibria sustained by punishment strategies.[6] Our evidence allows us to provide an assessment of these model restrictions.

[2] A common pool model related to our Autarky model and Fershtman and Nitzan [1991] has been recently applied by Harstad [2009] to study climate agreements.

[3] Recent contributions in dynamic bargaining are Baron [1996], Battaglini and Coate [2006, 2008, 2009], Baron, Diermeier and Fong [2009], Diermeier and Fong [2009], Duggan and Kalandrakis [2008], Kalandrakis [2004, 2005], Penn [2009].

[4] Diermeier and Gailmard [2006], Diermeier and Morton [2006], Frechette, Kagel, Lehrer [2003], Frechette, Kagel, Morelli [2005], and McKelvey [1991] provide important experimental analyses of legislative bargaining à la Baron and Ferejohn [1989], but in a *static* setting with purely distributive policies. Frechette, Kagel, Morelli [2009] extend the experimental analysis to policy spaces with public goods using a model by Volden and Wiseman [2006]. All of this works, however, limit the analysis to static environments in which only a single policy outcome is decided. Battaglini and Palfrey [2007] study a simple *dynamic* model of legislative bargaining, but limit the analysis to purely distributive policies in which public goods cannot be accumulated.

[5] See Battaglini and Coate [2008, 2009], Baron, Diermeier and Fong [2009], Duggan and Kalandrakis [2008], Harstad [2009], Hassler, Rodriguez-Mora, Storesletten and Zilibotti [2003].

[6] See for example Acemoglu, Golosov and Tsyvinsky [2006], Yared [2007], Sleet and Yeltekin [2007].

Finally, our work contributes to the experimental literature on public good provision. This literature has traditionally focused on static environments and identified a number of behavioral biases in public good contributions. In our work we contribute to documenting and qualifying the extent to which these biases extend to dynamic settings, and to the provision of *durable* public goods, where current investment decisions have long term effects on welfare.

2 The model

Consider an economy in which a continuum of infinitely lived citizens live in n districts and each district contains a mass one of citizens. There are two goods: private good x and a public good g. An allocation is an infinite nonnegative sequence of public policies, $z = (x_\infty, g_\infty)$ where $x_\infty = (x_1^1, ..., x_1^n, ..., x_t^1, ..., x_t^n, ...)$ and $g_\infty = (g_1, ..., g_t, ...)$. We refer to $z_t = (x_t, g_t)$ as the *public policy* in period t. The utility U^j of a representative citizen in district j is a function of $z^j = (x_\infty^j, g_\infty)$, where $x_\infty^j = (x_1^j, ..., x_t^j, ...)$ We assume that U^j can be written as:

$$U^j(z^j) = \sum_{t=1}^{\infty} \delta^{t-1} \left[x_t^j + u(g_t) \right],$$

where $u(\cdot)$ is continuously twice differentiable, strictly increasing, and strictly concave on $[0, \infty)$, with $\lim_{g \to 0^+} u'(g) = \infty$ and $\lim_{g \to \infty^+} u'(g) = 0$. The future is discounted at a rate δ.

There is a linear technology by which the private good can be used to produce public good, with a marginal rate of transformation p equal to 1. The private consumption good is nondurable, the public good is durable, and the stock of the public good depreciates at a rate $d \in [0, 1]$ between periods. Thus, if the level of public good at time $t-1$ is g_{t-1} and the total investment in the public good is I_t, then the level of public good at time t will be

$$g_t = (1 - d)g_{t-1} + I_t.$$

Because all citizens in district j are identical, we refer collectively to the "behavior of a district" as described by the behavior of a representative citizen j. Henceforth we will simply refer to district j. In period t, each district j is endowed with w_t^j units of private good, and we denote $W_t = \sum_{i=1}^n w_t^j$. We will restrict attention in this paper to *symmetric* economies, where $w_t^j = \frac{W_t}{n}$ $\forall j$, and W_t=W $\forall t$. The initial stock of public good is $g_0 \geq 0$, exogenously given.

The *public policy* in period t is required to satisfy three feasibility conditions:

$$\begin{aligned} x_t^j &\geq 0 \;\; \forall j \\ I_t &\geq -(1-d)g_{t-1} \;\; \forall t \\ I_t + \sum_{j=1}^{n} x_t^j &\leq W_t \;\; \forall t \end{aligned}$$

The first two conditions guarantee that allocations are nonnegative. The third condition requires that the current economy-wide budget is balanced. These conditions can be rewritten slightly. If we denote $y \equiv g_t = (1-d)g_{t-1} + I_t$ as the new level of public good after an investment I_t when the last period's level of the public good is g_{t-1}, then the public policy in period t can be represented by a vector $(y, x_t^1, ..., x_t^n)$. Dropping the t subscripts and substituting y, the budget balance constraint $I_t + \sum_{j=1}^{n} x_t^j \leq W_t$ can be rewritten as:

$$\sum_{j=1}^{n} x^j + [y - (1-d)g] \leq W,$$

recalling that we use y to denote the post-investment level of public good attained in period t, and $(1-d)g$ for the pre-investment level of public good inherited from period $t-1$. The one-shot utility to district j from this public policy, $(y, x^1, ..., x^n)$, is $U^j = x^j + u(y)$.

Our interest in this paper is to compare the performance of different mechanisms for building public infrastructure, i.e., generating a feasible sequence of public policies, z. While more general formulations are possible, we will consider mechanisms that are time independent and have no commitment. That is, the mechanism is played in every period, the rules of the mechanism are the same in every period, and the outcome of the mechanism is a public policy for only the current period. The level of the state variable g, however, creates a dynamic linkage across policy making periods. In such mechanisms we will characterize the outcomes associated with symmetric Markov perfect equilibria.

3 The planner's problem

As a benchmark with which to compare the equilibria in mechanisms, we first analyze the sequence of public policies that would be chosen by a benevolent planner who maximizes the sum of utilities of the districts. This is the welfare optimum in this case because the private good enters linearly in each district's utility function. The planner's problem has a recursive representation in which g is the state variable, and $v_P(g)$, the planner's value function can be represented as:

$$v_P(g) = \max_{y,x} \left\{ \begin{array}{c} X + nu(y) + \delta v_P(y) \\ s.t \quad X + y - (1-d)g \leq W, \ X \geq 0 \ , y \geq 0 \end{array} \right\} \tag{1}$$

where $X = \sum_{j=1}^{n} x^i$ is the sum of private transfers to the districts, and $v_P(g)$ is the planner's value function. By standard methods (see Stokey and Lucas [1989]) we can show that a continuous, concave and differentiable $v_P(g)$ that satisfies (1) exists and is unique.

Since the budget constraint, $X+y-(1-d)g \leq W$, is binding and $y \geq 0$ is never binding,[7] we can rewrite the planner's problem as:

$$\max_{y} \left\{ \begin{array}{c} W + (1-d)g - y + nu(y) + \delta v_P(y) \\ X = W + (1-d)g - y \geq 0. \end{array} \right\} \tag{2}$$

The optimal policy $y_P(g)$ can be characterized by studying (2). Depending on whether the constraint is binding, there are two possible cases. In the first case, where it is binding, the planner would like to invest an amount I (which equals $y-$ $(1-d)g$, by definition) greater than W but cannot because of the constraint. Thus, the solution is this case is $y_P(g) = W + (1-d)g$, and $X_P(g) = 0$. In the second case, the constraint is not binding and the unconstrained optimization yields $y_P(g) \leq W + (1-d)g$ and $X_P(g) \geq 0$. In this case a necessary condition for $y_P(g)$ is characterized by the first order equation:

$$nu'(y_P(g)) + \delta v_P'(y_P(g)) = 1$$

By the concavity of u and v_P, the second order condition is satisfied, and furthermore the first order condition has a unique solution for $y_P(g)$, independent of g, which we denote y_P^*.

This implies the following simple rule-of-thumb optimal policy for investing in the public good as a function of its current level g. For any values of g such that $y_P^* - (1-d)g \leq W$, invest $I_P(g) = y_P^* - (1-d)g$ and total private good consumption is $X_P(g) = W + (1-d)g - y_P^*$. For any values of g such that $y_P^* - (1-d)g > W$, invest $I_P(g) = W$ and private good consumption is $X_P(g) = 0$. This second case is possible only if $y_P^* - (1-d)g \geq W$, i.e., if g is lower or equal to a threshold g_P^*:

$$g_P^* = \max \left\{ \frac{y_P^* - W}{1-d}, 0 \right\}$$

Figure 1 represents the optimal choice of the public good. We summarize the above argument as Proposition 1, below:

Proposition 1. *The optimal solution to the planner's problem is uniquely characterized by a* y_P^* *solving* $nu'(y_P^*) + \delta v_P'(y_P^*) = 1$ *and an investment policy function* $I_P(g) = \min\{W, y_P^* - (1-d)g\}$.

[7] The constraint $y \geq 0$ is never binding because $\lim_{g \to 0^+} u'(g) = \infty$.

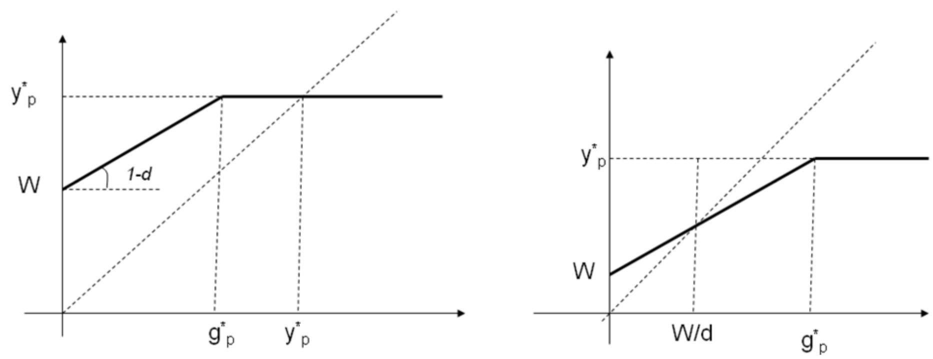

Figure 1: The Planner's Problem

This implies that the optimal public policy is time independent and given by $((X_P(g), y_P(g))$, where:

$$y_P(g) = \min\{W + (1-d)g, y_P^*\} \quad \text{and} \tag{3}$$

$$X_P(g) = W - \min\{W, y_P^* - (1-d)g\} \tag{4}$$

Clearly in a symmetric solution each district would receive $x_P^j(g) = \frac{X_P(g)}{n}$, but any other division is equally efficient for a utilitarian planner. Formulas (3) and (4) have a clear intuition. When the state g is sufficiently low, the planner invests all currently available resources (W) in the public good: this is because the marginal value of an additional unit of the public good is so high that each dollar invested in the public good yields more than a dollar in value. For $g > \frac{y_P^* - W}{1-d}$, however, the marginal value of the public good investment is lower than 1, so it is more valuable to leave some resources to the districts for private consumption. In this region of g, the planner expends resources just enough to cover depreciation and maintain the level of the public good at y_P^*, where the marginal value of investment is exactly 1. Note that if for some reason the stock of public good at period $t-1$ is equal to $g > \frac{y_P^*}{1-d}$,[8] then optimal investment is negative in period t. For future reference, when we will compare this solution with the equilibria in the political systems, it is interesting to note that this investment function $I_P(g)$ is (weakly) monotone decreasing in g. From Proposition 1, one can see that there can actually be two cases, depending on how high the optimal steady state, y_P^*, is relative to the parameters of the model $\{n, W, d, \alpha, \delta\}$. The steady state is at the intersection point between the 45^o line and the investment curve (3). This is illustrated in Figure 1.

The first case, shown in the left panel of the figure is when the steady state is $g_P^{SS} = y_P^* = y(y_P^*)$ and, therefore, $X(g_P^{SS}) > 0$. The second case is when the steady state g_P^{SS} satisfies

[8] For example, it could be that $g_0 > \frac{y_P^*}{1-d}$.

$g_P^{SS} = y(g_P^{SS}) < y_P^*$. In this case $W + (1-d)g = g$, so $g_P^{SS} = W/d$ and $X(g_P^{SS}) = 0$. If $g_P^* \leq y_P^*$, then $y_P(g)$ crosses the 45^o degree line on the right of g_P^*. In this case the steady state is y_P^*. If $g_P^* > y_P^*$, then the steady state is on the left of g_P^* and lower than y_P^*.

To illustrate the planner's solution, we derive the analytical solution for the case when the utility function is given by $u(y) = \frac{B}{\alpha}y^\alpha$. In the Appendix we show that:

Proposition 2. Let $u(y) = \frac{B}{\alpha}y^\alpha$. *If* $1-\delta(1-d) > Bn\left(\frac{d}{W}\right)^{1-\alpha}$*, the long run steady state in the planner's solution is* $g_P^{SS} = \left(\frac{Bn}{1-\delta(1-d)}\right)^{\frac{1}{1-\alpha}} = y_P^*$*. If* $1-\delta(1-d) \leq Bn\left(\frac{d}{W}\right)^{1-\alpha}$ *the steady state is* $g_P^{SS} = \frac{W}{d} < y_P^*$.

