INSTITUTO TECNOLOGICO AUTONOMO DE MEXICO

CENTRO DE INVESTIGACIÓN ECONÓMICA

Working Papers Series

Similarity-Based Mistakes in Choice

Fernando Payro
Centro de Investigación Económica, ITAM
Levent Ülkü
Centro de Investigación Económica, ITAM

February 2015
Working Paper 15-03

Av. Camino a Santa Teresa 930
México D.F 10700
MEXICO
Tel. +52 (55) 5628 4197

Similarity-Based Mistakes in Choice*

Fernando Payro† Levent Ülkü‡

This version: February 2015

Abstract

We characterize the following choice procedure. The decision maker is endowed with a preference and a *similarity* - a reflexive binary relation over alternatives. In any choice problem she includes in her choice set all options that are similar to her most preferred feasible alternative. Hence an inferior option may end up, by mistake, being chosen because it is similar to a better one. We characterize this boundedly rational behavior by suitably weakening the rationalizability axiom of Arrow (1959). We also characterize a generalization where the decision maker chooses alternatives potentially on the basis of their similarities to attractive yet infeasible alternatives. We show that if similarity-based mistakes cause a departure from rationalizability, they lead to cyclical behavior.

J.E.L. codes: D0.

Keywords: Bounded rationality, similarity, mistakes, cyclic choice

*We thank Sean Horan, Paola Manzini and Marco Mariotti for useful comments.

†Department of Economics and CIE, ITAM; fernando.payro@itam.mx

‡Department of Economics and CIE, ITAM; levent.ulku@itam.mx

1 Introduction

We study mistakes in choice that arise from similarities between alternatives. In our model the decision maker identifies her most preferred alternative among those that are feasible. However she may be misled by similarities between the alternatives during the act of choice, and select an inferior option on the basis of its similarity to the best. Hence her behavior is summarized by a choice correspondence, identifying in every set as choosable the best feasible alternative together with any other feasible alternative that is similar to it. We show that this choice procedure may lead to cyclic behavior. We characterize it using a single axiom which is a suitable weakening of a classical axiom due to Arrow (1959).

In our model, similarity is a binary relation over alternatives which substantiates the statement "x is similar to y" in the sense that x could be mistaken for y. Tversky (1977) asserts that similarity need not be transitive, nor symmetric. Hence similarity is very different in nature from preferences, in particular when compared with the indifference relation. Accordingly, we only impose the condition of reflexivity on similarity, i.e., that every alternative should be similar to itself. In general, similarity between alternatives need not be related to an underlying preference ordering. For example, a decision maker may mistake a cereal brand for another because they are shelved next to each other. Likewise the low-budget remake *Transmorphers* may be mistaken for the box-office hit movie *Transformers*.

Arrow's axiom requires the removal of alternatives from a menu to have predictable and rational consequences on choice: if any of the original choices are feasible in a smaller menu, the new choice set should coincide with them. Any other consequence of removing an alternative from a menu, whether a new alternative jumps in the choice set to join a previously choosable alternative which is still feasible, or one of the original choices jumps out of the choice set despite being available, is a choice reversal which is incompatible with rationalizability by the maximization of a preference. Our weakening of Arrow's axiom requires the existence of *some* element in the choice set, whose inheritance by subsets prohibits choice reversal. However we do not impose this property on every element in the choice set.

Our weak-Arrow condition allows for cyclical behavior. Furthermore, weak-Arrow and the no-binary-cycles axioms are together equivalent to Arrow's axiom. Hence, similarity-

based mistakes lead to cyclical behavior if they cause any departure from standard rationalizability.

We also study a more general type of mistake-making that allows alternatives to be included in the choice set because they are similar to an infeasible alternative that is better than the best available. It turns out that our weakening of Arrow's axiom, together with the classical α condition characterize this choice procedure.

The possibility of mistakes gives a novel justification for the use of choice correspondences rather than choice functions in the theory of decision making. Choice correspondences allow for some indeterminacy in behavior as any alternative in the choice set might end up being the choice. A choice function, on the other hand, identifies a unique alternative which is necessarily *the* choice. Choice correspondences are used to model behavior when the decision maker's preference ordering contains indifferences or incomparabilities between alternatives. Our model considers a decision maker whose preference is a linear order, which leaves no room for incomparabilities or indifferences between distinct alternatives. However her behavior is potentially indeterminate because she can make similarity-based mistakes, and therefore it is more aptly studied using a choice correspondence.

