Divergence in the Spatial Stochastic Model of Voting

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Abstract

This article uses the notion of a "Local Nash Equilibrium" (LNE) to model a vote maximizing political game that incorporates valence (the electorally perceived quality of the political leaders.) Formal stochastic voting models without valence typically conclude that all political agents (parties or candidates) will converge towards the electoral mean (the origin of the policy space.) The theorem presented here obtains the necessary and sufficient conditions for the validity of the "mean voter theorem". The conditions involve the party valences, and the electoral and stochastic variances. Since a pure strategy Nash Equilibrium (PNE), if it exists, must be a LNE , the failure of the necessary condition for an LNE at the origin also implies that PNE cannot be at the origin. The Theorem shows , when the necessary condition fails, that low valence agents will, in equilibrium, adopt positions far from the electoral origin. The theoretical conclusions appear to be borne out by empirical evidence from Israel for the elections of 1988-1996.

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1 Introduction

The electoral models based on the early work of Hotelling (1929) and Downs (1957) essentially supposed that the motivation of parties is to win a majority of the votes or seats. The predictions of the Downsian, vote maximizing, models vary, but they tend to suggest that parties converge to an electoral center. The simplest model assumes two parties and a one-dimensional policy space, X. If voters "deterministically" choose the party with the nearest policy position, then there will exist a Condorcet point, unbeaten under majority rule vote, at the median of the electoral distribution. In higher dimensions, such two party pure strategy Nash equilibria generally do not exist and instability may occur (Plott, 1967;McKelvey, 1976, 1979; Schofield, 1978, 1983,1985; Cohen and Matthews.1980; McKelvey and Schofield 1986, 1987; Saari 1997; Austen-Smith and Banks,1999). That is to say, whatever position, z_i, is picked by party i, there always exists a point z_j which will give party j a majority over party i.

On the other hand,when X has two or more dimensions, it is known that a Condorcet point exists when electoral preferences are represented by a spherically symmetric distribution of electorally preferred points. Even when the distribution is not spherically symmetric, an equilibrium can be guaranteed as long as the decision rule requires a sufficiently large majority (Schofield, 1984; Strnad, 1985; Caplin and Nalebuff, 1988) or when the electoral distribution has a concavity property (Caplin and Nalebuff, 1991) .Although a pure strategy Nash equilibrium generically fails to exist in competition between two agents under majority rule in high enough dimension, there will exist mixed strategy Nash equilibria (Kramer 1978) whose support lies within a subset of the policy space known as the *uncovered set* (McKelvey 1986; Cox 1987; Banks Duggan and LeBreton, 2002). All these "attractors" of the political process are centrally located with respect to the distribution of voters' ideal points. Such a conclusion seems at odds with empirical evidence that parties do not exhibit such strong convergence to the electoral center (Merrill and Grofman, 1999; Adams, 2001).

Extension of the deterministic model to the "multiparty" situation with more than two parties has also shown failure of existence of pure strategy equilibria (Eaton and Lipsey, 1975). Partly as a result of these theoretical difficulties with the deterministic model, and also because of the need to develop empirical models of voter choice (Poole and Rosenthal,1984), attention has focused on "stochastic" vote models. A formal basis for such models is provided by the notion of "Quantal response equilibria" (McKelvey and

Palfrey, 1995,). In such models, behavior of each voter is modeled by a vector of choice probabilities (Hinich 1977; Enelow and Hinich 1984,1989; Coughlin 1992; Lin, Enelow and Dorussen,1999; Banks and Duggan,2004). A standard result in this class of models is that all parties converge to the electoral origin when the parties are motivated to maximize vote share (McKelvey and Patty,2004) or plurality in the two party case (Banks and Duggan,2005).

However, this formal convergence result need not hold if there is an asymmetry in the electoral perception of the "quality" of party leaders (Stokes, 1992). The average weight given to the perceived quality of the leader of the j^{th} party is called the party's *valence.* In empirical models this valence is independent of the party's position, and adds to the statistical significance of the model . In general, valence reflects the overall degree to which the party is perceived to have shown itself able to govern effectively in the past, or is likely to be able to govern well in the future.(Penn,2003) The early empirical model of Poole and Rosenthal (1984) on US Presidential elections included these valence terms and noted that there was no evidence of candidate convergence . Formal models of elections incorporating valence have been developed recently (Ansolabehere and Snyder,2000; Aragones and Palfrey, 2002, Groseclose,2001), but results to date have been obtained only for the two party case. Extension to the multiparty case is of interest because of recent empirical models of voting in the Netherlands and Germany (Schofield,Martin, Quinn and Whitford,1998; Quinn and Martin, 2002; Schofield and Sened, 2005a), Britain (Alvarez and Nagler, 1998; Alvarez, Nagler and Bowler, 2000; Quinn, Martin and Whitford, 1999; Schofield, 2004, 2005a), Israel (Schofield, Sened and Nixon, 1998; Schofield and Sened, 2005b) and Italy (Giannetti and Sened, 2004). All these models have suggested that divergence is generic. Most of these empirical models have been based on the "multinomial logit"assumption that the stochastic errors had a "Type I extreme value distribution" (Dow and Endersby , 2004)

This paper will present a "classification theorem" for the formal vote model of voter choice based on the same stochastic distribution assumption..The "policy space" is assumed to be of dimension w,and there is an arbitrary number,p, of parties. The party leaders exhibit differing valence.

A " convergence coefficient", incorporating all the parameters of the model will be defined. Instead of using the notion of a Nash equilibrium, the Theorem is given in terms of a "local Nash equilibrium" It is shown that there are necessary and sufficient conditions for the existence of a "pure strategy vote maximizing local Nash equilibrium"(LNE) at the mean of the voter distribution. When the necessary condition fails, then parties , in equilibrium, will adopt divergent positions. In general, parties whose leaders have the lowest valence will take up positions furthest from the electoral mean. Moreover, because a pure strategy Nash equilibrium (PNE) must be a local equilibrium , the failure of existence of the LNE at the electoral mean implies non existence of such a centrist PNE. The failure of the necessary condition for convergence has a simple interpretation; if the variance of the electoral distribution is sufficiently large in contrast to the expected vote share of the lowest valence party at the electoral mean, then this party has an incentive to move away from the origin towards the electoral periphery. Other low valence parties will follow suit, and the local equilibrium will result will parties distributed along the principal electoral axis.

An empirical study of voter behavior for Israel for the elections of 1988, 1992 and 1996 is used to show that the necessary condition for party convergence failed for these elections. The equilibrium positions obtained from the formal result, under vote maximization, are in, general, comparable with, but not identical to, the estimated positions: the two highest valence parties were symmetrically located on either side of the electoral origin, while the lowest valence parties were located far from the origin Only one of the lowest valence parties were located off the principal electoral axis. It is suggested that the discrepancy between the formal and empirical model can be accommodated by considering the strategic calculation of the party with respect to post- election coalition negotiation.

2 The Formal Model of Elections

The data of the spatial model is a distribution, $\{x_i \in X\}_{i \in N}$, of voter ideal points for the members of the electorate, N, of size n. As usual we can assume that X is a compact convex subset of Euclidean space, $\mathbb{R}^w$, with w finite.

