CURRENCY SPECULATION IN A GAME-THEORETIC MODEL OF INTERNATIONAL RESERVES

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Abstract. This paper is concerned with the ability of speculation to generate a currency crisis. We consider a game-theoretic setting between a unit mass of speculators and a government that holds foreign currency reserves. We analyze conditions under which the speculators may be able to force the government to devaluate the currency. Among these conditions, we analyze the role of heterogeneous beliefs, transaction costs, the level of international reserves, and the widening of currency bands. The explicit consideration of international reserves in our model makes speculators' actions to be strategic substitutes—rather than strategic complements. This is a main analytical departure with respect to related global games of currency speculation not including reserve holdings [e.g., Morris and Shin (1998)]. Our simple framework with international reserves becomes suitable to review some long-standing policy issues.

1. Introduction

1.1. **Currency crises.** Foreign currency reserves allow governments to follow exchange rate policies by intervention in the foreign exchange market. In a currency peg, these international reserves are used to absorb balance of payments deficits and to provide a cushion against other market forces. But currency speculation may also occur: If a mass of trades considers that the stock of international reserves is too low then they may rush to short the currency. The stock of reserves may be depleted—and the government is forced to leave the peg and float the currency. A currency attack can result in a sudden devaluation with severe negative effects on the financial and real sectors; these effects may stem from collateral requirements and other financial frictions, and price rigidities. A currency crisis may then emerge.

There are numerous examples of currency attacks, and there are long-standing issues regarding the optimal amount of transparency, transaction costs, and other regulations, to protect the value of a currency. Some of these issues became apparent in the last three most important currency crises.

Since its inception in 1979, the European exchange rate mechanism (ERM) experienced constant tensions that translated into a substantial number of currency realignments. After a swing of devaluations affecting some major currencies (e.g., the French franc, British pound, and Italian lira) the ERM essentially collapsed in 1993 as it moved to a much broader currency band.

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Then, currency values stabilized. Most models of exchange rate determination are not suited to assess the influence of currency bands on currency speculation. We shall study a simple extension of our model in which it becomes harder to attack the currency under a broader currency band.

The 1994 currency crisis of the Mexican peso brought up some transparency issues. For instance, in several papers Calvo [e.g., see Calvo (1998)] argued that with uncertainty on the fundamentals, economic crises may spread by contagion and herding behavior. The International Monetary Fund (IMF) has set up the Special Data Dissemination Standards (SDDS) for all member countries. Disclosure practises of foreign currency reserves and other macro variables have varied over time and across member countries, but it is often argued that it is desirable to adhere to the highest possible standards of transparency (see *op. cit.*). Global games with heterogeneous beliefs provide a natural setting to deal with uncertainty on the fundamentals (Carlsson and Damme, 1993). Equilibrium effects from changes in heterogeneity of beliefs have already been addressed in Morris and Shin (1998) and several other papers. Our interest is to see how these conclusions from the existing literature may survive in our setting with explicit consideration of international reserves.

In the Asian crisis that started in 1997, the Thai government spent billions of dollars of its foreign currency reserves to defend its baht against speculative attacks. The lack of timely response by the IMF and other institutions such as the US Fed was blamed to be a hitch at the onset of the crisis. In this paper we address how the borrowing of international reserves could be effective to deter currency speculation. Indeed, the effectiveness of these interventions is going to depend on the degree of coordination of the speculators: If there is common knowledge about the fundamentals then we know from some simple models of exchange rate determination that over a certain region external aid from other central banks may be quite inoperative (e.g., see Figure 1 below). These models generate multiple equilibria over a wide range of parameter values and hence they lack predictive power. Transaction costs, taxes on capital gains, and the size of currency depreciation have actually no apparent effect. Hence, an obvious policy prescription of some of these models with multiple equilibria would be to shut down international capital markets. The current Greek debt crisis is another case in point. Massive coordination by European countries has proved to lower risk premia—albeit the reprieve may only last for a few years. The effectiveness of these coordination efforts is not clear since in some cases the whole private sector could short more currency than global entities can ever supply. Hence, the question is whether or not external borrowing of international reserves becomes more effective under asymmetric information to prevent a currency crisis.

1.2. **Self-fulfilling currency attacks.** In a fixed exchange rate, the government bears the risk of a speculative attack as it is willing to exchange the currency at a predetermined price. Although there are associated benefits of fixing the value of a currency, the costs could be prohibitive. For speculators it is of paramount importance that the government wants to resist the attack; it is precisely this foreseen resistance what motivates their actions in the first place. That is, speculators would like to short the domestic currency at the pegged price—and later undo the trading at a lower equilibrium price.

Currency attacks may be self-fulfilling. The mere belief on an imminent attack may induce speculators to flee from the currency. A frenzied rush of capital outflows is then vindicated by a devaluation that confirms the initial beliefs. The point has been neatly discussed in Obstfeld (1996). Obstfeld proposed a game in which two private holders of domestic currency must decide whether to sell or to hold the currency. The government owns reserves to sustain the peg, yet a 50% devaluation sets off if reserves are depleted. Let us assume the following conditions: (a) The government owns 10 units of reserves, (b) The pegged rate is 1-1, (c) Each holder has 6 units of currency, and (d) Each holder bears a cost of 1 upon selling. Then, we get the following pay-off matrix:

	$Hold$	$Sell$
$Hold$	$0,0$	$0,-1$
$Sell$	$-1,0$	$3/2,3/2$

FIGURE 1. The intermediate reserve game in Obstfeld (1996).

