Abstract

Governments devote considerable resources to providing goods and services. One observes considerable variation in public provision across goods, across countries (and other jurisdictions) and across time. Theoretical models of public provision typically assume only one good can potentially be publicly provided. Yet, an important element of the observed international and intertemporal variation is in the *proportion* of public spending that is devoted to various publicly provided goods and services, such as education, medical care, security, housing and social insurance. A model which seeks to explain that variation must allow for public provision of more than one good, and such a multi-dimensional issue space typically renders the Median Voter Theorem useless as a means of determining political outcomes. This paper embeds the 'citizen-candidate' approach to political decision-making into a model in which public provision of more than one good is possible, and derives its equilibria. The equilibrium set is shown to display a 'propensity toward public provision', in a well-defined sense.

The Political Economy of the Public Provision of Goods

Al Slivinski[1]
Department of Economics
University of Western Ontario
London, Canada N6A 5C2
aslivins@uwo.ca

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1 Introduction

There is a long-standing interest in understanding the public provision of goods. Any behavioral model of this phenomenon must include an account of the political decisions made regarding which goods to provide publicly, how to finance that provision, and the rules and restrictions on eligibility for such provision. The earliest work in this area took as given that a good, usually public, was to be provided publicly, and then asked what level of provision would be chosen. The typical means for integrating political decision-making into economic models of this phenomenon was by referral to some variant of the Median Voter Theorem. More recent work has noted that most of the goods provided publicly are in fact private goods. This work has analyzed political decisions regarding the level of provision and whether or not private supplementation of that level, or opting out of public provision, will be allowed.[Epple and Romano (1996a), (1996b), (1999), Gouveia (1997)]. This work also exploits the existence of a majority rule equilibrium (i.e., a Condorcet winner among the set of alternatives) as the determinant of the political outcome.

This latter work has provided useful analysis of such public decision-making, in particular demonstrating that majorities can be expected to prefer a regime of public provision with private supplementation over either pure private provision or public provision without private supplementation. However, the actual provision of goods across countries/jurisdictions or across time shows considerable variation in not just the level of public provision of particular goods but also in the shares of public expenditure that are allocated to those goods that are publicly provided. If this kind of variation is to be explained by a theoretical model, it is necessary that the model admit the possibility of public provision of more than one good. This paper develops a simple model in which that is the case.

A set of citizens each get utility from one or both of two goods, which are assumed in the first instance to be Samuelsonian pure public goods. They also get utility from private consumption, and all of this is financed from the fixed incomes of the citizenry. These incomes are assumed to be identical, so as to eliminate from this initial analysis any purely redistributive issues. Citizens vote for one of a set of candidates who run for public office. The winning candidate decides on the set of goods to be publicly financed, and the level of public funding for those that are. Goods whose provision is not publicly financed can still be provided via private, non-cooperative donations, as analyzed in Bergstrom, Blume and Varian (1984). Candidates are simply citizens who in equilibrium decide to incur the cost of standing for this office, as in Osborne and Slivinski(1996) and Besley and Coate(1997). The candidate who wins the plurality-rule election makes the public provision decision that is optimal for himself, an outcome that is correctly anticipated by all voters. Heterogeneity of tastes in the population is modeled in a very simple manner. Each citizen is of one of three types, depending on whether they get utility from either one or both of the public goods in question.

It is shown that if no single type of citizen constitutes a majority of the population, then in all equilibria the office-holder is a citizen who finances both public goods with taxes, and there is no ensuing private provision of either good. This result holds no matter how few in number are those citizens who get utility from both public goods, so long as neither of the other types is a majority. If one of the other types *does* constitute a majority of the population, then there are equilibria in which a citizen of that majority type becomes the office-holder, and so only one public good is publicly financed. The other good is provided solely through private donations, at a lower level than it would be if tax-supported. However, even in this case there can exist equilibria in which the office-holder is the type of citizen who finances both goods publicly. Thus, the model displays a sort of propensity toward public provision of public goods.

A second version of the model in which all three goods are private is developed in Section 3. Two of the goods are possible candidates for universal public provision, as above, and it is assumed that citizens can augment any publicly provision of these goods with private purchases. Here, somewhat surprisingly, the results are precisely the same. That is, if no single type is a majority, the office-holder is always someone who chooses to finance consumption of both candidate goods publicly. If, on the other hand, one of the single-good types does constitute a majority of the electorate, an office-holder of that type may become the office-holder, but having two private goods financed publicly is still a possible equilibrium outcome.

The next section of the paper lays out the public goods model, and derives its equilibria. The private goods version is developed in Section 3, and it is shown that the set of equilibrium outcomes is essentially the same as for the public goods version The proofs of all results not provided in the text are contained in an Appendix.

2 The public goods case

Only equilibria in which all agents play pure strategies will be considered throughout the analysis.

In stage I of the game, citizens decide whether or not they wish to run for office. Thus each citizen i chooses $r_i \in \{O, I\}$ representing the decision to enter (I) or stay out (O) of the election. $C(r) = \{i \in N | r_i = I\}$ is then the set of candidates implied by the vector $r = (r_i)_{i \in N}$. Entering citizens incur a utility cost $c \geq 0$. There are assumed to be n_t citizens of type t (types will be defined below) and n citizens in total.

At stage II, citizens observe the set of candidates who have entered, and vote for one of them. If $C(r)$ is the set of candidates that exist as a result of the decisions at Stage I, each citizen i now chooses a $v_i(r) \in C(r)$, which is a vote for one candidate in $C(r)$. $v(r) \in C(r)^n$ will be referred to as a *voting equilibrium,* and any $(r, v(r)) \in \{O, I\}^n \times C(r)^n$ will be a *political equilibrium.* A plurality rule electoral system is assumed, so the winner of the election is that candidate with the most votes, who becomes the office-holder. In the event of a tie vote, each tied candidate has an equal probability of being the winner.

In Stage III of the game the officeholder chooses a level of spending on each of the two public goods. These are denoted by $g = (g_a, g_b)$, and $G = g_a + g_b$ denotes total government spending. Further, it is assumed that this spending must be financed with a tax of G/n on each citizen. Since incomes are lump sum and there is no income heterogeneity, there is no reason to consider more elaborate taxation schemes. These assumptions on incomes are made for simplicity, but also to determine what political outcomes result in the absence of any pure income-redistribution.

At Stage IV, citizens observe the chosen g vector and their after-tax income of $y(g) = w - (G/n)$. All type t citizens then simultaneously choose a vector $d^t(g) = (d_a^t(g), d_b^t(g)) \in B(g) = \{d \in R_+^2 | d_a + d_b \leq y(g)\}$. This is a set of private donations to the provision of z_a and z_b, respectively. It will be true that in any equilibrium, all citizens of the same type choose the same donations, so we save notation by recognizing this at the outset. Let $\Delta(g) = (d_s^t(g))_{t=a,b,0;s=a,b}$ be the vector of such donations by all types of citizens, and D_s^t be the total donations made to z_s by donors of type t, while D_s is total private donations to s, so that $z_s = D_s + g_s$ for $s = a, b$.

The assumed payoff to citizen i of type t is:

$$U_t^i(g, \Delta, r_i) = v(y(g) - d_a^t(g) - d_b^t(g)) + \delta_a(t) m_a(g_a + D_a(g)) + \delta_b(t) m_b(g_b + D_b(g)) - \varepsilon(r_i)c$$

where $\delta_s(t) = 1$ if $t \in \{0, s\}$ and is 0 otherwise, and $\varepsilon(r_i) = 1$ if $r_i = I$ and 0 if $r_i = O$. It is assumed that the functions $v()$ and $m_s()$ are all strictly increasing and strictly concave.