In the concluding section, we briefly describe a model of choice of bundles given a set of objects which can be chosen in isolation or as part of a bundle. A rational decision maker would have a preference over bundles and choose the best available. However the set of bundles may be potentially very large and its ranking may not be easy. By reinterpreting similarity as complementarity, our procedure gives a boundedly rational way of choosing a bundle: the decision maker first identifies her most preferred object (rather than the most preferred bundle) and next bundles it with any other object which complements it.

Relation to literature Our choice procedure operates in two stages. First it identifies the best available alternative. Next it identifies all feasible alternatives which are similar to the best. There is an important contrast, however, between our work and the recent literature on multistage choice procedures. In this literature alternatives are eliminated in every step until a choice is made.[1] Even though the unique survivor of the first stage remains choosable in our procedure, some of the alternatives passed over in the first stage may end up in the choice set as well.

[1] See, for example, Manzini and Mariotti (2007), Masathoğlu et al. (2012) and Bajraj and Ülkü (2015).

Rubinstein (1988) analyzes a procedure in which a decision maker uses similarity in completing her otherwise incomplete preference over lotteries. The role for similarity highlighted in our analysis is different, as our decision maker, fully rational in her mental attitudes towards alternatives, is potentially behaviorally irrational because of the possibility of similarity-based mistakes.

2 Basic concepts

We consider a standard choice environment with a finite set of alternatives X. The set Σ contains all nonempty subsets of X, and any $A \in \Sigma$ is a *choice problem*. We will omit set brackets and denote sets as strings of alternatives whenever convenient. For example we will write xyz and x instead of $\{x, y, z\}$ and $\{x\}$. In the latter case, the context will clarify whether we are referring to the set or the alternative. For any binary relation R on X, we will denote by $\max(A, R)$ the set of maximal alternatives in A according to R. If R is a linear order, $\max(A, R)$ is a singleton and we will denote its unique element by $\max(A, R)$ as well.

A *choice correspondence* is a map $c : \Sigma \to \Sigma$ satisfying $c(A) \subseteq A$ for every choice problem A. We will call members of $c(A)$ the *choosable* alternatives in A. The interpretation is that any element in the choice set $c(A)$ might end up being the choice and definitely no alternative outside $c(A)$ could be the choice. Hence a choice correspondence is capable of indicating indeterminacy in behavior. A choice correspondence c is *rationalizable* if there exists a preference (complete and transitive binary relation) R on X such that $c(A) = \max(A, R)$. It is well-known that rationalizability is equivalent to the following form of the weak axiom of revealed preference due to Arrow (1959).[2]

Arrow's axiom: For every A and every $B \subset A$, if $B \cap c(A) \neq \varnothing$, then $c(B) = c(A) \cap B$.

Arrow's axiom requires the following consistency of choices. In any small choice problem containing some of the choosable alternatives of a larger choice problem, (i) no new alternative should become choosable, and (ii) no feasible alternative that was originally choosable should drop out of the choice set.

[2] See, for example, Moulin (1985).

3 Similarity-based mistakes

We are interested in a procedure for making choices which exhibits mistake-making on the basis of similarities between alternatives. We first clarify what we mean by similarity.

Definition 1 A *similarity* is any reflexive binary relation S on X.

The statement xSy indicates that x is similar to y in the sense that, for whatever reason, be it appearance, location, psychological factors, etc., x could be mistakenly chosen instead of y. Similarity need not be transitive, nor symmetric. However every alternative is similar to itself. Similarity may or may not be preference-related. We give some examples to illustrate.

Example 1 For some utility representation u of the decision maker's preferences and for some number $\varepsilon > 0$, say x is similar to y if $|u(x) - u(y)| < \varepsilon$.

Example 2 For some welfare-irrelevant linear order L which lists the alternatives, say x is similar to y if for any third alternative z neither $[xLz$ and $zLy]$ nor $[yLz$ and $zLx]$. In this case alternatives are similar if they are adjacent in L.

Example 3 Similarity could be asymmetric if, for example, some other alternatives could be mistaken for an attractive alternative x^* but not vice versa. In this case if xSy, then $x = y$ or $y = x^*$.