Note that none of the two holders can break the peg unilaterally. Hence, an individual holder alone cannot recover the transaction cost. But if both traders sell, then there is a capital gain from the 50% devaluation which outweighs the transaction cost. Therefore, this game has two pure-strategy equilibria; one in which both holders sell and another one in which no holder sells. The players' actions are *strategic complements* because selling is profitable only if the other holder sells. But this game also reflects the idea that the total gains from speculation depend on the amount of reserves released for sale by the government; or put it somewhat differently, in the $(sell, sell)$-equilibrium a trader would be better off if the other holder had only 4 units of currency.

As pointed out by Morris and Shin (1998), a main problem with various models with multiple equilibria is that the immediate reasons behind the actual onset of an attack are left unexplained.

1.3. **Our results.** At this venture it may be helpful to provide a cursory review of our results with those of Figure 1. In this figure there are two equilibria: $(hold, hold)$ and $(sell, sell)$. Many models of currency speculation have emphasized the existence of multiple equilibria, and the need of coordination devices over those equilibria. Coordination may actually come in the form of sound economic policies [cf. Kaminsky et al. (1998)] that direct traders to non-speculative equilibria. We must note, however, that in a corresponding extensive form representation of the game the $(sell, sell)$ equilibrium is sub-game perfect. Hence, this is a focal point of the game: For low transaction costs any sensible perturbation of the game will point towards this equilibrium. This simple observation seems to be absent in the so called second-generation models of currency crises that simply stress multiplicity of equilibria without regard to further properties of these equilibria. But the problem with the simple game of Figure 1 is that $(sell, sell)$ remains the preferred equilibrium outcome regardless of the size of transactions costs and the benefits from speculation. This is a really odd result that comes from perfect information. The only prescriptions of this game is for the government to meet the amount of reserves, or to shut down the economy from international capital flows. It should nevertheless be pointed out that the $(sell, sell)$ equilibrium pinpoints the inherent instability of fixed exchange rate regimes, since traders are motivated by the gains of speculation.

The above game allows each agent to be placed in the position of the other player: There is common knowledge about the fundamentals. Asymmetric information will certainly change the picture. Players with diffused information about the stock of reserves may not be so sure about shorting the currency and bear the transaction cost if there is a certain probability that other players may not move to short the currency. That is, each player has to guess the beliefs of other players, and everyone will be guessing about others' guesses, and so on. This is a complex topic that leads us to the literature of global games.

1.4. **The model of Morris and Shin.** These authors propose a two-stage game played by the government and a continuum of speculators. In the first stage, each speculator decides whether or not to sell short one unit of the domestic currency at a certain cost $t > 0$, whereas in the second stage the government decides whether or not to defend the peg e^*. If the government defends, the price stays at the original level e^* and the speculators who attack earn nothing and pay the cost of short-selling. If the government does not defend and floats the currency, the price falls to $f(\theta)$, where f is increasing in the state θ of the fundamentals, and the speculators exchanging the currency earn the price difference minus the cost: $e^* - f(\theta) - t$. The government's pay-off upon defending is written as,

$$v - c(\alpha, \theta).$$

This value increases with the state θ of the fundamentals and goes down with the mass α of speculators who attack.

As in the example above, Morris and Shin show that under common knowledge there is a wide range of θ in which the game has two equilibria; one in which no speculator attacks and the government maintains the peg, and another one in which all speculators attack and the government accommodates. Morris and Shin show that this multiplicity of equilibria is not robust: Asymmetric information about the state of the fundamentals induces a unique equilibrium.

Conditioning upon the government not defending, the pay-off that a speculator receives from attacking is independent of the mass of speculators who attack. The actions of the speculators are thus strategic complements because the chances of a devaluation increase with the mass of speculators who attack. Therefore, Morris and Shin assume that *all* speculators can sell the domestic currency at the pegged price if the government does not defend. In this model it is not really clear who buys the domestic currency from speculators since the pay-off of each speculator is independent of the mass of speculators who attack. Further, the domestic currency always depreciates if the government does not defend, which implies that there must be an excess supply of the domestic currency at the pegged price—even if no speculator attacks.

In our model below, speculators' actions are *strategic substitutes*: When the peg is abandoned the gains from trading that accrue to each speculator are inversely related to the mass of speculators shorting the currency. As is well known from the global games literature, strategic substitutability may generate additional technical problems for existence and uniqueness of equilibria.

1.5. **Other related work.**

1.5.1. *Bank runs.* Banks play an important role as providers of liquidity insurance. Demand-deposit contracts pool idiosyncratic risks to finance more attractive long-term investments. But the early interruption of long-term investments typically entails a loss. If idiosyncratic liquidity shocks are sufficiently uncommon and independent across the population, banks can improve upon the autarkic allocation.

Hence, banks are also vulnerable to runs that may cause them to vanish. As in the case of currency attacks, the fear of an imminent run may propel massive withdrawals—vindicating the initial beliefs. As is well known, Diamond and Dybvig (1983) provide a model of demand-deposit contracts in which there are two equilibria; an efficient equilibrium in which only investors facing liquidity shocks withdraw early, and a bank run equilibrium in which all investors withdraw and the bank fails.

The model of Diamond and Dybvig is subject to the same criticisms as models of currency attacks with multiple equilibria. Goldstein and Pauzner proposed a model with asymmetric information à la Morris and Shin and show that the multiplicity of equilibria in Diamond and Dybvig (1983) washes out. In the model of Goldstein and Pauzner (2005) the actions of the depositors are not strategic complements everywhere. This is because conditioning on the bank failing, as more depositors withdraw their funds, the lower is their share on the bank's liquidation value. There are, however, $one-sided\ strategic\ complementarities$, because if the bank survives then early withdrawals reduce the pay-offs to the depositors that stay with the bank. Goldstein and Pauzner build their proof of uniqueness upon this property of the pay-offs.