Note that citizens do not care about the identity of the winner of the election, beyond the fact that the g chosen by the winner affects their payoff.

We are looking for all possible pure strategy sub-game perfect Nash equilibria. As usual, we start with the last stage of the game. The public provision decision has been made by the elected office-holder, and individuals must decide if they wish to privately contribute to the provision of a good that is not publicly financed, or to augment the public financing of one that is. Earlier stages of the game are then analyzed using backward induction.

2.1 Stage IV: Private Provision

Given a choice of g by a winning candidate, the equilibrium contributions that will be made by each of the three types are denoted as $d^t(g) = (d_a^t(g), d_b^t(g))$. It is immediate that types a and b will never donate to the public good that gives them no utility, whereas a type 0 may donate to both. The amounts given by types a and b can be characterized as in a standard model of donations to a single public good, since the only effect of the existence of the other public good is that public financing of that good at stage III thereby reducing their disposable income. Note that with the simple utility functions assumed here the donations of a type $t \in \{a, b\}$ citizen in the last stage can be characterized by:

$$v'(y(g) - d_t^t) \geq m_t'(g_t + D_t^0 + D_t^t)$$

with equality holding if $d_t^t > 0$, while the donations of a type 0 must satisfy:

$$v'(y(g) - d_a^0 - d_b^0) \geq m_s'(g_s + D_s^0 + D_s^s), \text{ for } s = a, b.$$

Again, each of these latter conditions hold as an equality if a type 0 citizen donates a positive amount to the good in question.

Some simple facts about this stage of the game will be useful. Assuming normality of all goods, then for citizens of type a or b, there exists a locus of (y, g_s) combinations which can be described by the identity $v'(\gamma_s(g_s)) \equiv m'_s(g_s)$. These functions have the property that if a g such that $y(g) \leq \gamma_s(g_s)$ has been chosen by the winning candidate, citizens of type s will not contribute to z_s provision at the last stage of the game. The function $\gamma_s(g_s) = y$ is simply the inverse of the income expansion path for a type s citizen's demand for g_s. That is, letting $h_s(y) = \arg\max_{z_s}\{v(y - z_s) + m_s(z_s) | z_r \in [0, y]\}$, then $h_s(\gamma_s(g_s)) \equiv g_s$.

The characterization of the donation behavior of type 0 citizens is somewhat more complex. However, for each type s of public good, a function $\upsilon_s(g)$ can be defined that determines whether a type 0 individual will donate to z_s. However, it depends on the level of tax-financed provision of both goods, g.

Thus, for any given g, a type $t \in \{a, b\}$ citizen will make a donation of:

$$d_t^t(g) = \begin{cases} 0, \text{ if } \gamma_s(g_s) \leq y(g) \\ \delta_t^t(g), \text{ otherwise} \end{cases} \tag{1}$$

where $\delta_t^t(g)$ is the Nash equilibrium contribution of a type t individual to z_t when the amount g_t has been publicly financed, leaving him with disposable income of $y(g)$. If $\delta_t^t(g) > 0$, it will be characterized by the first order condition:

$$v'\left(y(g) - \delta_t^t(g)\right) = m'_t\left(g_t + \Sigma_s n_t^s(g)\delta_t^s(g)\right) \tag{2}$$

where $n_t^s(g)$ is the number of citizens of type s who donate to z_t given g. A type 0 citizen will make a donation to z_s also if and only if $y(g) > \upsilon_s(g)$. The amount given by type 0 individuals will be characterized by:

$$v'\left(y(g) - \delta_a^0(g) - \delta_b^0(g)\right) = m'_s\left(g_s + \Sigma_s n_t^s(g)\delta_t^s(g)\right), \text{ for } s = a, b. \tag{3}$$

The following lemma provides some preliminary results on what can occur in Stage IV of *any* equilibrium.

Lemma 1 *In any equilibrium:*

(i) if type 0 *citizens donate to* s*, then so do type* s *citizens.*

(ii) if type s *citizens donate to* s, *and type* 0 *citizens do not, it must be that type* 0 *citizens each donate more to the other good than each type* s *citizen donates to* s.

Note that from (i), if type 0 citizens are donating to both goods, then all citizens must be donating.

2.2 Public and private provision equilibria

We now consider the decisions regarding public provision of each z_s that will be made by each type of possible office-holder. The following result will be useful in this analysis.

Lemma 2 *No officeholder will choose a g that results in type 0 citizens donating to both goods at Stage IV.*

This follows essentially from the fact that if this were to happen, the office-holder could always make themselves better off by increasing the public funding of the z_s that matter to them, and increasing taxes to pay for this. Taxation is always preferable to private provision for someone who can choose the tax rate.

2.2.1 Type A and B office-holders

If a type A citizen is elected, it is clear that he will not use taxes to finance the provision of z_b. If there is no private giving to z_a in the last stage of the game, then he will choose $g_a = g_a^a$, where g_a^a is the solution to:

$$\max_{g_a}\{v(w - \frac{g_a}{n}) + m_a(g_a)\}. \tag{4}$$

Therefore g_a^a must satisfy the condition:

$$\frac{1}{n}v'(w - \frac{g_a^a}{n}) = m_a'(g_a^a) \tag{5}$$

It is irrelevant to a type A office-holder whether or not there will be private support for z_b in the last stage of the game. On the other hand, if the equilibrium continuation involves private giving to z_a, then g_a^a cannot be the equilibrium level of public provision, since g_a^a is only optimal if the office-holder ignores this later private provision. Suppose that in some equilibrium continuation after a type A candidate becomes the office-holder, there is private giving by type A individuals, (including himself). This would mean that the final bundle of goods consumed by the type A candidate is given by:

$$(x, z_a) = \left(w - \frac{g_a}{n} - d_a^a, g_a + n_a d_a^a + n_0 d_a^0\right) \tag{6}$$

where d_a^t is giving to z_a by type t. It is also easy to show that a type 0 individual will not give more to z_a than does a type A individual in the last stage. This means that the type A officeholder could get at least the same level of z_a by simply choosing $g_a = z_a$, and this would leave him with a greater level of x, and the option to donate nothing privately. Hence, in equilibrium, the type A officeholder will never choose a g_a that results in donations to z_a in the last stage by typeA citizens. Lemma 1(i) above then implies that type 0 citizens will not donate, either. So in any equilibrium continuation following the election of a type A office-holder, the only private provision will be of z_b, and the level of that private provision will be that implied by a Nash equilibrium of the standard private donation model, as in Bergstrom, Blume and Varian(1984). In fact the type 0 and type B citizens are in identical positions in the private provision game that follows an A office-holder's choice of g_a, and so we have the following.

Proposition 3 *If a type A citizen becomes the office-holder, in the equilibrium continuation he chooses a public provision vector of $g^a = (g_a^a, 0)$, where g_a^a is characterized by (5). At Stage IV, $D_a = 0$, and z_b is entirely privately financed, with the level of private financing being: $D_b^a = (n_0 + n_b)d_b^a$, where d_b is characterized by:*

$$v'(y(g^a) - d_b^a) = m_b' \left((n_0 + n_b)d_b^a\right) \tag{7}$$

There is an obvious analog to this result for the case in which the public decision-maker is of type B[1]. The final possibility is that a type 0 citizen is elected to make public provision decisions.

