Providing Managerial Incentives: Do Benchmarks Matter?*

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First draft: April 1998
This draft: October 2000

Abstract

In this paper we study delegated portfolio management when the manager's ability to short-sell is restricted. Within this constrained framework we ask whether performance-adjusted contracts provide portfolio managers with adequate incentives to gather information. We show that when portfolio managers are constrained in their ability to short-sell, performance-adjusted or *linear* 'benchmarked' contracts can provide incentives for gathering more precise information. The risk-averse manager's optimal effort is an increasing function of her share in the portfolio's return. The first-best, purely risk-sharing contract is shown to be suboptimal when the leverage capacity of the manager is bounded. Using numerical methods we show that under the optimal contract, the manager's share in the portfolio increases as the restriction to short-sell becomes tighter. When the constraint is relaxed the optimal contract converges towards the first-best risk sharing contract.

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*We thank comments from Alexei Goriaev, Ernst Maug, Neil Stoughton and seminar participants at the 1999 SED meetings in Sardinia, the 1999 Workshop in Mutual Fund Performance at EIASM, Brussels, the 2000 EFA meetings in London, the Bank of Norway and the Stockholm Schools of Economics. The usual caveats apply. Address all correspondence to Juan-Pedro Gómez, Norwegian School of Management, Elias Smiths vei 15, Postboks 580, N-1302 Sandvika, Norway. Phone:+47-6755 7398. Fax:+47-6755 7675.

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1 Introduction

In this paper we study delegated portfolio management under the condition that the manager's ability to leverage her portfolio (shorting risky assets or borrowing at the risk-free rate) is restricted. We analyze the implications of this restriction in the context of an agency relationship where an investor delegates her portfolio decision to a manager who has superior, private information on stock returns. The manager can improve the "quality" (meaning precision) of her information by putting in more effort. This effort decision, which is costly for the manager, is not observable to the investor; hence moral hazard arises.

Under the above mentioned scenario, we study the efficacy of providing portfolio managers with linear contracts. These contracts have a fixed payment component and a component which depends on the performance of the portfolio relative to some subset of the market portfolio. Our paper contributes three results which pertain to optimal linear contracts. First, linear performance-adjusted contracts are shown to provide the manager with adequate incentives for gathering better information. Second, the manager's share in the portfolio return is shown to be different from the first best. Third, using numerical methods we show that the manager's share in the portfolio increases as the leverage constraint becomes tighter.

1.1 Motivation and Intuition.

Investors delegate portfolio decisions to managers for a number of reasons. These include better risk diversification, lower transaction costs, provision of customer services (like record keeping) and security selection. Arguably, active managers charge considerably higher fees than index funds on the basis of their alleged superior information.[1] Portfolio managers devote time and resources to build this informational advantage. The higher the quality of the manager's information the higher the risk-adjusted expected return on the investor's portfolio. When the manager's activity is not fully observed by the investor, it is in the investor's interest to provide the manager with adequate incentives to gather better information.

Intuitively, one may argue that managers can be induced to gather better information by linking their rewards to the portfolio's performance. Perhaps because of such considerations, in 1985 the Security Exchange Commission (SEC) allowed the use of relative performance-adjusted fees to compensate managers.[2] According to this

[1] We leave aside the question of whether active management leads to *ex-post* superior performance (for example see Gruber (1996)).

[2] Amendment to the Section 205 of the Investment Advisers Act. In 1986, the Department of

regulation, the fee must be a "fulcrum" fee where the incentive (penalty) component rises or falls symmetrically with the performance of the fund. Additionally, performance must be measured against an appropriate independent index, rather than in absolute terms. In this paper we study the efficacy of such linear contracts and ignore the more theoretical question of contract optimality in the class of all contracts.

The above mentioned intuitive arguement about the efficacy of linear contracts has been challenged. Stoughton (1993) and, more recently, Admati and Pfleiderer (1997) have shown that linear performance-adjusted contracts fail to affect the manager's decision to gather information. In other words, the manager's optimal effort choice is independent of the contract she receives from the investor. Stoughton studies linear "raw" performance-adjusted contracts while Admati and Pfleiderer analyze contracts that include relative or benchmarked performance fees. The *non-incentive* result arises regardless of whether performance is measured in absolute or relative terms. If these results are to be true then there exist strong arguments to do away with performance-adjusted contracts. For example, there is a belief amongst practitioners that performance-adjusted contracts induce managers to invest in similar portfolios (at the cost of diversification) and this at times may be responsible for large market movements (see for example Maug and Naik, 1995). We, however, believe that the case against performance-adjusted contracts should not be closed without having a closer look at the genesis of the *non-incentive* result.

Both Stoughton (1993) and Admati and Pfleiderer (1997) assume that the manager's portfolio opportunity set is unbounded. In this unrestricted scenario, the manager's effort and the investor's optimal contract turn out to be independent decisions. We believe that the assumption of the manager's opportunity set being unbounded is unrealistic. The standard contract for portfolio managers usually includes a number of restrictions like, for instance, mandates on style (the type of securities they can invest on), short-selling constraints or limits to the portfolio's tracking error volatility. Moreover, in many countries, institutional investors, like pension plans or insurance companies, are legally constrained in their ability to short-sell. Many of these restrictions follow due to bankruptcy concerns. Also from a strategic perspective, one should at least allow for price changes as the manager borrows or lends large amounts. Our assumption is a simplification of this scenario. Assuming that managers' opportunity sets are bounded implies that managers face infinite prices after borrowing over a certain limit.

In this paper, we assume that the manager is bounded in her ability to short-sell (hence restricted in her portfolio choice). As a first cut on the problem, we take this bound as exogenously given in the same way as bankruptcy constraints are exogenously set in the literature on optimal contracts. As long as this bound is finite, i.e. it is not infinite, we show that the manager's effort is an increasing function of the contract's incentive fee (the percentage she gets to retain from the portfolio's return). Our paper generalizes the *non-incentive* result in the sense that, the non-incentive

Labor approved the use of performance fees for ERISA-governed pension funds.

result occurs as a special case when the short-selling constraints tend to infinity in the limit.

The intuition behind our result is as follows: In forming her optimal portfolio, a manager whose reward depends on a benchmarked compensation scheme takes into account her incentive fee. In the unrestricted case, a change in this incentive fee would not change her expected wealth because she can appropriately change the composition of her portfolio by borrowing at the risk-free rate or shorting risky assets. As a result, a change in her share of the portfolio's return cannot change any of her actions. However, when she is constrained in her ability to short-sell, she may no longer be able to form the portfolio she desires. Constrained in such a manner, the manager can increase her expected wealth only by increasing the precision of her signal (i.e. by reducing the variance of her returns). In our model, the manager can increase signal precision by putting in more effort. The extent to which she will want to expend more effort in gathering more precise information will depend on her compensation scheme, her risk aversion and her marginal disutility of effort.

We also study the investor's optimal contract problem. Under moral hazard, when the manager is unconstrained in her ability to short sell (the second best case), the manager's share in the optimal portfolio is proportional to the manager's risk-tolerance as a percentage of the overall (hers plus the investor's) risk-tolerance. Since, in this case effort is independent of the contract, the only role for the incentive fee is to *efficiently* share risk between the principal and the agent. But, when the manager is short-selling constrained we show that this "second best" sharing rule is suboptimal.

As it is common with moral hazard problems, it is difficult to provide an analytical solution for the optimal contract when the manager is constrained (the third best case). Using numerical methods, we show that under the optimal contract in this third best scenario the manager's share in the portfolio increases when the restriction becomes tighter. This can be interpreted as follows: In the third best scenario, the incentive fee plays an additional role beyond risk sharing, namely effort inducement. When the short-selling bounds shrink (making the restriction tighter) the volatility of the portfolio decreases as well since fewer extreme portfolios are feasible. If the investor does not increase the incentive fee the manager will be under-exposed to active management risk. As a consequence, effort will also decrease. The risk sharing and the effort inducement arguments are aligned in the same direction: the optimal incentive fee increases.

1.2 Related Literature.

Our point of departure is Bhattacharya and Pfleiderer (1985) who show that linear contracts can indeed be used by investors to extract private information from portfolio managers. Bhattacharya and Pfleiderer (1985) assume that managers could be endowed with better information *ex-ante*. But what incentives do managers have to collect this better information? Stoughton (1993) asked whether linear contracts could provide managers with enough incentives to collect better information. He, as well as Admati and Pfleiderer (1997), answered in the negative. We contend that this negative result is crucially dependent on the assumption of the manager being unconstrained in her ability to short-sell.

Our paper provides a rationale for the existence of linear benchmarked contracts. Hence we provide a micro foundation for several papers which study the implications such contracts. Maug and Naik (1995) show that relative performance evaluation may induce herd like behavior among portfolio managers endowed with private information on stock returns. Following on the seminal work of Brennan (1993), Gomez and Zapaptero (1999) show that benchmarked contracts induce portfolio managers to increase their active management risks in order to beat the benchmark. In equilibrium, this is reflected as an additional source of systematic risk priced by the market. These papers highlight the negative effects of linear benchmarked contracts. Our paper highlights a positive aspect. Future research could evaluate the net effect. On a related note, Cuoco and Kaniel (1998) compare the implications for equilibrium volatility and portfolio turnover of fulcrum and asymmetric performance-based contracts. They conclude that there are not significant differences between them. Das and Sundaran (1998) compare these two types of contracts on the basis of Pareto efficiency and volatility. They find little support for the current regulation on the compensation of mutual fund managers insofar as asymmetric fees provide Pareto-dominant outcomes with lower level of equilibrium volatility. Carpenter (2000) shows that asymmetric, option like compensation schemes do not necessarily lead to greater risk-seeking when the risk-averse manager cannot hedge her incentive fee.