Each of the parties,or agents, in the set $P = \{1, \ldots, j, \ldots, p\}$ chooses a policy, $z_j \in X$, to declare to the electorate prior to the election. Let $\mathbf{z} = (z_1, \ldots, z_p) \in X^p$ be a typical vector of agent policy positions. Given $\mathbf{z}$, each voter, i, is described by a vector $\mathbf{u}_i(x_i, \mathbf{z}) = (u_{i1}(x_i, z_1), \ldots, u_{ip}(x_i, z_p))$, where

$$\begin{aligned} u_{ij}(x_i, z_j) &= u^*_{ij}(x_i, z_j) + \epsilon_j \\ \text{and } u^*_{ij}(x_i, z_j) &= \lambda_j - \beta ||x_i - z_j||^2 \end{aligned}$$

Here,$u^*_{ij}(x_i, z_j)$ is the "observable" utility for i, associated with party j. The term λ_j is the valence of agent j, which we assume is exogenously determined. The term β is a positive constant and $||\cdot||$ is the usual Euclidean norm on X. The terms $\{\epsilon_j\}$ are the stochastic errors,whose cumulative distibution is denoted by Ψ. It is natural to assume that the valence of party j, as perceived by voter i is $\lambda_{ij} = \lambda_j + \epsilon_j$.

Because of the stochastic assumption, voter behavior is modeled by a probability vector. The probability that a voter i chooses party j is

$$\rho_{ij}(\mathbf{z}) = \Pr[[u_{ij}(x_i, z_j) > u_{il}(x_i, z_l)], \text{ for all } l \neq j].$$

Here Pr stands for the probability operator associated with Ψ. The expected vote share of agent j is

$$V_j(\mathbf{z}) = \frac{1}{n}\sum_{i \in N} \rho_{ij}(\mathbf{z}).$$

In the vote model it is assumed that each agent j chooses z_j to maximize V_j, conditional on $\mathbf{z}_{-j} = (z_1, ..z_{j-1}, z_{j+1}.., z_p)$.

The theorem presented here assumes that the exogeneous valences are given by the vector $\boldsymbol{\lambda} = (\lambda_p, \lambda_{p-1}, \ldots, \lambda_2, \lambda_1)$ and the valances are ranked $\lambda_p \geq \lambda_{p-1} \geq \cdots \geq \lambda_2 \geq \lambda_1$. The model is denoted $M(\boldsymbol{\lambda},\beta; \Psi)$.

In this model it is natural to regard λ_j as the "average" weight given by a member of the electorate to the perceived competence or quality of agent j. The "weight" will in fact vary throughout the electorate, in a way which is described by the stochastic distribution. In these models, the C^2-differentiability of the cumulative distribution, Ψ, is usually assumed, so that the individual probability functions $\{\rho_{ij}\}$ will be C^2-differentiable in the strategies $\{z_j\}$. Thus, the vote share functions will also be C^2-differentiable., and Hessians can be calculated.

Let $x^* = (1/n)\Sigma_i x_i$. Then the mean voter theorem for the stochastic model asserts that the "joint mean vector" $\mathbf{z}^*_0 = (x^*, \ldots, x^*)$ is a "pure strategy Nash equilibrium". Lin, Enelow and Dorussen (1999) used C^2-differentiability of the expected vote share functions, in the situation with zero valence, to show that the validity of the theorem depended on the concavity of the vote share functions. They asserted that a sufficient condition for this was that stochastic variance was "sufficiently large". Because concavity cannot, in general, be assured, I shall utilize a weaker equilibrium concept, that of "Local Strict Nash Equilibrium"(LSNE). A strategy vector $\mathbf{z}^*$ is a LSNE if , for each j, z^*_j is a critical point of the vote function

$V_j(z_1^*, ..z_{j-1}^*, z_j., z_{j+1}^*, ..z_p^*)$ and the eigenvalues of the Hessian of this function (with respect to z_j),are negative. Definition 1 gives the various technical definitions used here.

Definition 1. *Equilibrium concepts.*

(i) A strategy vector $\mathbf{z}^*=(z_1^*, ...z_{j-1}^*, z_j^*, z_{j+1}^*..z_p^*)$ is a "local strict Nash equilibrium" (LSNE) iff, for each agent j,there exists a neighborhood X_j of z_j in X such that

$$V_j(z_1^*, ...z_{j-1}^*, z_j^*, z_{j+1}^*..z_p^*) > V_j(z_1^*, ..., z_j..z_p^*) \text{ for all } z_j \in X_j - \{z_j^*\}$$

(ii) A strategy vector $\mathbf{z}^*=(z_1^*, ...z_{j-1}^*, z_j^*, z_{j+1}^*..z_p^*)$ is a "local (weak) Nash equilibrium" (LNE) iff, for each agent j,there exists a neighborhood X_j of z_j in X such that

$$V_j(z_1^*, ...z_{j-1}^*, z_j^*, z_{j+1}^*..z_p^*) \geq V_j(z_1^*, ..., z_j..z_p^*) \text{ for all } z_j \in X_j$$

(iii) A strategy vector $\mathbf{z}^*=(z_1^*, ...z_{j-1}^*, z_j^*, z_{j+1}^*..z_p^*)$ is a "(weak) pure strategy Nash equilibrium" (PNE) iff, for each agent j,

$$V_j(z_1^*, ...z_{j-1}^*, z_j^*, z_{j+1}^*..z_p^*) \geq V_j(z_1^*, ..., z_j..z_p^*) \text{ for all } z_j \in X$$

(iv) A strategy vector $\mathbf{z}^*=(z_1^*, ...z_{j-1}^*, z_j^*, z_{j+1}^*..z_p^*)$ is a "strict pure strategy Nash equilibrium" (PSNE) iff, for each agent j,

$$V_j(z_1^*, ...z_{j-1}^*, z_j^*, z_{j+1}^*..z_p^*) > V_j(z_1^*, ..., z_j..z_p^*) \text{ for all } z_j \in X - \{z_j^*\}$$

(v) The strategy z_j^* is termed a "local strict best response", a "local weak best response", a "global weak best response", a "global strict best response",respectively to $\mathbf{z}_{-j}^*=(z_1^*, ...z_{j-1}^*, z_{j+1}^*..z_p^*)$, .depending on which condition (i) to (iv) is satisfied.

Obviously if $\mathbf{z}^*$ is an LSNE or a PNE it must be an LNE, while if it is a PSNE then it must be an LSNE. We use the notion of LSNE to avoid problems with the degenerate situation when there is a zero eigenvalue to the Hessian. The weaker requirement of LNE allows us to obtain a necessary condition for $\mathbf{z}_0^* = (x^*, \ldots, x^*)$ to be a LNE and thus a PNE, without having to invoke concavity. The theorem below also gives a sufficient condition for the joint mean vector $\mathbf{z}_0^*$ to be an LSNE. A corollary of the theorem shows,in situations where the valences differ, that the necessary condition is likely to fail. In dimension w, the theorem can be used to show that, for $\mathbf{z}_0^*$ to be an LSNE, the necessary condition is that a "convergence coefficient ", defined in terms of the parameters of the model, must be strictly bounded above by w. Similarly, for $\mathbf{z}_0^*$ to be a LNE, then the convergence coefficient must be weakly bounded above by w. When this condition fails, then the joint mean vector $\mathbf{z}_0^*$ cannot be a LNE and therefore cannot be a PNE. Of course, even if the sufficient condition is satisfied, and $\mathbf{z}_0^* = (x^*, \dots, x^*)$ is an LSNE, it need not be a PNE.