2.2.2 Type 0 office-holder

A type 0 citizen might wish to publicly finance either or both goods. If in fact both are provided, and there is no private financing of either good ex-post, then it follows that the public funding vector chosen will be $g^0 = (g_a^0, g_b^0)$, characterized by:

$$\frac{1}{n} v' \left(w - \frac{g_a^0 + g_b^0}{n} \right) = m_a'(g_a^0) = m_b'(g_b^0) \tag{8}$$

[1] It is possible that in equilibrium the other good is not privately financed at all, because, for example, $y(g_{a,}0)$ is sufficiently low that in Stage IV there are zero donations to b. This 'cross-good crowding out' will not alter the behaviour of an office-holder of type A or B, however, and in what follows it will be assumed that there is some private financing of the other good.

These conditions imply that $v'\left(w - \frac{G^0}{n}\right) > m_s'(g_s^0)$, for $s = a, b$, since $v', m_s' > 0$. This in turn means that for any $d_s > 0$ and $D_s > 0$, it must be that $v'\left(w - \frac{G^0}{n} - d_s\right) > m_s'(g_s^0 + D_s)$, since $v'', m_s'' < 0$. So this public funding vector will in equilibrium be followed by no private giving. The other possibility is that the type 0 officeholder funds one or both z_s at a sufficiently low level that there is some private giving at stage IV. In fact, it is a direct corollary of Lemma 2 that she will not, and so we have the next proposition.

Proposition 4 *A type* 0 *office-holder will choose* $g^0 = (g_a^0, g_b^0)$ *characterized by (8). This will result in no donations at Stage IV, so that* $D^0 = (0, 0)$.

It follows then that the only private provision occurs when the office-holder is one who benefits directly from only one of the two public goods. The private funding in that case is all directed towards provision of the good that is not tax-financed, and is done by all citizens who derive utility from that public good.

The next two propositions characterize the relative levels of private and public funding of public goods that emerge from equilibria with alternative office-holders. The first does this for tax-financing of each good for each case.

Proposition 5 $g_s^s > g_s^0$, *while* $G^0 > g_s^s$ *for* $s = a, b$.

It is not surprising that those who are interested in only one public good spend more public money on it than does an office-holder who wishes to fund both public goods, when there is no income heterogeneity. It is less obvious how the level of public funding by a type 0 office-holder compares to the private funding of a publicly ignored good when a type a or b person is the office-holder. However, the simple model predicts that public funding is greater than private funding even when another good is also being tax-financed.

Proposition 6 $g_s^0 > D_s^+$ *for* $s = a, b$, *where* D_s^+ *is the level of private financing of* z_s *that occurs when it receives no tax financing and the other public good does.*

2.3 Citizen preferences over office-holders

The results above complete the analysis of the 'public finance' segment of the model. The decisions of each type of possible office-holder have been determined, along with the private provision - if any - that will follow from those decisions. This in turn means that each citizen knows what (x, z) outcome will result from the election of each type of candidate, and this is all that is needed to determine their preferences over those candidates. These preferences will then be used to determine the set of voting equilibria at Stage II, for any given set of candidates, $C(r)$.

Let $\omega^t = (x^t, z^t) = (x_0^t, x_a^t, x_b^t, z_a^t, z_b^t)$ indicate the outcome when a type t citizen is the decision maker. Citizen preferences over candidates will be induced by their preferences over the various outcomes that result.

It follows from the results above that each type of citizen most prefers the outcome that occurs when a citizen of their own type is the office-holder. It seems intuitive that type a citizens would next prefer that a type 0 citizen be the officeholder, which requires that;

$$v(w - \frac{g_a^0 + g_b^0}{n}) + m_a(g_a^0) > v(w - \frac{g_b^b}{n} - d_a) + m_a\left((n_a + n_0)d_a\right) \tag{9}$$

It is not obvious that this will hold, since with a type 0 office-holder, a type A citizen finds himself taxed at a higher rate, since $G^0 > g_b^b$, and the amount g_b^0/n of the taxes paid by a type a to a type 0 government is pure waste from his point of view. On the other hand, more of z_a is provided than when a type B holds office, and all of the taxes paid to a type B government are wasted in A's view. In fact, a revealed preference argument establishes that the inequality above holds. In the type 0 regime, it is as if a type a pays a lump-sum tax in the amount g_b^0/n, with this revenue effectively being thrown away. Then, a level of taxation to support provision of z_a is determined by the type 0 office-holder. This leads to a level of g_a that is by definition chosen by the type 0 to maximize $v(y^b - \frac{g_a}{n}) + m_a(g_a)$, and so g_a^a is set at the level that would be chosen by a type A individual with income of y^b, where in this case $y^b = w - \frac{g_b^0}{n}$.

On the other hand, a type B officeholder also taxes the type a and uses the revenue for something which type A's regard as worthless, and this tax is greater than that imposed by a type 0 office-holder. Following this, the type 0 and type A citizens are left to finance z_a without the use of the tax system.

Clearly then, a type 0 office-holder will do better than this (from his own point of view) even if he sets the level of funding to be $g_a = D_a$, and the fact that he will actually choose $g_a^0 > D_a$ indicates that this must be preferable. Given the allocation of g_b^0 to z_b, type A and 0 citizens have the same preferences over g_a choices, so this argument implies the following.

Proposition 7 *Type t citizens' most preferred electoral outcome is that a type t individual win the election. For $t = a, b$, the second-best outcome is that a type 0 citizen win.*

Note that the 'demographic composition' of the population is given by the vector $\eta = (n_0, n_a, n_b)$, giving the number of each type of citizen in the population. However, the results in Proposition 6 hold independently of changes in this vector, even though the equilibrium values of many variables will be influenced by changes in η. That is, even though G^0, g_s^t, D_s depend on η, it will be true for any nonzero vector η that $g_s^s > g_s^o > D_s$, $G^o > g_s^s$, etc. Further, the electoral preferences just derived are independent of η. Using obvious notation, we have established that:

$$\begin{aligned} \omega^a &\succ {}_a\omega^0 \succ_a \omega^b, \text{ and} \\ \omega^b &\succ {}_b\omega^0 \succ_b \omega^a. \end{aligned}$$

All that remains to determine is the relative preference of a type 0 citizen for type A or B candidate. If a type A is elected, a type 0 citizen is in a situation in which too much z_a and too little z_b is being publicly provided from his point of view. The opposite is true if a B candidate is elected. The type 0's preference could go either way, and is the only preference issue that depends on the make-up of the population of citizens. In fact, whether a type 0 citizen ranks a type A or type B office-holder second to a type 0 may be influenced by the relative numbers of each type, since this will influence the levels of private funding that result from having only one of the z_s publicly funded.

Consider the levels of funding that result when a type A citizen holds office. The funding of z_a is solely through taxes, in an amount g_a^a determined by the condition (5) above, which means that g_a^a depends only on the total number of citizens, n. The level of (purely private) funding of z_b in this case, however, is determined by the condition (7), which depends on $n_0 + n_b$. If one considers a demographic shift in the population such that n is constant, there are fewer type A individuals, and more of types 0 and B, this has no effect on the level of g_a^a, nor on the taxes levied by the type A officeholder to finance g_a^a. However, it does have the effect of increasing the level of private provision of z_b and reducing the amount contributed to that funding by each type 0 (and type B) citizen[2]. This demographic shift therefore increases the payoff to a type 0 citizen from the outcome ω^a.The same demographic shift has no effect on g_b^b for the same reason, and if $n_0 + n_a$ falls, it decreases the level of private provision of z_a when a type B citizen is in office. Such a demographic shift then has no effect on public funding levels under type A and B candidates, while increasing the private funding of the public good whose constituency grows. This leads to the following result.