1.5.2. *Bubbles.* Our work is also related to the theory of bubbles in behavioral finance. In Abreu and Brunnermeier (2003) a continuum of speculators must decide at each instant whether or not to sell an overpriced stock. They face a mass of behavioral traders who are responsible for the abnormal price growth. It is assumed that the price of the stock will continue to grow at the bubbly rate as long as the mass of speculators who sell remains below the mass of behavioral traders (who buy); once trading surpasses this threshold, the bubble bursts out immediately.

Although the problem is framed in a richer, dynamic setting, the similarities between their problem and ours are evident. A speculator should sell immediately in the belief of imminent collapse and wait otherwise; moreover, the belief of a sudden crash is self-confirming. However, the technical approach of Abreu and Brunnermeier to this problem was very different. They did not follow the line of the global games literature because, in their own words: "In the richer strategy set of our model strategic complementarity is not satisfied and the global games approach does not apply" (Abreu and Brunnermeier, 2003, page 177).

In their model, speculators have an incentive to preempt others because the pay-off from selling at the date of bursting of the bubble is decreasing in the mass of speculators who sell. The pay-offs exhibit strategic substitutability at the date of bursting because then the amount of speculators who sell outweighs the amount of behavioral traders who buy and the market must clear—behavioral traders play here the role played by the government in a model of currency crisis. Given the similarities between the two problems, we expect that our work will serve as a first step towards the incorporation of global games to the theory of bubbles.

2. The Model

The state of the world is given by the amount R of international reserves that the government has ready to defend the peg. The government operates here as a passive player who buys the domestic currency until it runs out of reserves. The amount of reserves may be interpreted as the government's degree of commitment to the exchange rate defense rather than as an exogenous limit (as in Obstfeld, 1996). That is, R must be thought of as the outcome of a previous, yet not modeled, deliberation by the government; e.g., its ability to draw in funds from international capital markets. This information is usually hard to guess by both the government and the traders as it may depend on unexpected external forces.

Intervention is necessary because the government's desired exchange rate, the pegged rate, differs from the equilibrium rate: There is an excess supply of s_e units of the domestic currency which would cause a devaluation if the government did not intervene.

We consider a simultaneous-move game played by a continuum of speculators of unit mass. Each speculator can short one unit of the domestic currency at a cost $c > 0$. If the mass of speculators who short the currency, s, plus the excess supply, s_e, exceed the government's reserves, $s + s_e > R$, the domestic currency depreciates by a fraction $\delta \in (0, 1)$. Otherwise, the peg survives.

In a devaluation, the total amount of reserves is shared among those who sell (or, equivalently, they all have equal chances to sell before the devaluation). Therefore, the pay-off to a speculator who attacks is $-c$ if the peg survives, and

$$\frac{R}{s + s_e}\,\delta - c$$

if it does not survive. A speculator who does not attack gets zero in any case.

In summary, the gains from speculation stem from selling short the domestic currency at the pegged rate and then purchasing back the same currency at the ensuing equilibrium rate after the depreciation. The overall gains from speculation are the total reserves times the rate of depreciation. The government will try to sustain the peg, but the amount of reserves is limited. We assume that these reserves are equally shared by all traders executing the transaction. Then, the actions of the speculators are strategic substitutes if $s + s_e > R$: Conditioning on the peg being abandoned, the pay-off to a speculator from attacking decreases with the mass of speculators who attack.

2.1. **Perfect information.** Let us begin with the simple case in which the amount of reserves held by the government is common knowledge among speculators. Depending on the size of R we can identify three different types of games (as in Obstfeld, 1996):

- If $R < s_e$ we are in a low reserve game. The government does not have enough reserves to defend the peg even if no speculator attacks. Therefore, a devaluation will come for sure. Assuming that the cost associated to short selling is sufficiently small;

$$c < \frac{R}{1 + s_e}\,\delta, \tag{1}$$

  we can ensure that attacking is a dominant strategy whenever $R < s_e$. In this case, there is a unique equilibrium in which all speculators attack.
- If $s_e \leq R < 1 + s_e$ we are in an intermediate reserve game. The peg will be abandoned depending on the mass of speculators who attack; $s \in [0, 1]$. Attacking is the optimal choice for all speculators who believe that $s + s_e > R$, and not attacking is the optimal choice for those who believe that $s + s_e \leq R$. Moreover, both beliefs are self-confirming because they end up being right if they are held equally across the population of speculators. There are, thus, two equilibria

in pure strategies within this range of reserves: One in which all speculators attack and the peg is abandoned, and another one in which no speculator attacks and the peg survives.

- If $R \geq 1 + s_e$ we are in a high reserve game. Here the government has enough reserves to defend the peg even if all speculators attack. The peg will thus survive, and so attacking becomes a strictly dominated strategy. There is a unique equilibrium in which no speculator attacks.

2.2. **Imperfectly observed reserves.** Let us now assume that speculators do not observe R directly, but hold certain beliefs. Suppose that each speculator holds a uniform prior over the interval $[\underline{R}, \overline{R}]$, where $R = \underline{R}$ fulfills (1) and $\overline{R} > 1 + s_e$. Each speculator receives a conditionally independent signal x which is also distributed uniformly over the interval $[R - \varepsilon, R + \varepsilon]$ (with $\varepsilon > 0$).[1] Under these assumptions, the posterior belief about R of a speculator who receives the signal x is uniform over the interval $[x - \varepsilon, x + \varepsilon]$.