This result gives us a complete characterization of the equilibrium dynamics and long term behavior. When the economy has relatively large resources as measured by W (i.e. $1-\delta(1-d) > Bn\left(\frac{d}{W}\right)^{1-\alpha}$) eventually the level of the public good will reach a saturation point at which its marginal value is equal to the marginal utility of consumption (*one*), and the steady state is y_P^*. When $1-\delta(1-d) \leq Bn\left(\frac{d}{W}\right)^{1-\alpha}$ this optimal saturation point, y_P^*, will never be reached if one starts at $g_0 = 0$. Depreciation is too high compared to W for the saturation point to ever be reached ($\frac{W}{d} < y_P^*$). The actual time path depends on initial conditions. In both cases investment in public goods is a non decreasing function of g: It is constant ($= W$) for low values of $g \leq g_P^* = \max\left\{\frac{y_P^*-W}{1-d}, 0\right\}$, and strictly decreasing above g_P^*.

An immediate corollary of Proposition 2 that will be useful in the experimental application is that if $d = 0$, then we are always in the first case, and the steady state is $\left(\frac{Bn}{1-\delta}\right)^{\frac{1}{1-\alpha}}$.

4 Political Mechanisms for Building Public Infrastructure

The set of possible mechanisms to implement sequences of public policies is obviously huge. We limit ourselves to two different types of mechanisms.

The first is a purely decentralized mechanism, which we call *Autarky*, whereby each district retains full property rights over a share of the endowment ($\frac{W}{n}$) and in each period chooses on its own how to allocate its endowment between investment in the public good (which is shared by all districts) and private consumption, taking as given the strategies of the other districts. The total economy-wide investment in the public good in any period is then given by the sum of the district investments.

The second type of mechanism we consider is a bargaining mechanism for a centralized economy-wide representative legislature, which we call the *Legislative* mechanism. In this mechanism, each district cedes its property right over its share of the economy wide endowment in exchange for $1/n$ representation in the legislature. In each period, the legislature decides on a uniform lump sum tax on all districts, which cannot exceed a district's endowment, W/n, and a level of investment in the public good. The legislative policy also includes an allocation of the budgetary surplus (tax revenue minus investment) to the districts, which is non-negative for all districts, but not necessarily uniform. Investment can be negative, but the amount of negative investment cannot exceed the current stock of public good. Thus, as before, we can represent a policy by the legislature at time t, by an public policy $(x_t^1, ..., x_t^n, y_t)$ that satisfies the same feasibility constraints as in the planner's problem. The bargaining protocol with which a public policy is chosen in a legislature is as follows. At the beginning of each period an agent is chosen by nature to propose a policy $(x^1, ..., x^n, y)$. Each legislator has the same probability to be recognized as proposer. If at least $q \in \{1, 2, ..., n\}$ legislators vote in favor of the proposal, it passes and it is implemented. The legislature then adjourns and meets in the following period with a new level of public good y. If instead the policy does not receive a qualified majority, then the status quo policy is implemented. We assume that the status quo is zero taxation, which implies zero investment in public goods and so $x^j = W/n$ for all j. The legislature, moreover, adjourns and meets in the following period with a new level of public good $(1-d)g$.[9].

We study the equilibria of these two equilibria in the next two subsections.

4.1 *Decentralized Provision*: The Autarky Mechanism

To study the properties of the Autarky mechanism we focus on symmetric Markov-perfect equilibria, where all districts use the same strategy, and these strategies are time-independent functions of the state, g. A strategy is a pair $(x_A(\cdot), i_A(\cdot))$: where $x_A(g)$ is the level of consumption in the district and $i_A(g)$ is a district's level of investment in the public good in state g. Given these strategies, by symmetry the public good in state g is $y_A(g) = (1-d)g + n i_A(g)$. Associated with any equilibrium is a value function $v_A(g)$ which specifies the expected discounted future payoff to a legislator when the state is g. In the remainder we focus on an equilibrium in which v_A is concave. Proposition 3 shows that such an equilibrium exists.

The optimization problem for district j if the current level of public good is g, and the district's value function is $v_A(y)$ is:

$$\max_{y,x} \left\{ \begin{array}{c} x + u(y) + \delta v_A(y) \\ s.t \;\; x + y - (1-d)g = W - (n-1)x_A(g) \\ W - (n-1)x_A(g) + (1-d)g - y \geq 0 \\ x \leq (1-d)g/n + W/n \end{array} \right\} \tag{5}$$

[9]This bargaining protocol differs from the protocol adopted in Battaglini and Coate [2008]. There, if no agreement is reached in the previous attempts, a new legislator is randomly selected to make a proposal for at least T times. In the last stage of the bargaining game, a legislator is chosen to make a default proposal.

where y is the new level of public good and x is private consumption: district j realizes that given the other districts' contributions, his/her investment ultimately determines y. The first constraint is a rewritten form of $I_A(g)+x=W$, substituting out for $I_A(g)$ (where $I_A(g)$ is the total investment $ni_A(g)$) The second constraint is derived from $x_A(g)\geq 0$. The third constraint is derived from $i_A(g)\geq -\frac{(1-d)g}{n}$: it requires that no legislator can reduce $y_A(g)$ by more than his share $(1-d)g/n$.[10] Note that since $x_A(g)$ is an endogenous variable that depends on v_A, (5) is not necessarily a contraction. District j, however, takes $x_A(g)$ as given.

Depending on the state g the solution of (5) falls in one of two cases: we may have $W+(1-d)g=y_A(g)+(n-1)x_A(g)$, so $x_A(g)=0$; or $W+(1-d)g>y_A(g)+(n-1)x_A(g)$, so $x_A(g)>0$.

If v_A is concave, then in the latter case the solution is characterized by a unique public good level y_A^* satisfying the first order equation:

$$u'\left(y_A^*\right)+\delta v_A'(y_A^*)=1 \tag{6}$$

The investment by each district is equal to $i_A(g)=\frac{1}{n}\left[y_A^*-(1-d)g\right]$ and per capita private consumption is $x_A(g)=\frac{W+(1-d)g-y_A^*}{n}$.

In the other possible case, if $x_A(g)=0$, then $y_A(g)=W+(1-d)g$ and investment by each district is $i_A(g)=\frac{W}{n}$. This second condition is possible only if and only if $W\leq y_A^*-(1-d)g_A$, that is if g is below some threshold g_A defined by:

$$g_A=\max\left\{\frac{y_A^*-W}{1-d},0\right\}$$

We summarize this in the following proposition, which also proves the existence of an equilibrium:

Proposition 3. *A concave equilibrium exists. In a concave equilibrium of the Autarky game, public investment is:* $y_A(g)=\min\left\{W+(1-d)g,y_A^*\right\}$ *where* y_A^* *is a constant with* $y_A^*<y_P^*$.

The public good function $y_A(g)$ is qualitatively similar to the corresponding planner's function $y_P(g)$. The main difference is that $y_A^*<y_P^*$ and $g_A<g_P$, so public good provision is typically smaller (and always smaller in the steady state). This is a dynamic version of the usual free rider problem associated with public good provision: each agent invests less than is socially optimal because he/she fails to fully internalize all legislators' utilities. Part of the free rider problem can be seen from (6): in choosing investment, legislators count only the marginal benefit to their district, $u'(y)+\delta v_A'(y)$, rather than $nu'(y)+\delta nv_A'(y))$, but all

[10] Legislators choose $y_A(g)\geq g$ for any g that is reached on the equilibrium path when the initial level g_0 is below the steady state. As in the legislative model, however, legislators can reduce g if they want. In a decentralized system as the VC game, $y\geq\frac{n-1}{n}g$ guarantees that (out of equilibrium) the sum of reductions in g can not be larger than the stock of g.

the marginal costs (-1). In this dynamic model, however, there is an additional effect that reduces incentives to invest, called *dynamic free riding.*[11] To see this, consider the value function for $g > g_A$ (where we have an interior solution):

$$\begin{aligned} v(g) &= W - (n-1)x_A(g) - (y_A^* - (1-d)g) + u(y_A^*) + \delta v_A(y_A^*) \\ &= \frac{W - (y_A^* - (1-d)g)}{n} + u(y_A^*) + \delta v_A(y_A^*) \end{aligned}$$

where the last equation follows by the fact that in a symmetric equilibrium: $x_A(g) = W - (n-1)x_A(g) - (y_A^* - (1-d)g)$. A marginal increase in g has two effects. An immediate effect, corresponding to the increase in resources available in the following period: $(1-d)g$. But there is also a delayed effect on next period's investment: the increase in g triggers a reduction in the future investment of all the other districts through an increase in $x_A(g)$: for any level of $g > g_A$, $y_A(g)$ will be kept at y_A^*. In a symmetric equilibrium, if district j increases the investment by 1 dollar, he will trigger a reduction in future investment by all future districts by $1/n$ dollars; the net value of the increase in g for j will be only δ/n.

In the experimental analysis it is convenient to have a unique equilibrium. The next result guarantees that this is the case when, as we assume in the experiment, $u(y) = \frac{B}{\alpha}y^\alpha$ and $d = 0$:

Proposition 4. *When $u(y) = \frac{B}{\alpha}y^\alpha$ and $d = 0$ in the Autarky game there is a unique equilibrium steady state $y_A^* = \left(\frac{Bn}{n-\delta(1-d)}\right)^{\frac{1}{1-\alpha}}$.*

In the experimental study of the Autarky game we always assume $\alpha = 0.5$, $d = 0$, $\delta = .75$, $B = 1$ (the same parametrization we use in the legislative game). We consider two treatments (corresponding to similar treatments in the legislative game): $n = 3$, $W = 15$; and $n = 5$, $W = 20$. For both these parameterization, we obtain that $g_A = 0$; moreover y_A^* is equal to $\left(\frac{4}{3}\right)^2$ in the $n = 3$ case, and to $\left(\frac{20}{17}\right)^2$ in the $n = 5$ case. By proposition 4, these two treatments have a unique steady state: $y_A^* = \left(\frac{4}{3}\right)^2$ in the $n = 3$ case, and $y_A^* = \left(\frac{20}{17}\right)^2$ in the $n = 5$.

4.2 *Centralized Provision*: The Legislative (L) Mechanism

As in the previous section, to characterize behavior when policies are chosen by a legislature we look for a symmetric Markov perfect equilibrium. In this type of equilibrium any representative selected to propose at some time t uses the same strategy, and this depends only on the current stock of public good (g). Similarly, the probability a legislator votes for a proposal depends only on the proposal itself and the state g. As is standard in the theory

[11] A similar dynamic free riding effect arises in the Fershtman and Nitzan [1991] model, though in the context of a differential game with quadratic utilities.

of legislative voting, we focus on weakly stage-undominated strategies, which implies that legislators vote for a proposal if they prefer it (weakly) to the status quo. Without loss of generality, we focus on an equilibrium in which proposals are accepted with probability one.

As it is easy to verify, in a symmetric Markov equilibrium, a proposer would either make no monetary transfer to the other districts, or would make a transfer only to $q-1$ legislators, selected randomly each with the same probability of being selected. An equilibrium can therefore be described by a collection of functions $\{y_L(g),\ s_L(g)\}$ that specifies the choice made by the proposer in a period in which the state is g. Here $y_L(g)$ is the proposed new level of public good and $s_L(g)$ is a transfer offered to the districts of the $q-1$ randomly selected representatives. The proposer's district receives the surplus revenues $x_L(g) = W - y_L(g) + (1-d)g - (q-1)s_L(g)$. Associated with any symmetric Markov perfect equilibrium in the L game is a value function $v_L(g)$ which specifies the expected future payoff of a legislator when the state is g.

Contrary to the Planner's case of the previous section, the policy is now chosen by a self interested proposer who maximizes the utility of his own district. Given v_L, the proposer's problem is:

$$\max_{x,y,s} \left\{ \begin{array}{c} x + u(y) + \delta v_L(y) \\ s.t \quad s + u(y) + \delta v_L(y) \geq \frac{W}{n} + u\left[g(1-d)\right] + \delta v_L(g(1-d)) \\ (q-1)s + x + y - (1-d)g \leq W \\ x \geq 0, s \geq 0 \end{array} \right\} \tag{7}$$

where x is the transfer to the proposer. This problem is similar to the planner's problem (2): the second inequality is the budget balance constraint, and the last two inequalities are the feasibility constraints.[12] The first inequality is however new: it is the incentive compatibility constraint that needs to be satisfied if a proposal is to be accepted by $q-1$ other districts.