To parallel the empirical applications we assume a Type I extreme value distribution (Train, 2003) for the errors.

The cumulative distribution,Ψ, takes the closed form

$$\Psi(h) = \exp\left[-\exp[-h]\right],$$

with variance is $\frac{1}{6}\pi^2$.It readily follows (Train, 2003,p.79) for the choice model given above that, for each i,

$$\rho_{ij}(\mathbf{z}) = \frac{\exp[u_{ij}^*(x_i, z_j)]}{\sum\limits_{k=1}^{p} \exp u_{ik}^*(x_i, z_k)}.$$

This implies that the model satisfies the independence of irrelevant alternative property (IIA) namely that for each voter i, and for each pair,j, k, the ratio

$$\frac{\rho_{ij}(\mathbf{z})}{\rho_{ik}(\mathbf{z})}$$

is independent of a third candidate l.

To state the theorem ,we first transform coordinates so that in the new coordinates, $x^* = 0$. I shall refer to $\mathbf{z}_0^* = (0,...0)$ as the *joint origin* in this new coordinate system. Whether the joint origin is an equilibrium depends on the distribution of voter ideal points. These are encoded in the voter variance/covariance matrix. We first define this, and then use it to characterize the vote share Hessians.

Definition 2: *The voter variance- covariance matrix,*∇. To characterize the variation in voter preferences, we represent in a simple form the variance covariance matrix (or data matrix, ∇) of the distribution of voter ideal points. Let X have dimension w and be endowed with a system of coordinate axes $(1, \dots, t, s, \dots, w)$. For each coordinate axis let $\xi_t = (x_{1t}, x_{2t}, \dots, x_{nt})$ be the vector of the t^{th} coordinates of the set of n voter ideal points. We use (ξ_s, ξ_t) to denote scalar product.

The symmetric $w \times w$ voter variance/covariance data matrix ∇ is then defined to be

$$\nabla = \begin{pmatrix} (\xi_1, \xi_1) & & & (\xi_1, \xi_w) \\ & (\xi_s, \xi_s) & & \\ & & (\xi_t, \xi_t) & \\ (\xi_w, \xi_1) & & & (\xi_w, \xi_w) \end{pmatrix}$$

The normalized covariance matrix is $\frac{1}{n}\nabla$.We write $v_s^2 = \frac{1}{n}(\xi_s, \xi_s)$ for the electoral variance on the s^{th} axis and

$$v^2 = \sum_{r=1}^{w} v_r^2 = \frac{1}{n}\sum_{r=1}^{w}(\xi_r, \xi_r) = trace(\frac{1}{n}\nabla)$$

for the total electoral variance. The normalized covariance between the r^{th} and s^{th} axes is $(v_r, v_s) = \frac{1}{n}(\xi_r, \xi_s)$.

The formal model is denoted $M(\boldsymbol{\lambda},\beta;\Psi,\nabla)$, though we shall usually suppress the reference to ∇.

Definition 3. *The Convergence Coefficient of the model* $M(\boldsymbol{\lambda},\beta;\Psi)$.

(i) At the vector $\mathbf{z}_0$ =(0,..0) the probability $\rho_{ik}(\mathbf{z}_0)$ that i votes for party,k,is

$$\rho_k = \left[1 + \sum_{j \neq k} \exp\left[\lambda_j - \lambda_k\right]\right]^{-1}.$$

(ii) The coefficient $A_k$for party k is

$$A_k = \beta(1 - 2\rho_k)$$

(iii) The Hessian for party k at $\mathbf{z}_0$ is

$$C_k = \left[2[A_k](\frac{1}{n}\nabla) - I\right]$$

where I is the w by w identity matrix.

(iv) The *convergence coefficient of the model* $M(\boldsymbol{\lambda},\beta;\Psi)$ is

$$c(\boldsymbol{\lambda},\beta;\Psi) = 2\beta[1 - 2\rho_1]v^2 = 2A_1v^2 \ .$$

The definition of ρ_k follows directly from the definition of the extreme value distribution. Obviously if all valences are identical then $\rho_1 = \frac{1}{p}$,as expected.The effect of increasing λ_j, for $j \neq 1$, is clearly to decrease ρ_1, and therefore to increase A_1,and thus $c(\boldsymbol{\lambda},\beta;\Psi)$.

Theorem . The condition for the joint origin be a LSNE in the model $M(\boldsymbol{\lambda},\beta;\Psi)$ is that the Hessian

$$C_1 = \left[2[A_1](\frac{1}{n}\nabla) - I\right]$$

of the party 1, with lowest valence,has negative eigenvalues.□

Comment on the Theorem. The proof of the Theorem depends on considering the first and second order conditions at $\mathbf{z}_0$ for each vote share function. The first order condition is obtained by setting $dV_j/dz_j = 0$ (where we use this notation for full differentiation, keeping $z_1, \ldots, z_{j-1}, z_{j+1}, \ldots, z_p$ constant). This allows us to show that $\mathbf{z}_0$ satisfies the first order condition. The second order condition is that the Hessian d^2V_j/dz_j^2 be negative definite at the joint origin. If this holds for all j at $\mathbf{z}_0$, then $\mathbf{z}_0$ is a LSNE. However, we need only examine this condition for the vote function V_1 for the lowest valence party. As we shall show, this condition on the Hessian of V_1 is equivalent to the condition on C_1, and if the condition holds for V_1, then the Hessians for $V_2, \ldots, V_p$ are all negative definite at $\mathbf{z}_0$. As usual, conditions on C_1 for the eigenvalues to be negative depend on the trace, trace(C_1), and determinant, det(C_1), of C_1. These depend on the value of A_1 and on the electoral variance/covariance matrix, ∇. Using the determinant of C_1, we can show that $2A_1v^2 < 1$ is a sufficient condition for the eigenvalues to be negative. In terms of the "convergence coefficient" $c(\boldsymbol{\lambda},\beta;\Psi)$ we can write this as $c(\boldsymbol{\lambda},\beta;\Psi) < 1$. In a policy space of dimension w, the necessary condition on C_1, induced from the condition on the Hessian of V_1, is that $c(\boldsymbol{\lambda},\beta;\Psi) \leq w$. This condition is obtained from examining the trace of C_1 . If this necessary condition for V_1 fails, then $\mathbf{z}_0$ can be a neither a LNE nor a LSNE.

Ceteris paribus, a LNE at the joint origin is "less likely" the greater are the parameters β, $\lambda_p - \lambda_1$ and v^2.