Our work also contributes to a growing literature on portfolio restrictions in the context of performance-adjusted contracts. Grinblatt and Titman (1989) study the need for restrictions like caps, penalties, and covenants on the manager's "personal" portfolio to mitigate the adverse risk incentives inherent in performance-adjusted fees. If not corrected, these incentives would lead the manager to leverage her portfolio in her own benefit above the "optimal" target level set by the client.[3] Our focus, however, is on effort incentives. Thus, portfolio restrictions are directly modeled in the form of a leverage target (an upper bound) fixed exogenously. This bound could possibly be set by the regulator to limit the manager's risk exposure.[4]

More recently, Dybvig, Farnsworth and Carpenter (2000) attempt to characterize the basic features of an optimal contract in the presence of moral hazard concerns. Their paper is more general in scope as they attempt to endogenize constraints on the manager's strategy (through truthful revelation of private information) as a part of the optimal contract. As they note, this is a difficult task because the necessity and sufficiency of such constraints are difficult to establish. Moreover, their optimal

[3]Interestingly, Grinblatt and Titman also warn against the incentives to misuse of private information ("front-running" strategies) embedded in this type of contracts.

[4]As it will be shown in Section 3, our restrictions on short-selling can be easily expressed as a bound on the portfolio's tracking error volatily, a very common restriction for portfolio managers

mechanism might induce extremely large punishments in some states (which occur with very small probabilities). Such large punishments may not be credible due to bankruptcy constraints. We, on the other hand, assume that managers are exogenously constrained in their ability to short sell. The assumption is implied (not by truthful revelation concerns but) by bankruptcy constraints for example. It also simplifies the problem and allows us to explicitly deal with the sensitivity of the manager's effort decision to changes in the manager's share in the portfolio.

1.3 Organization.

Our paper is organized as follows. Section 2 introduces the model and presents the standard, unrestricted second best results. The manager's effort function is shown to be "flat" in the incentive fee. Thus, the first best allocation of risk prevails. The assumption of limited leverage is introduced in Section 3. Effort is shown to be increasing in the incentive fee. Section 4 shows that, under limited leverage, the first best risk sharing rule is suboptimal. Section 4.1 presents numerical results on the optimal contract under limited leverage. Optimal contract variables are estimated for different values of the short-selling restriction and risk aversion coefficients. Section 5 concludes.

2 The model

There are two players: an investor (who represents, for instance, the board of a pension or mutual fund) and a portfolio manager. The investment opportunity set consists of two assets: a risky asset with gross rate of return $\tilde{x}$ and a bond paying a riskless gross rate of return R. The bond can be interpreted as a benchmark portfolio against which the returns on the manager's portfolio are measured.[5] The distribution of the risky asset return and the return on the bond are public information. The investor is interested in active management. That is, she is willing to pay for valuable private information she cannot learn herself.

There is only one consumption good in this economy which is taken as the "numeraire." The investment horizon is one period. At the beginning of the period (t_0) the investor is endowed with a unit of wealth. The manager has no wealth. There is no present consumption; all the consumption takes place at t_1. At t_0 the investor transfers her wealth to the manager. The investor also offers a linear performed-adjusted contract (α, F) to the manager. According to this contract the manager's fee consists of a percentage $\alpha \in (0, 1)$ of the return on the portfolio she manages on behalf of her client, the investor. Together with this performance-adjusted component, the contract may include a (performance-free) wage F. If the manager refuses

[5] Assuming a riskless benchmark portfolio does not affect our qualitative results. Alternatively, see Ou-Yang (1999) for a justification of the riskless asset as the optimal benchmark.

the contract the game ends and she receives her reservation value (normalized to -1). If she accepts the contract, she puts in some effort e which results in a signal $\tilde{y}$. Given $\tilde{y}$ she forms a portfolio. The real state of the world unveils in t_1 and the contract is honored.

To simplify the notation (and without loss of generality) let us assume that $\tilde{x}$, the gross rate of return on the risky asset, is distributed as a standard normal variable. That is $\tilde{x} \sim \mathcal{N}(0,1)$. For the signal $\tilde{y}$, let,

$$\tilde{y} = \tilde{x} + \tilde{\epsilon},$$

with $\tilde{\epsilon}$ the noise term. We will assume $\tilde{\epsilon} \sim \mathcal{N}(0,\sigma^2)$, with $\sigma^2 < \infty$ such that higher σ^2 implies a less precise signal. Finally, the return on the risky asset and the noise term are assumed to be uncorrelated. Given the definition of the signal and the assumptions on $\tilde{x}$ and $\tilde{\epsilon}$, $\tilde{y}$ is normally distributed:

$$\tilde{y} \sim \mathcal{N}\left(0, \frac{1}{P}\right), \tag{1}$$

with $P = (1+\sigma^2)^{-1} \leq 1$ the signal precision. Given y, the manager's updated beliefs about the conditional (on the signal realization) distribution of the risky asset is given by Bayes rule as,[6]

$$\tilde{x}\,|\,y :\sim \mathcal{N}\left(Py, 1-P\right). \tag{2}$$

In (2), $1-P$ represents the active management risk conditional on the private information of the manager. Given the normality assumption it is independent of the signal realization. Clearly, when the precision increases the active management risk decreases.

Following Stoughton (1993), the precision of the signal is assumed to be an increasing function of effort e according to

$$e = \frac{1}{\sigma^2}.$$

This implies that the (posterior) precision of the manager's private signal is an increasing and concave function of effort. More concretely, since $P = (1+\sigma^2)^{-1}$, we have

$$P(e) = \frac{e}{1+e}. \tag{3}$$

If the manager puts no effort, $P(0) = 0$ and (2) will be distributed as $\tilde{x}$. By putting in more effort the manager improves the signal's precision P and hence reduces the active management risk. Finally, given (3) equation (2) can be re-written as follows:

[6] We will follow the standard notation whereby a symbol with a tilde on top will represent the variable and the same symbol, without a tilde, its realization. The vertical bar reads as "conditional to."

$$\tilde{x}\,|\,y :\sim \mathcal{N}\left(\frac{e}{1+e}y, \frac{1}{1+e}\right). \tag{4}$$

After, the manager receives the signal and incorporates it into the stock's return distribution according to (2), she chooses a portfolio allocation that maximizes her expected utility of wealth. Let θ denote the amount of money invested by the manager in the risky asset. Then, given the contract (α, F), the conditional wealth of the manager and the investor can be written as, respectively: [7]

$$\begin{aligned} \tilde{W}_a(y) &= F + \alpha\tilde{W}(y), \quad (5)\\ \tilde{W}_b(y) &= (1-\alpha)\tilde{W}(y) - F. \quad (6) \end{aligned}$$

where,

$$\tilde{W}(y) = R + \theta(\tilde{x}\,|\,y - R)$$

is the final return on portfolio θ conditional to y. As we will see later on, θ is a function of the signal, $\theta(y)$. Thus, θ can be interpreted as the proportion (on the investor's wealth) actively managed, that is, with the goal of beating the benchmark's return.

We will assume that the unconditional expected return on the risky asset and the bond's return are equal. That is, on average and without any informational asymmetry, the investor could get the same return as the manager: the return on the benchmark. Since the expected return on the stock normalized to zero, this implies that $R = 0$. So we have

$$\tilde{W}(y) = \theta\,\tilde{x}\,|\,y,$$

with conditional moments:

$$\begin{aligned} E(\tilde{W}(y)) &= \theta\,Py,\\ \mathrm{Var}(\tilde{W}(y)) &= \theta^2\,(1-P). \end{aligned}$$

The investor and the manager are assumed to have preferences represented by exponential utility functions:

$$\begin{aligned} U_a(x) &= -\exp(-a\,x + V(a,e)),\\ U_b(x) &= -\exp(-b\,x). \end{aligned}$$

[7] We will use $a > 0$ ($b > 0$) to denote the manager (investor) and the corresponding risk aversion coefficient.

With constant absolute risk aversion a, $V(a,e)/a$ represents the monetary value of the manager's disutility of effort e. We assume the function $V(a,e)$ is continuous and twice differentiable, with continuous derivatives. Moreover, the function is assumed to satisfy: [8]

S1 $V(a,0) = V'(a,0) = 0$

S2 $V'(a,e) > 0$ for all $e > 0$

S3 $V''(a,e) > \frac{1}{(1+e)} \times V'(a,e)$

Assumptions (S1) and (S2) are standard in the literature. Assumption (S3) sets a (lower) bound on the convexity of the manager's disutility function: the marginal disutility of effort must increase *fast enough*. This will guarantee the existence of an optimal effort level for the manager.[9] This assumption discards, for instance, linear disutilities. Any quadratic function of effort that satisfies (S1) and (S2) will verify (S3) as well.

2.1 Unrestricted portfolio choice.

In this section we will study the manager's portfolio and effort choice when her ability to short-sell is unconstrained. Given the assumptions in the model, the manager's portfolio choice is, ultimately, a decision on her optimal level of active management (as a percentage of wealth). The investor offers a contract (α, F) to the manager. The manager puts in effort and realizes a signal. Conditioned on this signal y, the manager solves her optimal portfolio problem:

$$\theta(y) = \arg\max_{\theta} E\left[U_a\left(\tilde{W}_a(y)\right)\right].$$

This problem assumes that the ability of the manager to short-sell/leverage her portfolio is unbounded. That is, there are no restrictions to short-selling the stock $(\theta < 0)$ or borrowing $(\theta > 1)$ at the risk-free interest rate. This is the usual assumption in the literature.[10]

Solving this problem, and writing the precision in terms of e, we obtain the optimal (conditional) portfolio of the manager:

[8] Prime ($\prime$) and double prime ($\prime\prime$) denote, respectively, first and second derivative with respect to effort.