Proposition 8 *If n is constant, then:*

i) any shift in the population such that $n_0 + n_a$ rises (falls) causes the utility to a type 0 citizen from outcome ω^b to increase (decrease).

ii) any shift which increases (decreases) $n_0 + n_b$ causes the utility to a type 0 citizen from outcome ω^a to rise (fall).

It follows then, that type 0 citizens are more likely to regard a type A candidate as second-best whenever $n_a < n_b$. However, this does not mean that $n_a < n_b$ is necessary for type 0 citizens to prefer ω^a over ω^b, since the strength of preference embodied in the m_s functions also plays a role. In most of what follows, we will in fact assume that type 0 preferences are such that:

$$\omega^0 \succ_0 \omega^a \succ_0 \omega^b.$$

Any results that depend on this will of course have a mirror-image result that obtains when type 0 citizens rank ω^b second.

2.4 Voting equilibria in stage II

In what follows it is assumed that a type 0 citizen views having a type A office-holder as the second-best option. It will become apparent that the equilibrium outcomes do not depend on this.

One way to approach the analysis of the political outcome would be that used when the Median Voter Theorem is invoked in analyzing public decision-making. The public alternatives are assumed to be the 3 types of candidates,

[2] These results are easy to show; total differentiation of (7) does it, although this involves treating the n_t as continuous variables.

A, B and 0, and if the preference profile of the n citizens over the set $\{A, B, 0\}$ is single-peaked, there exists a Condorcet winner among the candidate types. The above analysis implies that citizen preferences are single-peaked, no matter whether an A or B office-holder is ranked second by type 0 citizens, and we have the following.

Remark 1 *If any single citizen type constitutes a majority of the population, then a voter of that type is the median voter, and hence an office-holder of that type is a Condorcet winner. Otherwise, an office-holder of type* 0 *is always a Condorcet winner.*

This approach predicts that a type 0 citizen will always win office, unless one of the other types makes up a majority of the citizenry. However, this result does not necessarily tell us anything about political outcomes in a world in which citizens are free to run for office, and citizens can vote strategically. To analyze political equilibria of this type, we first consider what the voting equilibria, $v(r)$, can look like for various sets of candidates, $C(r)$. The following assumption is made for this stage of the game.

A.1 - *No voter uses a weakly dominated strategy in any voting sub-game.*

This assumption implies that if there are two types of candidates in the election, citizens vote for one who is of the type they most prefer, in any equilibrium. It also implies that in any equilibrium no citizen votes for the type of candidate they like least[3], and this in turn implies that a type B candidate can never win a two-candidate election, unless type B citizens constitute a majority.

At the voting stage of the model (stage II), voters know what the public-private provision outcome will be if a candidate of any type wins the election, so they know the utility they will get from any of them becoming the office-holder. Given the assumption that no voter uses a dominated strategy, it is easy to determine the outcome of elections in which there are one or two candidates. If $C(r)$ contains three or more candidates, there are many $v \in C(r)^n$ that are possible voting equilibria. The following result will help to narrow down the set of possibilities.

Proposition 9 *For any* r*, the only voting equilibria* $v(r)$ *in which candidates tie for the lead are:*

[3] This would not follow with more than two candidates in an electoral system of majority rule, if a runoff election is held in the event that no candidate wins a majority.

i) all the tied candidates are of the same type,

ii) $C(r)$ contains candidates of at least two different types and $n_t = n/2$ for one of those types

iii) $C(r)$ contains candidates of every type and $n_t = n/3$ for all t.

This result implies that the only possible ties in any voting equilibrium require either that the candidates who tie are all the same, or that the demographics, η, be of a very particular nature. We will, in fact, assume that the demographic conditions in ii) and iii) of the Proposition never hold, as it seems of little interest to consider political outcomes that can occur only when the population is divided among types in such specific ways. The possibility of i) remains, but we will see later that the r at Stage I that is necessary for it to occur will never be part of a full equilibrium.

This effective elimination of ties resulting from any equilibrium v still leaves an immense and complicated set of v that can that can be equilibrium continuations for any r such that $C(r)$ has more than two candidates. However, the fact that r must itself be an equilibrium serves to greatly reduce the set of political equilibria, $(r, v(r))$. Thus, rather than lay out all of the many possible voting equilibria that can follow from the many possible vectors r, we will consider political equilibria as a whole.

2.5 Political equilibria

In determining the set of political equilibria, it is assumed that the utility cost of entry as a candidate is positive, but small, in the sense that a candidate who can enter and alter the electoral outcome to one that he prefers will always do so. That is, the utility cost of entry is less than

$$\min_{t=a,b,0} \min_{r \neq s} |U_t(\omega^r) - U_t(\omega^s)|.$$

The set of possible political equilibria, which we will denote as (r, v), depends in part on the demographic characteristics of the population, η. In what follows we will maintain the assumption that $\omega^a \succ_0 \omega^b$. It will become apparent how the results would differ if the opposite were assumed

Proposition 10 *If either $n_t < n/2$ for all t, or $n_0 \geq n/2$, then the only political equilibria (r, v) that can occur in a sub-game perfect equilibrium result in a type 0 candidate winning the election.*

The proof of this Proposition is simple, and does not require us to consider the many (r, v) combinations that are possible. Note first of all that we can ignore elections which result in a tie other than those in which all candidates tied for the lead are of the same type, by the discussion above. Now suppose, by way of contradiction, that (r, v) is a political equilibrium such that a type A candidate wins (including the possibility of a tie for the lead among multiple type A candidates). If there are any type B candidates in $C(r)$, the outcome cannot be worse for them if they exit instead, and they would thereby save the utility cost of entry, so there must be only type A and 0 candidates in any equilibrium $C(r)$. However, it then follows again that any type 0 candidate is better off choosing O, since the electoral outcome cannot be worse if he does so. This means there must be only type A candidates in the equilibrium $C(r)$. However if this is the equilibrium $C(r)$, a type 0 candidate can enter, get all the votes of the type B and 0 citizens, and win, since $n_a < n_0 + n_b$, by assumption. The assumption on c above implies a type 0 candidate would then prefer to enter. Hence, the supposition that a type A candidate is the winner leads to a contradiction. The argument that no type B candidate can win in equilibrium is symmetric.

The proposition above implies that although there may be equilibria with many candidates, they all have the same electoral outcome. A type 0 candidate must win, and therefore both public goods are tax financed in equilibrium, no matter how few citizens there are who get utility from both z_a and z_b, because the result requires no lower bound on n_0, other than that no other type constitutes a majority on its own.