The solution to (7) is complicated by the fact that the set of binding constraints is state dependent and the value function is not typically concave in g. Despite this, the next result shows a sufficient condition for the existence of a Markov equilibrium that is satisfied by the parameters of the experiment. We say that an equilibrium is *well behaved* if the associated value function is continuous, non decreasing and almost everywhere differentiable. We have:

Proposition 5. *Assume $d = 0$. There is a $\overline{\delta}$ and a $\overline{W}$ such that for $\delta > \overline{\delta}$, and $W > \overline{W}$ a well-behaved Markov equilibrium exists in which the public good level is given by*

$$y_L(g) = \left\{ \begin{array}{cc} y_1^* & g \leq g_1(y_1^*) \\ \widetilde{y}(g) & g \in (g_1(y_1^*), g_2(y_2^*)] \\ y_2^* & else \end{array} \right. \tag{8}$$

[12] It can be verified that the constraint $y \geq 0$ is never binding and therefore it can be ignored without loss of generality.

where y_1^* *and* y_2^* *are constants with* $y_2^* > y_1^*$*;* $g_1(y_1^*)$, $g_2(y_2^*)$ *are functions respectively of* y_1^* *and* y_2^**; and* $\widetilde{y}(g)$ *is an increasing function of* g*.*

Figure 2 provides a representation of the equilibrium for one of the two parameter configurations that we use in the experiment (the other case is qualitatively similar)[13]. There is an intuitive explanation for the shape of the investment function (8). For $g \leq g_1(y_1^*)$ the proposer acts as if the other districts did not exist: he diverts resources only toward his own district and chooses the investment without internalizing the other districts' welfare. This implies that the proposer can choose y such that

$$y_1^* \in \arg\max_y \{u(y) - y + \delta v_L(y)\} \tag{9}$$

The other districts accept this policy because the investment y_1^*, is sufficiently high to make this policy better than the status quo. When $g \geq g_1(y_1^*)$, the proposer can not afford to ignore the other districts. He first finds it optimal to "buy" their approval by increasing g and investing $\widetilde{y}(g) > y_1^*$ (in the interval $(g_1(y_1^*), g_2(y_2^*)]$): $\widetilde{y}(g)$ is chosen large enough to satisfy the incentive compatibility constraint as an equality. For $g > g_2(y_2^*)$, however, the proposer finds it optimal to provide pork to a minimal winning coalition of districts, and to invest y_2^*. In choosing y now the proposer must internalize the utility of q legislators, so:

$$y_2^* \in \arg\max_y \{qu(y) - y + \delta q v_L(y)\} \tag{10}$$

It is interesting to note that when the proposer's strategy is constant (at y_1^* or at y_2^*) we have a *dynamic free rider problem* similar to the one discussed in Section 4.1: an increase in investment above, say, y_2^*, at t would induce a proportional reduction in investment at $t+1$, and so discourage public good accumulation. This is an important reason for underinvestment in the steady state. When $y_L(g) = \widetilde{y}(g)$ the dynamic free riding problem is mitigated because an increase in g induces an increase in $\widetilde{y}(g)$. This occurs because the increase in g makes the incentive constraint at $t+1$ more binding, so it forces the proposer in the following period to increase the investment in public goods.

To compute an equilibrium we note that there is a two way relationship between the equilibrium value $v_L(g)$ and y_1^*,y_2^*. First, using $y_L(g)$ and $x_L(g)$ described in Proposition 5

[13] See Table 1 for details on the equilibrium values for both cases.

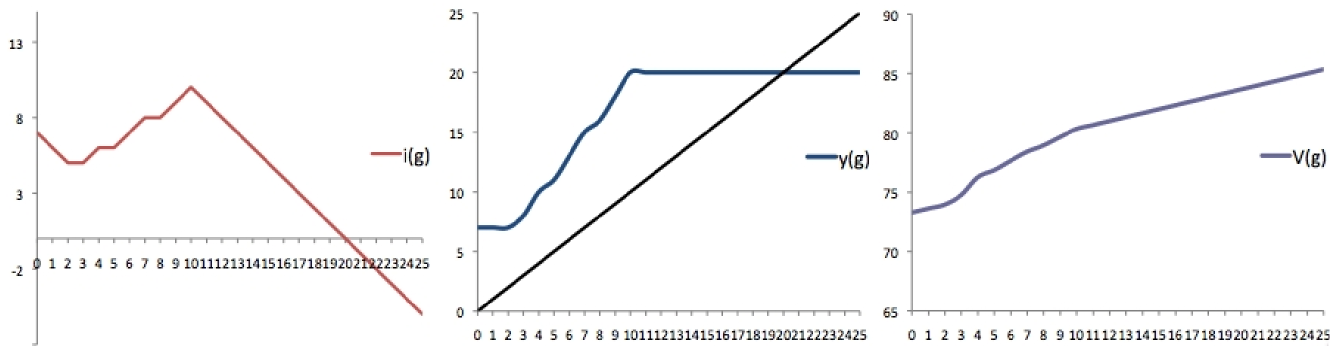

Figure 2: Legislative mechanism with n=3, i(g), y(g) and V(g)

we can represent the value function as a function only of y_1^*, y_2^*:[14]

$$v_L(g) = \begin{cases} \frac{1}{n}\left[W - [y_1^* - (1-d)g]\right] + u(y_1^*) + \delta v_L(y_1^*)) & g \leq g_1(y_1^*) \\ \frac{1}{n}\left[W - [\widetilde{y}(g) - (1-d)g]\right] \\ \quad +u(\widetilde{y}(g)) + \delta v_L(\widetilde{y}(g))) & g \in (g_1(y_1^*), g_2(y_2^*)] \\ \frac{1}{n}\left[W - [y_2^* - (1-d)g]\right] + u(y_2^*) + \delta v_L(y_2^*) & else \end{cases} \tag{11}$$

Second, given a value function v_L, we can find y_1^*, y_2^* by solving (9) and (10). In the experiment the state space G is finite, with m states. In this case, for a given y_1^*, y_2^*, (11) is a system of m equations in m unknowns: we can then easily solve for a function $v(g; y_1^*, y_2^*)$. Given this $v(g; y_1^*, y_2^*)$, we can find the (new) optimal y_1^*, y_2^* using (11). An equilibrium corresponds to a fixed point of this correspondence that maps $\Re_+^2$ to itself.

As for the Autarky case, here too it is convenient to have a unique equilibrium outcome for the experimental analysis. The next result guarantees that this is the case when, as we assume in the experiment, $u(y) = \frac{B}{\alpha} y^\alpha$ and $d = 0$:

Proposition 6. *When $u(y) = \frac{B}{\alpha} y^\alpha$ and $d = 0$ in the Legislative game there is a unique equilibrium steady state $y_L^* = \left(\frac{Bn}{\frac{n}{q} - \delta(1-d)}\right)^{\frac{1}{1-\alpha}}$.*

In all experiments we assume $u(y) = \frac{B}{\alpha} y^\alpha$ with $\alpha = 0.5$, $B = 1$, $d = 0$, and $\delta = .75$. The two treatments that we study in the laboratory are $n = 3$, $q = 2$, $W = 15$; and $n = 5$, $q = 3$,

[14] To write the value function for $g \geq g_2$ note that in this range the value function of a proposer is: $W - [y_2^* - (1-d)g] - (q-1)\left[\frac{W}{n} + \Psi((1-d)g) - \Psi(y_2^*)\right] + \Psi(y_A^*)$ and the probability of being a proposer is $1/n$. The value of a legislator who receive pork transfers, on the other hand, is $\left[\frac{W}{n} + \Psi((1-d)g) - \Psi(y_A^*)\right] + \Psi(y_A^*)$, where $\Psi(x) = u(x) + \delta v_L(x)$ and the probability of receiving a transfer $s(g)$ conditional on not being a proposer is $(q-1)/(n-1)$. Finally, the value of a legislator excluded from transfers is simply $\Psi(y_A^*)$, and the probability of being in this state conditional on not being a proposer is $1-(q-1)/(n-1)$. The expression in (11) follows from these expressions by taking expectations. The other cases can be computed in a similar way.

$W = 20$. By Proposition 6 the equilibria in these two cases will be associated with a unique steady state $y_2^* = 16$ when $n = 3$, and $y_2^* = \left(\frac{3}{3/2-.75}\right)^2 \approx 30$ when $n = 5$. It is interesting to note that although the steady state is lower than the efficient level, it is considerably higher than in the Autarky regime. The reason for this improvement is that the voting rule forces the proposer to internalize the utility of other $q - 1$ agents when choosing a policy.

The first panel of Figure 2 represents the investment function $I_L(g)$:

$$I_L(g) = \begin{cases} y_1^* - (1-d)g & g \leq g_1 \\ \widetilde{y}(g) - (1-d)g & g \in (g_1, g_2] \\ y_2^* - (1-d)g & else \end{cases} \tag{12}$$

(where for simplicity g_1 is the equilibrium value $g_1(y_1^*)$, and similarly for g_2). It is interesting to note that while in the planner's $I_P(g)$ is a monotonically (weakly) decreasing function, in the political equilibrium $I_L(g)$ is not monotonic (compare (12) with the expression in Proposition 1). The non-monotonicity of the investment function is a consequence of the fact that the incentive compatibility constraint is not always binding and that the value of the status quo is endogenous. When g is small the marginal value of the public good is high. The cost if the bargaining proposal fails is therefore high. In this case the proposer can implement his preferred policy ignoring the incentive compatibility constraint. When this happens (in $g \leq g_1$), the proposer will not accumulate more than y_1^* (except, of course, if forced by the incentive compatibility constraint). When $g \geq g_1$, however, the proposer is forced to internalize the utility of at least a minimal winning coalition of other legislators: and so it will have to invest until the marginal utility of g is at least $1/q$. The final range in which investment is declining linearly corresponds to the region in which accumulating more than y_2^* is not profitable even when the $q - 1$ utilities of the other members of the minimal winning coalition are internalized.

The second panel of Figure 2 shows the equilibrium proposed level of the public good, as a function of the state, $y_L(g)$. This curve fully describes the dynamics of public good provision and the steady state. The steady state level of public good g^* corresponds to the point where the 45^o line intersects the investment curve. As in the Autarky game, we can have different types of steady states and corresponding equilibria. If $g^* \leq g_2$, in the steady state the proposer can extract transfers from his district without paying any transfer to the other districts; if $g^* > g_2$, instead, in the steady state there is always a minimal winning coalition of legislators who receive positive transfers for their own districts. In Figure 2, as in all our experiments, the steady state corresponds to the case $g^* = y_2^*$.

The panels representing the value function and $y_L(g)$ makes clear the complications involved with computing and studying the equilibrium. When g passes from the region in which the incentive compatibility constraint is not binding (i.e., $g \leq g_1$) to the region with a binding incentive compatibility constraint ($g > g_1$), the expected marginal value of g increases, because the incentive compatibility constraint forces the proposer to internalize the utility of more agents. As can be seen from Figure 2, investment in g increases, thereby reducing the inefficiency, and the value function becomes non concave.

5 Experimental Design

The experiments were all conducted at the Social Science Experimental Laboratory (SSEL) using students from the California Institute of Technology. Subjects were recruited from a pool of volunteer subjects, maintained by SSEL. Eight sessions were run, using a total of 102 subjects. No subject participated in more than one session. Half of the sessions used the Legislative mechanism with simple majority rule, and half used the Autarky mechanism. Half were conducted using 3 person committees, and half with 5 person committees. In all sessions there was zero depreciation ($d = 0$), the discount factor was $\delta = 0.75$, and the current-round payoff from the public good was proportional to the square root of the stock at the end of that round ($\alpha = .5$). In the 3 person committees, we used the parameters $W = 15$, while in the 5 person committees $W = 20$. Payoffs were renormalized so subjects could trade in fractional amounts.[15]. Table 1 summarizes the theoretical properties of the equilibrium for the four treatments. It is useful to emphasize that, as proven in the previous sections, given these parameters the steady state is uniquely defined both for the Autarchy and Legislative game and for all treatments: so the theoretical predictions of the convergence value of g is independent of the choice of equilibrium.

Mechanism	**n**	**B**	**W**	$(\mathbf{g}_1, \mathbf{g}_2)$	$\mathbf{y}_1^*$	$\mathbf{y}_2^*$	$\mathbf{g}_A$	$\mathbf{y}_A^*$	$\mathbf{g}_P$	$\mathbf{y}_P^*$
Legislative	3	2	15	(1,7)	5	16				
Legislative	5	$\sqrt{3}$	20	(4.83,18.5)	7.83	29.83				
Autarky	3	2	15				0	1.77		
Autarky	5	2	20				0	1.38		
Planner	3	2	15						129	144
Planner	5	2	20						380	400

Table 1: Experimental parameters and equilibrium

[15] We do this in order to reduce the coarseness of the strategy space and allow subjects to make budget decisions in line with the symmetric Markov perfect equilibrium in pure strategies. This is particularly important for the Autarky mechanism where the steady state level of the public good is 1.77 for n=3 and 1.38 for n=5, and the equilibrium level of individual investment is, respectively, 0.59 and 0.28 in the first period and 0 in all following periods.

Discounted payoffs were induced by a random termination rule by rolling a die after each round in front of the room, with the outcome determining whether the game continued to another round (with probability .75) or was terminated (with probability .25). The $n = 5$ sessions were conducted with 15 subjects, divided into 3 committees of 5 members each. The $n = 3$ sessions were conducted with 12 subjects, divided into 4 committees of 3 members each.[16] Committees stayed the same throughout the rounds of a given match, and subjects were randomly rematched into committees between matches. A match consisted of one multiround play of the game which continued until one of the die rolls eventually ended the match. As a result, different matches lasted for different lengths. Table 2 summarizes the design.