Proof of the Theorem. At $\mathbf{z}_{-1} = (0, ..)$, let $\rho_{i1}(z_1)$ be the probability that i picks 1.Then

$$\rho_{i1}(z_1) = \Pr\left[[\lambda_1 - \beta||x_i - z_1||^2 - \lambda_j + \beta||x_i - z_j||^2 > \epsilon_j - \epsilon_1], \text{ for all } j \neq 1\right].$$

Using the extreme value distribution Ψ we obtain

$$\begin{aligned}
\rho_{i1}(x_i, z_1) &= [[1+\Sigma_{j=2}[\exp(f_j)]]^{-1} \\
\text{where } f_j &= \lambda_j - \lambda_1 + \beta||x_i - z_1||^2 - .\beta||x_i - z_j||^2 \\
\text{and} \frac{d\rho_{i1}}{dz_1} &= 2(\beta(z_1 - x_i)[\rho_{i1}^2 - \rho_{i1}]
\end{aligned}$$

At $z_1 = 0, \rho_{i1} = \rho_1$ is independent of i,so we obtain

$$\begin{aligned}
\frac{d\rho_{i1}}{dz_1} &= 2(\beta(z_1 - x_i)[\rho_1^2 - \rho_1] \\
\text{and} \frac{dV_1}{dz_1} &= \frac{1}{n}\sum_i \frac{d\rho_{i1}}{dz_1} = 0 \text{ at } z_1 = \frac{1}{n}\sum_i x_i,
\end{aligned}$$

giving the first order condition $z_1 = 0$. Obviously the condition $\frac{dV_j}{dz_j} = 0$ is satisfied at $.z_1 = \frac{1}{n}\sum\limits_i x_i = 0$. Thus $\mathbf{z}_0$ =(0,..0) satisfies the first order condition.

At $z_{-1} = (0, ..0)$ the Hessian of ρ_{i1} is

$$\frac{d^2\rho_{i1}}{dz_1^2} = \{\rho_{i1} - \rho_{i1}^2\}\{[1 - 2\rho_{i1}][\nabla_{i1}(z_1)] - 2\beta I\}.$$

Here $[\nabla_{i1}(z_1)] = 4\beta^2[(x_i - z_1)(x_i - z_1)^{\mathrm{T}}$ is the w by w matrix of cross product terms. Now $\Sigma_i[\nabla_{i1}(0)] = 4\beta^2\nabla$, where ∇ is the electoral covariance matrix given in Definition 2., Then the Hessian of V_1 at $z_1 = 0$ is given by

$$\frac{1}{n}\sum_i \frac{d^2\rho_i}{dz_1^2} = \{\rho_1 - \rho_1^2\}\{[1 - 2\rho_1][4\beta^2]\left[\frac{1}{n}\nabla\right] - 2\beta I\}.$$

Because the first term $\{\rho_1 - \rho_1^2\}$is positive, the eigenvalues of this matrix will be determined by the eigenvalues of

$$\begin{aligned}
C_1 &= \left[2[A_1](\frac{1}{n}\nabla) - I\right] \\
\text{where } A_1 &= \beta[1 - 2\rho_1]
\end{aligned}$$

as required.

Moreover,

$$\begin{aligned} \lambda_p &\geq \lambda_{p-1} \geq \cdots \geq \lambda_2 \geq \lambda_1 \\ \text{implies that } \rho_p &\geq \rho_{p-1} \geq \cdots \geq \rho_2 \geq \rho_1 \\ \text{so that} A_1 &\geq A_2 \geq \cdots \geq A_p. \\ \text{This implies that } trace(C_1) &\geq trace(C_2) \geq \cdots \geq trace(C_p) \\ \text{and } \det(C_1) &\geq \det(C_2) \geq \cdots \geq \det(C_p) \end{aligned}$$

Thus if C_1 has negative eigenvalues then so do $C_2, \ldots, C_p$, and this implies that $z_1 = z_2 = \cdots = z_p = 0$ will all be mutual local strict best responses. This shows that the stated condition is sufficient for $\mathbf{z}_0 = \mathbf{z}_0^* = (0, 0, \ldots, 0)$ to be an LSNE. Obviously, if C_1 does not have negative eigenvalues, then $\mathbf{z}_0$ cannot be a LSNE..□

Note that for a general spatial model with an arbitrary, non-Euclidean but differentiable metric $\Pi(x_i, z_j) = ||x_i - z_j||$, a similar expression for A_1can be obtained., but in this case the covariance term ∇ will not have such a ready interpretation. Note also that if the non-differentiable Cartesian metric $\Pi(x_i, z_j) = \Sigma_{k=1}^{w} |x_{ik} - z_{jk}|$ were used, then the first order condition would be satisfied at the median rather than the mean.

For the case of the Euclidean norm, the Theorem gives the following Corollaries.

Corollary 1. Assume X is two dimensional. Then, in the model $M(\boldsymbol{\lambda},\beta;\Psi)$, the sufficient condition for the joint origin to be a LSNE is that $c(\boldsymbol{\lambda},\beta;\Psi)$ be strictly less than 1. The necessary condition for the joint origin to be a LNE is that $c(\boldsymbol{\lambda},\beta;\Psi)$ be no greater than 2.

Proof. The condition that both eigenvalues of C_1 be negative is equivalent to the condition that $\det(C_1)$ is positive and trace(C_1) is negative. Now

$$\begin{aligned} \det(C_1) &= (2A_1)^2 \left[(v_1, v_1) \cdot (v_2, v_2) - (v_1, v_2)^2 \right] \\ &\quad +1 - (2A_1) \left[(v_1, v_1) + (v_2, v_2) \right]. \end{aligned}$$

By the triangle inequality,the term $\left[(v_1, v_1) \cdot (v_2, v_2) - (v_1, v_2)^2 \right]$ is non negative. Thus $\det(C_1)$ is positive if

$$\begin{aligned} 2A_1) \left[(v_1, v_1) + (v_2, v_2) \right] &< 1 \\ \text{or } 2\beta[1 - 2\rho_1] v^2 &< 1. \end{aligned}$$

This gives the sufficient condition that $c(\boldsymbol{\lambda},\beta;\Psi) < 1$ for $\mathbf{z}_0 = \mathbf{z}_0^*$.to be a. LSNE ..

The necessary condition for $\mathbf{z}_0$ to be an LNE is that the eigenvalues be non-positive. Since $trace(C_1)$ equals the sum of the eigenvalues we can use the fact that

$$trace(C_1) = (2A_1)[(v_1, v_1) + (v_2, v_2)] - 2,$$

to obtain the necessary condition $2\beta[1 - 2\rho_1]v^2 \leq 2$.

Thus $c(\boldsymbol{\lambda},\beta; \Psi) \leq 2$ gives the necessary condition. $\square$

Notice that the case with two parties with equal valence immediately gives a situation with $2\beta[1-2\rho_1]v^2 = 0$, irrespective of the other parameters. However, if $\lambda_2 >> \lambda_1$, then the joint origin may fail to be a LNE if βv^2 is sufficiently large. Note also that for the multiparty case ρ_1 is a decreasing function of $(\lambda_p - \lambda_1)$ so the necessary condition is more difficult to satisfy as $(\lambda_p - \lambda_1)$ increases.

If the sufficient condition fails, but the necessary condition is satisfied, then the eigenvalues may still be non-positive, and can be explicitly computed in terms of the model parameters and data. If the second condition fails then obviously at least one of the eigenvalues must be strictly positive, and so $\mathbf{z}_0^*$ cannot be an LNE. The condition $c(\boldsymbol{\lambda},\beta; \Psi) = 2$ includes the non-generic case where both eigenvalues are zero.