Proposition 11 *If $n_a > n/2$, then there are two possible types of political equilibria that can arise in a subgame perfect equilibrium. In the first, there is a single entrant of type A who wins the election by acclamation. In the second, there are at least 4 candidates, and the election is won by a candidate of type* 0*. In order for the second political equilibrium to arise, it must be the case that:* $\eta \in \mathfrak{N}_a = \{\eta | n_a \geq n/2,\ n_0 \geq 1,\ n_b > 1 + n/3\}$

An example of the sort of political equilibrium that can result in a type 0 candidate winning despite the majority status of the type A citizens is the following. $C(r)$ consists of two type A candidates and single candidates of type 0 and type B. The voting equilibrium that supports this outcome has enough citizens of every type voting for the single type 0 candidate to guarantee that she wins by at least 2 votes, so that no citizen can change the outcome by switching their vote. Since a type 0 candidate is at least the second choice of every citizen, this v is acceptable equilibrium behavior. However, it must also be true that no candidate prefers O to I, and that no citizen who chose O, and is therefore not in $C(r)$, would prefer to choose I. Thus, we must specify subgame perfect voting equilibria v that follow from any unilateral deviations by both candidate and non-candidate citizens.

To insure no non-candidates prefer I, we simply specify that any new entrant gets no votes.

The type 0 candidate will clearly not prefer to choose O in such an equilibrium, since the outcome must be worse. If the single type B candidate were to choose O, then the equilibrium continuation v has all type A voters switch to a single type A candidate, who therefore wins. This is a worse outcome for the type B candidate than is the equilibrium outcome, so the type B candidate prefers I. If one of the type A candidates exits, then the continuation voting equilibrium has the A and 0 voters vote evenly for the candidates of those two types, and the type b citizens all vote for candidate B. The restriction on η guarantees that this will result in a victory by the single type B candidate, and that the type A and 0 citizens cannot change the outcome by unilaterally changing their votes. This is worse for the type A citizen, so these candidates will not choose O.

This equilibrium may seem contrived, but the point is that it is possible to construct equilibria in which a type 0 candidate wins, even when the type A citizens constitute a majority, so long as they are not too much in the majority. No amount of contrivance makes any candidate but a type 0 the victor when type 0 citizens are the majority, **or** when *no* single type constitutes a majority of the voters.

Because we are assuming that the type 0 citizens regard a type A candidate as second-best, it is not obvious from this what would happen if the type B citizens constituted a majority of the population. In fact, the set of possible political equilibria is similar, as the following result states.

Proposition 12 *If $n_b > n/2$, then there are two possible types of political equilibria that can arise in a subgame perfect equilibrium. In the first, there* *is a single entrant of type* B *who wins the election by acclamation. In the second, there are at least 4 candidates, and the election is won by a candidate of type* 0. *In order for the second political equilibrium to arise, it must be the case that:* $\eta \in \mathfrak{N}_b = \{\eta | n_0 \geq 1,\ n/2 < n_b < 2n/3 - 1\}$.

The type o (r, v) that supports the second equilibrium again contains 4 candidates, but now there are two type B candidates, and a single candidate of each other type, and again, the key element in constructing the equilibrium is that if either of the two type B candidates exits, citizens react so as to elect the type A candidate. Once again this equilibrium is contrived, but the point remains that it is possible in this case to construct an equilibrium such that a type 0 candidate is the winner even though the type B citizens constitute a majority of the population, so long as they are not an overwhelming majority.

3 The private goods case

In this section a variant of the model in which the goods z_a, z_b are private goods is developed. Private goods can also be provided to individuals by a government that levies taxes to pay for such public provision, and often they are. Suppose then that the office-holder can decide to provide a quantity g_s of z_s for $s = a, b$, to any individual who wants it, at zero cost.[4]. The means to finance this public provision is a lump-sum tax in the amount G/n, once again. Since this is now a private good, we will assume that any such public provision is consumed only by those who get positive utility from from the good(s) provided. We also assume that the good can be provided by the government by some means such that it can not be re-sold. We assume that once again there are three types of individuals, each of whom gets utility from one or both of the private goods z_a, z_b, as well as the composite private good x. It follows from all this that $G = (n_0 + n_a)g_a + (n_0 + n_b)g_b$ for any g chosen by the ultimate office-holder. Now let z_s^t denote the quantity of z_s consumed by an individual of type s, since individuals will be in a position to purchase these goods privately if they are not provided publicly, and to augment any public provision with further private purchases. So, $z_s^t = g_s^r + q_s^t$, where g_s^r is the amount of z_s provided publicly when the officeholder is of type r, and q_s^t

[4]Clearly meeting an income-test in order to merit public provision is not a possibility here, but is an issue to be analyzed in a model with income heterogeneity

is the private purchases of that good by a type t individual[5].

We assume that the marginal cost and market price of both z_s are 1. It is easy to incorporate price differentials across goods into the model, however doing so does not alter any of the results derived below. Given that only those who get utility from a publicly provided good will demand it, and all such individuals will do so, the per-capita cost of providing the amount g_s of z_s freely to all who want it is $(\frac{n_s+n_0}{n})g_s$, and this is therefore also the per-person tax that will be levied to finance g_s. This means that the effective price of public provision is $p_s \equiv (\frac{n_0+n_s}{n}) < 1$ for good z_s, and this is lower the greater is the proportion of n who do not consume z_s.

A type $t \in \{a, b\}$ individual will, if elected to office, provide good $s = t$ in the amount

$$g_s^s = \arg\max_{g_s}\{v(w - p_s g_s) + m_s(g_s)\}$$

Because any further private purchases of s by anyone other than himself have no effect on the officeholder, and public provision is at a lower price than private provision, this is in fact what a type s office-holder will choose in any equilibrium. It follows then that the tax/person imposed by a type t office holder will be $\tau_t = p_t g_t^t$, for $t = a, b$. The type 0 office-holder will provide the two goods publicly in the amounts g_s^0, where

$$g^0 = \arg\max_{g}\{v(w - p_a g_a - p_b g_b) + m_a(g_a) + m_b(g_b)\},$$

and it follows immediately that per-capita taxes will be $\tau_0 \equiv p_a g_a^0 + p_b g_b^0) > p_s g_s^s \equiv \tau_s > p_s g_s^0$.

It is also easy to show that when any good is publicly provided by any type office-holder, that good is not purchased privately by anyone, given the lack of income heterogeneity. However, when a type a office-holder provides g_a^a publicly, both type b and 0 individuals purchase z_b privately in the amount $q_b^a = \arg\max_{q_b}\{v(y_a(g_a^a) - q_b) + m_b(q_b)\}$. Since $y_a < w$ and z_b is assumed to be normal, it follows that q_b^a is less than the amount of z_b that type b individuals would consume if z_a consumption were not publicly financed. Denoting this amount by z_s^s for each s, then, and letting z_s^0 be the private consumption of

[5]It is important that we have assumed here that it is not possible to re-sell the publicly provided good. Were this not so, those who get no utility from it could nonetheless acquire the amount that the government provides and resell it to those who believe that the government supplied amount is too little. The availability of this option would, in turn affect the levels of public provision set by office-holders.

z_s by a type 0 citizen if there is no public provision of any good, it is easy to demonstrate that;

$$g_s^s > z_s^s,\; g_s^s > g_s^0$$

As to the choices of a type 0 office-holder, it is possible that $g_s^0 < z_s^0$ for one s, even though the 'prices' of both goods are lower when purchased through the tax system. This depends on the relative sizes of the sub-populations n_a and n_b, since this determines the prices p_a, p_b. For the same reason, it is not possible to say in general whether a type 0 individual will spend more on the two goods together if he is the office-holder than he would spend if he had to buy them privately.