[9] Assumption (S3) can be re-written as follows $(e > 0)$:

$$\frac{V''(a,e)e}{V'(a,e)} > \frac{e}{1+e} = P(e).$$

That is, as an upper bound to the signal's precision.

[10] Admati and Pfleiderer (1997) and Stoughton (1993). Heinkel and Stoughton (1995) impose the restriction $\theta \in [0,1]$ in a risk neutral framework.

$$\theta(y) = \frac{e}{a\alpha} y. \tag{7}$$

Stoughton (1993) shows that the manager's unconditional expected utility can be written as follows:

$$E\left[U_a\left(\tilde{W}_a(F,e)\right)\right] = -\exp\left(-aF + V(a,e)\right) \times g(e), \tag{8}$$

$$g(e) = \left(\frac{1}{1+e}\right)^{1/2}. \tag{9}$$

Obviously, $g(0) = 1$. If the manager puts no effort in collecting better information, she will receive, on average, no incentive fee. Notice that *the manager's expected utility is independent of* α.

Before receiving the signal the manager decides her optimal, utility maximizing, effort, that is[11]

$$e_{SB} = \arg\max_{e \geq 0} E\left[U_a\left(\tilde{W}_a(F,e)\right)\right].$$

The first order condition can be rewritten as:

$$V'(a, e_{SB}) = \frac{1}{2(1+e_{SB})}. \tag{10}$$

As it is clear from (10) the second best effort choice is a function only of the manager's risk aversion coefficient; in particular, *it does not depend on* α *or* F. Assumptions (S1)-(S3) guarantee the existence of $e_{SB} > 0$ satisfying equation (10). This, in essence, is the non-incentive result.

2.2 The unrestricted optimal contract.

Though the manager's effort choice is not observed by the manager, the investor knows that she cannot influence the manager's choice of effort through (α, F). As a result she sets (α, F) to optimize risk sharing. And, unlike most moral-hazard problems, it turns out that the optimal risk allocation in the second best scenario is *Pareto-efficient*.

The investor offers the manager a contract (α, F) that maximizes her expected utility subject to the manager effort as a function of (α, F) (10) and the manager's participation constraint:[12]

[11] SB denotes a *second best.*

[12] The manager's reservation utility is set equal to minus one without loss of generality. Note that since e_{SB} is independent of (α, F) there is no incentive constraint.

$$\begin{array}{ll} \max\limits_{\alpha,F} & E\left[U_b\left(\tilde{W}_b(\alpha,F,e_{SB})\right)\right] \\ \text{s.t.} & E\left[U_a\left(\tilde{W}_a(\alpha,F,e_{SB})\right)\right] \geq -1 \\ & \\ & \alpha \in (0,1). \end{array}$$

Let us define the following functions:

$$\begin{aligned} m(\alpha) &= \frac{1-\alpha}{r\alpha}, \\ M(\alpha) &= m(\alpha)\left(2-m(\alpha)\right), \end{aligned}$$

where $r = \frac{a}{b}$ represents the manager's "relative" (to the investor) risk aversion. $m(\alpha)$ is convex and decreasing in α. $M(\alpha)$ has two roots:

$$M\left(\frac{1}{1+2r}\right) = M(1) = 0.$$

The function $M(\alpha)$ reaches its maximum value at one for $\alpha = 1/(1+r)$; it is concave for all $\alpha < 3/2(1+r)$, convex otherwise. Appendix A1 shows that the investor's expected utility can be written as:

$$\begin{aligned} E\left[U_b\left(\tilde{W}_b(\alpha,F,e)\right)\right] &= -\exp(aF/r)\int_0^\infty f(\alpha,e|s)\,ds, \\ f(\alpha,e|s) &= \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}s(1+eM(\alpha))\right)s^{-1/2}. \end{aligned} \tag{11}$$

Thus, to derive the optimal contract, the investor solves:

$$\max_{\alpha,F} \quad -\exp(aF/r)\times\int_0^\infty f(\alpha,e_{SB}\,|\,s)\,ds \tag{12}$$

$$\text{s.t.} \quad \exp(-aF+V(a,e_{SB}))\times g(e_{SB}) \leq 1 \tag{13}$$

$$\alpha \in (0,1).$$

Appendix A2 shows that at the optimal $\alpha^* = \frac{1}{1+r}$ and the investor's indirect utility can be expressed as a function of e_{SB} and r:

$$E\left[U_b\left(\tilde{W}_b(\alpha^*,e_{SB})\right)\right] = -\exp(V(a,e_{SB})/r)\times g(e_{SB})^{(1+r)/r}. \tag{14}$$

As indicated earlier, the optimal share α^* is indeed the first best (the contract is Pareto efficient): each agent participates on the portfolio's return according to her risk tolerance as a proportion of the aggregate risk-tolerance. Finally, as the manager's effort is independent of F, F_{SB} is set to satisfy

$$\exp(-aF_{SB} + V(a, e_{SB})) \times g(e_{SB}) = 1.$$

This (α^*, F_{SB}) maximizes the investor's expected utility.

3 Restricted portfolio choice

This section introduces the main concern of the paper. We will assume that the manager is restricted in her portfolio choice. This restriction, that we call "bounded short-selling" [BSS], can be expressed as follows:

$$|\,\theta| \leq \kappa,$$

$$1 \leq \kappa < \infty.$$

The investor's wealth at t_0 consist of one unit of the consumption good. Thus, κ $(1-\kappa)$ represents the relative size of the maximum short-selling (leverage) position the manager is allowed to take. The symmetry with respect to κ is convenient in order to simplify the algebra: $\theta(y)$ will be a symmetric function of y.[13] Note that κ can be any large number. All we require is that it should not be infinite. Our constraint can be also be interpreted as a bound K on the portfolio's tracking error volatility, such that:[14]

$$\mathrm{Var}(\tilde{W}) \leq K.$$

Given the manager's optimal portfolio in (7), assumption [BSS] can also be expressed as follows:

[13]Notice (equation (1)) that $\tilde{y}$ has a *gaussian* distribution. None of our results depends, qualitatively, on this assumption.

[14]Given the definition of $\mathrm{Var}(\tilde{W})$ this conditions can be written as

$$\begin{array}{rcl} \theta^2\,(1-P) & \leq & K, \\ & \text{or} & \\ |\,\theta\,| & \leq & \sqrt{\dfrac{K}{1-P}} = \kappa. \end{array}$$

$$|y|\, e \leq a\kappa\alpha.$$

The left-hand term represents the risky asset's conditional mean return (absolute value) weighted by its precision. The right-hand side term is the short-selling limit, κ, multiplied by the manager's risk aversion coefficient and α. Clearly, as long as $|y| < a\kappa\alpha/e$, the manager's optimal decision will not be affected by [BSS]. However, when the signal exceeds either bound then: (i) for all $a < \infty$, the higher the effort, the higher will be the "distortion" induced by [BSS] on the manager's optimal portfolio; (ii) such a distorting effect will diminish as α and/or the risk aversion a increase. Obviously, none of these effects would exist if the constraint [BSS] were to be removed.

Under [BSS], the manager's marginal utility of effort is smaller. Increasing effort expenditure implies that the signal's precision becomes sharper. However, for certain signals, the manager may not be able to form the portfolio of her choice. The net effect of this trade-off results in a decrease in the marginal utility of effort as compared to the case where [BSS] does not hold. As a consequence, α now plays an additional role: by increasing α the investor can "marginally" relax the restriction imposed by [BSS]. Hence, a higher α induces the manager to exhert higher effort.

Based on this intuition, it follows that the manager's optimal effort under [BSS] will be: (i) smaller than e_{SB} for all α and (ii) increasing in α. Also, the distortion between the two effort levels should be inversely related to the manager's risk aversion: i.e. the larger is a the smaller is the effect of [BSS]. In the limit, when a tends to infinity, the effect of the restriction should vanish and we should return to the second best. In what follows, we formalize this intuition.

The manager's optimal portfolio solves the following "constrained" problem:

$$\begin{aligned} \theta(y) = \arg\max_{\theta} : \quad & E\left[U_a\left(\tilde{W}_a(y)\right)\right] \\ \text{s.t.} \quad & \kappa \geq \theta \geq -\kappa. \end{aligned}$$

Let $\mathcal{L}(\theta, \lambda_0, \lambda_1)$ denote the *Lagrangian*. $\lambda_0, : \lambda_1 \geq 0$ will represent the corresponding multipliers. It is easy to check that the manager's problem is *concave* and the maximum satisfies:

$$\begin{aligned} \frac{\partial}{\partial\theta}\mathcal{L}(\theta^*, \lambda_0^*, \lambda_1^*) &= 0 \\ \kappa &\geq \theta^* \geq -\kappa; \end{aligned}$$

together with the *slack* constrains:

$$\begin{aligned} (\theta^* + \kappa)\lambda_0^* &= 0, \\ (\theta^* - \kappa)\lambda_1^* &= 0. \end{aligned}$$

This system of equations yields two "corner" solutions and one interior solution:

1. Lower corner solution: $\begin{cases} \theta^* &= -\kappa \\ \lambda_0^* &= -\frac{e}{1+e}\alpha\left(y + \frac{a\alpha\kappa}{e}\right) \\ \lambda_1^* &= 0 \end{cases}$

2. Interior solution: $\begin{cases} \theta^* &= \frac{1}{\alpha}\frac{e}{a}y \\ \lambda_0^* &= 0 \\ \lambda_1^* &= 0 \end{cases}$

3. Upper corner solution: $\begin{cases} \theta^* &= \kappa \\ \lambda_0^* &= 0 \\ \lambda_1^* &= \frac{e}{1+e}\alpha\left(y - \frac{a\kappa\alpha}{e}\right). \end{cases}$

The interior solution coincides with the manager's optimal portfolio (7) in the unconstrained problem. Neither slack constraint is binding ($\lambda_0^* = \lambda_1^* = 0$). The dollar amount invested in the risky asset is independent of α. Therefore, the marginal utility of α is zero.

