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M E X I C O

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Abstract

In Internet auctions bidders frequently bid in one of three ways: either only early, or late, or they revise their early bids. This paper rationalizes all three bidding patterns within a single equilibrium. We consider a model of a dynamic auction in which bidders can search for outside prices during the auction. We find that in the equilibrium bidders with the low search costs bid only late and always search, while the bidders with high search costs bid early or multiple times and search only if they were previously outbid. An important feature of the equilibrium is that early bidding allows bidders to search in a coordinated manner. This means that everyone searches except the bidder with the highest early bid. We also compare the static and dynamic auction and conclude that dynamic auction is always more efficient but not always more profitable.

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1 Introduction

Bidding behavior in Internet auctions displays intriguing regularities that are often difficult to rationalize given the complexity of the Internet environment. However, it is important to understand how people bid in these auctions, because in the end, this is what determines their efficiency and profitability properties. Bidding data from various auction sites suggest that frequently occurring bidding patterns can be divided into three broad categories: early bidding, late bidding and multiple bidding. Both early and late bidding refer to a single bid placed by a single bidder. In the former case the bid arrives in the few initial days and in the latter case on the last day of the auction. Multiple bidding occurs when a bidder revises his earlier bid later in the auction. Most of the literature addresses each of the three bidding patterns separately (see Roth and Ockenfels 2002, Rasmusen 2006, Hossain 2008, and Compte and Jehiel 2004) and only little attention has been devoted to the question whether these three patterns could arise jointly in a single equilibrium. The intuition would suggest that if, for example, incentives to bid late are superior to those for bidding early, we might expect that in time the late bidding would replace the early bidding in the data. But this has not happened. On the contrary, all three bidding patterns are very much present in the data. In what follows we present a model which illustrates that the equilibrium behavior can be consistent with all three bidding patterns. This provides a unified and intuitive explanation for major empirical regularities.

Central to our argument is the idea that during the auction some bidders may want to embark on a costly search for alternative outside prices. The value of early bidding is in that it allows "coordination" of search decisions of those bidders who find it difficult to search and want to avoid it. Consequently, the bidder with the highest early bid remains passive while everyone else searches. These incentives are responsible for early and multiple bidding. On the other hand, the late bidding is caused by bidders for whom searching is easy and who always want to look for outside prices before they bid in the auction. They do not want to bid early because having their early bid tied up in the auction could cause them to miss out on a good deals they may find on the outside. In the equilibrium of our model both of these incentives coexist.

Bidders in Internet auctions are not necessarily interested in buying the object at any price, but rather, they are looking for a good deal. In the auction, if the price rises too high, they might decide to look for a better deal elsewhere. What complicates the matter is that the price searching is costly in terms of time and effort. Bidders with different locations or time constraints may also have different search costs. For instance, a bidder looking for a car in Los Angeles is going to have many opportunities to shop around at different dealers compared to a bidder in Montana. Therefore, for the Montana bidder, the Internet auction may be the only way to find the car he wants. He can benefit from signaling his high search costs to the other bidders by making a high early bid. It is much easier for the bidder in Los Angeles to search for outside prices and therefore he may want to keep his options open by placing a small early bid. Once he is outbid, he realizes that he faces an opponent who is inflexible and this induces him to intensify his search. As a result, both bidders benefit. The Los Angeles bidder benefits from a low price that he finds at the dealer and the Montana bidder benefits from reduced competition in the auction. The model presented in this paper captures the essence of this story.

Internet auctions have two distinguishing features. The first is that they can last for several days. This allows bidders to come and go as they wish and revise their bids in discrete time intervals. The second feature is proxy bidding, which is implemented by numerous auction sites, including eBay. When a bidder enters a proxy bid, the auction automatically bids for the bidder up to the minimum amount needed to outbid the highest competing proxy bid. Proxy bidding effectively gives the auction the properties of a second-price auction. Our model contains both of these features. We consider a simple second-price auction with two discrete bidding rounds: early and late. In both rounds bidding occurs simultaneously and after the early bidding round the second highest early bid, or the "standing price," is publicly revealed. Between early and late bidding round all bidders get a chance to learn an outside price. We call this a searching round. A decision to search implies that a bidder gets his outside price but pays a search cost. In the model we focus on the heterogeneity in bidders' search costs rather than their valuations. Therefore, bidders have private search costs but value the object equally, at one.

We characterize an equilibrium in which bidding activity occurs in both bidding rounds. The equilibrium can produce all aforementioned bidding patterns, early late and multiple bidding. The early and multiple bidding are caused by bidders who have sufficiently high search costs. Early bids are increasing in search costs and allow "implicit coordination" of search decisions. The bidder with the highest search cost submits the highest early bid and becomes the high bidder. This means that, in equilibrium, he passes while all other bidders search. Similarly, all other bidders who are outbid in the early round infer that the high bidder passes, and hence, they search. A strategy which allows this type of coordination is the crucial part of the equilibrium and we refer to it as the searching-when-loosing strategy.

In the late bidding round the high bidder revises his bid up to his valuation (one). All other bidders have searched and bid their outside prices. But not all late bids are recorded. This accounts for the difference between early and multiple bidding. To illustrate the difference, a high bidder who has bid less than his value in the early round will revise his bid to one in the late round. This bidder has both bids recorded and becomes a part of multiple bidding pattern. On the other hand, a bidder who has the second highest early bid is the one that sets the standing price and then searches. If he finds an outside price which is below the standing price, then he opts out of the auction and purchases the object for the outside price. In his case only the early bid is recorded and he would become a part of an early bidding pattern.

The late bidding part of the equilibrium is qualitatively different. Late bidding is due to the bidders who have low search costs. These bidders bid just late and always search. To see the intuition, suppose that a bidder has a zero search cost, i.e., he can search effortlessly. Because there is always a chance that the outside price could be very low (e.g., the object is on sale at the store nearby) he will search irrespective of whether he expects the other bidders to search or not. Since it doesn't cost him anything to search, then why not do it. It makes no sense for this bidder to bid early in the auction, because if he does, then he would be risking winning the auction at the price which is higher than what the outside price could be. In equilibrium, this type of bidder always searches and bids his best outside alternative only late in the auction.

The predictions of our model are consistent with large body of empirical evidence. It specifically addresses two features of the bidding data that we find puzzling: the large proportion of bids that arrive quite early in the auction; and the coexistence of all three bidding patterns, early, late and multiple bidding, in similar proportions[1]. Shmueli *et al.* (2004) for example report that in their sample of 196 auctions on 189 Palm M515 device about

[1] Two other bidding patterns occur frequently in the data: "sniping" – bidding in the dying moments of the auction (Roth and Ockenfels 2002); and "nibbling" – revising own bid in small increments and short time intervals (Hossain 2008). Our model does not specifically address these two bidding practices.