The key issue, as in the public good case, has to do with the preferences of citizens regarding the different type of candidates. Again it is immediate that each citizen type regards a candidate of the same type as optimal. Further, it is still true that a citizen of type A or B regards a type 0 candidate as the second-best option. The reasoning is similar to that in the public good case. A type B office-holder will tax a type A citizen in the amount $p_b g_b^b$, and leave the citizen to purchase z_a at the market price. A type 0 office-holder will tax the same citizen in the amount $p_b g_b^0 < p_b g_b^b$, and then will provide him with an amount of z_a that the citizen regards as optimal, given its effective public price of $p_a < 1$, and that he has a disposable income of $w - p_b g_b^0$ remaining to spend on z_a. The second outcome is therefore clearly better for the A citizen. A similar argument holds for the type B citizens, hence it is once again true that the type 0 candidate represents a Condorcet winner.

The relative preferences of type 0 citizens for type A or B office-holders are dependent on the demographics of the population, but in a way that is more complex than for public goods. A type 0 citizen faced with a type A office-holder ends up with the bundle $(z_a, z_b, x) = (g_a^a, q_b^0, w - p_a g_a^a - q_b^0)$. The composition of the population enters through the public price variable p_a. Both g_a^a and q_b^0 depend on it, the latter only through the effect of changes in p_a on the income that can be spent on z_b. The effect of changes in p_a on the equilibrium utility of a type 0 citizen living under a type a office-holder is given by:

$$\frac{\partial U_0(a)}{\partial p_a} = \frac{\partial g_a^a}{\partial p_a}[m_a' - p_a v'] - v' g_a^a. \tag{10}$$

Because both z_s are assumed normal, $\frac{\partial g_a^a}{\partial p_a} < 0$ and of course $v' > 0$. The term in brackets, however, is also negative, since it is being evaluated at the

A - officeholder outcome, ω^a, and

$$m_a'(g_a^a) = p_a v'(w - p_a g_a^a) < p_a v'(w - p_a g_a^a - q_b^0)$$

Moreover, one can re-write (10) as:

$$\frac{\partial U_0(a)}{\partial p_a} = \frac{\partial g_a^a}{\partial p_a} m_a' - v'[g_a^a + p_a \frac{\partial g_a^a}{\partial p_a}]$$

which is negative if the demand for z_a by a type A citizen (i.e., for g_a^a) at ω^a, given p_a, is price-inelastic. (It may of course still be negative if that demand is not too elastic).[6] In this event, type 0 citizens will again display a relative preference for the candidate type that is a minority, since an increase in the number of type A citizens at the expense of the number of type B citizens has the effect of raising p_a and lowering p_b, for example.

In any case, it follows that the preferences of citizens over office-holder types have the same form as in the case of public goods, and therefore the set of voting equilibrium outcomes is the same. A type 0 office-holder will be elected, and will publicly provide both public goods, unless type A or B citizens constitute a majority (and perhaps even then).

4 Conclusions

This paper is a first step in the development of a political-economic equilibrium model which determines which of a set of public or private goods are to be publicly provided. The result that there is a propensity to provide both of a pair of public or private goods through the tax system, even if the set of individuals who consume both of them is small, is at first surprising. It is less surprising if one realizes that in that case a majority of the population does get utility from each good, separately. However, it is possible that both goods are publicly provided even when that is not the case, and this seems to hinge on the fact that office-holders willing to provide both goods represent a reasonable compromise for those who consume only one of them.

[6] Further, there is an argument for this elasticity being between 0 and -1. The price and income elasticities for z_a must sum to 0, and if z_s and x are all normal, then the price elasticities must sum to something in the interval $]-1, 0[$, hence with each being negative, each must be in that interval. In what follows price-inelasticity will therefore be assumed.

There are many extensions that seem worth thinking about, the first of them being to introduce heterogeneity in incomes. This has proved a useful device for generating predictions in models that consider public provision of only a single good, but the point here was to see what could be said in an environment that allowed minimal scope for re-distributive concerns. Such concerns are undoubtedly part of the motivation for public provision schemes, however, so introducing them is important in any model designed to help us understand the observed heterogeneity in the sets of goods that are publicly provided in different jurisdictions and at different times. A second, seemingly simpler extension is to introduce set of citizens who get utility from neither of the two goods. Citizens with these preferences have been absent from the one-good models of public provision, generally, also. For some pairs of goods this omission may be warranted, as it seems reasonable that there are no citizens who get utility from **neither** health care nor education, for example. For other pairs of goods for which this is not the case, however, the existence and relative numbers of such individuals is logically be an important influence on political decisions about public provision.

5 Appendix

Proof of Lemma 1

(i) If $d_s^0 > 0$, then $v'\left(y - d_a^0 - d_b^0\right) = m_s'\left(g_s + D_s\right)$, while $d_s^s = 0$ requires $v'(y) \geq m_s'(g_s + D_s)$ and this violates the fact that $v'' < 0$.

(ii) If $d_s^0 = 0$, then $v'\left(y - d_{\sim s}^0\right) \geq m_s'\left(g_s + D_s^s\right)$, where $d_{\sim s}^0$ is the contribution to 'not s'. However, $d_s^s > 0$ requires that $m^v(y - d_s^s) = m_s'(g_s + D_s^s)$ so that $v'' < 0$ implies that $d_{\sim s}^0 \geq d_s^s$.

■

Proof of Lemma 2

By (i) of Lemma 1, if this were to occur, it must be that all type s citizens are also each donating to s. It must therefore be true that, if y is the level of after-tax income of all types, the result of this taxation plus donations satisfies:

$$v'(y - d^0) = v'(y - d^s) = m_s'(g_s + D_s), \text{ for } s = a, b. \tag{11}$$

This in turn implies that in fact the office-holder could have imposed taxes that were higher by the amount d^t per capita, and publicly financed the same

$z_s = g_s + D_s$, since (11) implies that all the d^t are equal. Then, if this office-holder were a type 0, he could in fact increase funding to each of the two goods by a further small amount ε, increase taxes by the amount $2\varepsilon/n$ and this would have a total effect on his utility of $-v'(y-d^0)\frac{2\varepsilon}{n}+\varepsilon(m_a'(z_a)+m_b'(z_b))$ which is positive if $n > 1$, by (11). If the office-holder were a type A, then he could increase funding to only z_a by a small ε beyond this level, and a symmetric possibility exists for a B office-holder. So, in all cases the office holder has available a tax package that makes him better off in the absence of donations, and since donations are voluntary and non-negative, he cannot be made worse off by any that do occur afterwards.

■

Proof of Prop. 3

Lemma 3 already establishes that she will not choose a g that leads her to donate to both goods herself. Suppose then, that she donates only to z_a, which means that the type A citizens donate also, by Lemma 2(i), and both types of citizens donate the same amount, d_a'. It also follows that $d_a' \geq d_b^b$, from Lemma 2(ii), with a strict inequality if type b citizens don't donate at all. This implies that the office-holder could instead have levied taxes in an additional amount

$$\frac{n_b d_b^b + (n_0 + n_a)d_a'}{n} \tag{12}$$

on each citizen and achieved the same level of total funding only through taxes. Since $d_a' \geq d_b^b$, it follows that this leaves the type 0 with no less disposable income, and so no worse off than in the supposed equilibrium. However, he could increase the level of funding by an amount ε to $z_a$only, and levy commensurately higher taxes of $\frac{\varepsilon}{n}$, with the total effect on his utility of $-v'(y-d_a')\frac{\varepsilon}{n}+m_a'(z_a)\varepsilon$. If the type 0 were giving to z_a and not to z_b, then it must be that: $v'(y-d_a') = m_a'(z_a)$ so this must increase his utility even if there are no ex-post donations. (Note that the level of z_b cannot drop, as it is completely tax-financed.) The office-holder cannot then be made worse off by any ex-post donations that do occur, so this strategy is superior for him to one which results in his donating to z_a.