In the corner solutions the dollar amount $\alpha\theta$ invested ($\lambda_1^* > 0$) or sold short ($\lambda_0^* > 0$) in the risky asset is, in absolute value, increasing in α. The marginal utility of α is positive and the intuition is clear. When the signal y is "very good" or "very bad", the manager will not be able to form the portfolio of her choice as she is restricted by κ. In such situations, since her *own* portfolio is $\alpha\theta$, an increase in α increases her utility.

We can re-write the optimal portfolio as a function of the signal y:

$$\theta(y) = \begin{cases} -\kappa & \text{if } y < -\frac{a\kappa\alpha}{e} \\ \\ \frac{1}{\alpha}\frac{e}{a}y & \text{if } |\,y| \le \frac{a\kappa\alpha}{e} \\ \\ \kappa & \text{if } y > \frac{a\kappa\alpha}{e}. \end{cases}$$

Note that when $|y| > a\kappa\alpha/e$ the dollar investment in the risky asset, $\alpha\theta(y)$, will be increasing in α: the manager will "behave" indeed as an investor with decreasing absolute risk aversion.

Recall that the manager had accepted some contract (α, F) in the beginning of the game. To decide on how much effort to put in she uses the knowledge that for each y that she observes, she will form the portfolio $\theta(y)$. In other words, (α, F) along with $\theta(y)$ determines her (unconditional) expected utility as a function of effort. To express this utility in a simple manner we introduce some notation. Let us denote

$$\Phi(x) = \int_0^x \phi(s)\,ds,$$

$$\phi(s) = \begin{cases} \frac{1}{\sqrt{2\pi}}\, s^{-1/2}\exp(-s/2) & \text{when } s > 0, \\ 0 & \text{otherwise.} \end{cases}$$

So, $\Phi(\cdot)$ is the cumulative probability function of a Chi-square variable with one degree of freedom. $\phi(\cdot)$ is the corresponding density function. The following proposition now presents the manager's unconditional expected utility function.

Proposition 1 *Given the contract* (α, F) *and the constraint* $\kappa < \infty$*, the expected utility function of the risk-averse manager is:*

$$E[U_a\left(\tilde{W}_a(\alpha, F, e\,|\,\kappa)\right)] = -exp(-aF + V(a,e)) \times g_\kappa(e\,|\,\alpha), \tag{15}$$

with

$$g_\kappa(e|\alpha) = \left(\frac{1}{1+e}\right)^{1/2} \times \Phi\left(\frac{(a\kappa\alpha)^2}{e}\right) + exp\left(\frac{(a\kappa\alpha)^2}{2}\right) \times \left(1 - \Phi\left(\frac{(a\kappa\alpha)^2}{e}(1+e)\right)\right). \tag{16}$$

Proof: See Appendix B1.

Equation (15) confirms the intuition presented at the beginning of this section. The unconditional expected utility of the "constrained" manager (i.e. after introducing [BSS]) can be expressed as the weighted sum of two utility functions.[15] The first function corresponds to the "interior" expected utility in (9) where the manager is not affected by the constraint. The second function is the manager's expected utility when the constraint is binding. In that case the manager sets $|\theta| = \kappa$.

The function $g_\kappa(e|\alpha)$ is the counterpart of $g(e)$ in (9). Obviously, $g_\kappa(0|\alpha) = 1$. Both functions are decreasing and convex with respect to effort. The later converges to the former in the limit, when κ tends to ∞. These results, which will be needed later on, are summarized in the following corollary.

Corollary 2 *Function* $g_\kappa(e\,|\,\alpha)$ *satisfies:*

$$g'_\kappa(e\,|\,\alpha) < 0,$$
$$g''_\kappa(e\,|\,\alpha) > 0,$$

for all $\alpha \in (0,1)$*. Moreover,*

$$\lim_{a\kappa\alpha \to \infty} g_\kappa(e\,|\,\alpha) = g(e). \tag{17}$$

[15]The disutility function, $V(a,e)$, affects both terms. This is because the effort decision is taken *ex-ante*, before the signal is observed. Notice that the weights are not constant: they are a function of effort themselves.

Proof: See Appendix B2.

But most importantly, unlike in (9) $g_\kappa(e\,|\,\alpha)$ depends on α. We have seen that the (conditional) marginal utility of α is positive when $|\,y| > a\kappa\alpha/e$. In Corollary 4 we will show that the unconditional expected utility of the manager is increasing in α. We first need the following lemma:

Lemma 3 *For all* $0 < x < \infty$

$$\phi(x) - \frac{1}{2}\,(1 - \Phi(x)) > 0.$$

Proof: For all $x > 0$:

$$\frac{1}{2}\,(1 - \Phi(x)) = \\ \frac{1}{\sqrt{2\pi}}\exp(-x/2)\,x^{-1/2} - \frac{1}{2}\int_x^\infty \frac{1}{\sqrt{2\pi}}\exp(-s/2)\,s^{-3/2}\,ds.$$

Therefore,

$$\phi(x) - \frac{1}{2}\,(1 - \Phi(x)) \;=\; \frac{1}{2}\int_x^\infty \frac{1}{\sqrt{2\pi}}\exp(-s/2)\,s^{-3/2}\,ds \;> 0.$$

Q.E.D.

Corollary 4 *The unconditional marginal utility of* α *is positive.*

Proof: Given (15) the corollary will be proved if we can show that the function $g_\kappa(e\,|\,\alpha)$ is decreasing in α:

$$\frac{\partial}{\partial\alpha} g_\kappa(e\,|\,\alpha) = \\ -2(a\kappa)^2\alpha\left[\phi\left(\frac{(a\kappa\alpha)^2}{e}(1+e)\right) - \frac{1}{2}\left(1 - \Phi\left(\frac{(a\kappa\alpha)^2}{e}(1+e)\right)\right)\right] \times \exp\left(\frac{(a\kappa\alpha)^2}{2}\right) < 0 \tag{18}$$

for all $\alpha \in (0,1)$ given Lemma 3.

Q.E.D.

Proposition 1 and Corollary 4 summarize the main contributions in this section: assumption [BSS] implies that the manager's unconditional expected utility is increasing in α, the incentive component of her compensation scheme. Next we will analyze how [BSS] affects the manager's effort choice.

3.1 The third-best effort

This section will show that assumptions (S1)-(S3) guarantee the existence of an unique optimal third best effort for the constrained manager. We will see that such an effort will always be smaller than the second best effort. This will lead to the main result in the paper: effort will be shown to be an increasing function of α under the assumption [BSS].

The manager will choose that effort that maximizes her unconditional expected utility. More concretely, for each $\alpha \in (0,1)$ we define the third best effort, $e_{_{TB}}$, as follows:

$$e_{_{TB}} = \arg\max_{e \geq 0} \quad -\exp(-aF + V(a,e)) \times g_\kappa(e\,|\,\alpha).$$

Let us introduce

$$\mathcal{J}_\kappa(e\,|\,\alpha) = V'(a,e) \times g_\kappa(e\,|\,\alpha) + g'_\kappa(e\,|\,\alpha).$$

Proposition 5 *Given assumptions (S1)-(S3), for each $\alpha \in (0,1)$ there exists an unique $e_{_{TB}} > 0$ such that*

$$\mathcal{J}_\kappa(e_{_{TB}}\,|\,\alpha) = 0, \tag{19}$$
$$\mathcal{J}'_\kappa(e_{_{TB}}\,|\,\alpha) > 0. \tag{20}$$

Proof: See Appendix B3.

Once the third best effort has been characterized we ask how it compares to the second best effort. The following Corollary answers this question.

Corollary 6 *$e_{_{SB}} > e_{_{TB}}$ for all α. Both are equal, in the limit, when $a\kappa\alpha$ tends to infinite.*

Proof: According to (15) and equation (B2) in the Appendix, function $\mathcal{J}_\kappa(e\,|\,\alpha)$ can be written as follows:

$$\mathcal{J}_\kappa(e\,|\,\alpha) = \tag{21}$$

$$\mathcal{J}(e) \times \Phi\left(\frac{(a\kappa\alpha)^2}{e}\right) + V'(a,e) \times \exp\left(\frac{(a\kappa\alpha)^2}{2}\right) \times \left(1 - \Phi\left(\frac{(a\kappa\alpha)^2}{e}(1+e)\right)\right).$$

Plugging in the second best effort:

$$\mathcal{J}_\kappa(e_{SB}\,|\,\alpha) =$$
$$V'(a, e_{SB}) \times \exp\left(\frac{(a\kappa\alpha)^2}{2}\right) \times \left(1 - \Phi\left(\frac{(a\kappa\alpha)^2}{e_{SB}}(1 + e_{SB})\right)\right) > 0.$$

This implies

$$E'\left[U_a\left(\tilde{W}_a(\alpha, F, e_{SB}|\kappa)\right)\right] = -\exp(-aF + V(a, e_{SB})) \times \mathcal{J}_\kappa(e_{SB}\,|\,\alpha) < 0.$$

For the constrained manager, the marginal utility of effort at e_{SB} is negative, whatever the value of α. Equations (19) and (20) in Proposition 5 complete the proof.