30% of all bids arrive in the first half of the auction duration and 50% within one quarter of the auction duration to go. More detailed description of bidding behavior is given by Bapna and Gupta (2003) who split their sample of about 90 auctions for variety of objects into three groups that are closely related to our categorization. The early bidders are referred to as "evaluators," late bidders as "opportunists," and multiple bidders as "participators." On average, early bidders occur in about 40% of cases, late bidders in about 23% of cases and multiple bidders in the remaining 37% of cases. Shah *et al.* (2004) analyzed a large dataset consisting of 11 537 eBay auctions for Sony Playstation 2 and Nintendo consoles. They find significant proportions of early and late bidders, 28% and 38% respectively. The remaining portion of bidders bid multiple times.

The most closely related paper is by Rasmusen (2006). He considers a dynamic auction with an informed bidder who knows his private value and an uninformed bidder. The uninformed bidder can learn his value during the auction by paying a discovery cost. In equilibrium, the uninformed bidder bids early and learns his value only if he is outbid. Rasmusen highlights the fact that sometimes, depending on his valuation, the informed bidder has incentives to bid late or randomize his early bid (signal-jamming) because he can profit from keeping the other bidder uninformed. Our model is different in several respects and provides different incentives for late bidding. The key observation is that getting an outside price can only lower the maximum willingness to pay of a particular bidder, and hence, creates a positive externality for everyone else. Thus, a bidder never wants to keep his opponents uninformed. Late bidding in our model is due to bidders with very low search costs who simply do not want to have their bids tied up in the auction before they search for outside prices.

Other related papers are by Hossain (2007), Rezende (2005), and Compte and Jehiel (2004). Hossain, just like Rasmusen, considers a model with informed and uninformed bidders. Every time the standing price changes, each uninformed bidder obtains a better estimate of his private valuation by getting a free signal of whether he likes the object at that price or not. This induces the uninformed bidders to experiment by repeatedly revising their bids during the auction - a behavior Hossain calls "nibbling." Both Rezende and Compte & Jehiel look at ascending clock-auction environment. In the first paper a bidder can pay a cost to refine his private valuation during the auction and in the second paper he can discover the level of competition, i.e., how many other bidders are in the auction. Both papers demonstrate that information acquisition during the auction is valuable. The current paper is also partially related to various streams of literature on preemptive bidding pioneered by Fishman (1988), Engelbrecht-Wiggans (1988), on value learning and entry, McAffee and McMillan (1987), Levine and Smith (1994), and on value learning and efficiency, Persico (2000), Bergemann and Välimäky (2002) and, Schwarz and Sonin (2005).

There has been a growing interest in comparing a static and a dynamic auction formats. Both types of auctions, short (static) and long (dynamic), are used frequently on the Internet. Several studies have argued that in variety of environments dynamic auctions achieve greater efficiency and revenue than static auctions, see Compte & Jehiel (2007) and Rezende (2005). The current paper also contributes to this debate by arguing, that in our setting, the dynamic auction is always more efficient than the static auction. Furthermore, we confirm the previous claims that the dynamic auction is more profitable for large number of bidders, but we also illustrate by using an example that in some cases the static auction may generate a higher revenue.

The rest of the paper proceeds as follows. In the next section we set up the model. Section 3 characterizes equilibrium behavior, first in the simpler common outside prices environment and then later in the more appropriate environment with independent and private outside prices. In section 4 we compare the welfare and revenue of a static and a dynamic auction format. Lastly, we conclude by a short discussion of our results.

2 The Model

The model is a simplified version of an Internet auction environment. The primary differences between the Internet-type auctions and standard "textbook" auction formats are the multiple rounds of bidding, the proxy-bidding and the availability of outside buying opportunities. We integrate these features into our model.

There is a single object offered in the auction. The object is worthless to the seller, i.e., $v_0 = 0$. In addition, there are other units of the same object offered in the outside market at posted prices. There is $n \geq 2$ number of bidders. All bidders are risk neutral and have a common valuation for a single unit of the object, $v = 1$. Bidders bid for the object in the auction but before it closes, any bidder may invoke a private offer (an outside price), q_i. The outside price is a random draw from an atomless distribution $F[0, 1]$

with density f. To get q_i, a bidder has to pay a search cost $c_i \backsim G[0,\bar{c}]$, which is his private type, and is drawn independently for each bidder before the auction. We assume that G is atomless, strictly increasing and that the upper bound $\bar{c}$ is sufficiently large, $\bar{c} \geq 1 - E[q_i]$, so that some types in the support will not find it worthwhile to learn the outside price at all.

The game has three rounds: *the early (bidding) round*, *the searching round*, and *the late (bidding) round*, i.e., $r \in \{e, s, l\}$. The bidding format is a dynamic (two-round), second-price auction. Bidding takes place in the early and late round. Each bidding round, $r \in \{e, l\}$, begins with all bidders simultaneously placing their bids $b_{i,r} \in [0,1]$. Let b_r be the vector of all bids placed in round r and denote by $b_r^{(k)}$ the k-th highest bid. If several bids are tied then their relative rank is decided randomly. A bidder cannot lower his bid between rounds, i.e., $b_{i,e} \leq b_{i,l}$. At the beginning of each round all bidders observe the current auction price. The opening price in the auction is set to zero. Since no bidding takes place in the searching round, the same *standing price*, p, is observed at the onset of both the searching round and the final bidding round. The standing price equals to the second highest early bid, $p = b_e^{(2)}$. Finally, after the auction closes, the final auction price equals to the second highest overall bid[2], $b_l^{(2)}$.

The auction begins with the early bidding round. All bidders simultaneously place bids in the auction. Before the searching round, each bidder first observes the standing price, p, and who is the current high bidder[3], W,

$$W(b_e) = \begin{cases} i & \text{if} \quad b_{i,e} = b_e^{(1)} \\ \oslash & \text{if} \quad b_{j,e} = 0 \text{ for all } j \end{cases} .$$

Note that if $b_{i,e} = 0$, then $W \neq i$. Bidding zero has the same effect and could be interpreted as not having bid at all. The pair $\{p, W\}$ we refer to as history. In the searching round all bidders decide whether they want to search for an outside price, q_i. A bidder who searches incurs a search cost, c_i, and gets a price draw, q_i. In the late bidding round bidders submit another round of bids. Then, the auction closes and any bidder who has searched can purchase the object at his outside price.

[2] At any point in the auction a single bidder can have only a single valid bid placed in the auction. Hence, if a bidder raises his early bid then his old lower bid is automatically canceled and he is committed to his new higher bid. With this rule it cannot happen that a bidder would set a price for himself.

[3] The current high bidder is the one who would be awarded the object if the auction ended at that moment.

To examine the payoffs let $\tau_i \in \{0,1\}$ indicate whether bidder i has searched, ($\tau_i = 1$), or passed, ($\tau_i = 0$). Similarly, let $\gamma_i \in \{0,1\}$ indicate whether i has won the auction ($\gamma_i = 1$) or not, ($\gamma_i = 0$). The ex-post payoff to bidder i if he had won the auction is given by

$$V_i(\tau_i, \gamma_i) = \gamma_i(1 - b_l^{(2)} - \tau_i c_i) + (1 - \gamma_i)\tau_i(1 - q_i - c_i).$$

A strategy for a bidder i is a triple

$$(\beta_{i,e}(c_i),\ \tau_i(c_i; p, W),\ \beta_{i,l}(c_i, q_i; p, W)),$$

where $\beta_{i,e}$ is the early round bidding function; τ_i is the probability of searching and $\beta_{i,l}$ is the bidding function in the late round.