We now introduce the choice procedure of main interest in this paper. For any linear order P giving a decision maker's preferences and any similarity S, define a choice correspondence $c_{P,S}$ by

$$c_{P,S}(A) = \{x \in A : xS \max(A, P)\} \tag{1}$$

for every choice problem A. Say c has the similarity-based mistakes (SBM) representation (1) if $c = c_{P,S}$ for some linear order P and similarity S.

If c admits SBMs as in (1), all feasible alternatives which are similar to the best feasible alternative are deemed choosable. Note that since S is reflexive the best feasible alternative is always in the choice: $\max(A, P) \in c_{P,S}(A)$. We follow with two remarks on choice correspondences which admit such SBMs.

Remark 1 *Symmetry versus asymmetry of similarity.* Any similarity that matters in this formulation is that relating the less preferred alternative to the more preferred alternative. To be precise for any P and any three similarities S,S' and S'' such that for any distinct x and y

$$\begin{aligned} xS'y &\Leftrightarrow [xSy \textit{ and } yPx] \\ xS''y &\Leftrightarrow [xS'y \textit{ or } yS'x] \end{aligned}$$

then $c_{P,S} = c_{P,S'} = c_{P,S''}$. Note that S' is an asymmetric subset of S. Any similarity lost in using S' instead of S has no impact on behavior as it would have to relate an alternative to some other which is worse according to P. Moving from S' to S'' establishes symmetry but does so by only introducing irrelevant similarities. Hence we could impose similarity to be symmetric (equally, asymmetric) without any loss of generality.

Remark 2 *Identification of the preference.* Suppose that two similar alternatives x and y are also adjacent in the preference ordering. Then choices made by a choice correspondence with the SBM representation (1) cannot reveal which alternative is better. To be precise, take any x, y, P and S such that ySx, xPy and no z exists such that xPz and zPy. Define a new linear order P' as follows: $aP'b$ whenever aPb and $(a, b) \neq (x, y)$ and yPx. Then $c_{P,S} = c_{P',S}$.

In an attempt to axiomatize (1), let us rephrase Arrow's rationalizability axiom from the previous section.

Arrow's axiom rephrased: For every A and *for every* $a \in c(A)$, if $a \in B \subset A$ then $c(B) = c(A) \cap B$.

We will weaken Arrow's axiom as follows.

w-Arrow: For every A *there exists some* $a \in c(A)$ such that if $a \in B \subset A$ then $c(B) = c(A) \cap B$.

The difference between w-Arrow and the rephrasing of Arrow's axiom above is precisely the quantifiers in italics. In order to assess the exact behavioral consequences of our weaker condition, recall the following classical axiom.

No binary cycles (NBC): If $x \in c(xy)$ and $y \in c(yz)$, then $x \in c(xz)$.

It is straightforward to show that w-Arrow and NBC are not logically connected. The next result shows that if a choice correspondence satisfies both w-Arrow and NBC, then it is rationalizable.

Theorem 1 *A choice correspondence satisfies Arrow's axiom if and only if it satisfies NBC and w-Arrow.*

Proof. In one direction, it is straightforward to argue that Arrow's axiom implies w-Arrow and NBC. In the other direction, suppose, towards a contradiction that c satisfies w-Arrow and NBC, but fails Arrow's axiom. Hence there exist sets A and B and an alternative $x \in c(A)$ such that $x \in B \subset A$ however $c(B) \neq c(A) \cap B$. Let a and b be the special alternatives in $c(A)$ and $c(B)$ respectively which are identified by w-Arrow. Clearly $a \notin B$, in particular, $a \neq b$ and $a \neq x$. If $b \notin c(A)$, then $b \neq x$ and furthermore notice that $a = c(ab)$, $b \in c(bx)$ and $ax = c(ax)$, which is a binary cycle. Hence we must have $b \in c(A)$. There are two cases to consider.

Case 1: Suppose $y \in c(A) \cap B$ but $y \notin c(B)$. In this case $y \neq a$ and $y \neq b$. Furthermore $b = c(by)$ and $ay = c(ay)$ so $a \neq b$ as well. Finally, note that $ab = c(ab)$, which gives a binary cycle, a contradiction.

Case 2: Suppose $y \in c(B)$ but $y \notin c(A)$. In this case $y \neq a$ and $y \neq b$. Furthermore, $ab = c(ab)$, $by = c(by)$ and $a = c(ay)$, which is a binary cycle, a contradiction. ■

We now give a characterization of $c_{P,S}$ given in (1).