Note that under this specification parameter ε is both a measure of the precision of each signal and the degree of informational asymmetry among speculators since signals are conditionally independent; varying the degree of dependence between the signals would allow us to disentangle both features. More importantly, it is crucial to realize that only the event $[\underline{R}, \overline{R}]$ is common knowledge among speculators, no matter how small ε might be. Note that an event $E \subset [\underline{R}, \overline{R}]$ is nth-order mutual knowledge at $R \in E$ only if $E \supseteq [R - 2n\varepsilon, R + 2n\varepsilon] \cap [\underline{R}, \overline{R}]$, which means that there is always some n for which the last inclusion fails to hold.

As shown below, small departures from common knowledge lead to very different results. Indeed, the two pure strategy equilibria of the intermediate reserve game in the previous subsection require a high degree of coordination among speculators. A speculator must predict the behavior of speculators who receive signals which are an ε away from this speculator, which in turn depends on their beliefs about the behavior of speculators who are an ε away from them, and so on. This is how a small seed of noise infects the whole range of signals.

A strategy for a speculator is now a function from the set of signals to the set of actions. Let $\pi(x)$ denote the proportion of speculators who attack from those who have received the signal x. Adding up across signals, the aggregate short sales under the stock of reserves R we get:

$$s(R, \pi) = \frac{1}{2\varepsilon} \int_{R-\varepsilon}^{R+\varepsilon} \pi(x)\, dx.$$

Given π, the peg is abandoned in the event:

$$A(\pi) := \{R : s(R, \pi) + s_e > R\}.$$

[1] The limits of this interval should obviously be adjusted if $x < \underline{R} + \varepsilon$ or $x > \overline{R} - \varepsilon$.

And the expected pay-off from attacking to a speculator who receives the signal x must be:

$$u(x,\pi) = \frac{1}{2\varepsilon}\int_{A(\pi)\cap[x-\varepsilon,x+\varepsilon]} \frac{R}{s(R,\pi)+s_e}\,\delta\,dR - c. \tag{2}$$

An equilibrium of the game occurs if $\pi(x) = 1$ whenever $u(x,\pi) > 0$, and $\pi(x) = 0$ whenever $u(x,\pi) < 0$.

3. Results

3.1. **Threshold equilibrium.** A threshold equilibrium is an equilibrium in which there is a R^* such that: (a) The peg is abandoned for all $R < R^*$ and (b) The peg survives for all $R \geq R^*$. We will see below that functions π and s have both a particularly simple form in a threshold equilibrium. This will be of great help in order to show that there is a unique threshold equilibrium.

Suppose that R^* characterizes a threshold equilibrium. For every signal $x \leq R^* - \varepsilon$ we have that $u(x,\pi) > 0$ because speculator x believes that the peg will be abandoned with probability one. For every signal $x \geq R^* + \varepsilon$ we have that $u(x,\pi) = -c$ because speculator x believes that the peg will survive with probability one. Moreover, $u(x,\pi)$ is strictly decreasing in x in the interval $(R^* - \varepsilon, R^* + \varepsilon)$ since as we move to the right within this interval, the integral in (2) adds up states in which the pay-off is $-c$ and leaves off states in which it is positive. By the continuity of the integral, there is a unique x^* fulfilling $u(x^*,\pi) = 0$. Therefore, we have shown that, in any threshold equilibrium, π must have the form:[2]

$$I_x(z) = \begin{cases} 1 & \text{if } z \leq x \\ 0 & \text{if } z > x. \end{cases} \tag{3}$$

If $\pi = I_x$, we know that $s(R,\pi)$ is equal to one if $R \leq x - \varepsilon$ and equal to zero if $R > x + \varepsilon$; moreover, it decreases at the rate of $1/2\varepsilon$ between these two points. In short:

$$s(R, I_x) = \begin{cases} 1 & \text{if } R \leq x - \varepsilon \\ \frac{1}{2} - \frac{1}{2\varepsilon}(R - x) & \text{if } x - \varepsilon < R \leq x + \varepsilon \\ 0 & \text{if } R > x + \varepsilon. \end{cases}$$

Consequently, event $A(\pi)$ becomes $A(I_x) = [\underline{R}, \rho(x))$, where

$$\rho(x) = \frac{1}{1+2\varepsilon}[x + (1 + 2s_e)\varepsilon].$$

Every x fulfilling $u(x, I_x) = 0$ characterizes an equilibrium. We show now that there is exactly one such x. Considering $u(x, I_x)$ as a function of x alone, we see that if ε is not too big[3] this function is positive at the lower end of the set of signals and negative at the upper end. As one moves to the right, two opposite effects are in action: (i) More speculators are required to cause

[2] The value of $I_x(x)$ can be chosen arbitrarily from [0,1].

[3] A sufficient condition is $2\varepsilon < \min\{s_e - \underline{R}, \overline{R} - (1 + s_e)\}$.

a devaluation; and (*ii*) The individual benefit from shorting the currency goes down with the mass of speculators attacking the currency. The first effect is not present in models of bank runs (Goldstein and Pauzner, 2005), the second is not present in models with global strategic complementarities (Morris and Shin, 1998). The expression for $u(x, I_x)$ is

$$u(x, I_x) = \delta \left\{ (1+2\varepsilon)\rho(x) \log\left(\frac{1+s_e}{\rho(x)}\right) - [\rho(x) - (x-\varepsilon)] \right\} - c.$$

Taking its second derivative with respect to x,

$$\frac{\partial^2 u(x, I_x)}{\partial x^2} = -\frac{\delta}{(1+2\varepsilon)\rho(x)},$$

we see that $u(x, I_x)$ is strictly concave, which, in turn, implies that there is a unique x^* fulfilling $u(x^*, I_{x^*}) = 0$. We have just proved the following proposition.