3 Equilibrium

In this section we characterize the equilibria. Our solution concept is the Perfect Bayesian Equilibrium. We restrict our attention to equilibria in symmetric and pure strategies. Notice, that due to symmetry restriction, a strategy has no bidder subscript, i.e., $(\beta_e(c_i), \tau(c_i; p, W), \beta_l(c_i; p, W))$. Furthermore, since we consider only pure strategies in the searching round, then each bidder either searches or passes with probability one, i.e., $\tau \in \{0, 1\}$.

Finally, we do not allow the use of dominated strategies in the late bidding round. The standard argument, due to Vickrey (1962), by which value bidding is a (weakly) dominant strategy in the second-price sealed-bid auction applies to the late bidding round. If the bidder did not search, then his ex-post payoff belongs to $\{1 - b_l^{(2)}, 0\}$, since he gets nothing if he loses the auction. If he did search, then his outside price is drawn from the unit interval, i.e., $q_i \in [0, 1]$, and his ex-post payoff belongs to $\{1 - b_l^{(2)}, 1 - q_i\}$. The value of the outside price determines the maximum willingness to pay for the object in the late bidding round. This puts us to the Vickrey's world in which bidding q_i is an undominated strategy (see Wang, 2006). Hence, in the late bidding round, each bidder revises his early bid upwards whenever his best outside alternative is higher than his early bid:

$$b_{i,l} = \begin{cases} 1 & \text{if} \quad \text{passed} \\ \max[b_{i,e}, q_i] & \text{if} \quad \text{searched} \end{cases}. \tag{1}$$

Notice that if the outside price is lower than the standing price, then $b_{i,l} = b_{i,e}$, which effectively means that the bidder has chosen to drop out of the auction, i.e., his bid cannot win the auction. This goes back to the difference between early and multiple bidding. Since from this point on we are mostly going to refer to bidding behavior in the early bidding round, for the sake of exposition we will drop the subscript indicating the round and rather use b and β when referring to the early bid and the early bidding function respectively.

3.1 Common Price Draws

This section presents a simpler version of the model in which price draws are common, i.e., $q_i = q$ for all i, and $q \backsim F[0,1]$. All bidders get the same draw if they decide to search. In order to understand the incentives for early bidding we begin by looking at the expected payoffs in the searching round. Here, a bidder's payoff from searching or passing will depend on whether he was outbid after the early bidding round. There can be two cases: either bidder i is the high bidder after the early round, $W = i$, or he is not, $W \neq i$. We look at either of these cases in turn.

Suppose first that in the searching round bidder i is the high bidder and p is the standing price. If i searches, then he pays his search cost c_i, learns q and possibly revises his bid to $\max[b_i, q]$ in the final bidding round. With probability $\tau_{-i} = \prod_{j \neq i} \tau_j$ each of the remaining bidders searches and bids q in the final round. If $q > p$, then no matter whether i wins or looses[4], he gets $1 - q$. If, on the other hand, $q \leq p$, then bidder i wins the auction and gets $1 - p$. With probability $(1 - \tau_{-i})$ at least one of the remaining bidders does not search and bids 1. Then, i loses the auction and buys the object for the outside price q. This gives him an expected payoff from searching

$$\tau_{-i} \int_0^1 (1 - \max[p, q]) f(q) dq + (1 - \tau_{-i}) \int_0^1 (1 - q) f(q) dq - c_i. \qquad (2)$$

When bidder i passes, then he revises his bid to 1 in the late bidding round. The only way how he can get a positive payoff is when all other bidders search, i.e., with probability τ_{-i}. In that case he wins the auction

[4]He can loose if $q > b_i$. Then, all bidders bid q in the final bidding round and the winner is determined by chance.

and pays $\max[p, q]$. The payoff from passing is

$$\tau_{-i} \int_0^1 (1 - \max[p, q]) f(q) dq. \quad (3)$$

Now suppose that i is not the high bidder, $W \neq i$, after the early round. First notice that when another bidder is the high bidder, $W = j$, then j has the highest early bid, $b_j = b^{(1)}$, and in our auction only he knows its value. However, to keep things simple, let us suppose for a moment that b_j were publicly observable[5]. When i searches, then he pays the search cost, c_i, gets the outside price and possibly revises his final round bid to $\max[b_i, q]$. Then, if he looses the auction, he pays the outside price, q, and, if he wins, he pays the auction price equal to q. In either case, he gets $1-q$. Hence, the expected payoff from searching is

$$1 - E[q] - c_i. \quad (4)$$

In the opposite case, when i does not search, he raises his late round bid to 1 and outbids j's high early bid. The auction's price in the final bidding round jumps to b_j. Then, if all other bidders search, i gets $1 - \max[b_j, q]$ and if at least one of them passes, he gets zero. His expected payoff[6] is

$$\tau_{-i} \int_0^1 (1 - \max[b^{(1)}, q]) f(q) dq. \quad (5)$$

Notice that (3) and (5) are essentially the same. The only difference is that in (3) i is the high bidder after the early bidding round and $p = b^{(2)} = b_j$ is publicly observable, while in (5) $b^{(1)} = b_j$ is observed only by the high bidder. Importantly, i's own bid does not directly influence his payoff form passing. Secondly, since (4) is greater than (2), then if i is going to search, he prefers to be the low bidder rather than the high bidder. But again i's early bid b_i does not enter any of the value functions explicitly. Bidding in the early round matters only to the extend that it determines whether i is the high or the low bidder. This difference is important. Being a high bidder implies certain commitment to buying the object at the going

[5]It will be seen later that whether b_j is observable or not is irrelevant for our results.

[6]Here we make use of the temporary assumption that i is able to observe the high bid $b_j^{(1)}$. However, as was already mentioned, this assumption was made for convenience purposes only and none of the results depend on it.

price while being the low bidder carries no such commitment. Thus, for the same standing price, searching is relatively less attractive when one is the high bidder (the expected final price is $E[\max[p, q]]$) than when one is the low bidder (the expected final price is $E[q]$). This possibility of commitment by placing high early bid is what allows a bidder to signal his search cost to the other bidders as discussed in the introduction. A searching strategy[7] which most naturally reflects this payoff asymmetry between being a high versus a low bidder is what we call the searching-when-loosing strategy.

Definition: A strategy by which bidder i *searches* when he is the low bidder ($W \neq i$) and *passes* otherwise ($W = i$) we call the *searching-when-loosing* strategy.

Next we show that this strategy is not only intuitively appealing but also a part of an *early bidding* equilibrium in which all bidders bid early.