Mechanism	n	# Committees	# Subjects
Legislative	3	70	21
Legislative	5	60	30
Autarky	3	70	21
Autarky	5	60	30

Table 2: Experimental design

Before the first match, instructions[17] were read aloud, followed by a practice match and a comprehension quiz to verify that subjects understood the details of the environment including how to compute payoffs. The current round's payoffs from the public good stock (called *project size* in the experiment) was displayed graphically, with stock of public good on the horizontal axis and the payoff on the vertical axis. Subjects could click anywhere on the curve and the payoff for that level of public good appeared on the screen.

For the bargaining/voting mechanism each round had two separate stages, the proposal stage and the voting stage. At the beginning of each match, each member of a committee was randomly assigned a committee member number which stayed the same for all rounds of the match. In the proposal stage, each member of the committee submitted a *provisional budget* for how to divide the budget between the public good, called *project investment*, and private allocations to each member. After everyone had submitted a proposal, one was randomly selected and became the *proposed budget*. Members were also informed of the committee member number of the proposer, but not informed about the unselected provisional budgets. Each member then cast a vote either for the proposed budget or for the *backup budget* with zero public investment and equal private allocations. The proposed budget passed if and only if it received at least $\frac{n+1}{2}$ votes. Payoffs for that round were added to each subject's earnings and a die was rolled to determine whether the match continued

[16] Two of the $N = 3$ sessions used 9 subjects.

[17] Instructions for all the sessions are available from the authors.

to the next round. If it did continue, then the end-of-round project size became the next round's beginning-of-round project size.

At the end of the last match each subject was paid privately in cash the sum of his or her earnings over all matches plus a showup fee of \$10. Earnings ranged from approximately \$20 to \$50, with sessions lasting between one and two hours. There was considerable range in the earnings and length across sessions because of the random stopping rule.

6 Experimental Results

6.1 Time series of the stock of public good

6.1.1 Median values

Figure 3 shows the time series of the stock of public good by treatment.[18] The horizontal axis is the time period and the vertical axis is the stock of the public good. In order to aggregate across committees, we use the median level of the public good from all committees in a given treatment. Similar results hold if we use the mean or other measures of central tendency. Superimposed on the graphs are the theoretical time paths (represented with solid lines), corresponding to the Markov perfect equilibrium.

These time paths exhibit several systematic regularities, which we discuss below in comparison with the theoretical time paths.

FINDING 1. The Legislative mechanism leads to much greater public good production than the Autarky mechanism. The median stock of public good is greater in the L mechanism than the A mechanism in every single period in the n=3 and n=5 treatments.[19] With three districts, the median stock of public good is more than five times greater in the L3 treatment than the A3 treatment, averaged across all 13 rounds for which we have data (31.3 vs. 5.6). In 4 of the 13 rounds, the stock of public good in the L3 is more than 10 times greater than in the A3 treatment. The difference is also very large for the five district treatments. The median stock of public good is more than three times greater in the L5 treatment than the A5 treatment, averaged across all 10 rounds for which we have data (34.7 vs. 9.6). The differences between the L and the A mechanisms are relatively small in

[18] These and subsequent figures show data from the first ten rounds. Data from later rounds are excluded from the graphs because there were so few observations. The data from later rounds are included in all the statistical analyses.

[19] The difference between the stock of public good in the two mechanisms is statistically significant at the 1% level (p-value$<$0.01) according to the results of a t-test on the equality of means and a Kolmogorov-Smirnov test on the equality of distributions.

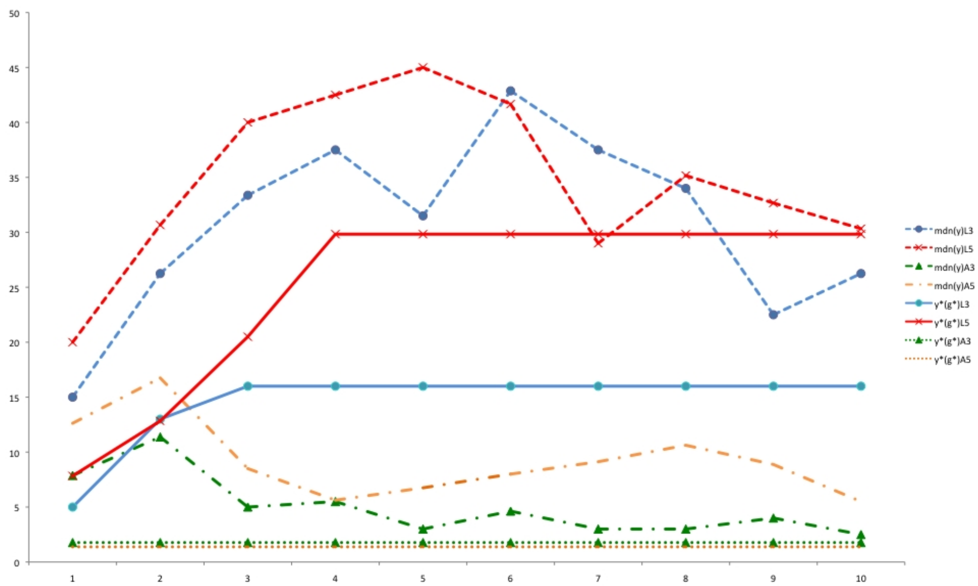

Figure 3: Median time paths of the stock of g, all mechanisms, all treatments

the initial round, but they increase sharply as more rounds are played. By round 10, the gaps in the median stock of public good are very large (26.2 vs. 2.5 for three districts and 30.3 vs. 5.5 for five districts).

FINDING 2. Both mechanisms lead to public good levels significantly below the optimal steady state.[20] The optimal steady state is y*=144 for the three district treatments and y*=400 for the five district treatments. The optimal investment policy is the fastest approach: invest W in every period until y* is achieved. In the L mechanism, the stock of public good levels out at about 30 in both treatments. The median stock averages 30.1 in rounds 7-10 in L3, and 31.8 in rounds 7-10 in L5. *These very inefficient long run public good levels in the L treatment occur in spite of initial round median investment that is fully efficient, with I=W in both treatments.* In the A mechanism, the stock of public good levels out in the single digits in both treatments. The median stock averages 3.8 in rounds 3-10 for A3, and 7.9 in rounds 3-10 for A5.

FINDING 3. In both mechanisms, there is overinvestment relative to the equilibrium in the early rounds, followed by significant disinvestment, approaching the steady state. The median investment in the first three rounds of L3 are 15,11.2,

[20]In both mechanisms the stock of public good in the last rounds (rounds 8 to 13) is significantly smaller than the level predicted by the social planner solution (the optimal steady state for n=3 and the level attainable investing W each round for n=5) according to the results of a t-test on the equality of means and a Kolmogorov-Smirnov test on the equality of distributions (p-value<0.01 for both).

and 7.2. As a result the median public good stock by the end of round 3 equals 33.4. This compares with the equilibrium investment policies in the first three rounds equal to 5,8, and 3, and a level of stock equal to 16. Thus, in L3, committees overshoot the equilibrium in early rounds by a factor of two. The scenario in early rounds is similar in L5. The median investment in the first three rounds of L5 are 20,10.7, and 9.3. As a result the median public good stock by the end of round 3 equals 40. This compares with the equilibrium investment policies in the first three rounds of L5 equal to 7.8,5, and 7.7, and a level of stock equal to 20.5. Thus, in L5, committees also overshoot the equilibrium in early rounds by a factor of two. This overshooting is largely corrected in later rounds. The stock of public good then declines over the later rounds. In the L5 treatment, convergence is especially close to equilibrium, with the difference between the median public good levels and the equilibrium public good levels in the last 4 rounds of data measuring less than 2 units of the public good (31.79 vs. 29.83). A similar pattern of overshooting in the A mechanisms is also evident. The median aggregate investment in the first two rounds of A3 are 7.9 and 3.5. As a result the median public good stock by the end of round 2 equals 11.4. This compares with the equilibrium aggregate investment in the first two rounds equal to 1.8 and 0, with a equilibrium level of stock at the end of round 2 equal to 1.8. The median aggregate investment in the first two rounds of A5 are 12.6 and 4.1. As a result the median public good stock by the end of round 2 equals 16.8. This compares with the equilibrium aggregate investment in the first two rounds equal to 1.4 and 0, with a equilibrium level of stock at the end of round 2 equal to 1.4. Beginning in round 3, the stock of public good in both A treatments declines sharply, with the median public good stock averaging 3.8 in rounds 3-10 of the A3 treatment and 7.9 in rounds 3-10 of the A5 treatment.

FINDING 4. The L mechanism leads to lower levels of the durable public good for n=3 than n=5. The stock of public good in L3 is predicted to be less than L5 in every round, and we find that the median stock of public good is less in 8 out of 10 rounds.[21] For L5, equilibrium is equal to 29.8, and this closely approached in the long run (median in round 10 equals 30.3). For L3, equilibrium is equal to 16, and after overshooting, declines to about 26 in round 10. This is less than what we observe in the L5 groups, but still somewhat above the equilibrium long run stead state of 16.

FINDING 5. The A mechanism leads to lower levels of the durable public good for n=3 than n=5. The median level of public good is less in all of the first 10

[21] The difference between the stock of public good in L3 and in L5 is statistically significant (p-value<0.01) in all rounds but one according to the results of a t-test on the equality of means (all differences significant except for round 7) and a Kolmogorov-Smirnov test on the equality of distributions (all differences significant except for round 10).

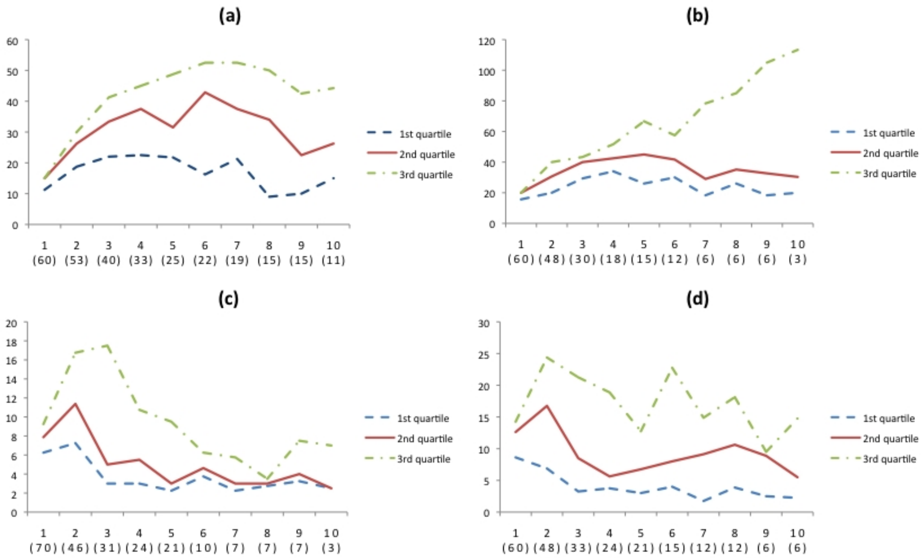

Figure 4: Quartiles of time paths of g, (a) 3-district L mechanism, (b) 5-district L mechanism, (c) 3-district A mechanism, (d) 5-district A mechanism. The number of observations (committees) per round is reported on the x axis below the round number

rounds of the A3 treatment, compared to the A5 treatment.[22] The differences, however, are not large in magnitude. In the last three rounds for which we have data for both treatments (rounds 11-13), the difference is negligible (less than 1.2 units of the public good).

6.1.2 Variation across committees

Because of possibility of nonstationary equilibria it is natural to expect a fair amount of variation across committees. Figure 3, by showing the median time path of the stock of public good, masks some of this heterogeneity. Do some committees reach full efficiency? Are some committees at or below the equilibrium? We turn next to these questions.

L committees Figure 4 illustrates the variation across committees by representing, for each round, the first, second and third quartile of investment levels for the L3 (panel (a)) and L5 game (panel (b)).

There was remarkable consistency across committees, especially considering this was a

[22] The difference between the stock of public good in A3 and in A5 is statistically significant (p-value<0.01) in almost all rounds according to the results of a t-test on the equality of means (all differences significant except for rounds 5 and 10) and a Kolmogorov-Smirnov test on the equality of distributions (all differences significant except for round 4).

complicated infinitely repeated game with many non-Markov equilibria.[23] There were a few committees who invested significantly more heavily than predicted by the Markov perfect equilibrium, but this only happened rarely, and nearly always such cooperation fell apart in later rounds. The most efficient committee in $L5$ invested W in each of the first 7 rounds, resulting in a public good level of 140. That committee did not invest anything for the remaining 2 rounds. Recall that the first best level of $L5$ is 400, so even this very successful committee did not come close to efficiency. In $L3$ only two committees reached levels above 60 (first best is 144) and not a single committee contributed W for more than 4 consecutive rounds.