Proposition 1. *There is a unique threshold equilibrium. In this equilibrium there is a signal x^* such that: (a) All speculators who receive a signal $x < x^*$ attack, (b) All speculators who receive a signal $x > x^*$ do not attack, (c) The peg is abandoned for all $R < \rho(x^*)$ and (d) The peg survives for all $R \geq \rho(x^*)$.*

Remark 1. Strictly speaking, there is a continuum of threshold equilibria which only differ in a set of measure zero (at x^*).

3.2. **Iterated deletion of dominated strategies.** A remarkable property of the model of Morris and Shin is that the equilibrium strategies are the only ones surviving iterated deletion of strictly dominated strategies. This property is a direct consequence of global strategic complementarities; we now show that it does not hold for more general pay-off structures.

Attacking is a dominant action for all speculators who receive a signal below $s_e - \varepsilon$ because they believe that the peg will be abandoned for sure. This fact has an effect on the behavior of speculators who receive signals above $s_e - \varepsilon$, since they now know that $\pi(x) = 1$ for all $x < s_e - \varepsilon$. That is, some speculators to the right of $s_e - \varepsilon$ may find that because all speculators below $s_e - \varepsilon$ attack then this is a sufficient condition for them to attack as well. More generally, we are interested in the lowest pay-off that speculator x can expect from attacking, provided that $\pi(z) = 1$ for all $z < x$. If such expected pay-off is positive, we know that speculator x will attack.

Proposition 2. *Not attacking does not survive the iterated deletion of strictly dominated strategies for all signals below x^*.*

Proof. The proof proceeds in three steps:
Step 1: We first show that the expected pay-off for speculator x, provided that $\pi(z) = 1$ for all $z < x$, is bounded below by the one derived from some threshold function I_{x_0}. If $\pi(z) = 1$ for all $z < x$, we know that $s(R, \pi)$ is weakly decreasing in the interval $(x - \varepsilon, x + \varepsilon)$. Since we are looking for the

minimum expected pay-off, it has to be the case that $s(R_0, \pi) + s_e = R_0$ for some R_0 in $(x-\varepsilon, x+\varepsilon)$. Now, choose the x_0 that makes $s(R_0, I_{x_0})+s_e = R_0$. We must have that $u(x, I_{x_0}) \leq u(x, \pi)$ since I_{x_0} lies above π on $(x-\varepsilon, R_0)$.
Step 2: The next step is to show that $u(x, I_x) \leq u(x, I_{x_0})$. The first integrates over the interval $[x-\varepsilon, \rho(x)]$, whereas the second integrates over $[x-\varepsilon, \rho(x_0)]$, which is larger. What we will do is to compare, moving to the left from the right limit of each interval, the pay-offs at each state. The pay-offs for I_x can be written as:

$$\frac{R}{\rho(x) + \frac{1}{2\varepsilon}[\rho(x) - R]}\,\delta - c. \tag{4}$$

Pairing each state in $[x-\varepsilon, \rho(x)]$ with the corresponding state in $[x-\varepsilon, \rho(x_0)]$ (recall that we are moving to the left from the right end of each interval), the pay-offs for I_{x_0} are:

$$\frac{R + \rho(x_0) - \rho(x)}{\rho(x_0) + \frac{1}{2\varepsilon}[\rho(x) - R]}\,\delta - c \tag{5}$$

if $R > x_0 - \varepsilon + \rho(x) - \rho(x_0)$, and

$$\frac{R + \rho(x_0) - \rho(x)}{1 + s_e}\,\delta - c \tag{6}$$

otherwise. Subtracting (4) from (5) gives us

$$\frac{2\varepsilon(1+2\varepsilon)[\rho(x) - R][\rho(x_0) - \rho(x)]}{[(1+2\varepsilon)\rho(x) - R][\rho(x) + 2\varepsilon\rho(x_0) - R]}\,\delta > 0.$$

Since (6) is greater than (5), the proof is complete.
Step 3: We have just shown that the least that speculator x can get from attacking, provided that $\pi(z) = 1$ for all $z < x$, is $u(x, I_x)$. Since $u(x, I_x) > 0$ for all $x < x^*$, this implies our result. □

In the previous proof we have shown that, if $\pi(z) = 1$ for all $z < x$, the case in which speculator x gets the least from attacking is when $\pi(z) = 0$ for all $z > x$. If we had global strategic complementarities, the converse would be true when $\pi(z) = 0$ for all $z > x$. Then, it would be immediate to see that the threshold equilibrium is the unique equilibrium. This is not our case, however. If we look now for the most that speculator x can get from attacking if $\pi(z) = 0$ for all $x > z$, we see that the right answer is not given by the threshold function I_x. The reason is that, conditioning on the peg being abandoned, the more speculators who attack, the lower is the pay-off that they get from attacking.

Proposition 3. *There is a minimal $x^\diamond$, with $x^\diamond > x^*$, such that attacking does not survive the iterated deletion of strictly dominated strategies for all signals above $x^\diamond$.*

Proof. The proof proceeds in three steps:
Step 1: We first construct an upper bound for the expected pay-off of speculator x whenever $\pi(z) = 0$ for all $z > x$. If $\pi(z) = 0$ for all $z > x$, then