Proposition 1: There is an early bidding equilibrium such that:
(i) All bidders with cost $c_i < 1 - E[q]$ bid b_i in the early bidding round, which is determined uniquely as a solution to

$$\int_0^1 \max[b_i, y] f(y) dy - E[q] = c_i. \tag{6}$$

In the searching round, the low bidder searches and the high bidder passes,

$$\tau(c_i; p, W) = \begin{cases} 0 & \text{if} \quad W = i \\ 1 & \text{if} \quad otherwise \end{cases}.$$

(ii) All bidders with cost $c_i \geq 1 - E[q]$ bid their value, 1, in the early bidding round and pass in the searching round, $\tau_i = 0$.
In the late bidding round all bidders bid $b_{i,l}$.
Proof: Appendix.

There are a few notable features of this equilibrium. The early round bidding function is strictly increasing and concave up to a threshold, $1 - E[q]$, at which point there is a kink and the function becomes flat. For all types $c_i < 1 - E[q]$, the optimal bid b_i^* equates the benefits from becoming the low bidder who searches (4), with the benefits of becoming the high bidder who

[7] Here we refer to a behavioral strategy in the searching round.

passes (3). The kink occurs at $1-E[q]$ where the expected benefit of searching is zero. All bidders with higher costs bid their value 1 and do not search. We refer to cost interval $[0, 1-E[q]]$ as the *relevant cost range*, because only on this interval bidders face nontrivial decision whether to search or pass.

All bidders with costs exceeding $1-E[q]$ never search. Therefore, nothing that could happen in the early bidding round would affect their searching behavior. But these types are still useful in the model, because their bidding and searching behavior affects the decisions of the others.

An interesting feature of this equilibrium is that bidders in the relevant cost range ($c_i \leq 1-E[q]$) implicitly coordinate[8] their search decisions through early bidding. The high bidder passes while the low bidder searches. Because the early round bidding function is strictly increasing it is the bidder with the lower search cost that is outbid in the early round and then searches. As will be seen later, this type of coordinated search has a positive impact on efficiency.

What makes such coordination possible is the use of the searching-when-losing strategy. It makes the (early round) equilibrium bidding function robust in the following sense. Conditional on players following the equilibrium play in the continuation game (in the searching and the final bidding round), the early round equilibrium bid weakly dominates any other. As a result, the early round bid does not depend on the number of bidders n nor on the distribution of their types G. A quick look at (6) reveals that neither n nor G appears in it. These features of the equilibrium are best illustrated by the following example.

Example 1: Consider just two bidders $i \in \{A, B\}$. Suppose that $c_i \sim U[0,1]$ and $q \sim U[0,1]$. Then, searching is valuable for any $c_i < 1 - E[q] = 1/2$, i.e., the relevant cost range is $[0, 1/2]$. Hence, all $c_i < 1/2$ use an early round equilibrium bidding function $\sqrt{2c_i}$ which then allows them to coordinate their search decisions. All $c_i \geq 1/2$ on the other hand would never search and hence they bid 1 in the early round.

[8] The word "coordinate" usually implies some sort of communication on the part of the bidders. Since our environment is purely non-cooperative we want to emphasize that the type of coordination we refer to is implicit - or, in other words, in the equilibrium bidders act as if they coordinated their search.

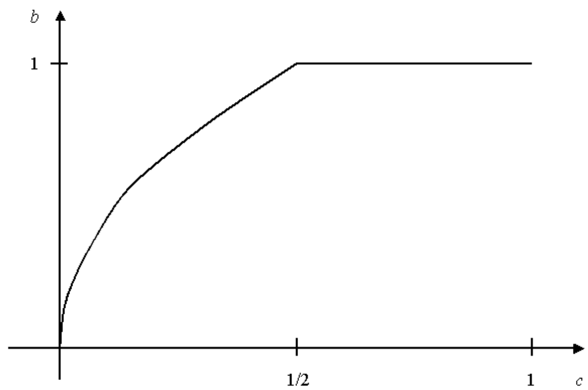

Figure 1

Notice that conditional on sticking to the equilibrium in the continuation game bidding $\sqrt{2c_i}$ (weakly) dominates any other bid. In other words, it is the best response to an arbitrary bid by the opponent. To see the intuition behind this consider what happens when A bids slightly more or slightly less, i.e., when $\sqrt{2c_A} \pm \Delta$ for an arbitrarily small and positive Δ. We will only illustrate the upward deviation, $\sqrt{2c_A} + \Delta$, but the same logic applies to the opposite case of $\sqrt{2c_A} - \Delta$. Take an arbitrary bid b_B. If $b_B < \sqrt{2c_A}$, then both bids, $\sqrt{2c_A}$ and $\sqrt{2c_A} + \Delta$, win the auction and in both cases A gets the same payoff given by (3) where $\tau_B = 1$. Alternatively, if $\sqrt{2c_A} + \Delta < b_B$, then A looses the auction in both cases and his payoff from both bids is the same, i.e., given by (4) where $\tau_B = 0$.

The only case in which payoffs from bidding $\sqrt{2c_A}$ versus $\sqrt{2c_A} + \Delta$ differ is when $\sqrt{2c_A} \leq b_B < \sqrt{2c_A} + \Delta$. In this case, bidding b_A causes A to become the low bidder after the early round, $W = B$, and his expected payoff is given by (4) where $\tau_B = 0$,

$$\frac{1}{2} - c_A = \frac{1}{2} - \frac{(\sqrt{2c_A})^2}{2}. \tag{7}$$

Now consider what happens when A bids $\sqrt{2c_A} + \Delta$ instead. Then, he becomes the high bidder, $W = A$, and his payoff is given by (3), where $\tau_B = 1$,

$$\frac{1}{2} - \frac{b_B^2}{2}. \tag{8}$$

The optimality of b_A requires that (7) is as least as big as (8) which is true since $\sqrt{2c_A} \leq b_B$. $\triangle$

The early bidding equilibrium provides a rationale for the early bidding behavior. But the equilibrium is not unique. One can construct other equilibria but these would involve rather unrealistic features[9], such as, pooling on a single bid, which cannot be easily reconciled with the observed bidding behavior on the Internet. We do not pursue these equilibria further here but rather focus on another important question of late bidding. In the introduction we stressed the fact that there are three distinct bidding patterns in the data. But the early bidding equilibrium can rationalize only early and multiple bidding. Therefore, it tells only part of the story. In the next section we show that by relaxing the somewhat restrictive assumption of common outside prices, we obtain an equilibrium in which the behavior is richer and generates all three bidding patterns.

3.2 Private Price Draws

This section presents our main result. As mentioned before, we drop the assumption of the common price draws and rather focus on the case of independent and private outside prices, $q_i \sim F[0, 1]$. Private outside prices seem more realistic. One could argue that all people do not shop in the same store which was one of the interpretations of the common price draws. We will illustrate the impact of private price draws on the behavior in the previously discussed equilibria. A remarkable result of this section is that the heterogeneity of price draws gives rise to an equilibrium in which a certain proportion of low-cost types bid just late and the rest bid early.