This leaves then only the possibility that other citizens donate privately when type 0 citizens do not. Suppose then it were true that type A citizens give privately to z_a in the amount d_a following the choice of g^0 by the type 0 candidate. Then it would have to be true that $v'(y(g^0) - d_a) = m_a'(g_a^0 + n_a d_a) \leq v'(y(g^0))$ which requires that $y(g^0) - d_a \geq y(g^0)$, which cannot be.

■

Proof of Prop. 5

Suppose, bwoc, that $g_a^a \le g_a^0$, for example. Then since $\frac{1}{n}v'(w - \frac{G^0}{n}) = m_a'(g_a^o)$, and $\frac{1}{n}v'(y - \frac{g_a^a}{n}) = m_a'(g_a^a)$, it must be that, with $m_a'' < 0$ that $\frac{1}{n}v'(w - \frac{G^0}{n}) \le \frac{1}{n}v'(y - \frac{g_a^a}{n})$, and then $v'' < 0$ implies that $g_a^0 < G^0 \le g_a^a$, a contradiction.

As to the second claim, using obvious notation, we know that:

$$\frac{1}{n}v'(y^0) = m_s'(g_s^0), \text{ for } s = a, b \tag{13}$$

and

$$\frac{1}{n}v'(y^b) = m_b'(g_b^b) \tag{14}$$

Suppose then that $G^0 \le g_b^b$, which would imply that $y^0 \ge y^b$, and so $\frac{1}{n}v'(y^b) \ge \frac{1}{n}v'(y^0)$ which in turn implies that $m_b'(g_b^b) \ge m_b'(g_b^0)$, and therefore $g_b^b \le g_b^0$, contradicting the first part of this result. ■

Proof of Prop. 6

Suppose, bwoc, that $D_a^+ \ge g_a^0$, for example. Than it is true that

$$\frac{1}{n}v'(w - \frac{g^0}{n}) = m_a'(g_a^0) \tag{15}$$

and:

$$v'\left(w - \frac{g_b^b}{n} - d_a^+\right) = m_a'(D_a^+). \tag{16}$$

The assumption would mean that $m_a'(D_a^+) \le m_a'(g_a^0)$, since $m_a'' < 0$, and this in turn implies that $v'(w - \frac{g_b^b}{n} - d_a^+) \le \frac{1}{n}v'(w - \frac{G^0}{n}) < v'(w - \frac{G^0}{n})$. So $v'' < 0$ implies then that $\frac{g_b^b}{n} + d_a^+ < \frac{G^0}{n}$, and multiplying both sides by n, gives $g_b^b + nd_a^+ < g_a^0 + g_b^0$, and Prop. 1.3 implies $g_b^b > g_b^0$, so this implies $D_a^+ = (n_0 + n_a)d_a^+ < nd_a^+ < g_a^0$, a contradiction. The proof for b is analogous. ■

Proof of Prop.9

Let r be the vector of I, O choices, let $C(r)$ be the resulting set of candidates, and $c(r)$ be the number of candidates. Let $W(r, v)$ be the set of candidates who are tied for the lead, given, $r \in \{I, O\}^n$ and the citizens' profile of voting strategies, $v \in C(r)^N$. Further, let $k(r) = \#C(r)$ and

$w(r,v) = \#W(r,v)$. Recall that by assumption, each candidate in $W(r,v)$ wins with probability $1/w(r,v)$.

First note that there are equilibria in which all candidates in $W(r,v)$ are of the same type. In particular, suppose that $C(r)$ consists of $k(r) \geq 3$ candidates of a single type. Then citizens can divide their votes up so as to have some subset of size $k(r) - 1$ of them tied for the lead, and any v that produces this result is a voting equilibrium, since no voter can change the outcome by changing their vote.

Suppose then for the remainder of the proof that $W(r,v)$ contains at least two types of candidates.

Remarks: When this is the case, then all citizens must be voting for a candidate in $W(r,v)$, since if they are not, they can change their vote to a candidate in $W(r,v)$ and guarantee she wins for sure by doing so. Since citizens are not indifferent between outcomes, this has to improve the outcome for the switching citizen, from a nondegenerate lottery over $W(r,v)$ to one in which their most-preferred type of candidate in $W(r,v)$ wins for sure. Further, each citizen must be voting for a candidate of the type they most prefer in the set $W(r,v)$, since again, if they are not, they can guarantee an outcome they prefer by switching their vote to a candidate in $W(r,v)$ of their most-preferred type.

Claim: There cannot be more than one candidate of the same type in $W(r,v)$.

Proof: Suppose, bwoc, that there are two or more candidates of type τ in W. Then all citizens of type τ must be voting for one of the type τ candidates, by the remarks above. However, this means any type τ citizen can switch their vote to a different type τ candidate and guarantee that candidate wins. Doing so results in outcome ω^τ for sure, which is a better outcome than any lottery in which the probability of outcome ω^τ is less than one. This proves the claim.

This now implies there can't be more than one candidate of any type in $W(r,v)$, so $w(r,v) \leq 3$. If $w(r,v) = 3$, this implies they must all be of different types, which can only be an equilibrium outcome if $n_t = n/3$ for all t, since the Remarks imply all citizens vote for their most-preferred type in $W(r,v)$. If $w(r,v) = 2$, then again the Remarks imply all citizens vote for their most-preferred type, meaning one candidate gets the votes of only citizens of their own type. Thus, a tie requires that $n_t = n/2$ for some type t.

The remaining possibility is that $W(r, v)$ contains candidates of a single type.

■

Proof of Prop. 10

We are interested in determining what are the possible pairs $(r, v) \in \{I, O\}^n \times C(r)^n$ that can arise in a sub-game perfect equilibrium, given the equilibrium continuations that follow from the election of each type of candidate . We will refer to such a pair (r, v) as a 'political equilibrium'. Any such equilibrium produces a 'winner', or a set $W(r, v)$, as above. However, the term winner will refer only to a candidate who wins the election with probability 1 in the political equilibrium.

We proceed by proving a series of results.

LA0: There are no political equilibria in which a type B candidate wins.

Proof: Suppose a B candidate does win. Then no type A's are in $C(r)$, since the outcome cannot be worse for them if they choose O instead. Then $C(r)$ must consist only of type 0 and B candidates, which means all type 0 candidates prefer to exit for the same reason. Thus only B candidates can have chosen I, but then a type A or 0 candidate could choose I, get the votes of all type A and 0 citizens, and win for sure, since $n_a > n/2$.

■

LA1. If in any political equilibrium $C(r)$ consists of one type of candidate, it must be type A.

Proof: If all candidates are of either of the other type, a type A candidate could choose I and win for sure, and will increase her payoff by doing so. ■

CA1. If $k(r) = 1$, then the single candidate must be a type A.

LA2. Assume that for all t we have $n_t \neq n/2$. Then there are no political equilibria in which $k(r) = 2$.

Proof: If $k(r) = 2$ and both candidates are of the same type, it must be type A by CA1, but then each of them is better off choosing O, so this cannot be a political equilibrium. If the candidates are of different types, assuming $n_t = n/2$ cannot hold implies one of them must lose for sure, and is therefore better off choosing O.