These findings are perhaps surprising since the planner's solution can indeed be supported as the outcome of a subgame perfect equilibrium of the game:[24]

Observation 1. *When $d = 0$, the efficient investment path characterized in Proposition 1 is a subgame perfect equilibrium in weakly undominated strategies of both the L3 and L5 games for any $\delta > 0$.*

Figure 3 and 4, therefore, make clear that the predictions of the Markov equilibrium are substantially more accurate than the prediction of the "best" subgame perfect equilibrium (that is the Pareto superior equilibrium from the point of view of the agents), even when this best equilibrium is unique and reasonably focal (being the efficient solution). This observation may undermine the rationale for using the "best equilibrium" as a solution concept.

Many committees overshoot and then fall back to approximately equilibrium levels. However, it is also true that some committees never exceed the long run steady state. One $L5$ committee, in a 6 round match, starts out at a public good level of 20 and declines from there. One $L3$ committee, also in a 6 round match, starts out at a public good level of approximately 3 and gradually increases, but only reaches 11 by the end of round 6.

A committees In the A mechanism, there was also a lot of uniformity across committees, again with a few exceptions. See Figure 4, panels (c) and (d), for the $A3$ and $A5$ treatments, respectively. Here too the Markov equilibrium predicts behavior better than the "best subgame perfect equilibrium". We have not characterized the entire pareto frontier of the equilibrium set. The following observation, however, makes clear that an almost Pareto efficient equilibrium is a subgame perfect equilibrium of the game.

[23] The only treatment in which there is a substantial deviation is L5, for periods after the sixth. This effect is due to the fact that starting from $t = 6$, only 6 committees remained in the game. In periods 6-9, only 2 committees are in the top quartile; in period 10 only one. A look at the trajectories of all committees makes clear that these committees were outliers.

[24] A proof of observations 1 and 2 presented below is available from the authors.

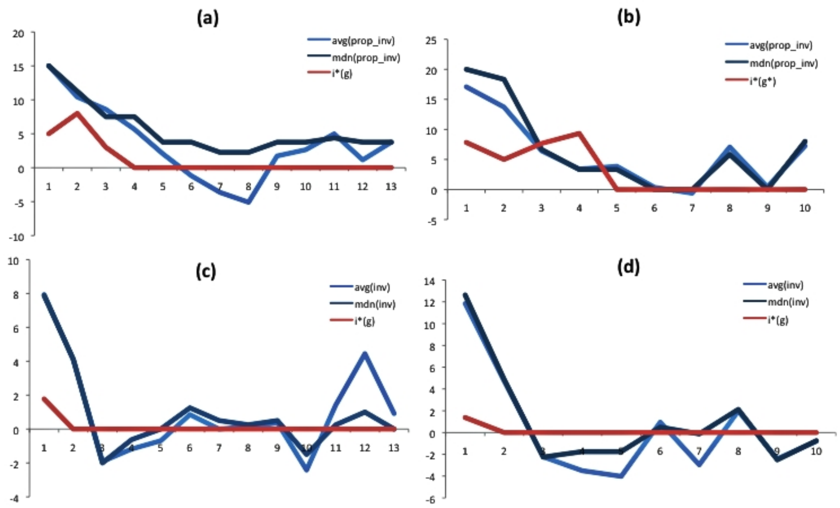

Figure 5: Investment per round, (a) 3-district L mechanism, (b) 5-district L mechanism, (c) 3-district A mechanism, (d) 5-district A mechanism.

Observation 2. *When* $d = 0$*, there exists a subgame perfect equilibrium in weakly undominated strategies in which all players invest* W *until a steady state level of* g *equal to* 141 *in A3 districts and* 351 *in A5 is reached.*

6.2 Proposed public good investment

The data consisting of proposed public good investment is somewhat richer than the data for the stock of public good, because our design was able to elicit proposals from non-proposers as well as proposers. (The proposal data also includes some failed proposals.). The results mirror the data for the state variable, y. Early round proposals offer significant overinvestment relatively to equilibrium, declining to equilibrium levels in later rounds. See figure 5.

6.3 Coalitions: Types of proposals

We now turn to the analysis of the proposed allocation of pork, as a function of g and n in the L mechanism. For this analysis we focus primarily on the number of members receiving significant amounts of pork in the proposed allocation, and whether the proposals had negative investment in the public good. We break down the proposed allocations into 4 canonical types. These types are: (1) *Invest W*: 100% allocation to the public investment; (2)*Proposer only*: The allocation divided between public investment and private consumption of the proposer only; (3)*Minimum Winning Coalition (MWC)*: The allocation divided between public investment and a minimum winning coalition that includes the proposer (2 if n=3; 3 if n=5); (4) *Universal*: Positive private allocations to all n members.[25] The last two categories are further broken down by whether investment in the public good is positive, zero, or negative.

Proposal Type	**Observations**	**% Accepted**
INVEST W	277(97)	0.95
PROPOSER ONLY	21(8)	0.63
MWC		
* with positive inv	66(20)	0.90
* with no inv	14(1)	1.00
* with negative inv	30(11)	0.55
UNIVERSAL		
* with positive inv	428(138)	0.97
* with no inv	52(14)	0.93
* with negative inv	57(26)	0.73

Table 3: L3 Proposal Types

Proposal Type	**Observations**	**% Accepted**
INVEST W	431(88)	0.98
PROPOSER ONLY	57(7)	0.71
MWC		
* with positive inv	71(16)	0.69
* with no inv	48(13)	0.54
* with negative inv	57(9)	0.56
UNIVERSAL		
* with positive inv	212(40)	0.93
* with no inv	24(6)	0.17
* with negative inv	52(9)	0.44

Table 4: L5 Proposal Types

Table 3 shows the breakdown of proposals for the L3 committees and Table 4 shows the breakdown for the L5 committees. The first column of each table lists the various proposal types. The second column lists the number of observations of each proposal type. There are

[25] For L5, there is a fifth residual category, not shown in the table, where pork is offered to exactly 4 members. There were only 14 such proposals observed and the acceptance rate was 100%.

two numbers in this column: the number in parenthesis gives the total number of proposals of this type that were actually voted on. The other number is the total number of provisional budget proposals of this type. Because of the random recognition rule, in L3 there are three times as many provisional budget proposals as budget proposals that are actually voted on. For L5 there are five times as many provisional budget proposals as budget proposals that are actually voted on. The final column gives the proportion of proposals of each type that passed when they were voted on.

FINDING 6. Most proposals are either (i) invest the entire budget; or (ii) universal private allocations with positive investment. In both L3 and L5, most proposals were to either invest W or universal allocations with a positive amount of investment. In L3, these two proposal types account for 75% of all budget proposals (including provisional budget proposals); in L5, these two types account for 63%. Of the remaining proposals, approximately half were MWC proposals (17% of all provisional budgets in L5 committees and 12% in L3 committees). Proposals that offered private allocation to the proposer only were quite rare in both treatments. Proposals with zero or negative investment occurred 21% of the time in L5 committees and 16% of the time in L3 committees. In contrast to the data, the Markov perfect equilibrium proposals should have been concentrated in the two categories: "proposer only" and MWC. However, it should be noted that even when pork is provided to more than a minimum winning coalition, most of the pork is concentrated on a minimum winning coalition.

FINDING 7. In universal allocations a minimal winning coalitions of players receives a more than proportional share of transfers. In the L3 committees, when positive pork is allocated, 75% of the allocated pork on average goes to the proposer and a single other coalition partner. In the L5 committees, when positive pork is allocated, 80% of it goes to the proposer and two other coalition partners and over 90% goes to the proposer and three other committee members. Thus, universal allocations are not equitable in the sense of giving non-proposers the same amount of pork.

6.4 Voting Behavior

Tables 3-4 also display the probability the proposal passes for each type of proposal. Tables 5-6 display additional results about voting outcomes in the $L3$ and $L5$ treatments respectively. The top part of the each table shows the proposal passage probabilities as a function of round. The middle part of each table shows the proposal passage probabilities as a function of g. The observed frequencies and acceptance rates of proposals are broken down by three ranges of g are in bold. In the first range, beginning at $g = 0$, the equilibrium proposal type assigns strictly positive allocation to investment in the public good and private allocation to the proposer, but zero private allocation to all other committee members. This corresponds to region of the state space below g_2 in which the proposer does not provide pork to the other members. Between g_2 and y^*, the proposer is constrained and finds it optimal to "buy off" the other committee members by investing up to y^* and also paying off some to minimum winning coalition of other committee member(s). After y^*, the equilibrium involves negative investment of the public good, and becomes a divide the dollar ultimatum game, which requires the proposer to give sidepayments to a minimum winning coalition (one other member in $L3$ and two other members in $L5$).

Round	**Observations**	**% Accepted**
1	210(70)	0.97
2	159(53)	0.94
3	120(40)	0.93
4	9(33)	0.91
5	75(25)	0.88
6	66(22)	0.77
7	57(19)	0.89
8	45(15)	0.80
9	45(15)	0.93
10	33(11)	0.91
11	12(4)	1.00
12	12(4)	0.75
13	12(4)	1.00
Overall	945(315)	0.91

g	**Observations**	**% Accepted**
$0 \leq g \leq 7(g_2)$	264(88)	0.97
$7 \leq g \leq 16(y^*)$	69(23)	1.00
$g > 16$	612(204)	0.88
Overall	945(315)	0.91

g	**Observations**	**% Accepted w/ inv<0**
$0 \leq g \leq 7(g_2)$	2(0)	.
$7 \leq g \leq 16(y^*)$	7(2)	1.00
$g > 16$	78(35)	0.66
Overall	87(37)	0.68

Table 5: L3 Proposal Acceptance Rates

Round	Observations	% Accepted
1	300(60)	0.95
2	240(38)	0.90
3	150(30)	0.60
4	90(18)	0.83
5	75(15)	0.93
6	60(12)	0.50
7	30(6)	0.83
8	30(6)	0.83
9	30(6)	0.83
10	15(3)	1.00
Overall	1020(204)	0.84

g	Observations	% Accepted
$0 \leq g \leq 18.5(g_2)$	465(93)	0.92
$18.5 \leq g \leq 29.83(y^*)$	240(48)	0.85
$g > 29.83$	315(63)	0.70
Overall	1020(204)	0.84

g	Observations	% Accepted w/ inv<0
$0 \leq g \leq 18.5(g_2)$	9(3)	0.33
$18.5 \leq g \leq 29.83(y^*)$	32(4)	0.50
$g > 29.83$	83(15)	0.67
Overall	124(22)	0.59

Table 6: L5 Proposal Acceptance Rates

FINDING 8. The vast majority of proposals pass. Overall, 84% of the L5 proposals and 91% of the L3 proposals receive majority committee support. Many of these are unanimously supported, especially the "invest W" proposals and the universal proposals with positive investment.[26] Furthermore, the probability of acceptance declines with g.

[26]In L5, 67% of the "invest all" proposals pass unanimously, and 82% of such proposals pass unanimously in L3. The corresponding percentages of unanimous ballots for universal proposals with positive investment are 40% and 65%.

When $g \leq y^*$, proposals are accepted 98% of the time by L3 committees and 90% of the time in L5 committees. In contrast, when $g > y^*$, proposals are accepted 88% of the time by L3 committees and only 70% of the time in L5 committees.

Acceptance rates differ by type of proposal. Some kinds of proposals are rejected somewhat frequently. This is particularly true for proposals with negative investment. In L3 committees, only 68% of proposals with negative investment pass and in L5 committees, only 59% pass. Proposals that give private allocation only to the proposer also fare relatively poorly, passing 63% of the time in L3 committees and 71% of the time in L5 committees. The most common proposal types, "invest W" and "universal with positive investment" nearly always pass. The acceptance rates for proposals to invest everything are 98% and 95% for the L5 and L3 treatments, respectively. The corresponding acceptance rates for universal proposals with positive investment are 93% and 97%. One surprise in the data is the relatively low acceptance rates for MWC proposals in L5.

Table 7 looks at the voting data through a different lens, and displays the results from logit regressions where the dependent variable is vote (0=no; 1=yes). An observation is a single voter's vote decision on a single proposal. The proposer's vote is excluded.[27] The data is broken down according to the treatment (n=3 or n=5). The independent variables are: *EU(status quo)*, the expected value to the voter of a "no" outcome (including the discounted theoretical continuation value); *EU(proposal)*, the expected value to the voter of a yes outcome; and *pork*, the amount of private allocation offered to the voter under the current proposal. Theoretically, a voter should vote yes if and only if the expected utility of the proposal passing is greater than or equal to the expected utility of the status quo. This would imply a negative coefficient on *EU(status quo)* and a positive coefficient on *EU(proposal)*, with the magnitudes of these coefficients being approximately equal. The effect of pork should be fully captured by *EU(proposal)* and therefore, we do not expect a significant coefficient on *pork*

	(1)	(2)
Treatment	*L3*	*L5*
EU(status quo)	-0.082***(0.02)	-0.22***(0.02)
EU(proposal)	0.080*** (0.02)	0.22*** (0.02)
pork	0.007(0.007)	0.07***(0.02)
constant	1.06(0.81)	0.1(0.97)
Pseudo-R2	0.1233	0.2905
Observations	490	576

[27] Proposers vote for their own proposals nearly 100% of the time (517 times out of 519).