Proposition 2: Suppose that outside prices are identically and independently distributed. There is a *free bidding* equilibrium in which:

[9] One can construct an equilibrium in which all bidders with types in $[0, t]$ (where $t < 1 - E[q]$) use an increasing bidding function in the early round and then follow searching-when-loosing strategy in the searching round (e.g., just like described by Proposition 1); and the remaining types in $[t, \bar{c}]$ pool on the same bid $\beta(t)$ and search only when they are the high bidder and pass otherwise. This type of equilibrium is not very intuitive mostly because it relies on pooling at a particular bid. Furthermore the equilibrium is not robust to larger number of bidders and to distributions G that put more mass on the higher end of the support. For these reasons we do not pay further attention to such equilibria.

(i) All bidders with cost $c_i < \hat{c}$ bid zero, $b_i = 0$, in the early bidding round and search $\tau_i = 1$ in the searching round.

(ii) All bidders with cost $\hat{c} \leq c_i < 1 - E[q]$ bid b_i as determined by

$$\int_0^1 \max[b_i, y] f^{(1,n-1)}(y) dy - E[q] = c_i. \tag{9}$$

In the searching round the low bidder searches and the high bidder passes,

$$\tau(c_i; p, W) = \begin{cases} 0 & \text{if} \quad W = i \\ 1 & \text{if} \quad otherwise \end{cases}.$$

(iii) All bidders with cost $c_i \geq 1 - E[q]$ bid their value, $b_i = 1$, in the early bidding round and pass in the searching round, $\tau_i = 0$.

In the late bidding round all bidders bid $b_{i,l}$.

The threshold $\hat{c}$ is given by

$$\hat{c} = E[q^{(1,n-1)} - \min[q_i, q^{(1,n-1)}]].$$

Off-equilibrium path, when $p \in (0, \beta(\hat{c}))$, beliefs are set in the following way: if $W = i$, then $c_{-i} \sim G[0, \beta^{-1}(p)]$ and if $W = j$, then $c_j \sim G[\beta^{-1}(p), \bar{c}]$ and $c_k \sim G[0, \beta^{-1}(p)]$ for all $k \neq i, j$.

Proof: Appendix.

The significance of this of equilibrium is that it is intuitive and encompasses all three types of bidding patterns discussed in the introduction: early (parts i and ii of the equilibrium), late (part iii), and multiple bidding (parts i and ii). Heterogeneity of the outside prices impacts the early bidding equilibrium from the previous section in three different ways. The first is the emergence of the late bidding part, part (i) in Proposition 2. Secondly, the bidding function now depends on the number of bidders. As the number of bidders grows, the proportion of late bidders in part (i) of the equilibrium increases ($\hat{c}$ increases) and the bidding function in part (ii) gets steeper.[10]

Interestingly, there is always a jump at $\hat{c}$. The reason is that a bidder with cost $\hat{c}$ is exactly indifferent between bidding zero followed by searching and biding positive amount followed by passing. Conditional on winning the

[10] It is also worth pointing out that the bidding function in part (ii) is "well-behaved" in the sense that it is continuous and increasing on $[\hat{c}, 1 - E[q]]$ and all bids are within bounds $[0, 1]$, i.e., one can easily verify that $\beta(\hat{c}) > 0$ and $\beta(1 - E[q]) = 1$.

auction he would get $1-\min[q_i, q_{-i}^{(2)}]-\hat{c}$ in the first case and $1-q_{-i}^{(1)}$ in the second case. Thus, there is a trade-off between paying a lower final auction price (the second highest outside price) on one hand and avoiding paying the search cost on the other hand.

The third and the last difference is that the late bidders always search. Recall that with common outside prices it could have never happened that all bidders searched in equilibrium. Here it can happen in part (i) of the Proposition 2. All these differences are tied to the extra private gains from searching when outside prices are private and independent. The intuition behind this is illustrated in the following example which is a continuation of Example 1.

Example 2 (late bidding behavior): With the parameters from Example 1 the early round bidding function has the following structure: $\hat{c}=1/6$ and $1-E[q]=1/2$,

$$\beta(c_i)=\begin{cases} 0 & \text{if} \quad c_i\in[0,1/6] \\ \sqrt{2c_i} & \text{if} \quad c_i\in[1/6,1/2] \\ 1 & \text{if} \quad c_i\in[1/2,1] \end{cases},$$

To see the rationale behind the late bidding behavior let us illustrate why couldn't it be an equilibrium for all bidders with cost $c_i \leq 1-E[q]$ to bid $\sqrt{2c_i}$ in the early bidding round (just like in Proposition 1). If this were the case, then some low search cost types would find it profitable to deviate. Consider bidder A with search cost c_A very close to zero. In the supposed equilibrium he bids $\sqrt{2c_A}$. The other bidder B sets the standing price by bidding $\sqrt{2c_B}=p$. Then, if A is the high bidder, $\sqrt{2c_A}\geq p$, he passes and B searches which gives A

$$1-E[\max[p,q_B]]. \tag{10}$$

However, a deviation to searching would give him

$$1-\int_0^1\int_0^1\left(\mathbf{1}_{[q_B\leq p]}p+\mathbf{1}_{[p<q_B\leq\sqrt{2c_A}]}q_B+\mathbf{1}_{[\sqrt{2c_A}<q_B]}\min[q_B,q_A]\right)dq_A dq_B - c_A$$

which we can rewrite more concisely as

$$1-E[\max[p,q_B]\mid q_B\leq\sqrt{2c_A}]-E[\min[q_B,q_A]\mid q_B>\sqrt{2c_A}]-c_A. \tag{11}$$

To see that deviation could be profitable, we send $c_A \longrightarrow 0$. Since $p \leq \sqrt{2c_A}$ the payoff from passing (10) goes to $1 - E[q_B] = 1 - 1/2 = 1/2$. On the other hand, in the deviation payoff (11), the second term ($E[\max[p, q_B] \mid q_B \leq \sqrt{2c_A}]$) and the last term ($c_A$) go to zero and the whole expression approaches $1 - E[\min[q_B, q_A]] = 1 - 1/3 = 2/3$. Hence, a bidder A with sufficiently small search cost would want to deviate from the supposed equilibrium and search regardless of whether he is the high or the low bidder.

This implies that if we had an equilibrium in which all $c_i \leq 1 - E[q]$ bid $\sqrt{2c_i}$, then there must be a certain proportion of types who always search. But we can further show that this could also not be an equilibrium, because then all types $c_i < \hat{c}$ who always search would want to deviate to bidding zero in the early round, i.e., to bidding just late. To see this, suppose $c_A < \hat{c}$. Then, the payoff from searching when A is the low bidder $\sqrt{2c_A} \leq b_B$ is independent of his own bid,

$$1 - E[q_A \mid q_A \leq b_B] - E[\min[q_A, b_{B,l}] \mid q_A > b_B],$$

where if you recall, $b_{B,l}$ denotes the final round bid of B which could be either 1, if B has passed, or $\max[q_B, b_B]$, if he has searched. Since this payoff is independent of b_A, when A is the low bidder, he is indifferent between bidding $\sqrt{2c_A}$ and 0. On the other hand, if A is the high bidder $b_A = \sqrt{2c_A} \geq p$, then his payoff is given by (11) which is decreasing in his early bid b_A. This makes him strictly prefer to bid zero. Hence, A maximizes his expected payoff by mimicking the behavior of the zero cost type and bids zero in the early round. $\triangle$

The previous example has shown that in the free bidding equilibrium there will be some types with sufficiently low search costs who will search with certainty. Moreover, given that they always search, they strictly prefer to bid zero in the early round. These incentives give rise to the late bidding behavior. In the following example we will explore how bidding behavior in the early round depends on the number of bidders.