Finally, given equation (B4) in the Appendix, $\mathcal{J}_\kappa(e_{SB}\,|\,\alpha)$ tends to zero when $a\kappa\alpha$ tend to infinity.

Q.E.D.

Given equation (21), the marginal utility of effort for the manager (22) can be expressed as the marginal utility of effort without any restriction on short-selling/leverage (times the probability that $\lambda_0^* = \lambda_1^* = 0$) minus the marginal disutility of effort when constraint [BSS] is binding, weighted by the corresponding probability. Given assumption (S2), $V'(a, e) > 0$ for any $e > 0$. The marginal utility of effort will be lower under [BSS] than in the unconstrained scenario.

Finally, how does α affect the manager's optimal effort? In Section 2.1 the answer was simple: it has no effect. Without any restriction on the manager's short-selling/leverage ability the effort decision is independent of α. We will show now that, after imposing [BSS], α will influence the manager's effort choice.

We have shown that introducing [BSS] lowers the marginal utility of effort. As a consequence, $e_{TB} < e_{SB}$. How can the investor use this result in her favour? When α increases the CARA manager will reduce her (optimal) investment in the risky asset, thus, bringing it marginally closer to the unconstrained portfolio. Therefore, the investor is, indirectly, mitigating the distorting effect of the restriction. The marginal utility of effort will increase. Given (20) the optimal effort will be higher. We move towards the second best.

The following proposition resumes this intuition and concludes this section.

Proposition 7 *Given the assumption of BSS, the manager's effort is a continuous and differentiable function of α for all $\alpha \in (0, 1)$. Moreover, it is increasing in α.*

Proof: According to (19) in Proposition 5, given $\hat{\alpha} \in (0, 1)$ there exists $e_{TB}\,|\,\hat{\alpha}$, $e_{TB} > 0$, such that

$$\mathcal{J}_\kappa(e_{TB}\,|\,\hat{\alpha}) = 0. \tag{22}$$

The function $\mathcal{J}_\kappa$ is continuous and differentiable with respect to (α, e). Given (20), the implicit function theorem allows us to "locally" solve (22); that is, to express e as a function of α in a neighborhood of $(\hat{\alpha}, e_{TB})$.

More formally: given $\hat{\alpha} \in (0,1)$ there exists a function $e(\alpha)$, continuous and differentiable, and an open ball $B(\hat{\alpha})$, such that,

$$\begin{aligned} e(\hat{\alpha}) &= e_{TB}, \\ \mathcal{J}_\kappa(e(\alpha)\,|\,\alpha) &= 0, \end{aligned}$$

for all $\alpha \in B(\hat{\alpha})$. Deriving the last equation with respect to α and valued at $\hat{\alpha}$:

$$\frac{\partial}{\partial\alpha}e(\hat{\alpha}) = -\frac{\partial}{\partial\alpha}\mathcal{J}_\kappa(e_{TB}\,|\,\hat{\alpha}) \times \mathcal{J}_\kappa'^{-1}(e_{TB}\,|\,\hat{\alpha}). \tag{23}$$

¿From (20),

$$\mathcal{J}_\kappa'(e_{TB}\,|\,\hat{\alpha}) > 0.$$

Therefore, the proposition will be proved if we show

$$\frac{\partial}{\partial\alpha}\mathcal{J}_\kappa(e_{TB}\,|\,\hat{\alpha}) = V'(a, e_{TB}) \times \frac{\partial}{\partial\alpha}g_\kappa(e_{TB}|\,\hat{\alpha}) + \frac{\partial}{\partial\alpha}g_\kappa'(e_{TB}|\,\hat{\alpha}) < 0 \tag{24}$$

for all $\hat{\alpha} \in (0,1)$. From (S2): $V'(a, e_{TB}) > 0$. From Corollary 4:

$$\frac{\partial}{\partial\alpha}g_\kappa(e_{TB}|\,\hat{\alpha}) < 0.$$

Given equation (B3) in the Appendix:

$$\frac{\partial}{\partial\alpha}g_\kappa'(e_{TB}|\,\hat{\alpha}) < 0.$$

Since (23) is satisfied for any $\hat{\alpha} \in (0,1)$, the Proposition is proved.

Q.E.D.

4 The third-best contract

In this section we introduce the investor's unconditional expected utility function when the manager is constrained by [BSS].

We will show that the first best share, α^*, is not optimal under the assumption of [BSS]. We will present a numerical solution to the investor's problem when the manager is constrained in her portfolio choice. The results in this exercise will illustrate how the optimal share changes with respect to the first best split when the assumption [BSS] is introduced. The numerical exercise will also quantify the change in effort expenditure and expected utility with respect to the (suboptimal) first best share in the constrained scenario.

When the manager is short-selling constrained, she solves the restricted problem in Section 3 and her optimal portfolio is (**??**). Given (6) and the investor's utility function:

$$E\left[U_b\left(\tilde{W}_b(y\,|\,\kappa)\right)\right] =$$
$$-\exp(aF/r)\times\begin{cases}\exp\left(\frac{e}{(1+e)}\,a\kappa\alpha\,m(\alpha)\left(y+\frac{a\kappa\alpha}{2e}m(\alpha)\right)\right) & \text{if } y<-\frac{a\kappa\alpha}{e}\\ \exp\left(-\frac{e^2}{2(1+e)}y^2M(\alpha)\right) & \text{if } |y|\le\frac{a\kappa\alpha}{e}\\ \exp\left(-\frac{e}{(1+e)}\,a\kappa\alpha\,m(\alpha)\left(y-\frac{a\kappa\alpha}{2e}m(\alpha)\right)\right) & \text{if } y>\frac{a\kappa\alpha}{e}.\end{cases}$$

Following the same procedure we used to derive the manager's unconditional expected utility function, we arrive at the investors's expected utility function. It's stated in the following proposition.

Proposition 8 *The expected utility function of the risk-averse investor who offers the contract (α, F) to the risk-averse manager subject to the short-selling constraint κ is:*

$$E\left[U_b\left(\tilde{W}_b(\alpha,F,e\,|\,\kappa)\right)\right] = -exp(aF/r)\times\left[\int_0^{\frac{(a\kappa\alpha)^2}{(1+e)e}} f(\alpha,e\,|\,s)\,ds + f_\kappa(\alpha,e\,|\,\kappa)\right], \quad (25)$$

with

$$f_\kappa(\alpha,e\,|\,\kappa) = exp\left(\frac{(a\kappa\alpha\,m(\alpha))^2}{2}\right)\times\left(1-\Phi\left(\frac{(a\kappa\alpha)^2}{e}\frac{(1+e\,m(\alpha))^2}{1+e}\right)\right).$$

Proof: See Appendix B4.

Once we know the investor's expected utility function, we define the third-best contract as the solution to the following problem:

$$\max_{\alpha,F} \quad -\exp(aF/r) \times \left[\int_0^{\frac{(a\kappa\alpha)^2}{(1+e)e}} f(\alpha, e\,|\,s)\,ds + f_\kappa(\alpha, e\,|\,\kappa)\right]$$

$$\text{s.t.} \quad e(\alpha) = \arg\max_{e\geq 0} -\exp(-aF + V(a,e)) \times g_\kappa(e\,|\,\alpha), \tag{26}$$

$$-\exp(-aF + V(a,e)) \times g_\kappa(e\,|\,\alpha) \geq -1, \tag{27}$$

$$\alpha \in (0,1).$$

In the last section we showed that when the manager is not facing any short-selling/leverage cap, the first-best allocation (α^*) is optimal even when moral hazard is present. This result is triggered by the failure-incentive nature of linear contracts in the absence of short-selling constrains, as we saw in Section 2.1. However, Proposition 7 shows that, under [BSS], effort is increasing in α.

In this section we will show that, under [BSS], α^* is not optimal. This result suggests that when the short-selling/leverage ability of the manager is bounded, α plays an additional role over risk-sharing. There will be a tradeoff between efficiency in risk allocation and effort inducement such that in the new equilibrium (i.e. the third-best contract) α^* is not optimal.

Let us define

$$E\left[U_b\left(\tilde{W}_b(\alpha, e\,|\,\kappa)\right)\right] = \tag{28}$$
$$-\exp(V(a,e)/r) \times g_\kappa(e\,|\,\alpha)^{1/r} \times \left[\int_0^{\frac{(a\kappa\alpha)^2}{(1+e)e}} f(\alpha, e\,|\,s)\,ds + f_\kappa(\alpha, e\,|\,\kappa)\right],$$

the investor's (unconditional) expected utility function when the participation constraint (27) is binding. Let us denote $\lambda \geq 0$ the corresponding multiplier. It can be seen straightforwardly that (27) will be binding if and only if

$$\int_0^{\frac{(a\kappa\alpha)^2}{(1+e)e}} f(\alpha, e\,|\,s)\,ds + f_\kappa(\alpha, e\,|\,\kappa) > 0.$$

We present two corollaries of Proposition 8. The first shows that if $\alpha = \alpha^*$ then the participation constraint (27) will be binding for all $e > 0$. The second states that the marginal utility of α^* is zero.