Theorem 2 *A choice correspondence has the similarity-based mistakes representation (1) if and only if it satisfies w-Arrow.*

Proof. In one direction, suppose that $c = c_{P,S}$ as in (1). Take a choice problem A and let $x = \max(A, P)$. Take any $B \subset A$ such that $x \in B$. Since S is reflexive $x \in c(A)$. Furthermore $x = \max(B, P)$, so $x \in c(B)$ as well. Now w-Arrow follows since

$$\begin{aligned} c(B) &= \{b \in B : bSx\} \\ &= \{a \in A : aSx\} \cap B \\ &= c(A) \cap B. \end{aligned}$$

In the other direction, take a choice correspondence c which satisfies w-Arrow. First define a binary relation S by

$$xSy \Leftrightarrow [x = y \text{ or } xy = c(xy)].$$

Note that S is reflexive (and symmetric) by definition. Next define a binary relation P_0 by

$$xP_0y \Leftrightarrow [x \neq y \text{ and for some } A, y \in A \text{ and } c(A\backslash x) \neq c(A)\backslash x].$$

We will show that P_0 is acyclic. Take $n > 1$ and $x_1, ..., x_n$ such that $x_iP_0x_{i+1}$ for $i < n$. Set $x_{n+1} = x_1$ and suppose, towards a contradiction, that $x_nP_0x_{n+1}$. Then, by definition, there exist sets $A_1, ..., A_n$ such that for every i, $x_{i+1} \in A_i$ and $c(A_i\backslash x_i) \neq c(A_i)\backslash x_i$. Let $A = \bigcup_i A_i$. By axiom, there is an alternative $a \in c(A)$ such that whenever $a \in B \subset A$, $c(B) = c(A) \cap B$. Now if $a = x_{i+1}$ for some i, then since $x_{i+1} \in A_i$, $c(A_i) = c(A) \cap A_i$, and since $x_{i+1} \in A_i\backslash x_i$, $c(A_i\backslash x_i) = c(A) \cap A_i\backslash x_i = c(A_i)\backslash x_i$, a contradiction. Hence $a \in A\backslash x_1...x_n$. But then, since for some i, $a \in A_i$ and $a \in A_i\backslash x_i$, we get an analogous contradiction: $c(A_i\backslash x_i) = c(A) \cap A_i\backslash x_i = c(A_i)\backslash x_i$.

Since P_0 is acyclic, its transitive closure is a strict partial order and, by the Szpilrajn (1930) Theorem, it can be extended to a linear order P. We need to show that $c(A) = c_{P,S}(A)$ for every A.

Fix A and let $a = \max(A, P)$. We will establish that a is the alternative in A identified by w-Arrow, i.e., that $a \in c(A)$ and if $a \in B \subset A$, then $c(B) = c(A) \cap B$. If for some $y \in A$, $y = c(ay)$, then yP_0a, an impossibility. Hence $a \in c(xy)$ for every $y \in A$. This implies that $a \in c(A)$ as well, since otherwise, removing alternatives one-by-one from A, we can find a set $B \subseteq A$ and an alternative $y \in B\backslash a$ such that $a \notin c(B)$ but $a \in c(B\backslash y)$. This would mean, again, that yP_0a, an impossibility. Now take a set $B \subset A$ with $a \in B$. Then $c(B) = c(A) \cap B$, since otherwise an alternative $y \in A\backslash B$ exists such that yP_0a, an impossibility.

To see that $c(A) \subseteq c_{P,S}(A)$, pick $x \in c(A)$. If $x = a$, there is nothing to show so suppose $x \neq a$. Then, by w-Arrow $c(xa) = xa$ and therefore xSa. Hence $x \in c_{P,S}(A)$. Finally to see that $c_{P,S}(A) \subseteq c(A)$, pick $x \in c_{P,S}(A)\backslash a$. We must have, then, xSa, i.e., $xa = c(xa)$. Now $c(xa) = c(A) \cap xa$ by w-Arrow, and therefore $x \in c(A)$. This finishes the proof. ■

From Theorems 1 and 2, we get

Corollary 1 *Suppose c has the similarity-based mistakes representation (1). If c fails rationalizability, then it admits a binary cycle.*

Similarity-based mistakes can explain some but not all cyclical behavior. Consider the cycle $c(xy) = x$, $c(yz) = y$ and $c(xz) = z$. If c was to have the representation (1), we would have to conclude that, since all choice sets are singletons, there is no similarity between any pair of these three alternatives. Hence behavior would have to be driven purely by maximization of P, contradicting a cycle.