$s(R,\pi)$ is weakly decreasing in $(x-\varepsilon, x+\varepsilon)$. Since we are looking for the maximum expected pay-off, it has to be the case that $s(R_0,\pi)+s_e = R_0$ for some R_0 in $(x-\varepsilon, x+\varepsilon)$, and also that $s(R,\pi)$ is constant in $(x-\varepsilon, R_0)$. Conditioning on $s(R,\pi)+s_e = R_0$ in $(x-\varepsilon, R_0)$, the expected pay-off at x is

$$\frac{1}{2\varepsilon}\int_{x-\varepsilon}^{R_0} \frac{R}{R_0}\,\delta\,dR - c, \tag{7}$$

which is strictly increasing in R_0. Then, the maximum expected pay-off is attained at the maximum R_0, namely, $\rho(x)$ (recall that $\pi(z)=0$ for all $z>x$). Hence, any π fulfilling $\pi(z)=0$ for $z\in(x-2\varepsilon, \rho(x)-\varepsilon)$, and $\pi(z)=1$ for $z\in(\rho(x)-\varepsilon, x)$, attains the maximum expected pay-off.
Step 2: Substituting $\rho(x)$ for R_0 in (7), we see that the maximum expected pay-off is positive if x is sufficiently small and negative if it is sufficiently large. We can compute its second derivative with respect to x,

$$\frac{-2(1+s_e)^2\varepsilon(1+2\varepsilon)}{[x+(1+2s_e)\varepsilon]^3}\,\delta,$$

and see that it is strictly concave. This means that there is exactly one $x^\diamond$ at which the maximum expected payoff is zero, that it is positive for $x<x^\diamond$, and that it becomes negative for $x>x^\diamond$. Therefore, attacking does not survive the iterated deletion of strictly dominated strategies if, and only if, $x>x^\diamond$.
Step 3: That $x^\diamond > x^*$ comes directly from the fact that the maximum expected pay-off constructed in Step 1 for each x is always larger than $u(x, I_x)$. □

3.3. **Uniqueness of the threshold equilibrium.** Our main result concerns the uniqueness of the threshold equilibrium.

Proposition 4. *The equilibrium in Proposition 1 is the only equilibrium.*

Proof. The proof is by contradiction. We suppose that there exists an equilibrium which is not a threshold equilibrium and then show that this is impossible. Suppose that π characterizes an equilibrium. Define $\overline{x}$ as

$$\overline{x} := \sup\{x|\pi(x)>0\}$$

and $\underline{x}$ as

$$\underline{x} := \begin{cases} \overline{x} & \text{if } \pi(x)=1 \text{ for all } x<\overline{x} \\ \sup\{x<\overline{x}|\pi(x)<1\} & \text{otherwise.} \end{cases}$$

Note that, if the equilibrium is not a threshold equilibrium, we must have that $\underline{x}<\overline{x}$. Also, by continuity, we must have that $u(\underline{x},\pi)=u(\overline{x},\pi)=0$. We presently show that this is impossible.
Step 1: The peg survives for all $R>\rho(\overline{x})$. We know that $s(R,\pi)$ must be weakly decreasing in $(\overline{x}-\varepsilon, \overline{x}+\varepsilon)$. Since $u(\overline{x},\pi)=0$, there must be a R_0 in $(\overline{x}-\varepsilon, \overline{x}+\varepsilon)$ at which $s(R_0,\pi)+s_e=R_0$. The expected pay-off $u(x,\pi)$ is strictly positive within the interval $[R_0-\varepsilon, \overline{x})$ since, as we move to the

left from its right end, we are excluding states in which the peg survives (and adding some in which it is abandoned). Therefore, $\pi(x) = 1$ for all x in $[R_0 - \varepsilon, \overline{x})$ which, in turn, implies that $R_0 = \rho(\overline{x})$. Furthermore, for the same reason, $s(R, \pi)$ must start decreasing at the fastest rate before $\rho(\overline{x})$.

Step 2: If $\underline{x} < \overline{x} - 2\varepsilon$, then $\underline{x} = \overline{x}$. If $\underline{x} < \overline{x} - 2\varepsilon$, we have that $u(\overline{x}, \pi) = u(\overline{x}, I_{\overline{x}})$, which is zero only if $\overline{x} = x^*$ (Proposition 1). But, then, what we have is a threshold equilibrium.

Step 3: If $\underline{x} \geq \overline{x} - 2\varepsilon$, then $u(\underline{x}, \pi) > u(\overline{x}, \pi)$. In order to compute $u(\underline{x}, \pi)$ and $u(\overline{x}, \pi)$ we must integrate over the intervals $[\underline{x}-\varepsilon, \underline{x}+\varepsilon]$ and $[\overline{x}-\varepsilon, \overline{x}+\varepsilon]$. Both intervals overlap and, therefore, we only need to compare the pay-off accumulated at both sides of the common subinterval.

Step 3.1: To accomplish this task, we first find an lower bound to the pay-off accumulated on the left-hand side subinterval. From (2) we know that such a lower bound can be obtained by (a) reducing the size of the set $A(\pi)$, and (b) substituting the denominator $s(R, \pi) + s_e$ by a larger quantity at each state R. Let $s_0 = s(\overline{x} - \varepsilon, \pi)$ and let $R_0 = \max\{R < \overline{x} - \varepsilon | s(R, \pi) + s_e = R\}$. First, we reduce the set $A(\pi)$ by assuming that the peg survives for all states in $[\underline{x} - \varepsilon, R_0)$. Second, we find an upper bound for the denominator $s(R, \pi) + s_e$ within $[R_0, \underline{x} + \varepsilon]$. We know that $s(R, \pi)$ is weakly increasing in $(\underline{x} - \varepsilon, \overline{x} - \varepsilon)$, which means that $s(R, \pi) \leq s_0$ within this interval. The pay-off accumulated in $[\underline{x} - \varepsilon, \overline{x} - \varepsilon]$ is bounded below by

$$\frac{1}{2\varepsilon}\left[\int_{R_0}^{R_1} \frac{R}{R_0 + \frac{1}{2\varepsilon}(R - R_0)}\,\delta\, dR + \int_{R_1}^{\overline{x}-\varepsilon} \frac{R}{s_0}\,\delta\, dR - c\right], \tag{8}$$

where $R_1 = \min\{R_0 + 2\varepsilon(s_0 - R_0), \overline{x} - \varepsilon\}$. The denominator inside the first integral corresponds to an upward sloping line, starting at the point R_0 on the 45-degree line and growing at the maximum feasible rate until the upper limit s_0 is reached; in the second integral the curve becomes flat. Thus, we have constructed the largest admissible denominators given the constraints: $s(R_0, \pi) + s_e = R_0$ and $s(R, \pi) \leq s_0$ in $[\underline{x} - \varepsilon, \overline{x} - \varepsilon]$.