Corollary 9

$$\int_0^{\frac{(a\kappa\alpha^*)^2}{(1+e)e}} f(\alpha^*, e\,|\,s)\,ds + f_\kappa(\alpha^*, e\,|\,\kappa) = g_\kappa(e\,|\,\alpha^*) > 0,$$

for all $e > 0$. Therefore, the participation constraint is binding for α^*.

Proof: See Appendix B5.

Therefore, given equation (28) and Corollary 9:

$$E\left[U_b\left(\tilde{W}_b(\alpha^*, e\,|\,\kappa)\right)\right] = -\exp(V(a,e)/r) \times g_\kappa(e\,|\,\alpha^*)^{(1+r)/r}. \tag{29}$$

Corollary 10

$$\frac{\partial}{\partial\,\alpha} E\left[U_b\left(\tilde{W}_b(\alpha^*, e\,|\,\kappa)\right)\right] = 0 \textit{for all } e > 0.$$

Proof: See Appendix B6.

After presenting corollaries 9 and 10, we introduce the main analytical result in this section.

Proposition 11 *Let us assume that neither the effort nor the signal are observed by the investor. Under the assumption [BSS] ($\kappa < \infty$), the efficient risk-sharing, $\alpha^* = 1/(1+r)$, is not optimal.*

Proof: According to (19) in Proposition 5, there exists $e_{TB} \mid \alpha^*$, $e_{TB} > 0$, such that

$$\mathcal{J}_\kappa(e_{TB}\,|\,\alpha^*) = 0,$$

that is, the third-best effort that solves restriction (26) for $\alpha = \alpha^*$. Thus, as stated in Corollary 9, the participation constraint (27) will be binding: α^* will be optimal (necessary condition) only if

$$\left.\frac{\partial}{\partial\,\alpha} E\left[U_b\left(\tilde{W}_b(\alpha, e(\alpha)\,|\,\kappa)\right)\right]\right|_{\alpha=\alpha^*} = 0,$$

where $e(\alpha)$ is, according to Proposition 7, a continuous and differentiable function, increasing in α with $e(\alpha^*) = e_{TB}$. We derive (28) with respect to α:

$$\begin{aligned}\frac{\partial}{\partial\,\alpha} E\left[U_b\left(\tilde{W}_b(\alpha, e(\alpha)\,|\,\kappa)\right)\right] = \\ \frac{\partial}{\partial\,\alpha} E\left[U_b\left(\tilde{W}_b(\alpha, e)\,|\,\kappa)\right)\right] + \frac{\partial}{\partial\,e} E\left[U_b\left(\tilde{W}_b(\alpha, e)\,|\,\kappa)\right)\right] \times \frac{\partial}{\partial\,\alpha} e(\alpha).\end{aligned} \tag{30}$$

¿From Corollary 10:

$$\frac{\partial}{\partial\,\alpha} E\left[U_b\left(\tilde{W}_b(\alpha^*, e_{TB})\,|\,\kappa)\right)\right] = 0.$$

Deriving (29) with respect to e and valuing it at e_{TB}:

$$\frac{\partial}{\partial e} E\left[U_b\left(\tilde{W}_b(\alpha^*, e_{TB}\,|\,\kappa)\right)\right] = \\ -\frac{1}{r}\exp(V(a, e_{TB})/r) \times g_\kappa(e_{TB}\,|\,\alpha^*)^{1/r} \times \left[\mathcal{J}_\kappa(e_{TB}\,|\,\alpha^*) + r g'_\kappa(e_{TB}\,|\,\alpha^*)\right].$$

By definition, $\mathcal{J}_\kappa(e_{TB}\,|\,\alpha^*) = 0$. From Corollary 6, $e_{TB} < e_{SB} < \infty$. Substituting α^* y e_{TB} in (B2):

$$g'_\kappa(e_{TB}\,|\,\alpha^*) < 0.$$

Therefore, given the definition of $g_\kappa(\cdot)$ in Proposition 1 and Proposition 7:

$$\frac{\partial}{\partial \alpha} E\left[U_b\left(\tilde{W}_b(\alpha, e(\alpha)\,|\,\kappa)\right)\right]\Big|_{\alpha=\alpha^*} = \\ -\exp(V(a, e_{TB})/r) \times g_\kappa(e_{TB}\,|\,\alpha^*)^{1/r} \times g'_\kappa(e_{TB}\,|\,\alpha^*) \times \frac{\partial}{\partial \alpha} e(\alpha^*) > 0.$$

Q.E.D.

4.1 A numerical solution to the third best contract

In this section we present a numerical solution for the third best contract. We will estimate the investor's optimal share when the manager is short-selling constrained. We will compare the suboptimal first best share with the optimal third best share on two different grounds: (i) effort expenditure and (ii) expected utility. We will analyze how "costly" is for the investor to keep the suboptimal pure risk-sharing first best contract for different values of the manager's risk aversion coefficient.

We assume a cuadratic disutility function of effort, $V(a, e) = a\, e^2$. The investor's *risk-tolerance* coefficient $(1/b)$ is set equal to 8. We will consider four different values for the manager's risk-tolerance coefficient $1/a = \{3, 8, 15, 24\}$. Given the assumption on the distuility function, condition (10) implies that the second best effort will be $e_{SB}(a) = \{1/2, 1, 3/2, 2\}$, respectively.

With respect to the short-selling/leverage constraint, κ, we will study 10 different values, from 1 (tightest restriction, no leverage) through 10 (loosest restriction).

For each κ, the algorithm creates a grid of 99 values of α from 0.01 through .99. Condition (B5) is solved for each pair (α, κ). That gives a numerical value of e_{TB} for each pair (α, κ). The resulting matrix of third best efforts confirms the predictions of Corollary 6 and Proposition 7: for all risk-aversion coefficients and all leverage bounds, the third best effort is (i) smaller than the corresponding second best effort and (ii) increasing in α.

For each κ, the investor's expected utility (28) is evaluated across α. The maximum expected utility for each risk-aversion coefficient and leverage bound is attained at the corresponding values of α reported in Table 1 in the Appendix C. As a reference, the corresponding first best values are: $\alpha^*(3) = 0.27$, $\alpha^*(8) = 0.5$, $\alpha^*(15) = 0.65$ and $\alpha^*(24) = 0.75$.

Table 1 illustrates an important conclusion of the numerical analysis: the optimal share increases, relative to the first best, in the constrained scenario. When κ increases (hence, loosening the constraint) the third best alpha converges smoothly towards the first best value. The effect of the manager's risk-aversion on the change in alpha is analyzed in Table 2. The first row in each panel, $\dot{\alpha}/\alpha$, shows the percentage change in alpha for the corresponding manager's risk-aversion coefficient. It is clear that, for each κ, the percentage increase is higher the higher the manager's risk-aversion. The difference can be very dramatic, ranging from almost 80% for $(a = 1/3, \kappa = 1)$ to barely 3% for $(a = 1/24, \kappa = 10)$.

We interpret this result as evidence in favour of the new (additional) role of the incentive fee in the third best scenario: effort inducement. When the short-selling bounds decrease (making the restriction tighter) the volatility of the portfolio decreases as well since fewer extreme portfolios are feasible. If the investor does not increase the incentive fee the manager will be under-exposed to active management risk. As a consequence, effort will also decrease. The risk sharing and the effort inducement arguments are aligned in the same direction: the optimal incentive fee increases. The change in alpha due to the incentive role is more visible the smaller the manager's risk-aversion in that case the former pure risk-sharing, first best alpha was relatively smaller.

The second and third columns in Table 2 report the percentage change in effort $(\dot{e}/e)$ and certainty equivalent wealth $(\dot{C}/C)$ when we compare the new third best share with the (suboptimal) first best share within the constrained scenario. The figures can be interpreted as the "cost" in terms of effort and utility that the manager would incur if she maintained the suboptimal first best share in the constrained scenario.

The first column represents the case of a total bound on leverage $(\kappa = 1)$. The change in effort expenditure for the risk averse manager can be very high (up to 42%) and it decreases with the manager's risk tolerance. An analogous result follows when we study the change in effort across κ.

Finally, with respect to the change in the manager's indirect utility function, the percentage change in the certainty equivalent wealth shows that the potential "efficiency" loss that arises from compensating the manager through the suboptimal first best sharing is almost negligible when the manager is sufficiently risk-tolerant $(a = 1/24)$. However, in the standard situation where the manager is assumed to be more risk-averse than the investor this loss can represent up to 9% of the investor's first best certainty equivalent wealth, depending on the leverage bound. When this bound increases (the constraint loosens) the loss decreases. In the limit, when the constraint vanishes, the third best scenario converges into the unconstrained, second best scenario and the first best share is optimal.

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Appendix A: The unrestricted problem

A1 The investor's unconditional expected utility

Given her utility function and the definition of her conditional wealth in (6), the investor's (conditional) expected utility function could be written as a function of $M(\alpha)$ as follows:

$$E\left[U_b\left(\tilde{W}_b(y)\right)\right] = -\exp(aF/r) \times \exp\left(-\frac{e^2}{2(1+e)}y^2 M(\alpha)\right). \tag{A1}$$

We define the investor's unconditional expected utility as:

$$E\left[U_b\left(\tilde{W}_b(\alpha, F, e)\right)\right] = \int_{-\infty}^{\infty} E\left[U_b\left(\tilde{W}_b(y)\right)\right] dF(y).$$

Given the distribution signal in (1) and equation (3):

$$E\left[U_b\left(\tilde{W}_b(a, F, e)\right)\right] =$$
$$-\exp(aF/r) \times \left(\frac{e}{1+e}\right)^{1/2} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-(1/2)y^2 e\,\frac{1+eM(\alpha)}{1+e}\right) dy.$$

Substituting $s = \frac{e}{1+e}y^2$ in the last equation we arrive at equation (11).