In the two dimensional case there is one situation where computation of eigenvalues is particularly easy. If the covariance (ξ_1, ξ_2) of the electoral data on the two axes is (close to) zero, then the voter covariance matrix is (approximately) diagonal and the two policy dimensions can be treated separately. In this case we obtain two separate necessary conditions

$$2\beta[1 - 2\rho_1]v_t^2 \leq 1$$

for both $t = 1, 2$, for $\mathbf{z}_0^*$ to be a LNE. If $v_t^2 \leq v_s^2$ then we obtain essentially identical necessary and sufficient conditions in terms of v_s^2. In the two-dimensional case we can compute the eigenvalues explicitly.

Corollary 2. In the two dimensional case, let $v_t^2 = \frac{1}{n}(\xi_t, \xi_t)$ be the electoral variances on the two axes t=1,2. Then the two eigenvalues of C_1 are

$$\begin{aligned} a_1 &= (A_1)\{\left[v_1^2 + v_2^2\right] + [[v_1^2 - v_2^2]^2 + 4(v_1, v_2)^2]^{\frac{1}{2}}\} - 1 \\ a_2 &= (A_1)\{\left[v_1^2 + v_2^2\right] - [[v_1^2 - v_2^2]^2 + 4(v_1, v_2)^2]^{\frac{1}{2}}\} - 1 \end{aligned}$$

*Proof.*This follows immediately from the proof of Corollary 1, using the fact that $a_1 + a_1 = trace(C_1) = c(\boldsymbol{\lambda},\beta; \Psi) - 2$. $\square$

When the covariance term $(v_1, v_2) = 0$,then the eigenvalues are obviously $a_t = A_1)\{v_t^2\}, t = 1, 2$. The more interesting case is when the covariance (v_1, v_2) is significant. By a transformation of coordinates, we can choose v_t, v_s to be the eigenvectors of the Hessian matrix for agent 1, and let these be these new "principal components " of the electoral covariance matrix. If $v_t^2 \leq v_s^2$ then the sthcoordinate can be termed " the principal electoral axis". The two dimensional empirical analysis of Israel, discussed below, shows that the valence differences implied that the eigenvalue associated with the principal electoral s-axis was large and positive, while the eigenvalue on the minor axis was negative. This immediately implies that, with other agents at the electoral origin, the position $z_1 = 0$ is a saddlepoint of the vote share function for agent 1. The eigenspace associated with the large eigenvalue can be identified with the principal electoral axis, and the smaller eigenvalue is associated with the orthogonal minor axis. Consequently, the gradient of agent 1's vote share function near the origin points in a direction away from the origin,and is aligned with the principal axis. It follows that, in local equilibrium, all agents will be located on (or close to) the principal axis, with the lowest valence agents farthest from the origin. This formal result is matched by the simulation of the vote maximizing model.

In the general w-dimensional situation, it is obvious that $trace(C_1)$ involves the dimension, w, so we obtain the necessary condition

$$\begin{aligned} trace(C_1) &= (2A_1)[trace(\frac{1}{n}\nabla) - w \leq 0, \\ \text{or } 2\beta[1 - 2\rho_1]v^2 &\leq w \end{aligned}$$

for the joint origin to be a LNE in this case.

Corollary 3. In the case that X is w-dimensional. then the necessary condition for the joint origin to be a LNE is that $c(\boldsymbol{\lambda},\beta; \Psi) \leq w$. □

Computation of the eigenvalues is more difficult, but presents no fundamental theoretical problem.When the necesary condition fails, then the origin will be a minimum or saddle of of the low valence party. As in the two dimensional case, if the electoral variance is much larger on a principal electoral component, then this axis will coincide with the eigenvector of the Hessian of party 1, and we expect parties to allign themselves on this axis.

Previous formal analyses of the stochastic vote model (Banks and Duggan, 2005; McKelvey and Patty,2004) have focused on conditions sufficient for a "coincident " vector, $\mathbf{z}^* = (x^*, ...x^*)$ to be a Nash equilibrium. Generally this has involved assuming that the vote share functions are concave. Obviously, if the necessary condition,given in Corollary 2, fails at the joint origin, then so must concavity. This casts doubt on the existence of PNE in these vote models. The natural question is whether there can be multiple LSNE but no Nash equilibria., Generic existence of LSNE can be shown by more abstract arguments (Schofield and Sened, 2002) Such arguments suggest that, in general, there will exist many different, non-convergent LSNE. In the simulation of the model for the Israel elections,discussed below, the various LSNE that were found were essentially permutations of one another. Most importantly, none of these equilibria involved parties adopting positions very close to the electoral origin.

3 Empirical Analysis for Israel.

Consider the case of the election of Israel in 1996. Figure 1 shows the estimated positions of the parties at the time of the 1996 election,

while Table 1 presents summary statistics of the 1996 election, together with valence estimates based on a multinomial logit model, and therefore on the Type I extreme value distribution on the errors. The two dimensions of policy deal with attitudes to the PLO (the horizontal axis) and religion (the vertical axis. The policy space was derived from voter surveys (obtained by Arian and Shamir, 1999) and the party positions from analysis of party manifestos (Schofield, Sened and Nixon, 1998; Schofield and Sened 2005a). Using the formal analysis,we can readily show that one of the eigenvalues of the low valence party, the NRP, is positive. Indeed it is obvious that there is a principal component of the electoral distribution, and this axis is the eigenspace of the positive eigenvalue. It follows that low valence parties should then position themselves on this eigenspace as illustrated in the simulation given below in Figure 2.

[Insert Table 1 and Figure 1 about here]

In 1996, the lowest valence party was the NRP with valence –4.52. The spatial coefficient is $\beta = 1.12$,so.for the extreme value model $M(\Psi)$ we compute $\rho_{NRP} \simeq 0$.and $A_{NRP} = 1.12$

$$\begin{aligned}
\rho_{NRP} &\simeq \frac{1}{1 + e^{4.15+4.52} + e^{3.14+4.52}} \simeq 0. \\
\text{Thus } A_{NRP} &= \beta = 1.12. \\
C_{NRP} &= 2(1.12)\begin{pmatrix} 1.0 & 0.591 \\ 0.591 & 0.732 \end{pmatrix} - I = \begin{pmatrix} 1.24 & 1.32 \\ 1.32 & 0.64 \end{pmatrix} \\
c(\Psi) &= 3.88
\end{aligned}$$

Then the eigenvalues are 2.28 and -0.40, giving a saddlepoint, and a value for the convergence coefficient of 3.88. . The major eigenvector for the NRP is (1.0,0.8), and along this axis the NRP vote share function increases as the party moves away from the origin.The minor, perpendicular axis is given by the vector (1,-1.25) and on this axis the NRP vote share decreases.. Figure 2, gives one of the local equilibria in 1996, obtained by simulation of the model..The Figure makes it clear that the vote maximizing positions lie on the principal axis through the origin and the point (1.0,0.8).In all, five different LSNE were located. However, in all the equilibria, the two high valence parties, Labor and Likud, were located at precisely the same positions. The only difference between the various equilibria were that the positions of the low valence parties were perturbations of each other.