We now show that (8) is decreasing in R_0. A sufficient condition for (8) to be decreasing in R_0 is that the first integral is so when $R_1 = R_0 + 2\varepsilon(s_0 - R_0)$. The derivative of the first summand in (8) in this case is negative if

$$\log\left(\frac{s_0}{R_0}\right) \leq \frac{1}{1 - 2\varepsilon}.$$

Since $s_0 < 1 + s_e$ and $R_0 > \rho(x^*)$, a sufficient condition for this to be true is that

$$\rho(x^*) \geq \frac{1 + s_e}{e}, \tag{9}$$

where $e = 2.7182\ldots$ On the other hand, we know that the derivative of $u(x, I_x)$ with respect to x,

$$\frac{\partial u(x, I_x)}{\partial x} = \delta\left[\log\left(\frac{1 + s_e}{\rho(x)}\right) - \frac{1}{1 + 2\varepsilon}\right],$$

has to be negative at $x = x^*$. Since this implies (9), we have shown that (8) decreases with R_0. Therefore, substituting R_0 by a larger number in (8) gives us a lower bound for the pay-off accumulated on the left-hand side subinterval.

Step 3.2: Let $\varrho(x)$ be the point at which $s(R, 1 - I_x) + s_e = R$, i.e.:

$$\varrho(x) = \frac{1}{1-2\varepsilon}[x - (1+2s_e)\varepsilon].$$

We know that $\varrho(\underline{x}) \geq R_0$ because $s_0 \geq \rho(\overline{x})$. Then, the pay-off accumulated on the left-hand side is bounded below by

$$\frac{1}{2\varepsilon}\left[\int_{\varrho(\underline{x})}^{\overline{x}-\varepsilon} \frac{\varrho(\underline{x})}{\frac{1}{2} + \frac{1}{2\varepsilon}(R - \underline{x}) + s_e}\,\delta\,dR - c\right]. \tag{10}$$

On the other hand, the pay-off accumulated on the right-hand side is bounded above by

$$\frac{1}{2\varepsilon}\left[\int_{\underline{x}+\varepsilon}^{\rho(\overline{x})} \frac{\rho(\overline{x})}{\frac{1}{2} - \frac{1}{2\varepsilon}(R - \overline{x}) + s_e}\,\delta\,dR - c\right]. \tag{11}$$

We have that (10) is larger than (11) if

$$\varrho(\underline{x}) \log\left(\frac{\frac{\overline{x}-\underline{x}}{2\varepsilon} + s_e}{\varrho(\underline{x})}\right) \geq \rho(\overline{x}) \log\left(\frac{\frac{\overline{x}-\underline{x}}{2\varepsilon} + s_e}{\rho(\overline{x})}\right). \tag{12}$$

But the function

$$x \log\left(\frac{a}{x}\right)$$

is decreasing if

$$x \geq \frac{a}{e}.$$

In our case

$$\varrho(\underline{x}) \geq \frac{\frac{\overline{x}-\underline{x}}{2\varepsilon} + s_e}{e},$$

and so

$$\varrho(\underline{x}) \geq \frac{1+s_e}{e}$$

suffices for (12) to be true. Combining $\varrho(\underline{x}) > \rho(x^*)$ and (9) we have that $u(\underline{x}, \pi) > u(\overline{x}, \pi)$. □

From these results we can now show:

Proposition 5. *In the limit, as ε goes to zero, x^* is obtained as the solution of the following equation:*

$$x \log\left(\frac{1+s_e}{x}\right) = \frac{c}{\delta}.$$

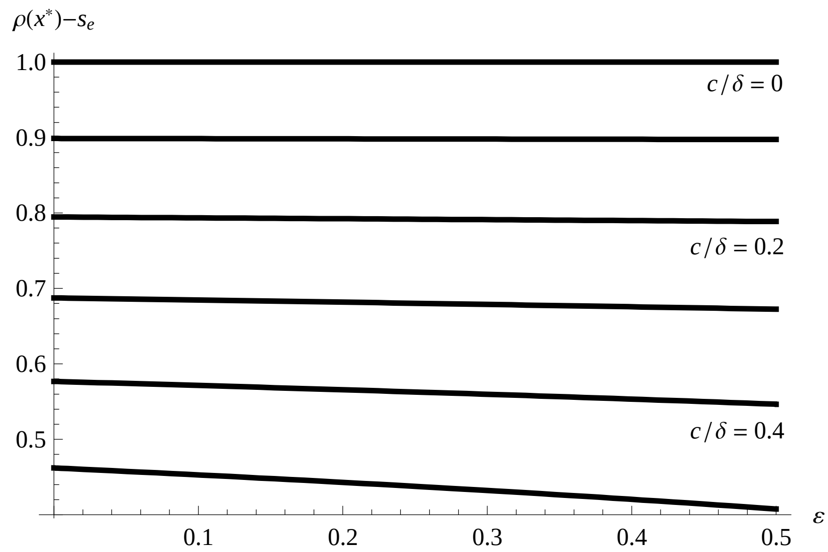

FIGURE 2. Various plots of $\rho(x^*) - s_e$ as a function of ε for $s_e = 3$ and $c/\delta < 1/2$.