A2 The first best risk allocation

Let us assume (13) is binding in the optimal. Let us denote $\lambda \geq 0$ the corresponding multiplier. Given the objective function (12) it follows straightforwardly that the restriction is binding ($\lambda > 0$) if and only if:

$$\int_0^{\infty} f(\alpha, e_{SB}\,|\,s)\, ds > 0. \tag{A2}$$

If (13) is binding:

$$\begin{array}{ll} \max\limits_{\alpha} & -\exp(V(a, e_{SB})/r) \times g(e_{SB})^{1/r} \times \displaystyle\int_0^{\infty} f(\alpha, e_{SB}\,|\,s)\, ds \\ \text{s.t.} & \alpha \in (0,1). \end{array} \tag{A3}$$

Considering the definition of function f in (11),

$$\alpha^* = \arg\min_{\alpha} f(\alpha, e_{SB}\,|\,s)$$

for all $s > 0$. From the last problem we obtain the following characterization of α^*:

$$\frac{\partial}{\partial \alpha} M(\alpha^*) = 0.$$

Since function $M(\alpha)$ satisfies:

$$\begin{aligned} M\left(\frac{1}{1+r}\right) &= 1, \\ \frac{\partial}{\partial \alpha} M\left(\frac{1}{1+r}\right) &= 0, \end{aligned}$$

then

$$\begin{aligned} \alpha^* &= \frac{1}{1+r}, \\ f(\alpha^*, e_{SB} \,|\, s) &= \left(\frac{1}{1+e_{SB}}\right)^{1/2} \frac{1}{\sqrt{\pi}} \, s^{-1/2} \exp(-s). \end{aligned}$$

Given the definition of the *Gamma* function,

$$\Gamma(1/2) = \int_0^\infty s^{-1/2} \exp(-s)\, ds = \sqrt{\pi},$$

it follows that

$$\int_0^\infty f(\alpha^*, e_{SB} \,|\, s)\, ds = g(e_{SB}). \tag{A4}$$

¿From (A4) and condition (A2), $\lambda > 0$ for all $e < \infty$. Plugging (A4) into (A3) we arrive at (14).

Appendix B: The restricted problem

B1 Proof of Proposition 1

The manager's conditional expected utility is:

$$E[U_a\left(\tilde{W}_a(y)\right)] =$$

$$-\exp(-aF + V(a,e)) \times \begin{cases} \exp\left(\frac{e}{1+e} a\kappa\alpha \left(y + \frac{a\kappa\alpha}{2e}\right)\right) & \text{if } y < -\frac{a\kappa\alpha}{e} \\ \exp\left(-\frac{e^2}{2(1+e)} y^2\right) & \text{if } |\,y| \leq \frac{a\kappa\alpha}{e} \\ \exp\left(-\frac{e}{(1+e)} a\kappa\alpha \left(y - \frac{a\kappa\alpha}{2e}\right)\right) & \text{if } y > \frac{a\kappa\alpha}{e}. \end{cases}$$

Averaging across y we obtain the manager's unconditional expected utility:

$$
\begin{aligned}
E[U_a\left(\tilde{W}_a(\alpha, F, e\,|\,\kappa)\right)] &= -\exp(-aF + V(a,e)) \times \left(\frac{e}{1+e}\right)^{1/2} \times \\
&\left[\exp\left(\frac{(a\kappa\alpha)^2}{2}\right) \int_{-\infty}^{-\frac{a\kappa\alpha}{e}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{e}{2(1+e)}(y - a\kappa\alpha)^2\right) dy + \right. \\
&\int_{-\frac{a\kappa\alpha}{e}}^{\frac{a\kappa\alpha}{e}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{e}{2}\, y^2\right) dy \; + \\
&\left. \exp\left(\frac{(a\kappa\alpha)^2}{2}\right) \int_{\frac{a\kappa\alpha}{e}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{e}{2(1+e)}(y + a\kappa\alpha)^2\right) dy \right].
\end{aligned} \tag{B1}
$$

We propose the following change of variable:

$$
s = \begin{cases} \frac{e}{(1+e)}(y - a\kappa\alpha)^2 & \text{if } y < -\frac{a\kappa\alpha}{e}, \\ e(1+e)\, y^2 & \text{if } |\,y| \leq \frac{a\kappa\alpha}{e}, \\ \frac{e}{(1+e)}(y + a\kappa\alpha)^2 & \text{if } y > \frac{a\kappa\alpha}{e}. \end{cases}
$$

Replacing the new variable in (B1):

$$
\begin{aligned}
E[U_a\left(\tilde{W}_a(\alpha, F, e\,|\,\kappa)\right)] &= -\exp(-aF + V(a,e)) \times \\
&\left[\left(\frac{1}{1+e}\right)^{1/2} \int_0^{\frac{(a\kappa\alpha)^2}{e}} \frac{1}{\sqrt{2\pi}}\, s^{-1/2} \exp(-(1/2)s)\, ds + \right. \\
&\left. \exp\left(\frac{(a\kappa\alpha)^2}{2}\right) \int_{\frac{1+e}{e}(a\kappa\alpha)^2}^{\infty} \frac{1}{\sqrt{2\pi}}\, s^{-1/2} \exp(-(1/2)s)\, ds \right].
\end{aligned}
$$

Given the last equation and the definition of $\Phi(\cdot)$ we arrive at (15).

B2 Proof of Corollary 1

The first derivative of $g_\kappa(e|\alpha)$ with respect to e is:

$$
g'_\kappa(e\,|\,\alpha) \;=\; -\frac{1}{2}\left(\frac{1}{1+e}\right)^{3/2} \times \Phi\left(\frac{(a\kappa\alpha)^2}{e}\right) < 0. \tag{B2}
$$

Deriving again with respect to e:

$$g''_\kappa(e\,|\,\alpha) = \tag{B3}$$
$$\frac{1}{2}\left(\frac{1}{1+e}\right)^{3/2}\left[\frac{3}{2}\left(\frac{1}{1+e}\right)\times\Phi\left(\frac{(a\kappa\alpha)^2}{e}\right)+\left(\frac{a\kappa\alpha}{e}\right)^2\times\phi\left(\frac{(a\kappa\alpha)^2}{e}\right)\right]>0.$$

By definition, $\lim_{x\to\infty}\Phi(x)=1$. Therefore, to prove (17) we have to show

$$\lim_{x\to\infty}\left[\exp\left(x/2\right)\times\left(1-\Phi\left(x\,\frac{1+e}{e}\right)\right)\right]=0. \tag{B4}$$

Let us re-write (B4) as follows:

$$\lim_{x\to\infty}\frac{1-\Phi\left(x\,\frac{1+e}{e}\right)}{\exp\left(-x/2\right)}.$$

Both functions (exponential and $\Phi(\cdot)$) are continuous and differentiable. Deriving numerator and denominator with respect to x, the limit in (B4) and the following should be equal:

$$\lim_{x\to\infty}\frac{\exp(-x/e)}{x}=0.$$

B3 Proof of Proposition 2

The function (15) is continuous and differentiable. Clearly,

$$E'\left[U_a\left(\tilde{W}_a(0\,|\,\alpha,F)\right)\right]>0.$$

Then, provided it exists, $e_{_{TB}}>0$. Condition (19) can be written as follows:

$$V'(a,e_{_{TB}})=-\frac{g'_\kappa}{g_\kappa}(e_{_{TB}}\,|\,\alpha). \tag{B5}$$

According to (16) and equation (B2), the domain of the second term (B5) is $(0,1/2]$. In order to prove the existence and uniqueness of $e_{_{TB}}$ it is enough to show that such a term is monotonous decreasing in e. Given (16) and equations (B2) and (B3):

$$g''_\kappa(e\,|\,\alpha)\times g_\kappa(e\,|\,\alpha)-(g'_\kappa(e\,|\,\alpha))^2 \;\;>\;\; \frac{1}{2}\left(\frac{1}{1+e}\right)^3\times\Phi^2\left(\frac{(a\kappa\alpha)^2}{e}\right).$$

Thus,

$$-\frac{g'_\kappa}{g_\kappa}(e\,|\,\alpha)$$

is (monotonous) decreasing in e. Condition (20) can be written as follows:

$$V''(a,e) \quad > \quad -\frac{g'_\kappa}{g_\kappa}(e\,|\,\alpha) \times V'(a,e) - \frac{g''_\kappa}{g_\kappa}(e\,|\,\alpha).$$

Since

$$\begin{aligned} -\frac{g'_\kappa}{g_\kappa}(e\,|\,\alpha) &\quad < \quad \frac{1}{2(1+e)}, \\ \frac{g''_\kappa}{g_\kappa}(e\,|\,\alpha) &\quad > \quad 0, \end{aligned} \tag{B6}$$

for all α, assumption (S3) implies (20).