However cycles which are *less robust*, in the sense of being part of some indeterminate behavior (i.e., non-singleton choice sets) can be explained by similarity-based mistakes. Take, for example, the cycle $c(xy) = xy$, $c(yz) = y$ and $c(xz) = z$. If $c = c_{P,S}$ as in (1), then $yPzPx$ and xSy. Hence we would have to have $c(xyz) = xy$ as well. If, on the other hand, $c = c_{P,S}$ and contains the cycle $c(xy) = xy$, $c(yz) = yz$ and $c(xz) = z$, then zPx, ySx, ySz and $x \not S z$. Either the P-maximal among the three alternatives is z, in which case $c(xyz) = yz$, or it is y and $c(xyz) = xyz$. In the former case, the preference ranking between x and y can not be determined.

4 A generalization

In this section we allow for a more general class of similarity-based mistakes. In representation (1), similarities matter only within the set, specifically when an alternative is similar to the best available alternative. It is also perceivable that a decision maker could mistakenly select an alternative similar to an attractive alternative which is not feasible. To address this issue, let us define for any linear order P and any similarity S,

$$\begin{aligned} P(A) &= \{x \in X : xPa \text{ for all } a \in A\}, \text{ and} \\ S(x, A) &= \{a \in A : aSx\}. \end{aligned}$$

Hence $P(A)$ is the set of alternatives are not strictly dominated by any alternative in A, and $S(x, A)$ gives the set of alternatives in A which could be mistaken for x. Note that $\max(A, P) \in P(A)$. Consider the choice correspondence $c^{P,S}$ defined by

$$c^{P,S}(A) = \bigcup_{x \in P(A)} S(x, A) \tag{2}$$

for every A. Say c has the similarity-based mistakes (SBM) representation (2) if $c = c^{P,S}$ for some linear order P and similarity S.

Representation (2) says that an alternative is choosable if and only if it is similar to the best feasible alternative or to an infeasible alternative which is better than the best feasible. Hence the decision maker can falsely identify an infeasible alternative as feasible and end up choosing the inferior alternative which is similar to it. Clearly, $c_{P,S}(A) \subseteq c^{P,S}(A)$.

Our characterization of (2) will rely, together with w-Arrow, on the following classical axiom.

α: If $x \in c(A)$ and $x \in B \subset A$, then $x \in c(B)$.

Notice that α is stronger than one of the set inclusions required by w-Arrow. It implies, in fact, that for every A and every $B \subset A$, $c(B) \supseteq c(A) \cap B$. However, $c(B)$ could contain alternatives which are not in $c(A)$. This is prevented by w-Arrow if B contains a specific choosable alternative of A.

Theorem 3 *A choice correspondence has the similarity-based mistakes representation (2) if and only if it satisfies α and w-Arrow.*

Proof. In one direction, suppose that $c = c^{P,S}$ for some linear order P and reflexive S. Fix A and let $a = \max(A, P)$. Take any B such that $a \in B \subset A$. Then $a = \max(B, P)$ as well. Furthermore $P(A) = P(B)$ and $S(x, B) = S(x, A) \cap B$. This gives

$$c(B) = c^{P,S}(B) = c^{P,S}(A) \cap B = c(A) \cap B.$$

Hence c satisfies w-Arrow. To see that it satisfies α, fix A, B and x satisfying $x \in c(A)$ and $x \in B \subset A$. Let $y \in P(A)$ be such that xSy. Since $P(B) \supseteq P(A)$, $y \in P(B)$ as well and consequently $x \in c(B)$.

In the other direction, take a choice correspondence c satisfying w-Arrow and α and define S and P_0 exactly as in the proof of Theorem 2. Note S is reflexive, and by w-Arrow, P_0 is acyclic exactly as in the proof of Theorem 2. Let, once again P be a linear order containing P_0. Fix any A. Let $a = \max(A, P)$. Note that a is the special alternative in $c(A)$ identified by w-Arrow.

Claim 1: $c^{P,S}(A) \subseteq c(A)$. To see this, take $x \in c^{P,S}(A)$. If xSa, then $x \in c(A)$ by w-Arrow and the definition of S, exactly as in the proof of Theorem 2. Suppose xSy for some $y \notin A$ such that yPa. Then $xy = c(xy)$ and consequently $x \in c(axy)$, since otherwise aP_0y, a contradiction. Now α dictates $x \in c(xa)$ and by w-Arrow $x \in c(A)$ as well.