Table 7: Logit estimates. Dependent variable: Pr {vote=yes}. Standard errors in parentheses; * significant at 10% level; ** significant at 5% level; *** significant at 1% level

FINDING 9. Voters are forward looking. The results of the vote regression are clear. The main effect on voting is through the difference between the expected utility of the status quo and the proposal. The signs of the coefficients are highly significant, large in magnitude, and not significantly different from each other in absolute value. The residual effect of pork is nonexistent in $L3$ committees, and significant but small in magnitude in $L5$ committees. The constant term is not significantly different from zero, suggesting that voters are not a priori inclined to favor or disfavor proposals.

6.5 Evidence of non-Stationary strategies

6.5.1 L mechanism

While we observe only small departures from the predicted stationary equilibrium behavior in the L games, at least two findings suggest a deeper analysis.

The first is the overinvestment in the public good, especially in early periods. For instance, in the L3 treatment, the median level of public good peaked in round 6 at nearly three times the equilibrium long run steady state, before declining to slightly above equilibrium levels by rounds 9 and 10.

The second interesting observation is that most proposals are either to invest everything or, if not, to give positive allocations to all members of the committee. In fact, we rarely see proposals where the proposer is the only member receiving a private allocation, even though the equilibrium path predicts such proposals in the early rounds. One possible explanation that may be consistent with both of these observations is that, rather than playing a stationary equilibrium, some committees are supporting more efficient allocations by using non-stationary strategies. Because this is an infinitely repeated game with a low-probability random stopping rule, a natural conjecture is that there are equilibria that can support higher levels of public good provision than the Markov equilibrium we characterize in the theoretical section of the paper.

Similarly, there could be punishment strategies imposed on proposers who do not share the residual budget with any other committee members. Such proposals would be rejected as part of the punishment, or possibly even accepted, but then punished by ostracism in the future.

We next take a look at the data to see if there is evidence of punishment strategies. We look at both voting behavior and proposal behavior. Table 8 reports the results of a logit regression of voting behavior on the same variables a table 3, but includes three additional variables that could in principle indicate some degree of punishment or reward behavior being used to affect proposals and support equilibrium outcomes that differ from the computed stationary solutions, in the two ways described above. The logic behind this regression is that, under the null hypothesis that the equilibrium is Markovian, behavior should be independent of these variables. Any evidence of dependence, therefore, is against the original assumption.

First, we find evidence of overinvestment in g with respect to the equilibrium prediction, especially in the early rounds in the data. This behavior could be supported by voting strategies that punish proposals that do not offer sufficient public good. To verify this conjecture, we include the proposed investment level I_t in the vote regression, and expect the sign to be positive if this sort of behavior is occurring.

Second, the distribution of pork tends to be more egalitarian than predicted by the theory. While this is not a big effect, it is clearly seen in the data. We include two variables that capture different notions of fairness. The first is a Herfindahl index, h, to indicate how unequal the proposed division of pork is across committee members. We expect the sign on this to be negative, in the sense that proposals with greater dispersion of the private proposals receive more negative votes.[28] The second variable is "greed" which is measured by the amount of own-private allocation by the proposer, an indicator of how slanted the private allocations are toward the proposer. We expect the sign on this to be negative also, in the sense that greedier proposals are punished with more negative votes.

	(1)	(2)
Treatment	*L3*	*L5*
EU(status quo)	0.01(0.02)	-0.15*** (0.03)
EU(proposal)	-0.02(0.03)	0.14*** (0.03)
pork	0.14***(0.03)	0.17***(0.04)
I	0.06***(0.01)	0.03***(0.01)
h	-2.81***(0.80)	-1.20(1.07)
greed	-3.02***(0.95)	-2.05***(0.71)
constant	1.48***(0.01)	0.62(0.99)
Pseudo-R2	0.2143	0.3230
Observations	490	576

Table 8: Logit estimates. Dependent variable: Pr {vote=yes}.Including i, h, and greed. Standard errors in parentheses; * significant at 10% level; ** significant at 5% level; *** significant at 1% level

[28] Note however, that the sign on the herfindahl index is automatically negative if there are more members in the winning coalition, so this may not be an indication of punishment/reward at all, but simply myopic selfish optimizing.

The results are presented in Table 8. For the $L3$ treatment, all the "nonstationary" variables are significant with the expected sign. More efficient proposals receive greater support, as do proposals that are more fair or less greedy. The size of the positive sign of the effect of I_t, however, seems too small to provide evidence in favor of an equilibrium in which non Markovian strategies reward efficient behavior, especially since a positive sign is consistent with equilibrium behavior.[29] Because of this, we are reluctant to conclude that the significant coefficient on I is indicative of nonstationary behavior. On the other hand the significance of the coefficients on the fairness variables demonstrates the existence of voting behavior that rewards exactly the types of proposals we see more of relative to the equilibrium predictions (invest W and universal). The coefficients on $EU(statusquo)$ and $EU(proposal)$ still have the correct (opposite) signs and are not significantly different from each other, but they are no longer significantly different from 0. The coefficient on $pork$ is now highly significant, and together with I, has soaked up most of the effect of $EU(statusquo)$ and $EU(proposal)$. This is not surprising. $pork$ and I are the main determinants of the difference between $EU(statusquo)$ and $EU(proposal)$.

Results for the $L5$ treatment are similar, all three of the new variables have the expected sign, and two are highly significant (I and $greed$). As in $L3$, more efficient proposals receive greater support, as do proposals that are more fair or less greedy. The coefficients on $EU(statusquo)$ and $EU(proposal)$ still have the correct (opposite) signs and they are highly significant: their magnitude, however, has dropped by about one-third. This parallels the finding in $L3$: the coefficient on $pork$ is now highly significant and much greater in magnitude than in $Table$ 3.

As a final check for nonstationary strategies, we look at how current proposals treat the proposer of the previous round, depending on how a current proposer was treated by the last proposer. The hypothesis is that how well the current proposer treats the previous proposer is increasing in how well the previous proposer treated him. Because the only way the current round's proposer can target a punishment or reward for the previous round's proposer is with pork, we run a regression where the dependent variable is the current proposal's private allocation to the previous round's proposer. For observations, we use all current round provisional budgets beginning in round 2, excluding the provisional budget of the previous round's proposer. The key independent variable we use for how well the previous round's proposer treated the current round proposer is $EUratio_{t-1}$, which is the lagged ratio of $EU(proposal)$ and $EU(statusquo)$, and we control for the current level of

[29] In equilibrium we should expect a higher level of investment to be associated with a higher probability that a random voters votes yes to a proposal. This because when g is small, the equilibrium predicts a high investment level and a unanimous yes vote; when g is high investment is predicted to be smaller, and proposal are predicted to pass by minimal winning coalitions.

public good, g. We report two different regressions in table 5. The first two columns include only the above variables. The second column checks for more detailed punishment/reward effects, by including lagged versions of the I, h, and $greed$ variables. In other words, we check whether there are lagged effects of efficiency or fairness of the previous proposal on the current proposal's private allocation to the previous proposer. The logic is the same as above: under the assumption of a Markov equilibrium, we should find no effect.

	(1)	(2)	(3)	(4)
Treatment	*L3*	*L5*	L3	L5
g	0.118***(0.02)	0.014(0.013)	0.112***(0.02)	0.01(0.01)
EUratio$_{t-1}$	293.49***(98.95)	265.29**(119.21)	282.85***(103.41)	232.09**(113.06)
I$_{t-1}$			-0.02(0.02)	-0.015(0.024)
h$_{t-1}$			-9.74(8.21)	6.06(10.1)
greed$_{t-1}$			-10.44(13.2)	-27.01***(7.09)
constant	-101***(33.42)	-53.45**(23.85)	-93.03***(35.62)	-41.27*(22.67)
Pseudo-R2	0.0032	0.0222	0.0239	0.0080
Observations	384	384	384	384

Table 9: TOBIT estimates for L treatments. Dependent variable: Private allocation offered to previous round's proposer. Standard errors in parentheses; * significant at 10% level; ** significant at 5% level; *** significant at 1% level

The coefficient on $EUratio_{t-1}$ is significant in both treatments. There is a significant effect of the the lagged greed variable in $L5$. We conclude from this analysis that there is significant evidence of the use of nonstationary strategies.

FINDING 10. There is evidence of nonstationary behavior in the L mechanism. In voting behavior, controlling for their own private allocation, voters punish proposals that are either too greedy or too unfair. We find this in both the $L3$ and the $L5$ committees. In proposal behavior, current proposals discriminate against previous proposers who were too greedy and reward previous proposers who treated them well. This nonstationarity in behavior seems to be motivated by fairness rather than by efficiency considerations. There is evidence that voting behavior rewards proposals that have higher investment levels: but this effect is consistent with equilibrium behavior, and too small to support an efficient outcome.

6.5.2 A mechanism

Under the A mechanism, public good levels also exhibited a time path of early overproduction followed by negative investment, converging toward the equilibrium steady state. In nearly all groups, the public good levels are consistently very low. However, because the general pattern is qualitatively similar to what we observed under the L mechanism, it is suggestive of some small amount of cooperation that may be accountable in part by non-stationary strategic behavior involving punishments and rewards.

The tools by which players in the A mechanism can reward or punish the other districts are more limited than with the L mechanism. The main difference is that punishments cannot be "targeted". Under the L mechanism a proposal specifies an individual side payment to each legislator, which allows current proposers to punish specific other members of the committee (for example by giving nothing to a previously greedy proposer). In contrast, in the A mechanism, an individual district can only punish/reward other districts collectively by investing less/more in the public good in future periods. With this in mind, we regress current individual investment decisions on last period's average investment in their group (lagAVE), controlling for the level of public good (g) and experience, measured by how many games they have played so far (EXP). A positive coefficient would be consistent with some sort of nonstationary behavior such as collective punishments and rewards. We also include last period's *variance* of investment decisions (lagVAR) in their group, as a high variance will indicate the presence of shirkers in their group, which could trigger (untargeted) punishments. A negative coefficient would be consistent with untargeted punishment of individual shirking behavior. Table 10 shows the results for the A3 and A5 treatments.

	(1)	(2)
Treatment	*A3*	*A5*
lagAVE	0.234*** (0.062)	0.140* (0.076)
lagVAR	-0.025***(0.007)	-0.045***(0.006)
g	-0.044 (0.029)	0.016 (0.023)
EXP	-0.210 (0.238)	0.374 (0.232)
constant	4.278*** (1.568)	1.591 (1.374)
Pseudo-R2	0.0178	0.0242
Observations	495	930

Table 10: TOBIT estimates for A treatments. Dependent variable: Individual investment decisions. Standard errors in parentheses; * significant at 10% level; ** significant at 5% level; *** significant at 1% level

The results are virtually identical for the two treatments. Both "punishment" variables, lagAVE and lagVAR, have the predicted sign and are statistically significant. There are no significant experience effects (EXP) nor significant effects of the level of the public good, g. This seems to suggest non-stationary behavior that may be consistent with strategic attempts to maintain higher-than-equilibrium investment levels. As with the the L mechanism, to the extent that these attempts may have increased investment levels, the magnitude of such an increase is rather small.

7 Discussion and Conclusions

In this paper we have studied the dynamic provision of durable public goods. Despite the fact that most, if not all, public goods are durable, very little is known on this subject, both from a theoretical and empirical point of view. We have attempted to provide a first answer to three very basic questions that should be the starting point of any further research.

First, is the free rider problem associated with dynamic public goods substantial? There are reasons to think that the free rider problem may not be as serious as suggested by theoretical predictions. A number of laboratory experiments have suggested that agents may be less selfish than commonly assumed in static environments, and even achieve efficiency in some settings. Is this type of cooperative behavior present in environments with repeated interactions and public good accumulation? In a dynamic environment agents may be able to use trigger strategies to punish inefficient behavior, so players may even have "selfish" incentives to behave cooperatively. Our results, however, suggest there is little hope for significant voluntary cooperation. The theoretical predictions of the Markov equilibrium that we use as benchmark suggest substantial underinvestment, independently of the institutional details: an investment even lower than in a static model because of the dynamic free rider problem described in sections 4.1 and 4.2. The experimental analysis supports this prediction. Although we observe more investment than predicted in the model, overinvestment is concentrated in the early stages of the game, and does not persist: in the long run, the public good levels approximate the Markov equilibrium steady state. In all treatments investments are orders of magnitude below the Pareto efficient levels.

The second question naturally follows from the first. Do institutions matter for efficiency? This question is obviously very important and preliminary to any type of normative analysis or any attempt to improve public good providing institutions. Our model predicts that a majoritarian Legislative institution delivers a lower inefficiency level than a decentralized Autarky institution. Here too the experiment clearly confirms the prediction. Under common assumptions on utilities and other relevant parameters, the majoritarian Legislative system induces from 3 to 5 times more public good provision than in the Autarky system. This result implicitly confirms from an experimental point of view the importance of institutions in public good provisions, and the fact that incentives matter in a way predicted by theoretical models.