B4 Proof of Proposition 4

Given the investor's indirect utility function in Section 4 the investor's unconditional expected utility will be:

$$\begin{aligned} &E[U_b\left(\tilde{W}_b(\alpha, F, e\,|\,\kappa)\right)] = -\exp(aF/r) \times \left(\frac{e}{1+e}\right)^{1/2} \times \\ &\left[\exp\left(\frac{(a\kappa\alpha\, m(\alpha))^2}{2}\right) \int_{-\infty}^{-\frac{a\kappa\alpha}{e}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{e}{2(1+e)}(y - a\kappa\alpha\, m(\alpha))^2\right) dy + \right. \\ &\int_{-\frac{a\kappa\alpha}{e}}^{\frac{a\kappa\alpha}{e}} \frac{1}{\sqrt{2\pi}} \exp\left(-(1/2)y^2 e\,\frac{1+eM(\alpha)}{1+e}\right) dy \; + \\ &\left.\exp\left(\frac{(a\kappa\alpha\, m(\alpha))^2}{2}\right) \int_{\frac{a\kappa\alpha}{e}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{e}{2(1+e)}(y + a\kappa\alpha\, m(\alpha))^2\right) dy\right]. \end{aligned} \tag{B7}$$

We propose the following change of variable:

$$s = \begin{cases} \frac{e}{(1+e)}(y - a\kappa\alpha\, m(\alpha))^2 & \text{if } y < -\frac{a\kappa\alpha}{e}, \\ \frac{e}{1+e}\, y^2 & \text{if } |\,y| \le \frac{a\kappa\alpha}{e}, \\ \frac{e}{(1+e)}(y + a\kappa\alpha\, m(\alpha))^2 & \text{if } y > \frac{a\kappa\alpha}{e}. \end{cases}$$

Replacing the new variable in (B7) we obtain (25).

B5 Proof of Corollary 4

Provided the definitions of $m(\alpha)$ y $M(\alpha)$:

$$m(\alpha^*) = M(\alpha^*) = 1.$$

Therefore, by definition,

$$f_\kappa(\alpha^*, e \,|\, \kappa) = \exp\left(\frac{(a\kappa\alpha^*)^2}{2}\right) \times \left(1 - \Phi\left(\frac{(a\kappa\alpha^*)^2}{e}(1+e)\right)\right). \tag{B8}$$

Additionally, given (11):

$$f(\alpha^*, e \,|\, s)\, ds = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}s(1+e)\right) s^{-1/2},$$

and then

$$\int_0^{\frac{(a\kappa\alpha^*)^2}{(1+e)e}} f(\alpha^*, e \,|\, s)\, ds = \int_0^{\frac{(a\kappa\alpha^*)^2}{(1+e)e}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}s(1+e)\right) s^{-1/2}.$$

Recall the definition of $g_k(e \,|\, \alpha)$ in Proposition 1. Substituting $x = s(1+e)$ in the last equation and adding (B8) we arrive at Corollary 9.

B6 Proof of Corollary 5

Given the definition (28):

$$\begin{aligned}
&\frac{\partial}{\partial \alpha} E\left[U_b\left(\tilde{W}_b(\alpha, e \,|\, \kappa)\right)\right] = -\exp(V(a,e)/r) \times g_\kappa(e \,|\, \alpha)^{1/r} \times \\
&\quad \left[\frac{1}{r}\, g_\kappa(e \,|\, \alpha)^{-1} \times \frac{\partial}{\partial \alpha} g_\kappa(e \,|\, \alpha) \times \left(\int_0^{\frac{(a\kappa\alpha)^2}{(1+e)e}} f(\alpha, e \,|\, s)\, ds + f_\kappa(\alpha, e \,|\, \kappa)\right) + \right. \\
&\quad \left. \int_0^{\frac{(a\kappa\alpha)^2}{(1+e)e}} \frac{\partial}{\partial \alpha} f(\alpha, e \,|\, s)\, ds + f\left(\alpha, e \,|\, \frac{(a\kappa\alpha)^2}{(1+e)e}\right) \frac{2\alpha(a\kappa)^2}{(1+e)e} + \frac{\partial}{\partial \alpha} f_\kappa(\alpha, e \,|\, \kappa)\right].
\end{aligned}$$

Valuing the last equation in α^* (and given Corollary 9):

$$\begin{aligned}
&\frac{\partial}{\partial \alpha} E\left[U_b\left(\tilde{W}_b(\alpha^*, e \,|\, \kappa)\right)\right] = -\exp(V(a,e)/r) \times g_\kappa(e \,|\, \alpha^*)^{1/r} \times \\
&\quad \left[\frac{1}{r} \frac{\partial}{\partial \alpha} g_\kappa(e \,|\, \alpha^*) + \right. \\
&\quad \left. \int_0^{\frac{(a\kappa\alpha^*)^2}{(1+e)e}} \frac{\partial}{\partial \alpha} f(\alpha^*, e \,|\, s)\, ds + f\left(\alpha^*, e \,|\, \frac{(a\kappa\alpha^*)^2}{(1+e)e}\right) \frac{2\alpha^*(a\kappa)^2}{(1+e)e} + \frac{\partial}{\partial \alpha} f_\kappa(\alpha^*, e \,|\, \kappa)\right].
\end{aligned} \tag{B9}$$

Deriving $g_\kappa(e \,|\, \alpha)$ with respect to α and valuing it at α^*:

$$\frac{\partial}{\partial \alpha} g_\kappa(e \,|\, \alpha^*) = -2 \exp\left(\frac{1}{2}\left(\frac{a\kappa}{1+r}\right)^2\right) \frac{(a\kappa)^2}{1+r} \times \\ \left[\phi\left(\frac{1+e}{e}\left(\frac{a\kappa}{1+r}\right)^2\right) - \frac{1}{2}\left(1 - \Phi\left(\frac{1+e}{e}\left(\frac{a\kappa}{1+r}\right)^2\right)\right)\right]. \tag{B10}$$

Deriving equation (11) with respect to α:

$$\frac{\partial}{\partial \alpha} f(\alpha^*, e | s) = -\frac{1}{2} f(\alpha^*, e | s) \times s \times e \times \frac{\partial}{\partial \alpha} M(\alpha^*) = 0, \tag{B11}$$

according to (A4). Finally:

$$f\left(\alpha^*, e | \frac{(a\kappa\alpha^*)^2}{(1+e)e}\right) \frac{2\alpha^*(a\kappa)^2}{(1+e)e} + \frac{\partial}{\partial \alpha} f_\kappa(\alpha^*, e | \kappa) = \\ \frac{1}{r} 2 \exp\left(\frac{1}{2}\left(\frac{a\kappa}{1+r}\right)^2\right) \frac{(a\kappa)^2}{1+r} \times \\ \left[\phi\left(\frac{1+e}{e}\left(\frac{a\kappa}{1+r}\right)^2\right) - \frac{1}{2}\left(1 - \Phi\left(\frac{1+e}{e}\left(\frac{a\kappa}{1+r}\right)^2\right)\right)\right]. \tag{B12}$$

Replacing (B10)-(B12) in (B9) we obtain Corollary 10.

Appendix C: Tables

Table 1

κ	1	2	3	4	5	6	7	8	9	10
				Manager's risk aversion $a = 1/3$						
$\alpha_{TB}(3)$	0.49	0.43	0.40	0.37	0.36	0.34	0.33	0.32	0.32	0.31
				Manager's risk aversion $a = 1/8$						
$\alpha_{TB}(8)$	0.68	0.63	0.61	0.59	0.58	0.57	0.56	0.56	0.55	0.55
				Manager's risk aversion $a = 1/15$						
$\alpha_{TB}(15)$	0.79	0.75	0.73	0.72	0.72	0.70	0.70	0.69	0.69	0.69
				Manager's risk aversion $a = 1/24$						
$\alpha_{TB}(24)$	0.85	0.83	0.80	0.80	0.79	0.78	0.78	0.79	0.78	0.77

Table 2

κ	1	2	3	4	5	6	7	8	9	10
				Manager's risk aversion $a = 1/3$						
$\dot{\alpha}/\alpha$	79.7	57.7	46.7	35.7	32.0	24.7	21.0	17.4	17.4	13.7
$\dot{e}/e$	41.7	29.4	22.9	16.9	14.3	10.5	08.4	06.4	05.8	04.2
$\dot{C}/C$	08.8	06.5	05.2	04.3	03.6	03.0	02.5	02.0	01.6	01.3
				Manager's risk aversion $a = 1/8$						
$\dot{\alpha}/\alpha$	36.0	26.0	22.0	18.0	16.0	14.0	12.0	12.0	10.0	10.0
$\dot{e}/e$	19.6	13.9	11.5	09.3	08.2	07.0	05.9	05.8	04.7	04.6
$\dot{C}/C$	02.7	01.9	01.5	01.3	01.1	01.0	00.9	00.8	00.8	00.7
				Manager's risk aversion $a = 1/15$						
$\dot{\alpha}/\alpha$	21.1	15.0	11.9	10.4	10.4	07.3	07.3	05.8	05.8	05.8
$\dot{e}/e$	11.6	08.0	06.2	05.4	05.3	03.8	03.7	02.9	02.9	02.9
$\dot{C}/C$	01.0	00.7	00.5	00.4	00.3	00.3	00.3	00.2	00.2	00.2
				Manager's risk aversion $a = 1/24$						
$\dot{\alpha}/\alpha$	13.3	10.7	06.7	06.7	05.3	04.4	04.4	05.3	04.0	02.7
$\dot{e}/e$	07.3	05.7	03.5	03.4	02.7	02.1	02.0	02.7	02.0	01.3
$\dot{C}/C$	00.5	00.3	00.2	00.2	00.1	00.1	00.1	00.1	00.1	00.1