Example 3 (arbitrary number of bidders): Consider the parameters from Example 1 but now suppose that the number of bidders is arbitrary, n. Since F is uniform we have that $f^{(1,n-1)}(x) = (n-1)x^{n-2}$. Then, $\hat{c}(n) = \frac{n-1}{2(n+1)}$ and $1 - E[q] = 1/2$. Substituting this into (9) we can express the early round bidding function as

$$\beta(c_i) = \begin{cases} 0 & \text{if} \quad c_i \in [0, \frac{n-1}{2(n+1)}] \\ \sqrt[n]{\frac{2-n(1-2c_i)}{2}} & \text{if} \quad c_i \in [\frac{n-1}{2(n+1)}, 1/2] \\ 1 & \text{if} \quad c_i \in [1/2, 1] \end{cases} .$$

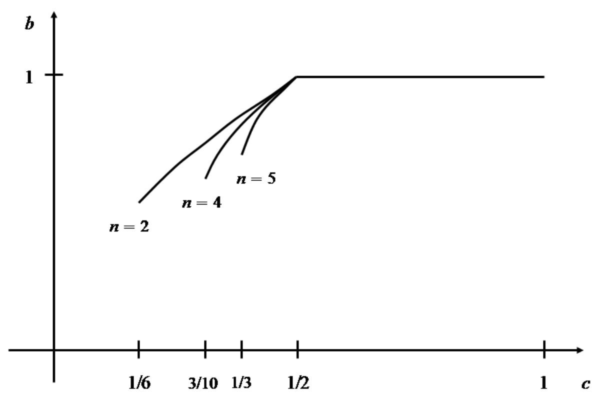

Figure 2

Notice that both, the bidding function and the threshold $\hat{c}$, depend on the number of bidders. The proportion of late bidders, $[0, \hat{c}(n)]$, grows and the bidding function in the region $[\hat{c}(n), 1/2]$ also gets steeper as n increases. This makes an intuitive sense. When higher number of bidders draw outside prices and bid them in the auction, this raises the expected final auction price. Thus, the expected surplus from winning the auction decreases. On the other hand, the expected surplus from searching is independent of number of bidders. Therefore, when number of bidders is high, being caught up in the auction with a high early bid is more likely to be become less valuable than bidding zero and searching.

The free bidding equilibrium helps us understand the incentives for early bidding in the dynamic auction. Bidders who are inflexible have incentives to avoid searching and bid for the right to stay passive early in the auction. The remaining bidders, who are flexible, would not find this strategy profitable. Rather, these types prefer to search even in the case when they are in the position of the high bidder. This however defeats the purpose of bidding early. From the perspective of the flexible types early bidding only reduces the surplus in the auction, which is the reason why they search and bid just late.

4 Comparison to a Static Auction Format

The environment modeled in this paper is dynamic. Naturally, early bidding can only arise in auction that is sufficiently long. Short auctions provide different incentives for acquisition of information. Bidders have to decide before the auction whether to learn outside prices without being able to coordinate their decisions. The difference in incentives will have implications for the social surplus and revenues that sellers can capture in the dynamic versus static auction format. In this section we demonstrate that the dynamic auction always generates higher social surplus than the static auction. However the question of revenue is ambiguous. Both Rezende (2005) and Compte & Jehiel (2007) argue that for a large number of bidders dynamic auction is more profitable. In addition Compte & Jehiel note that this revenue ranking may not hold in general. We provide an explicit example in which the static auction generates higher revenue for a particular distribution of bidders' search costs.

The first step is to describe the environment and characterize the bidding equilibrium. The static auction has only a single bidding round. Within the framework of our model this means that the game begins with the searching round, where all bidders can simultaneously decide to search. This is followed by the bidding round, in which all bidders simultaneously place a single round of bids. We begin by characterizing bidding equilibrium for the static auction. In doing so we refer to both cases of private and common outside prices.

Proposition 3: Suppose that outside prices are identically and independently distributed. In the equilibrium of the static auction, bidders use a threshold strategy in the searching round,

$$\tau(c_i) = \begin{cases} 1 & \text{if} \quad c_i \in [0, \check{c}(n)] \\ 0 & \text{if} \quad c_i \in [\check{c}(n), 1 - E[q]] \end{cases},$$

where the threshold $\check{c}$ is given by

$$\int_0^{\bar{c}} (\mathbf{1}_{[z \le \check{c}]} E[q_{-i}^{(1)} - \min[q_i, q_{-i}^{(1)}]] + \mathbf{1}_{[z > \check{c}]} E[1 - q_i]) g^{(1,n-1)}(z) dz = \check{c}.$$

In the final bidding round bidders bid their best outside alternative, $b_{i,l}$.

Proof: Appendix.

For common outside prices the equilibrium looks very similar.

Corollary 2: Suppose that outside prices are common. The bidding equilibrium is equivalent to that described by the Proposition 3 with the exception that the threshold $\check{c}$ is given by

$$\int_0^{\bar{c}} \mathbf{1}_{[z>\check{c}]} E[1-q] g^{(1,n-1)}(z) dz = \check{c}.$$

Proof: Appendix.

The equilibrium is fairly simple and intuitive. It involves threshold strategies in the searching round whereby the low search cost types, who are below the threshold, $\check{c}$, search and the high search cost types, those above the threshold, pass. There is very little difference between the cases of common and independent outside prices. With independent outside prices searching is a bit more valuable than in the case of common outside prices. The reason is that the only time when the expected payoff of an individual bidder differs between these two environments is in the case when all bidders search. When prices are independent, the expected price a bidder pays is $E[\min[q_i, q_{-i}^{(1)}]]$, i.e., either he looses the auction and pays his outside price q_i or he wins the auction and pays the highest price of the remaining bidders, $q_{-i}^{(1)}$. When prices are common, then the expected price is higher, $E[q]$.

4.1 Comparing Efficiency

Let us now compare the two auction formats. First we address the issue of efficiency. What determines the social surplus is the amount of trade less the sum of the search costs. Let $t_i \in \{0,1\}$ indicate whether bidder i has traded and $s_i \in \{0,1\}$ whether he has searched. Then, the ex-post surplus is given by

$$W_{ep} = \sum_{i=2}^{n} s_i(1-c_i) + (1-s_i)t_i.$$

The ex-ante surplus is defined in a similar way. Let α_i be the probability of winning the auction. Then,

$$W_{ea} = \sum_{i=2}^{n} \tau_i(1 - c_i) + (1 - \tau_i)\alpha_i.$$

We index by $\{D, S\}$ the social surplus for the dynamic and the static auction respectively. The next proposition establishes that the dynamic auction dominates the static auction in terms of social surplus.