The final questions we attempt to address are: to what extent the models that we use are adequate to study this problem? What equilibrium concepts should be used? This is a particularly important question since, depending on the equilibrium concept we can have very different predictions for the same model. It is clearly difficult to identify the equilibrium adopted by players, but the analysis provides some interesting insights. First, as discussed in Section 6.5.3, we observe a consistent pattern of behavior across committees, despite the fact that we have multiplicity of potential equilibria. The Markov equilibrium that we have adopted as benchmark does not fully capture the complexity of the agents' strategies, that are non stationary and depend on payoff irrelevant variable as the distribution of pork in the previous periods. But these effects seem to be very weak, especially for larger committees ($n = 5$). The aggregate evolution of g, moreover, is described reasonably well by the Markovian prediction. The Markovian prediction is certainly superior to the prediction of the best subgame equilibrium sustainable with nonstationary strategies, a equilibrium refinement that is sometime used in applied work. In our setting, this equilibrium would predict efficient or nearly efficient outcomes, which is far off the mark.

There are many possible directions for the next step in this research. On the experimental side, our design was intentionally very simple and used a limited set of treatments. The theory has interesting comparative static predictions about the effect of other parameters of the model that we have not explored in this work, such as: the size of the qualified majority; the discount factor; the production technology; preferences; endowments; and depreciation rates. We have also limited the analysis to only two polar types of institutions that differ on the degree of centralization of decisions. Our political process does not have elections and parties, there is no executive branch or "president" to oversee the general interest common to all districts. Elections, parties, and non-legislative branches are all important components of most political systems, and incorporating such institutions into our framework would be a useful and challenging direction to pursue. Finally, it would be interesting to allow for a richer set of allocations, such as allowing debt financing or multiple public goods.

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Appendix

Proof of Proposition 2

Two cases are possible. The first case is when in the steady state $y(y_P^*) = y_P^*$ and $x(y_P^*) > 0$. Since $y(g)$ is constant for $g \geq \max\left\{\frac{y_P^* - W}{1-d}, 0\right\}$, it is straightforward to show that the derivative of the value function in this region is $v'(g) = \frac{\partial}{\partial g}\left[W + (1-d)g - y_P^* + B\frac{1}{\alpha}n\left(y_P^*\right)^{\alpha} + \delta v_P(y_P^*)\right] = (1-d)$. Using the first order condition we must have $Bn\left(y_P^*\right)^{\alpha-1} + \delta(1-d) = 1$, so

$$y_P^* = \left(\frac{Bn}{1-\delta(1-d)}\right)^{\frac{1}{1-\alpha}} \tag{13}$$

for such an equilibrium to exist we need that $y_P^* > g_P^*$, so:

$$(1-d)\left(\frac{Bn}{1-\delta(1-d)}\right)^{\frac{1}{1-\alpha}} > \left(\frac{Bn}{1-\delta(1-d)}\right)^{\frac{1}{1-\alpha}} - W$$

that is $\left(\frac{Bn}{1-\delta(1-d)}\right)^{\frac{1}{1-\alpha}} < \frac{W}{d}$, or $1-\delta(1-d) > Bn\left(\frac{d}{W}\right)^{1-\alpha}$. Assume now that the steady state g_P^{SS} satisfies $g_P^{SS} = y(g_P^{SS}) \leq y_P^*$. In this case $W + (1-d)g = g$, so $g_P^{SS} = W/d$. For this case to be possible we need that $\left(\frac{Bn}{1-\delta(1-d)}\right)^{\frac{1}{1-\alpha}} \geq W/d.$, or $1-\delta(1-d) \leq Bn\left(\frac{d}{W}\right)^{1-\alpha}$. ■

Proof of Proposition 3

The fact that a concave equilibrium has the property stated in the proposition follows from the discussion in the text. Here we prove existence. We proceed in two steps.

Step 1. Let $y_A^* = \left[u^{-1}\right]'(1-\delta\frac{1-d}{n})$, and $g_A^1 = \max\left\{0, \frac{y_A^* - W}{1-d}\right\}$. Assume first that $d < \frac{W}{y_A^*}$, so $\frac{y-W}{1-d} < y$ for any $y \leq y_A^*$. For any $g > g_A^1$ define a value function $v_A^1(g) = \frac{W-(y_A^* - (1-d)g)}{n} + \frac{u\left(y_A^*\right)}{1-\delta}$. Note that this function is continuous, non decreasing, concave, and differentiable with respect to g, with $\frac{\partial}{\partial g}v_A^1(g) = \frac{1-d}{n}$. Let $g_A^2 = \max\left\{0, \frac{g_A^1 - W}{1-d}\right\}$, and define:

$$v_A^2(g) = \begin{cases} v_A^1(g) & g \geq g_A^1 \\ u((1-d)\,g + W) + \delta v_A^1((1-d)\,g + W) & g \in [g_A^2, g_A^1) \end{cases}$$

Note that $v_A(g)$ is continuous and differentiable in $g \geq g_A^2$, except at most at g_A^1 To see that it is also concave in this interval, note that it is concave for $g \geq g_A^1$. Moreover, for any $g \in [g_A^2, g_A^1)$ and $g' \geq g_A^1$ we have:

$$\begin{aligned} \frac{\partial}{\partial g}v_A^2(g) &= u'(((1-d)\,g + W) + \delta v_A^{1\prime}((1-d)\,g + W) \\ &> u'(y_A^*) + \delta v_A^{1\prime}(y_A^*) = 1 > \frac{1-d}{n} = \frac{\partial}{\partial g}v_A^2(g') \end{aligned}$$

The first inequality derives from $y_A^* > (1-d)\,g + W$ (which is true, by definition of g_A^1 and g_A^2, for all $g \in [g_A^2, g_A^1)$), and concavity of $u(g)$. So $v_A^2(g)$ is concave in $g \geq g_A^2$. Assume that for all $g \geq g_A^n$, with $g_A^n \geq 0$ and either $g_A^n < g_A^2$ or $g_A^n = 0$, we have defined a value function $v_A^n(g)$ that is concave and continuous, and that is differentiable in $g > g_A^1$. Define $g_A^{n+1} = \max\left\{0, \frac{g_A^n - W}{1-d}\right\}$, and

$$v_A^{n+1}(g) = \begin{cases} v_A^n(g) & g \geq g_A^n \\ u((1-d)\,g + W) + \delta v_A^n((1-d)\,g + W) & g \in \left[g_A^{n+1}, g_A^n\right) \end{cases}$$

We can easily show that this function is concave, continuous in $g \geq g_A^{n+1}$, and differentiable for $g > g_A^1$. Moreover, either $g_A^{n+1} = 0$ or $g_A^{n+1} < g_A^n$. We can therefore define inductively a value function $v_A(g)$ for any $g \geq 0$ that is continuous and concave, and that is differentiable at least for $g > g_A^1$ and so, in particular, at y_A^*. Define now the following strategies:

$$y_A(g) = \min\{W + (1-d)g, y_A^*\}\,, \text{ and } x_A(g) = \frac{W + (1-d)g - y_A(g)}{n}. \tag{14}$$

We will argue that $v_A(g), y_A(g), x_A(g)$ is an equilibrium. To see this note that by construction, if the agent uses strategies $y_A(g), x_A(g)$, then $v_A(g)$ describe the expected continuation value function of an agent. To see that $y_A(g), x_A(g)$, are optimal given $v_A(g)$ note that for $g \geq g_A^1$, $\left\{y_A^*, \frac{W+(1-d)g-y_A^*}{n}\right\}$ maximizes (5) when all the constraints except the second are considered; and for $g \geq g_A^1$, $W + (1-d)g > y_A^*$, so the second constraint is satisfied as well. For $g < g_A^1$, we must have $y_A(g) = W + (1-d)g$, $x_A(g) = 0$. We conclude that $y_A(g)$, $x_A(g)$ is an optimal reaction function given $v_A(g)$.

Step 2. Assume now that $d \geq \frac{W}{y_A^*}$. In this case $g(1-d) + W \leq y_A^*$ for any $g \leq y_A^*$. Define:

$$v_A^1(g) = \max_y \left\{ \begin{array}{c} \frac{W+(1-d)g-y}{n} + u(y) + \delta v_A^1(y) \\ W + (1-d)g - y \geq 0. \end{array} \right\} \tag{15}$$

This is a contraction with a unique continuous, concave and differentiable fixpoint $v_A^1(y)$. By the envelope theorem, we have $\frac{\partial}{\partial g} v_A^1(g) = 1 + \lambda \geq \frac{1-d}{n}$, where λ is the Lagrangian multiplier of the constraint in 15. Define:

$$y_A^{**} = \arg\max_y \left\{u(y) - y + \delta v_A^1(y)\right\}$$

Because for any g $\frac{\partial}{\partial g} v_A^1(g) \geq \frac{1-d}{n}$, $y_A^{**} > y_A^*$. Define now:

$$v_A(g) = \begin{cases} v_A^1(g) & g \leq \frac{y_A^{**} - W}{1-d} \\ \frac{W+(1-d)g-y_A^{**}}{n} + u(y_A^{**}) + \delta v_A^1(y_A^{**}) & else \end{cases} \tag{16}$$

We now argue that the strategies $y_A(g), x_A(g)$ defined by

$$y_A(g) = \min\{W + (1-d)g, y_A^{**}\}\,, \text{ and } x_A(g) = \frac{W + (1-d)g - y_A(g)}{n},$$

and the value function $v_A(g)$ defined in (16) are an equilibrium. For any $g \leq \frac{y_A^{**}-W}{1-d}$, the strategy $y_A(g)$ is $W+(1-d)g \leq y_A^{**}$. Since $d > \frac{W}{y_A^*}$, we have $d > \frac{W}{y_A^{**}}$, and so $\frac{y_A^{**}-W}{1-d} > y_A^{**}$ It follows that starting from an initial state $g_0 = 0$, a state $g \geq \frac{y_A^{**}-W}{1-d}$ is never reached in equilibrium, and the value function can be represented as (15). In a state $g > \frac{y_A^{**}-W}{1-d}$, y_A^* is chosen. In that period, therefore the utility is $\frac{W+(1-d)g-y_A^{**}}{n} + u(y_A^{**})$. From that period onward, the expected value function is $v_A^1(g)$. By construction, therefore, when players uses strategies $y_A(g), x_A(g)$, then (16) is the expected continuation value function.

To see that $y_A(g), x_A(g)$, are optimal given $v_A(g)$ note that for $g \geq \frac{y_A^{**}-W}{1-d}$, $\left\{y_A^{**}, \frac{W+(1-d)g-y_A^{**}}{n}\right\}$ maximizes (5) when all the constraints except the second are considered; and for $g \geq \frac{y_A^{**}-W}{1-d}$, $W+(1-d)g \geq y_A^{**}$, so the second constraint is satisfied as well. For $g < \frac{y_A^{**}-W}{1-d}$, the solution of (5) must be $y_A(g) = W+(1-d)g$. We conclude that $y_A(g)$, $x_A(g)$ is an optimal reaction function given $v_A(g)$. ■

Proof of Proposition 4

Two cases are possible. The first case is when in the steady state $y(y_A^*) = y_A^*$ and $x(y_A^*) > 0$. Since $y(g)$ is constant for $g \geq \max\left\{\frac{y_A^*-W}{1-d}, 0\right\}$, it is straightforward to show that the derivative of the value function in this region is $v'(g) = \frac{(1-d)}{n}$. Using the first order condition we must have $B\left(y_A^*\right)^{\alpha-1} + \delta\frac{(1-d)}{n} = 1$, so

$$y_A^* = \left(\frac{Bn}{n-\delta(1-d)}\right)^{\frac{1}{1-\alpha}} \tag{17}$$

for such an equilibrium to exist we need that $y_A^* > g_A$: so $n-\delta(1-d) > Bn\left(\frac{d}{W}\right)^{1-\alpha}$, that is always true when $d=0$. In the second possible case $y(y_A^*) < y_A^*$ (and so the steady state is lower than y_A^*). This case is possible only if $W+(1-d)g = g$: but when $d=0$ this implies $W=0$, a contradiction. We conclude that when $d=0$, y_A^* is given by (17), and the equilibrium steady state is unique. ■

Proof of Proposition 5

Define a function

$$\begin{aligned} v_L^1(g) &= \frac{W-(y_2^*-g)}{n} + u(y_2^*) + \frac{\delta}{1-\delta}\left[\frac{W}{n} + u\left(y_2^*\right)\right] \\ &= \frac{1}{1-\delta}\left[\frac{W}{n} + u\left(y_2^*\right)\right] + \frac{g-y_2^*}{n}. \end{aligned}$$

where $y_2^* = \left[u^{-1}\right]'\left(\frac{1}{q} - \delta\frac{1-d}{n}\right)$. Note that this function is continuous, increasing, concave, and differentiable with respect to g, with $\frac{\partial}{\partial g}v_L^1(g) = \frac{1}{n}$. Now define $\widetilde{y}(g)$ implicitly by the equation:

$$u(\widetilde{y}(g)) + \delta v_L^1(\widetilde{y}(g)) = W/n + u(g) + \delta \left[\frac{W - \widetilde{y}(g) + g}{n} + u(\widetilde{y}(g)) + \delta v_L^1(\widetilde{y}(g))\right]$$