3.4. **Comparative statics.** The previous proposition becomes fundamental to perform comparative static exercises. As suggested in the introduction, most important results refer to variations in the size ε of the noise term, which can be interpreted as a measure of the lack of transparency of the monetary policy. In particular, we shall be interested in the behavior of the quantity $\rho(x^*) - s_e$, which gives the proportion of states in which the peg is abandoned, as the noise ε becomes small. Our results show that, in general, this quantity gets bigger as the size of the noise decreases. That is, an increase in the transparency of the monetary policy tends to enlarge the set of states in which currency attacks succeed. In any case, this effect is of quantitative little importance for the majority of parameter values (in fact, it is almost zero for the most interesting cases).

Figure 2 presents several plots of $\rho(x^*) - s_e$ as a function of ε for different values of the ratio c/δ and $s_e = 3$. We have chosen the maximum value of ε to be 0.5 because this is the value that makes speculator $s_e + 1/2$ believe that all states in the intermediate reserve region are possible. The values for the ratio c/δ are all smaller that 1/2 in order to fulfill our parameter's restrictions ((1) and $\underline{R} < s_e - 2\varepsilon$).

Our results are in sharp contrast with the findings of Morris and Shin, who write on the same issue:[4]

> Above all, our analysis suggests an important role for public announcements by the monetary authorities, and more generally, the transparency of the conduct of monetary policy

[4]Morris and Shin, 1998, page 595.

> and its dissemination to the public. If it is the case that the onset of currency crises may be precipitated by higher-order beliefs, even though participants believe that the fundamentals are sound, then the policy instruments which will stabilize the market are those which aim to restore transparency to the situation, in an attempt to restore common knowledge of the fundamentals.

Finally, we see that for fixed ε, both a lower cost and a higher depreciation rate imply a larger range of states in which the peg is abandoned. But he general picture does not change for variations in s_e; in this case the game becomes roughly an invariant translation: An increase in s_e must come forth with the same increase in R.

4. Extensions

4.1. **Determinants of exchange rates.** So far we have considered that at the prevailing exchange rate e there is an excess supply s_e. Besides the own exchange rate, e, this excess supply could actually be a function of some fundamental value, θ. Variable θ can be conformed by various internal and external market forces. A change in θ may trigger a move away from the exchange rate fundamental value e^* as reflected by a variation in the excess supply s_e. In light of the preceding analysis, there is no loss of generality to assume that both θ and s_e are common knowledge, since the case of heterogeneous beliefs on θ can be easily accommodated under our framework.

We may even suppose that the fundamental value of the currency e^* depends on the amount of speculation, s, and the amount of reserves, R. The idea is that extensive speculation may lead to an undershooting of the currency, and an increased amount of reserves may instill confidence in the economy. Again, these considerations can easily be integrated into the above framework.

4.2. **Currency bands.** In a currency band, the exchange rate is allowed to fluctuate within certain margins. In our simple model, the risk of speculation becomes smaller the longer the currency is away from the floor, say $\underline{e}$. Let us assume that the value of the currency is at some point $e_0 > \underline{e}$. Then, in a speculative attack the exchange rate would have to move first from e_0 to $\underline{e}$. This move puts downward pressure on both the excess supply, s_e, and the gains from speculation, δ, as the government would trade reserves at the lower exchange rate, $\underline{e}$. Likewise, other managed float schemes may seem appropriate to minimize the benefits of speculation and boost currency stability.

4.3. **Large trader, and sequential games.** These important extensions are considered in Corsetti et al. (2004). If the actions of the large trader are not observed, it does not seem to be so obvious what would be their effects on the behavior of the small traders. Nevertheless, Proposition 5 above suggests that for very small transaction costs, speculators will attack the currency whenever $s_e + 1 > R$. Hence, in those cases it seems that the existence of a large trader will not change the results. But as Figure 2 shows the degree of asymmetric information matters when transaction costs are larger.

5. Concluding Remarks

In this paper we study a game-theoretic model of currency speculation with asymmetric information. There is a continuum of speculators that can short the currency and a government that holds a stock of international reserves to sustain a currency peg. Unlike most global games, speculators' actions are global substitutes—rather than global complements. We nevertheless establish existence of a unique threshold equilibrium, and this the only equilibrium of the game.

In various numerical exercises we observe that variations in the degree of asymmetric information have mild effects in equilibrium outcomes. The strategic substitutability condition embedded in the game seems to lead to more active speculation behavior as the degree of asymmetric information vanishes. These asymmetric information effects are more pronounced under large transaction costs (or small gains from currency depreciation) where traders with diffused priors become less forthcoming about the benefits of the currency attack.

Under asymmetric information, both transactions costs and capital gains influence the required amount of reserves to deter a currency attack. But as seen by Proposition 5 above, for small transaction costs, as the noise goes to zero, such required amount of reserves has to exceed the borrowing capacity of speculators. Furthermore, our numerical experiments suggests that this required quantity remains invariant to changes in the degree of asymmetric information. Therefore, our results point at the inherent instability of fixed exchange rate regimes. A shock to the economy that generates an excess supply of currency may need to be accommodated by a corresponding increase in international reserves: If international reserves cannot be spared then a speculative attack would be the likely outcome. Therefore, policy coordination among central banks and other global institutions would be the most effective tool to avoid currency attacks by enlarging the borrowing capacities of the economy. Of course, these attacks will be more intensive the further away is the peg from the fundamental value; likewise, the so called "sand-in-the-wheels" in the way of taxes or other transaction costs will have mitigating effects on currency speculation.

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