[Insert Figure 2 about here]

We next analyse the situation for 1992, by computing the eigenvalues for the Type I extreme value distribution,Ψ (See Figure 3). From the empirical model we obtain $\lambda_{shas} = -4.67, \lambda_{likud} = 2.73, \lambda_{labor} = 0.91, \beta = 1.25$. When all parties are at the origin, then the probability that a voter chooses Shas is

$$\begin{aligned}
\rho_{shas} &\simeq \frac{1}{1 + e^{2.73+4.67} + e^{0.91+4.67}} \simeq 0. \\
\text{Thus } A_{shas} &= \beta = 1.25. \\
C_{shas} &= 2(1.25)\begin{pmatrix} 1.0 & 0.453 \\ 0.453 & 0.435 \end{pmatrix} - I = \begin{pmatrix} 1.5 & 1.13 \\ 1.13 & 0.08 \end{pmatrix} \\
c(\Psi) &= 3.6
\end{aligned}$$

Then the two eigenvalues for Shas can be calculated to be +2.12 and -0.52 with a convergence coefficient for the model of 3.6. Thus we find that the origin is a saddlepoint for the Shas Hessian. The eigenvector for the large, positive eigenvalue is the vector $(1.0, 0.55)$. Again,this vector coincides with the principal electoral axis. The eigenvector for the negative eigenvalue is perpendicular to the principal axis. To maximize vote share, Shas should adjust its position but only on the principal axis. This is exactly what the simulation found. Notice that the probability of voting for Labor is $[1+e^{1.82}]^{-1} = 0.14$, and $A_{labor} = 0.9$, so even Labor will have a positive eigenvalue at the origin.Clearly, if Likud occupies the mean voter position, then Labor as well as all low valence parties would find this same position to be a saddlepoint. In seeking local maxima of vote shares all parties other than Likud should vacate the electoral center. Then, however, the first order condition for Likud to occupy the electoral center would not be satisfied. Even though Likud's vote share will be little affected by the other parties, it too should move from the center. This analysis predicts that the lower the party's valence, the further will its equilibrium position be from the electoral mean. This is illustrated in Figures 2 and 4.

Calculation for the model $M(\Psi)$ for 1988 gives eigenvalues for Shas of +2.0 and -0.83 with a convergence coefficient of 3.16, and a principal axis through (1.0,0.5). Again, vote maximizing behavior by Shas should oblige it to stay strictly to the principal electoral axis. The simulated vote maximizing party positions indicated that there was no deviation by parties off the pricipal axis or eigenspace associated with the positive eigenvalue.

Thus the simulation was compatible with the predictions of the formal model based on the extreme value distribution. All parties were able to increase vote shares by moving away from the origin, along the principal axis, as determined by the large , positive principal eigenvalue . In particular, the simulation confirms the logic of the above analysis. Low valence parties, such as the NRP and Shas, in order to maximize vote shares must move far from the electoral center . Their optimal positions will lie either in the "north east" quadrant or the "south west" quadrant The vote maximizing model, without any additional information, cannot determine which way the low valence parties should move. As noted above, the simulations of the empirical models found multiple LSNE essentially differing only in permutations of the low valence party positions.

In contrast, since the valence difference between Labor and Likud was relatively low in all three elections, their optimal positions would be relatively close to, but not identical to, the electoral mean. The simulation figures for all three elections are also compatible with this theoretical inference. The figures also suggest that every party, in local equilibrium, should adopt a position that maintained a minimum distance from every other party. The formal analysis, as well as the simulation exercise, suggests that this minimum distance depends on the valences of the neighboring parties. Intuitively it is clear that once the low valence parties vacate the origin, then high valence parties, like Likud and Labor will position themselves almost symmetrically about the origin, and along the major axis. It should be noted that the positions of Labor and Likud, particularly, closely match their positions in the simulated vote maximizing equilibria.

Clearly, the configuration of equilibrium party positions will fluctuate as the valences of the large parties change in response to exogenous shocks. The logic of the model remains valid however, since the low valence parties will be obliged to adopt relatively "radical" positions in order to maximize their vote shares.

The correlation between the two electoral axes was much higher in 1988 ($r^2 = 0.70$) than in 1992 or 1996 (when $r^2 \simeq 0.47$). It is worth observing that as r^2 falls from 1988 to 1996, a counter-clockwise rotation of the principal axis that can be observed,. This can be seen in the change from the eigenvalue (1.0,0.5) in 1988, to (1.0,0.55) in 1992 and then to (1.0,0.8) in 1996. Notice also that the total electoral variance increased from 1988 to 1992 and again to1996. Indeed, in 1996, Figure 3 indicates that there is evidence of bifurcation in the electoral distribution in 1996.

In comparing Figure 1, of the estimated party positions, and Figure 2, of simulated equilibrium positions, there is a notable disparity particularly in the position of Shas. In 1996, Shas was pivotal between Labor and Likud, in the sense that to form a winning coalition government, either of the two larger parties required the support of Shas. It is obvious that the location of Shas in Figure 1suggests that it was able to bargain effectively over policy, and presumably perquisites. Indeed, it is plausible that the leader of Shas was aware of this situation, and incorporated this awareness in the utility function of the party.

The relationship between the empirical work and the formal model, together with the possibility of strategic reasoning of this kind, suggests the following conclusion.

Conclusion . The close correspondence between the simulated LSNE based on the empirical analysis and the estimated actual political configuration suggests that the true utility function for each party j has the form $U_j(\mathbf{z}) = V_j(\mathbf{z}) + \delta_j(\mathbf{z})$, where $\delta_j(\mathbf{z})$ may depend on the beliefs of party leaders about the post election coalition possibilities, as well as the effect of activist support for the party.

This conclusion leads to the further conjecture , for the set of feasible strategy profiles in the Israel polity, that $\delta_j(\mathbf{z})$ is "small" relative to $V_j(\mathbf{z})$. A formal model to this effect could indicate that the LSNE for $\{U_j\}$ would be close to the LSNE for $\{V_j\}$.

If this were valid in general, then it would be possible to use a combination of multinomial logit electoral models, simulation of these models and the formal electoral model based on exogeneous valence to study general equilibrium characteristics of multiparty democracies.

4 Concluding Remarks.

Most of the early work in formal political theory focused on two-party competition, and generally concluded that there would be strong centripetal electoral forces causing parties to converge to the electoral center (Ordeshook and Riker,1973). The extension of this theory to the multiparty context, common in European polities, has proved very difficult, because of the necessity of dealing with coalition governments (Riker,1962). However, the symmetry conditions developed by McKelvey and Schofield (1987) showed that a large centrally located party could dominate policy, if it occupied what is known as a "core position" Thus, in situations where there is a stable policy core, there would be certainty over the post-election policy outcome of coalition negotiation (Laver and Schofield, 1998). Absent a policy core, the post-election outcome will be a lottery across various possible coalitions, all of which are associated with differing policy outcomes and cabinet allocations. Modeling this post election "committee game" can be done either with cooperative game theoretical concepts, such as the "competitive solution" (McKelvey, Ordeshook and Winer,1978),or the "uncovered set" (McKelvey,1986; ; Banks, Duggan, Le Breton, 2002). Other recent analyses have utilized non-cooperative game-theoretical techniques to model coalition bargaining (Banks and Duggan,2000).