Proposition 4: In both environments, with common and with independent outside prices,

$$\begin{array}{llll} \text{(i)} & W_{ep}^{D} & \geq & W_{ep}^{S} \text{ and} \\ \text{(ii)} & W_{ea}^{D} & > & W_{ea}^{S}. \end{array}$$

Proof: Appendix.

To see the intuition behind this result let us focus on the case when all bidders have their search costs in the relevant cost range, $c_i \in [0, 1 - E[q]]$. In the equilibrium of the dynamic auction the early bidding function plays the role of a coordinating device. Therefore, the high bidder in the early bidding round is also the bidder with the highest search cost. This bidder passes and gets the object in the auction. Everyone else searches and buys the object for the outside price. Thus, in equilibrium of the dynamic auction all bidders with the costs in the relevant cost range trade. Moreover, it is the bidder with the highest cost that passes. On the other hand, in the static auction, bidders use threshold strategy in the searching round. This necessarily implies that there will be some miscoordination of searching decisions. The miscoordination can be of two kinds: (i) at least two bidders pass or (ii) all bidders search. Both cause inefficiencies. In the first case one of the bidders ends up not trading when getting the outside price would increase social surplus, $1 - c_i \geq 0$. In the second case all bidders trade, but had one of them passed (for example the one with the highest search cost) he would have traded in the auction while saving the search cost which would of course increase social surplus.

4.2 Comparing Revenue

Next we turn to the question of the revenue. This is certainly more involved. For the purposes of simplicity we limit our view to the simpler case of common valuations. From the seller's perspective, in the static auction, the expected auction price is $E[q]$ as long as at most one bidder passes. Otherwise, the seller gets 1. The expected revenue is

$$\pi^S = \int_0^{\check{c}} E[q] g^{(2,n)}(z) dz + \int_{\check{c}}^{\bar{c}} 1 g^{(2,n)}(z) dz,$$

where $\check{c}$ is determined by[11]

$$1 - \frac{\check{c}}{1 - E[q]} = G^{(1,n-1)}(\check{c}). \tag{12}$$

Similarly, in the dynamic auction, the seller expects to get $E[\max[b_i, q]]$ if at most a single bidder passes and gets 1 otherwise. Hence, the revenue is

$$\pi^D = \int_0^{1-E[q]} E[\max[\beta(z), q]] g^{(2,n)}(z) dz + \int_{1-E[q]}^{\bar{c}} 1 g^{(2,n)}(z) dz.$$

In order to compare the two revenues we express π^D in the form of π^S by introducing a threshold $\tilde{c}$ such that

$$\begin{aligned} \pi^D &= \int_0^{\tilde{c}} E[q] g^{(2,n)}(z) dz + \int_{\tilde{c}}^{\bar{c}} 1 g^{(2,n)}(z) dz \\ \int_0^{1-E[q]} E[\max[\beta(z), q]] g^{(2,n)}(z) dz &= \int_0^{\tilde{c}} E[q] g^{(2,n)}(z) dz + \int_{\tilde{c}}^{1-E[q]} 1 g^{(2,n)}(z) dz \\ \int_0^{1-E[q]} (E[q] + z) g^{(2,n)}(z) dz &= \int_0^{\tilde{c}} E[q] g^{(2,n)}(z) dz + \int_{\tilde{c}}^{1-E[q]} 1 g^{(2,n)}(z) dz \\ \int_0^{1-E[q]} z g^{(2,n)}(z) dz &= (G^{(2,n)}(1 - E[q]) - G^{(2,n)}(\tilde{c}))(1 - E[q]), \end{aligned}$$

where we substituted for π^D and used the fact that $\tilde{c} \in [0, 1 - E[q]]$ to get form step 1 to step 2. The upper bound on $\tilde{c}$ is implied by the fact that any type with cost higher than $1 - E[q]$ would never search. Hence, the revenue cannot be lower than when all types in the relevant cost range search, i.e.,

[11] This is just a rearranged expression from Corrolary 2.

when $\tilde{c} = 1 - E[q]$. To get from step 2 to step 3 we substituted from (6). Now one can integrate the left-hand side by parts and rearrange to obtain

$$G^{(2,n)}(\tilde{c})(1 - E[q]) = \int_0^{1-E[q]} G^{(2,n)}(z)dz. \tag{13}$$

The next proposition establishes the relationship between the two thresholds ($\check{c}$, $\tilde{c}$), and revenues in the two auction formats.

Proposition 5: The dynamic auction generates higher revenue than the static auction when $\tilde{c}$ is lower than $\check{c}$,

$$\pi^D > \pi^S \text{ if and only if } \tilde{c} < \check{c},$$

where $\check{c}$ is determined by (12) and $\tilde{c}$ by (13).

The Proposition 5 recasts the comparison of revenues in terms of comparison of thresholds. The advantage of this is that the thresholds have a nice graphical representation (see Figure 3). This allows us on one hand to verify some claims in the literature stating that the dynamic auction dominates the static auction in terms of revenue for standard parametrization (e.g., uniform distributions) and also asymptotically as $n \to \infty$ (see Example 4a). On the other hand, we are also able to identify a case where static auction can be more profitable than the dynamic auction (see Example 4b).

Example 4a: We adopt the parameters from the Example 1. There are two bidders and both F and G are uniform on $[0, 1]$. We can represent the thresholds $\check{c}$ and $\tilde{c}$ as shown in the Figure 3 (part a) below.

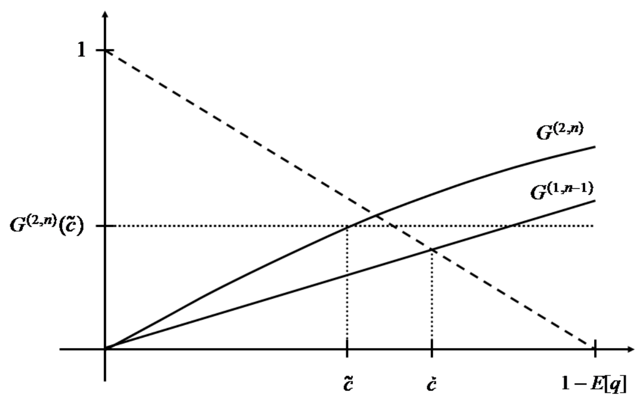

Figure 3

The threshold for the static auction is given by the intersection of the downward-sloping dashed line (left-hand side of (12)) and the distribution $G^{(1,n-1)}(z) = z$. The threshold for the dynamic auction is given by the intersection of the horizontal dotted line (left-hand side of (13)) with the distribution $G^{(2,n)}(z) = z(2-z)$. From the picture it is clear that $\tilde{c} < \check{c}$. Indeed, numerically, $\pi^D = 19/24 > 13/18 = \pi^S$.