This equation can be rewritten as:

$$u(\widetilde{y}(g))\,(1-\delta) + \frac{\delta}{n}\widetilde{y}(g) = u(g) + \frac{\delta^2}{n}g + \frac{W}{n} - \delta\left(u(y_2^*) - (1-\delta)\,\frac{y_2^*}{n}\right) \tag{18}$$

Note that (18) implicitly defines a differentiable and increasing function of g with $\widetilde{y}(g) > g$. To see this note that differentiating (18) with respect to $\widetilde{y}$ and g we have:

$$\widetilde{y}'(g) = \frac{u'(g) + \frac{\delta^2}{n}}{(1-\delta)\,u'(\widetilde{y}(g)) + \frac{\delta}{n}} > 0 \tag{19}$$

We can therefore define a point $g_L^2 = \min\left[g \geq 0\,|\widetilde{y}(g) \geq y_2^*\right]$. This point has the property that for any $g \geq g_L^2$, we have $\widetilde{y}(g) \geq y_2^*$; moreover, $g_L^2 < y_2^*$. Now define the function:

$$v_L^2(g) = \begin{cases} v_L^1(g) & g > g_L^2 \\ \frac{W-\widetilde{y}(g)+g}{n} + u(\widetilde{y}(g)) + \delta v^1(\widetilde{y}(g)) & else \end{cases} \tag{20}$$

Let $\overline{g} > 0$ be defined by $\overline{g} = u'^{-1}(1)$. We have:

Lemma A.1. *There is a $\overline{\delta}$ such that for $\delta > \overline{\delta}$, $\widetilde{y}(g)$ and $v_L^2(g)$ are increasing and continuous and concave respectively in $g \in [\overline{g}, g_L^2]$, and in $g \geq g_L^2$.*

Proof. We showed above that $\widetilde{y}'(g) > 0$. Furthermore, differentiating (19) with respect to g, we have:

$$\widetilde{y}''(g) = \frac{u''(g)\left[(1-\delta)\,u'(\widetilde{y}(g)) + \frac{\delta}{n}\right] - \left[u'(g) + \frac{\delta^2}{n}\right](1-\delta)\,u''(\widetilde{y}(g))\widetilde{y}'(g)}{\left[(1-\delta)\,u'(\widetilde{y}(g)) + \frac{\delta}{n}\right]^2} \tag{21}$$

It is clear that there is a $\overline{\delta}$ such that for $\delta > \overline{\delta}$, $\widetilde{y}''(g) < 0$ for any $g \in [\overline{g}, g_L^2]$. To see this note that for $\delta = 1$ we have $\widetilde{y}''(g) < 0$, as the numerator of (**??**) is smaller than 0 and its denominator greater than 0, and recall that $\widetilde{y}(g)$ is continuous. For $v_L^2(g)$, note that for $g \geq g_L^2$ the function is linear. For $g \leq g_L^2$

$$\begin{aligned} v_L^2(g) &= \frac{W - \widetilde{y}(g) + g}{n} + u(\widetilde{y}(g)) + \delta v_L^1(\widetilde{y}(g)) \\ &= \frac{W - \widetilde{y}(g) + g}{n} + u(\widetilde{y}(g)) + \delta\left[\frac{W - (y_2^* - \widetilde{y}(g))}{n} + \frac{\delta}{1-\delta}\left[\frac{W}{n} + u\left(y_2^*\right)\right]\right] \\ &= \frac{W + g}{n} + u(\widetilde{y}(g)) + (\delta - 1)\,\frac{\widetilde{y}(g)}{n} + \delta\left[\frac{W - y_2^*}{n} + \frac{\delta}{1-\delta}\left[\frac{W}{n} + u\left(y_2^*\right)\right]\right] \end{aligned}$$

so concavity in $[\overline{g}, g_L^2]$ follows from the concavity of $\widetilde{y}(g)$ for δ sufficiently large. Concavity in $g \geq g_L^2$ follows from the fact that in this range $v_L^2(g)$ is differentiable everywhere except at most at g_L^2, and its derivative is non increasing in g. ■

Define $y_1^* = \arg\max_{y'} \{u(y') - y' + \delta v_L^2(y')\}$ and $g_L^1 = \min\left[g \geq 0 \,|\widetilde{y}(g) \geq y_1^*\right]$. Note that $y_1^* < y_2^* = \left(\frac{nB}{\frac{n}{q}-\delta}\right)^{\frac{1}{1-\alpha}}$, an upper bound that is independent of W, and $g_L^1 \leq g_L^2$; moreover $g \leq g_L^1$, implies $\widetilde{y}(g) \leq y_1^*$. We can now construct the following value function:

$$v_L^*(g) = \left\{ \begin{array}{cc} v_L^2(g) & g \geq g_L^1 \\ \frac{W - y_1^* + g}{n} + u(y_1^*) + \delta v^1(y_1^*) & else \end{array} \right.$$

which is a continuous and non decreasing function of g. We can also construct the strategies:

$$y_L^*(g) = \left\{ \begin{array}{cc} y_1^* & g \leq g_L^1 \\ \widetilde{y}(g) & g \in (g_L^1, g_L^2] \\ y_2^* & else \end{array} \right.$$

and $x_L^*(g) = W - y_L(g) + (1-d)g - (q-1)s_L(g)$. We now show that the value function $v_L^*(g)$ and the strategies $y_L^*(g)$ and $x_L^*(g)$ are an equilibrium for a sufficiently large W and δ. Consider the Proposer's problem (7). One of two cases is possible. First, the incentive compatibility constraint is not binding, so the proposer can effectively ignore the other legislators. Second, the incentive compatibility constraint binds and so the proposer has either to modify the level of public good, or provide pork transfers to a minimal winning coalition or both.

Case 1: non binding IC. Assume first that we can ignore the incentive compatibility constraint and set $s = 0$. The problem becomes:

$$\max_{y} \left\{ \begin{array}{c} W - [y - (1-d)g] + u(y) + \delta v_L^*(y) \\ s.t. \;\; W - y + (1-d)g \geq 0 \end{array} \right\} \tag{22}$$

If we ignore the constraint in (22), then it is optimal (without loss of generality) to choose y such that:

$$y \in \arg\max_{y'} \{u(y') - y' + \delta v_L^*(y')\} \tag{23}$$

It is useful to have the following result:

Lemma A.2. *The threshold g_L^1 is a non increasing continuous function of W and for any ε there is a W_ε such that for $W > W_\varepsilon$, then $g_L^1 < \varepsilon$.*

Proof. Let k be defined as before by $u'(k) = 1$. Then since $v_L^2(y)$ is non decreasing in y, $y_1^* \geq k > 0$. Let $f(W)$ be defined by

$$u(y_1^*) + \frac{\delta^2}{n} y_1^* = u(f(W))\,(1-\delta) + \frac{\delta}{n} f(W) - \frac{W}{n} + \delta\left(u(y_2^*) - (1-\delta)\,\frac{y_2^*}{n}\right)$$

So $g_L^1 = \max\{0, f(W)\}$. Since $f(W)$ is a continuous decreasing function of W, it is then immediate that g_L^1 is a continuous and monotonically non increasing function of W. It is also immediate to verify that for any $\varepsilon > 0$ there is a W_ε such that $g_L^1 < \varepsilon$ for $W > W_\varepsilon$. ■

By Lemma A.2 we can find a W_1 such that for $W > W_1$, g_L^1 is sufficiently small to guarantee that $u'(y) + \delta v_L^{*\prime}(y) > 1$ for any $g \leq g_L^1$, so

$$y \in \arg\max_{y'} \{u(y') - y' + \delta v_L^*(y')\}$$

implies $y > g_L^1$. Lemma A.1 then guarantees that (22) has a unique solution y_1^* for $\delta \geq \overline{\delta}$. It is easy to see that in correspondence to y_1^* we have $x_L(g) \geq 0$ if and only if g is greater than or equal to $\max\left\{\frac{y_1^* - W}{1-d}, 0\right\}$. Since y_1^* is bounded, this is verified for any $g \geq 0$ when $W > W_1$, and W_1 is chosen to be sufficiently large. The incentive compatibility constraint is satisfied if and only if $\widetilde{y}(g) \leq y_1^*$ that is if $g \leq g_L^1$. We can therefore conclude that, for $\delta > \overline{\delta}$ and $W > W_1$, when $g \leq g_L^1$ the optimal policy is $y_L^*(g)$ and $x_L^*(g)$.

Case 2: binding IC constraint. When $g > g_L^1$ the incentive compatibility constraint can not be ignored. In this case, the problem solved by the proposer is:

$$\max_{y,s} \left\{ \begin{array}{c} [W - [y - (1-d)g] - (q-1)s] + u(y) + \delta v_L^*(y) \\ s.t. s + u(y) + \delta v_L^*(y) \geq \frac{W}{n} + u(g) + \delta v_L^*(g) \\ s \geq 0 \end{array} \right\} \tag{24}$$

Note that we can assume without loss of generality that the solution to this problem is larger or equal than y_1^* (if this were not the case, by increasing y the proposer would increase his utility and relax the constraint, a contradiction). By Lemma A.1, it follows that we can treat (24) as a concave maximization problem when $\delta \geq \overline{\delta}$. There are two possibilities. First, the proposer continues to provide no consumption to the districts of other legislators, but he increases the provision of the public good $y_L(g)$ in order to satisfy the incentive compatibility constraint (no transfer case). Second, he provides consumption to the districts of $q-1$ other legislators and to his own district (transfers case).

Consider the second case first, assuming $s > 0$. We can write (24) as:

$$\max_{y} \left\{ \begin{array}{c} W - [y - (1-d)g] \\ -(q-1)\left[\frac{W}{n} + \Psi((1-d)g) - \Psi(y)\right] + \Psi(y) \end{array} \right\} \tag{25}$$

where $\Psi(x) = u(x) + \delta v_L(x)$. Choosing an optimum in problem (25) is equivalent to choosing an optimum in problem: $\max_y \{q\Psi(y) - y\}$. So an optimal choice for the proposer is to propose $y_L(g) = y_2^*$ This case is feasible only if $s = \frac{W}{n} + \Psi(g(1-d)) - \Psi(y_2^*) \geq 0$, that is if and only if $g \geq g_L^2$. In the case in which $g \in [g_1, g_2]$ then we must have $u(y) + \delta v_L(y) = \frac{W}{n} + u[g] + \delta v_L(g)$, so the chosen y is $\widetilde{y}(g)$. It follows that in this range the optimal proposal is $y_L^*(g)$ and $x_L^*(g)$.

To see that $v_L^*(g)$ is the expected utility of a player when the strategies are $y_L^*(g)$, $x_L^*(g)$, is immediate for $g > g_L^2$. For $g \in [0, g_L^2]$, observe that, by a similar argument as in Lemma A.2, for for any γ there is a W_γ such that for $W > W_\gamma$, then $\widetilde{y}(y_1^*) > \gamma$. It follow that when $W > W_{g_L^2}$, $y_L^*(g) = \widetilde{y}(g) > g_L^2$ for any $g \in (g_L^1, g_L^2]$, so in this range the value function is given by (20). Finally it is easy to see that for $W > W_{g_L^2}$, $\widetilde{y}(\widetilde{y}(g)) \geq \widetilde{y}(y_1^*) > g_L^2$, so the value function is $v_L^*(g)$ in $[0, g_L^2]$. We conclude that there is a $\overline{\delta}$, $\overline{W}$ such that for $\delta > \overline{\delta}$ and $W > \overline{W}$ the value function $v_L^*(g)$ and the strategies $y_L^*(g)$ and $x_L^*(g)$ are an equilibrium.

Proof of Proposition 6

Consider an equilibrium $v_L(g), y_L(g), x_L(g)$ when there is no depreciation, that is $d = 0$. In this case the incentive compatibility constraint in state g if policy $y(g)$ is chosen becomes:

$$\begin{aligned} s(y_L(g)) &\geq \frac{W}{n} + \Psi((1-d)g) - \Psi(y_L(g)) \\ &= \frac{W}{n} + \Psi(g) - \Psi(y_L(g)) \end{aligned}$$

In the steady state, this condition becomes: $s(y_{SS}) \geq \frac{W}{n} + \Psi((y_{SS}) - \Psi(y_{SS}) = W/n > 0$. Without loss of generality we can assume that this constraint is satisfied with equality. The proposer's policy must therefore solve

$$\max_y \left\{ \begin{array}{c} W - [y - (1-d)y_{SS}] \\ -(q-1)\left[\frac{W}{n} + \Psi(y_{SS}) - \Psi(y)\right] + \Psi(y) \end{array} \right\} \tag{26}$$

Moreover in a neighborhood of y_{SS}, $s(y_{SS}) > 0$, so $v_L(g) = \frac{1}{1-\delta}\left[\frac{W}{n} + u\left(y_{SS}\right)\right] + \frac{g - y_{SS}}{n}$ around y_{SS}, implying $v_L'(g) = \frac{1}{n}(1-d)$. This fact together with the first order necessary condition of (26) implies $y_{SS} = \left(\frac{Bn}{\frac{n}{q} - \delta(1-d)}\right)^{\frac{1}{1-\alpha}}$. ∎