Although the non-cooperative stochastic electoral model presented here can give insight into the relationship between electoral preferences and beliefs (regarding the valences of party leaders), it is still incomplete. The evidence suggests that party leaders pay attention not only to electoral responses, but also to the post election coalition consequences of their choices of policy positions.(Cox, 1984,1987,1990,1997). Nonetheless, the combination of the electoral model and post-election bargaining theory suggests the following:

(i) Parties with high valence will be attracted towards the electoral center, but if there are two such competing parties, neither will locate quite at the center.

(ii) Under proportional electoral rule, there may be many low valence parties, whose equilibrium, vote maximizing positions will be far from the electoral center.

(iii) In order to construct winning coalitions, one or other of the high valence parties must bargain with more "radical" low valence parties,and this could induce a degree of coalitional instability.

In an attempt to model this complex political game, this paper has introduced the idea of local Nash equilibrium. The underlying premise of this notion is that party principals will not consider "global" changes in party policies, but will instead propose small changes in the party leadership in response to changes in beliefs about electoral response and the likely consequences of policy negotiations. It is also evident that the electoral model depends on the notion of "valence". Although the empirical models provide a justification for the inclusion of this variable, no attempt has been made to provide a formal justification. Valence can, however, be regarded as that element of a voter's choice which is determined by judgement rather than preference. This accords well with the arguments of James Madison in "Federalist 10" of 1787 and of Condorcet in his treatise of 1785 on social choice theory (Schofield,2005b).

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Table 1
Seats and votes in the Knesset

Party	1996 *National %*	1996 Sample %	1996 *Seats*	1996 *Valence*[1]		1996 %correct prediction	1992 Seats
Others Left	7.3	0	9	-			5
Meretz	7.6	6.0	9	0		28.5	12
Labor	27.5	44.0	34	4.15		71.7	44
3rd Way	3.2	1.8	13	-2.34		28.7	
Likud	25.8	43.0	30	3.14		70.1	32
Shas	8.7	2.0	10	-2.96		30.9	6
NRP	8.0	5.1	9	-4.52		40.8	6
Molodet	2.4	1.8	2	-0.89	.	78.0	3
Others Right	3.7	1.8	4	-			12

[1]The β coefficient for the MNL model with valence is 1.12 (confidence interval ± 0.15)

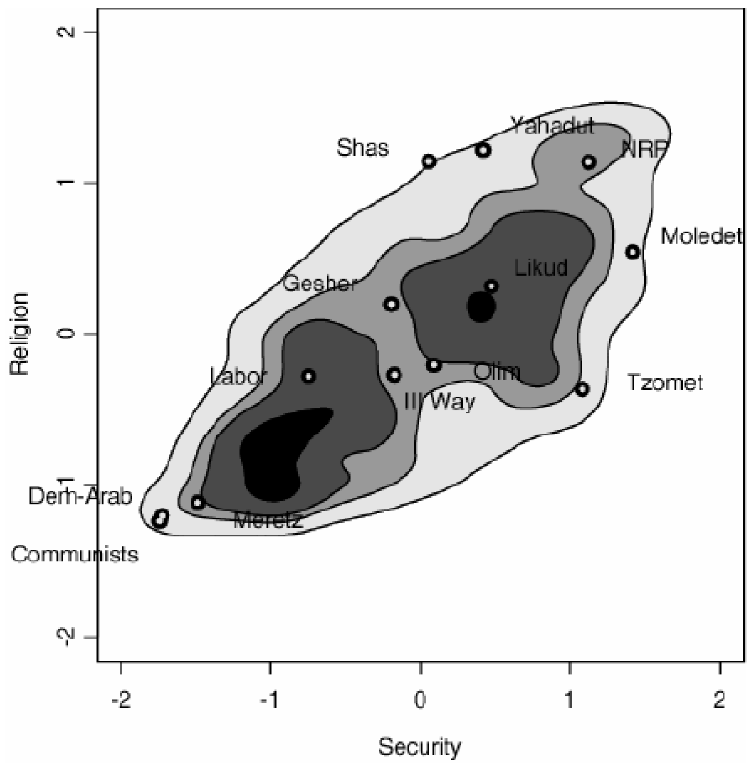


Figure 1: The Electoral Distribution and Party Positions in the Knesset in 1996.

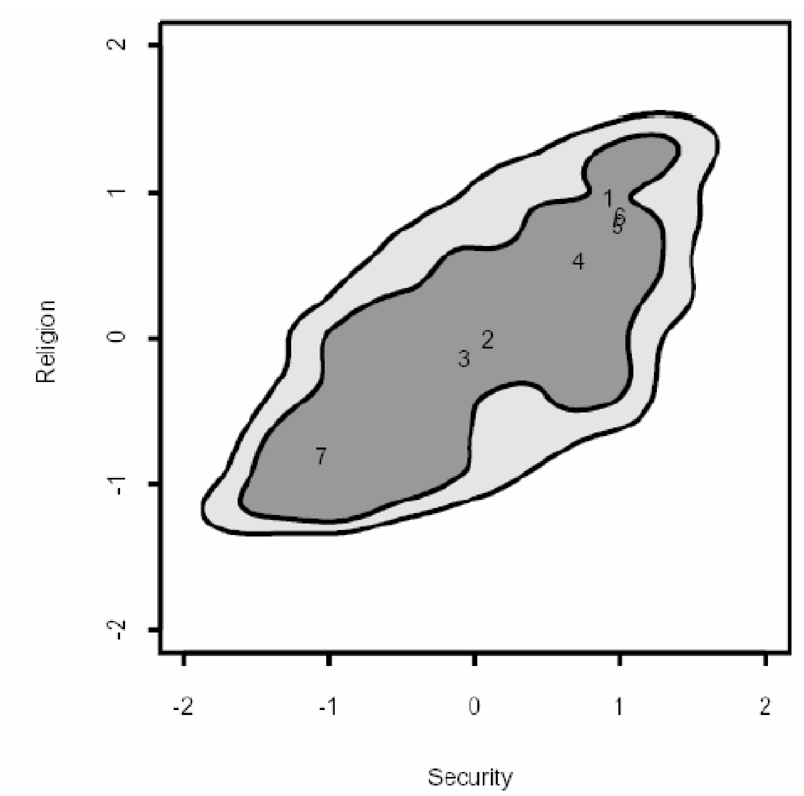


Figure 2: A Representative Local Nash Equilibrium of the Vote Maximizing Game in the Knesset for the 1996 Election.Key: 1 = Shas, 2 = Likud, 3 = Labor, 4 = NRP, 5 = Molodet, 6 = Third Way , 7= Meretz.