■

LA3 Assume it is not the case that $n_t = n/3$ for all t. Then there are no political equilibria in which $k(r) = 3$.

Proof: Suppose first that $C(r)$ contains a single type A candidate, and she wins the election. Then there can be no type B candidates in $C(r)$, as they are better off choosing O. This means there can be no type 0 candidates either, since they are better off choosing O. So a single type A candidate cannot win for sure in equilibrium, which means some type A citizens must not be voting for the single type A candidate, since $n_a > n/2$. Given $A1$, this can only happen if the other two candidates are of type 0 and B, so that some of the type A citizens are voting for the type 0 candidate.

Suppose then that this is the case, and the type 0 candidate wins. However, if the type 0 candidate wins, the type A candidate is better off choosing O, since the type 0 candidate must necessarily win in the resulting 2-candidate political equilibrium, as she will get all the votes of the type A and 0 citizens, and the type A candidate saves c.

Then if there is a single type A candidate in $C(r)$, it must be that the type B candidate wins, and since she gets only the votes of the type B citizens, she must beat both other candidates by more than one vote. However, then again the type A candidate will prefer to choose O, since the type 0 candidate will necessarily win in the resulting voting equilibrium.

Thus, $C(r)$ cannot contain a single type A candidate if $k(r) = 3$.

Suppose then that $C(r)$ contains two type A candidates. Then both of them would prefer to choose O, since the remaining type A must win in the resulting $k = 2$ voting equilibrium, and they save the cost, c.

We already know there are no $k = 3$ political equilibria in which three candidates of a single type enter, so the only remaining possibilities are two type 0 and one type B or the opposite. In the second case, the type 0 gets the votes of all type 0 and A voters, thereby winning for sure, so the type B candidates prefer O. If the first holds, then both type 0 candidates prefer O, as this guarantees a type 0 wins, and saves them from paying the cost c.

■

LA4 In any equilibrium with $k(r) \geq 4$, it must be that a type 0 candidate wins.

Proof: LA0 implies a type B cannot win, and Prop. 9 implies $W(r, v)$ is a singleton. Suppose then, bwoc, that a type A wins. Since things cannot be worse if a type B leaves, there must be no type B candidates in $C(r)$.

However, the same reasoning implies there are no type 0 candidates in $C(r)$, and we know there cannot be 4 type A candidates, so if there is such a political equilibrium, it must be won by a type 0.

■

Proof of Prop.11

Now suppose that $n_b > n/2$, but it is still true that $\omega^a \succ_0 \omega^b$. Again, we proceed by proving a series of results.

LB0 There are no equilibria in which a type A candidate wins.

Proof: Suppose this is the case. Then no type B citizens choose I, as the outcome cannot be worse if they choose O instead. There must therefore be only type A and 0 candidates in $C(r)$. If a type A candidate wins in this case for sure, however, a type 0 candidate is better off choosing O, so there can only be type A candidates in equilibrium, but then a type B citizen who chooses I will win for sure.

■

It is obvious that the only political equilibrium (r, v) with $k(r) = 1$ has a single type B candidate enter, and that this is always an equilibrium.

LB2 Assuming $n_t \neq n/2$ for all t, then there are no political equilibria with $k(r) = 2$.

Proof: Since there can be no ties, by Prop. 9, the losing candidate will always prefer to choose O rather than I.

■

LB3 Assuming that it is not the case that $n_t = n/3$ for all t, then there are no political equilibria in which $k(r) = 3$.

Proof: Suppose first that $C(r)$ contains a single type B candidate, and she wins the election. Then there can be no type A candidates in $C(r)$, as they are better off choosing O. This means there can be no type 0 candidates either, since they are better off choosing O if the B candidate wins. So a single type B candidate cannot win for sure in equilibrium, which means some type B citizens must not be voting for another candidate, since $n_b > n/2$. Given **A1**, this can only happen if the other two candidates are of type 0 and A, and some of the type B citizens are voting for the type 0 candidate.

Suppose then that this is the case, and the type 0 candidate wins. However, if the type 0 candidate wins, the type B candidate is better off choosing O, since the type 0 candidate must necessarily win in the resulting 2-candidate political equilibrium, as she will get all the votes of the type B and 0 citizens, and the type B candidate saves c.

Then if there is a single type B candidate in $C(r)$, it must be that the type A candidate wins, and there is a type 0 candidate in $C(r)$, also. However, if the type B candidate instead chooses O, all the type B citizens vote for the type 0 candidate in the resulting voting equilibrium, and since $n_b > n/2$, the type 0 candidate wins. Thus, the type B candidate must prefer to choose O.

Thus, $C(r)$ cannot contain a single type B candidate if $k(r) = 3$.

Suppose then that $C(r)$ contains two type B candidates. Then both of them would prefer to choose O, since the remaining type B must win in the resulting $k = 2$ voting equilibrium, and they save the cost, c.

We already know there are no $k = 3$ political equilibria in which three candidates of a single type enter, so the only remaining possibilities are two type 0 and one type A, or the opposite.

If the first holds, the type 0 candidates will both prefer to choose O, since then the remaining type 0 wins for sure with the votes of all type B and 0 citizens, and the cost c is saved

If the opposite holds, the single type 0 wins for sure with all the votes of the 0 and B citizens, so the type A candidates prefer O to I.

■

LB4 In any equilibrium with $k(r) \geq 4$, it must be that a type 0 candidate wins.

Proof: LA0 implies a type A cannot win, and P10 implies $W(r, v)$ is a singleton. Suppose then, bwoc, that a type B wins. Since the outcome cannot be worse for a type A, any type A in $C(r)$ prefers O, so there must be no type A candidates in $C(r)$. However, the same reasoning implies there are no type 0 candidates in $C(r)$, and we know there cannot be 4 type B candidates, so if there is such a political equilibrium, it must be won by a type 0.

■

Example: A political equilibrium in which a type 0 candidate wins for sure, when $n_b > n/2$, and $n_b \leq 2n/3 - 1$.

Let r be such that $C(r) = \{0, A, B_1, B_2\}$ and let $v \in C(r)^N$ be such that $V_0(v) > V_j(v) + 1$ for all $j \in \{A, B_1, B_2\}$. Since type A and B citizens can vote for the type 0 candidate, this is a possible voting equilibrium, with the type 0 candidate winning.

We again specify that any other entering citizen gets no votes, and it is again clear that if the 0 candidate leaves he gets a worse outcome.

If the type A candidate chooses O instead of I, we specify a continuation voting equilibrium v' such that all type B citizens vote for candidate B_1, which guarantees that candidate wins.

If either type B candidate chooses O, we specify a continuation voting equilibrium v'' such that all citizens of type A and 0 vote for the A candidate, so $V_a(v'') = n_a + n_0 = N - n_b$, while the type B citizens split their votes equally between the O and remaining B candidate. Given our assumption on n_b, this guarantees that the type A candidate wins, so the type B candidates prefer I to O.

This (r, v) is therefore a political equilibrium. We can construct political equilibria (r^k, v^k) with $k(r) > 4$ by adding more candidates of every type, and specify continuation voting equilibria after deviations by each type of candidate similar to the above. If there is more than one type 0 candidate in $C(r^k)$, then we assume only one of them gets any votes in v^k and if any of them leaves, all type B citizens vote for one of the type B candidates, who then necessarily wins.

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