Claim 2: $c(A) \subseteq c^{P,S}(A)$. Take $x \in c(A)$. If $x = a$, then $x \in c^{P,S}(A)$. If $x \neq a$, then $xa = c(xa)$ by α (or by w-Arrow) and xSa. Hence $x \in c^{P,S}(A)$. ■

The following corollary to Theorems 1 and 3 is immediate.

Corollary 2 *Suppose c has the similarity-based mistakes representation (2). If c fails rationalizability, then it admits a binary cycle.*

Hence both types of choice correspondences admitting SBMs which we have considered are rationalizable if they do not contain binary cycles. The behavioral difference between the two representations (1) and (2) is that (1) could fail α whereas (2) has to satisfy α.

5 A reinterpretation

To finish we would like to give a reinterpretation of our choice procedure in (1) in a different model of choice, based on complementarities rather than similarities. Change the interpretation of $c(A) \subseteq A$ from the set of choosable alternatives to the set of chosen alternatives. Hence the decision maker is understood to choose the bundle $c(A)$ out of the set of bundles 2^A.[3] This interpretation is not novel. It is found in the recent literature on school matching with choice functions, where given the set of applicants A, a school's choice $c(A)$ is the set of admitted students. (See, for example, Chambers and Yenmez [2013].)

[3] An equivalent way to describe tis model is to start with a set X and to consider choice functions f on 2^X with a particular *domain restriction*. Let $D = \{2^A : \varnothing \neq A \subseteq X\} \subset 2^{(2^X)}$ be the set of admissible choice problems. Consider choice *functions* of the form

$$f : D \to 2^X$$

satisfying $f(2^A) \in 2^A$. Each such f gives rise to a map $c : 2^X \to 2^X$ by $c(A) = f(2^A)$. Since $f(2^A)$ is a nonempty subset of A, the map c is a choice correspondence.

In this framework, rational choice requires the decision maker to have a preference (say, a linear order) $\succeq$ on 2^X, the set of all subsets of X, and, in any set A, find the best subset of A. In other words, $c_{\succeq}(A) = \max(2^A, \succeq)$. However 2^X is generally a very large set compared to X and ranking its members may be burdensome. Instead consider the following boundedly rational choice procedure which only requires ordering X. Imagine that the decision maker is endowed with two binary relations:

- a preference P (a linear order) on elements of X rather than sets in 2^X, and
- another binary relation C over X, which gives complementarities between alternatives.

Suppose that in any set A she first finds her most preferred alternative, then combines it with anything else in the set that complements it. Consider choosing from a menu in a restaurant. The decision maker first identifies the most preferred food item, say the steak. Then she builds a meal around the steak by ordering it together with any other feasible item that complements it. Supposing that the complementarity relation C is reflexive, this behavior is captured by the formula

$$c_{P,C}(A) = \{x \in A : xC \max(A, P)\}$$

precisely as in the representation (1). Consequently $c_{P,C}$ is characterized by w-Arrow.

References

[1] Arrow, K. (1959), "Rational choice functions and orderings." Economica 26,121-127.

[2] Bajraj, G. and L. Ülkü (2015), "Choosing two finalists *and* the winner." Social Choice and Welfare, forthcoming.

[3] Chambers, C. and M. B. Yenmez (2013), "Choice and matching." Working paper.

[4] Manzini, P. and M. Mariotti (2007), "Sequentially rationalizable choice."American Economic Review, 97(5): 1824-1839

[5] Masatlıoğlu, Y,. D. Nakajima and E. Özbay (2012), "Revealed attention", American Economic Review, 102(5): 2183-2205

[6] Moulin, H. (1985), "Choice functions over a finite set: a summary." Social Choice and Welfare 2,147-160.

[7] Rubinstein, A. (1988), "Similarity and decision-making under risk (Is there a utility theory resolution to the Allais paradox?)" Journal of Economic Theory 46 (1), 145-153.

[8] Szpilrajn, E. (1930), "Sur l'extension de l'ordre partiel." Fundamenta Mathematicae, 16, 386-389.

[9] Tversky, A. (1977), "Features of similarity." Psychological Review, 84 (4), 327-352.