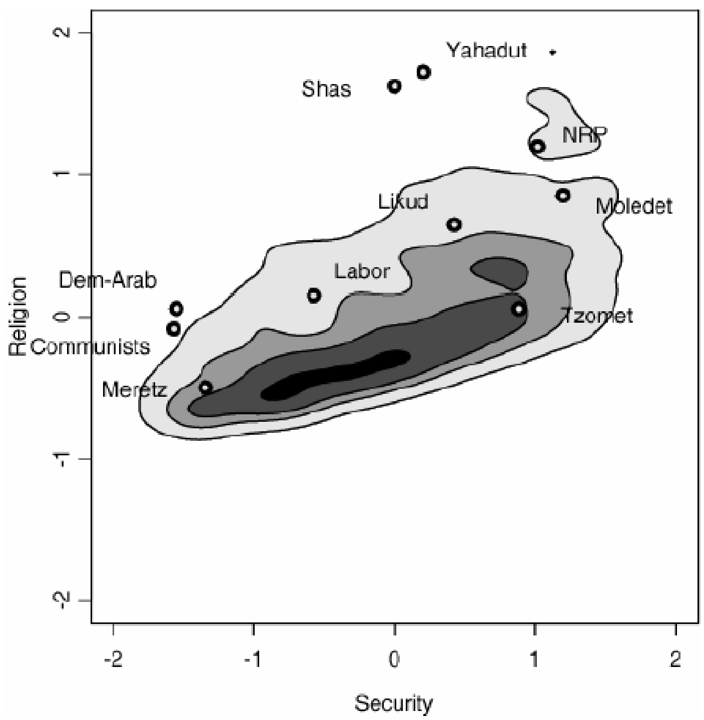
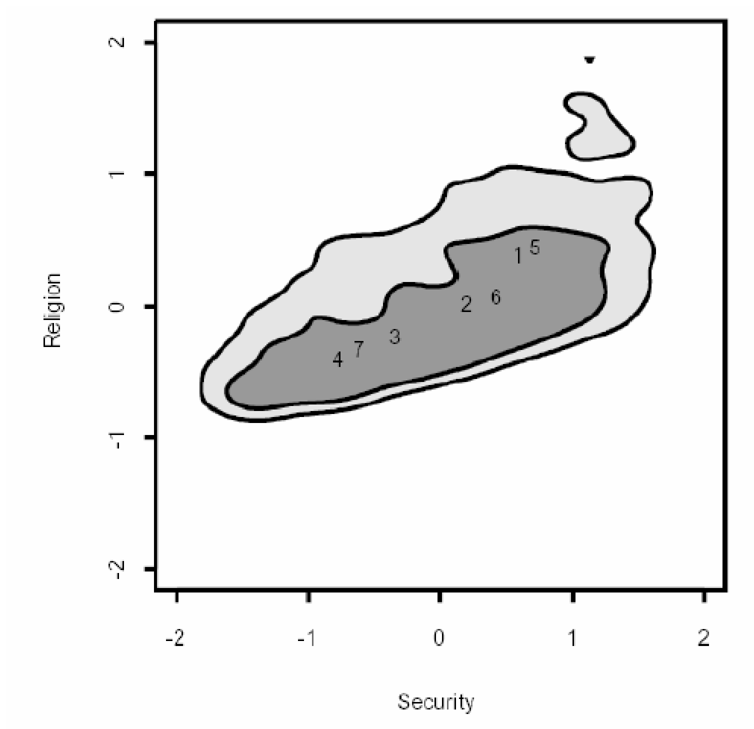


Figure 3: The Electoral Distribution and Party Positions in the Knesset in 1992.



Key: 1=Shas; 2=Likud; 3=Labor; 4=Meretz;5=NRP;6=Molodet;7=Tzomet.

Figure 4: A Representative Local Nash Equilibrium of the vote maximizing Game in the Knesset for the 1992 election.

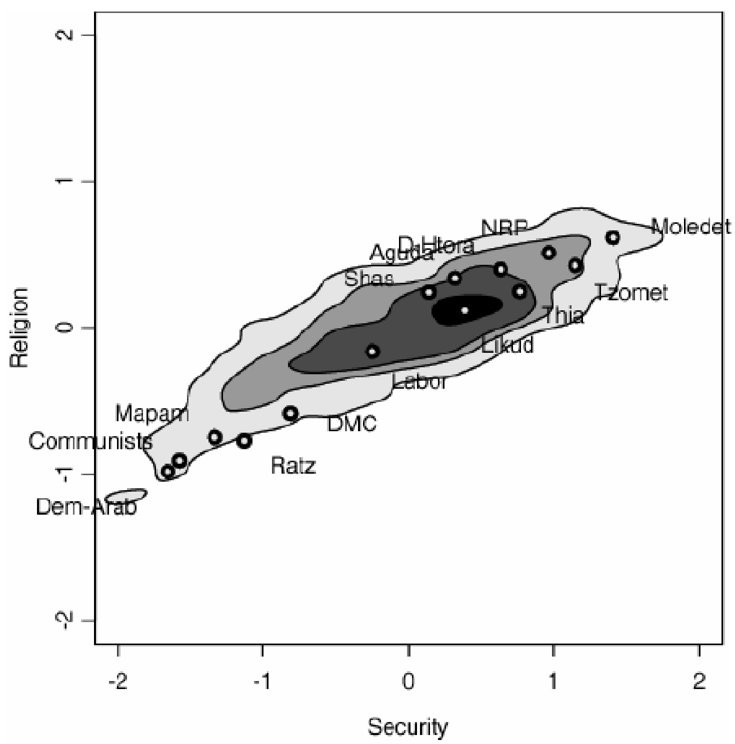
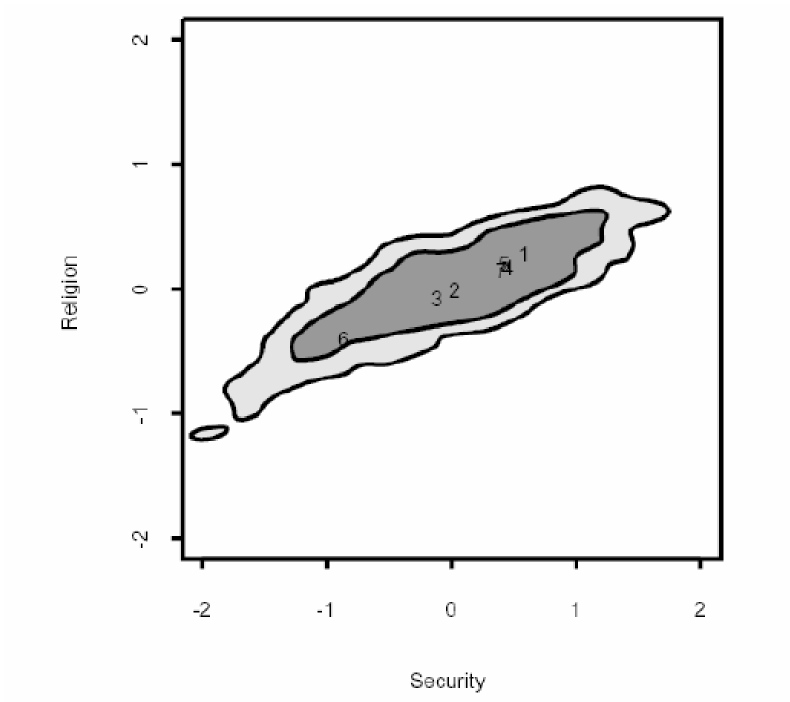


Figure 5: Electoral Distribution and Party Positions in the Knesset, 1988



Key:1=Shas;2=Likud;3=Labor; 4=Tzomet;5=NRP;6=Ratz;7=Thia.

Figure 6: A Representative Local Nash Equilibrium of the vote maximizing Game in the Knesset for the 1988 election.