Next we discuss the intuition for why does the dynamic auction dominate the static auction asymptotically. The more formal argument along with the picture can be found in the Appendix. Notice that since $\bar{c} > 1 - E[q]$, then $G^{(1,n-1)}(x)$ as an extreme value distribution converges to 0 on $x \in [0, 1-E[q]]$ as $n \to \infty$. Hence, the intersection with $1 - \frac{x}{1-E[q]}$ must occur at $x = \check{c} = 1$ in the limit. On the other hand, for any n, $G^{(2,n)}$ is a strictly increasing function. The area underneath it, on $[0, 1-E[q]]$, corresponds to $(1-E[q])G^{(2,n)}(\tilde{c})$, where $\tilde{c} \in (0,1)$. This implies that in the limit $\tilde{c} < \check{c}$ and hence $\pi^D > \pi^S$. $\triangle$

In the next example we show that it is possible to find an example of a distribution G for which the static auction generates higher revenue than its dynamic counterpart.

Example 4b: Figure 4 illustrates such possibility for the distribution with the full support which was the assumption throughout the paper.

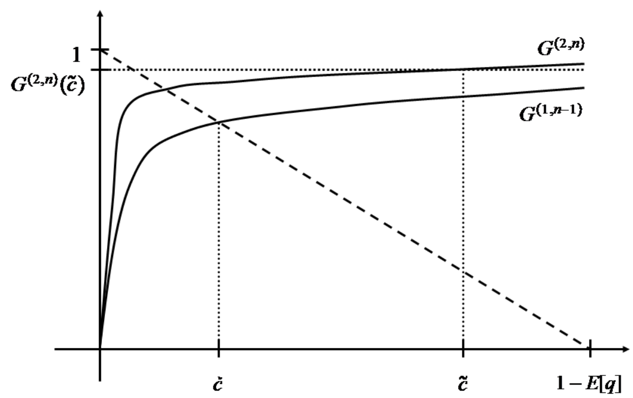

Figure 4

One could verify this analytically. Suppose there are two bidders. For the sake of simplicity we work with a distribution containing an atom[12] on $c = 0$. Consider F uniform on $[0, 1]$ and $G(x) = \frac{9}{10} + \frac{1}{10}x$ on $[0, 1]$. Furthermore assume that when bids are tied the winner of the auction is chosen randomly.

The dynamic auction: The early round bidding function is $\sqrt{2c_i}$ and $1 - E[q] = 1/2$. To compute revenue, notice that the expected payment of bidder A with cost $c_A > 0$ is

$$P_A(c_A) = \begin{cases} \int_0^{c_A} \left(\int_0^1 \max[\sqrt{2y}, q] dq \right) \frac{1}{10} dy & \text{if} \quad 0 < c_A \leq 1/2 \\ \begin{array}{c} \int_0^{1/2} \left(\int_0^1 \max[\sqrt{2y}, q] dq \right) \frac{1}{10} dy + \\ \int_{1/2}^1 \frac{1}{2}(1/2) \frac{1}{10} dy \end{array} & \text{if} \quad c_A > 1/2 \end{cases}.$$

Due to symmetry the auction revenue is

$$\frac{99}{100}\frac{1}{2} + 2\int_0^1 P_A(x)\frac{1}{10}dx = 0.503.$$

The static auction: The revenue for the static auction is computed in the similar manner. There, the threshold $\check{c}$ can be computed form (12), $\check{c} = 1/3$.

[12] Any distribution with an atom can be approximated arbitrarily closely by a distribution with a full support.

The expected payment of type $c_A > 0$ is

$$P_A(c_A) = \begin{cases} \int_0^{1/3} \left(1/2 \int_0^1 q dq\right) dy & \text{if} \quad c_A \leq 1/3 \\ \int_0^{1/2} \left(\int_0^1 q dq\right) dy + \int_{1/2}^{c_A} 1/2 dy & \text{if} \quad c_A > 1/3 \end{cases}.$$

Then, the revenue for the static auction is 0.505.

The static auction generates higher revenue in cases when the major portion of the mass is centered on types with low search costs. Recall, that in the dynamic auction, the seller benefits from early bidding, because of the extra surplus that it transfers from the high bidder to the seller. In the static auction, the seller benefits when bidders miscoordinate their search. If at least two bidders don't search, then the seller captures the highest possible surplus. The distribution that puts lots of weight on the low search cost types has two effects. On one hand, it lowers the profitability of the dynamic auction by minimizing the benefits from early bidding. On the other hand it increases the profitability in the static auction by allowing sufficient chances of search-miscoordination. Both effects combine to make the static auction preferable.

5 Discussion

In this paper we presented a model of an Internet-type auction, which provides a rationale for commonly observed bidding patterns. We decided to keep the model as simple as possible and focused primarily on describing clearly the underlying economic intuition. Nevertheless, further generalizations are not only possible but could also yield other useful insights. Here we briefly discuss two possible extensions that may be important[13]. The first is a richer modeling of bidder valuations. In this paper we have fully abstracted from modeling heterogeneity of bidders' valuations. Our primary objective here was to study how the heterogeneity in individual search abilities provides incentives for searching and bidding. One may have noticed that the common value for the object, 1, does not appear in any of the early round bidding functions in the relevant cost range. This indicates that a private value would not have a significant impact on the equilibrium behavior that

[13] Almost certainly one could find other relevant extensions.

we have described. The model could be extended to account for independent and private values, but of course, this would have to come at the cost of increasing the level of complexity and possibly obscuring the intuition.

The second extension may involve allowing for multiple searching and bidding rounds. This would be certainly interesting and should be pursued in the future work. It is reasonable to suppose that bidders could acquire information before the auction and that they arrive sequentially rather than bid simultaneously. Furthermore bidders might search multiple times and have the chance to revise their bids several times before the final bidding round. All of these variations are interesting and may alter results to teach us more about bidding behavior on the Internet. The current model is tailored specifically to address the idea that bidders bid early to avoid undertaking a possibly costly and lengthy search for alternatives. We believe that the model presented allows the most direct approach to studying these incentives.

To summarize, our objective was to examine bidding incentives when we allow bidders to search for outside prices. We found that in equilibrium, the private outside prices generate behavior which leads to three empirically relevant outcomes: early, late, and multiple bidding. The early and the multiple bidding arise from the incentives to avoid costly search. A crucial feature of the equilibrium is the searching-when-loosing strategy which allows early bidders to implicitly coordinate their search decisions. The late bidding on the other hand is caused by the spillover benefits that multiple price-draws have on the expected auction price. The winner of the auction pays a price which is lower than his best alternative. A contribution of this paper is in that it interprets frequently occurring bidding patterns in the data in the light of incentives to acquire information.

We were further interested in comparing the welfare and the revenue in our dynamic auction with the benchmark static auction. According to our expectations the dynamic auction dominates the static auction in terms of welfare. The reason is the usage of the searching-when-loosing strategy in the equilibrium, which allows least efficient bidder to stay passive. The question of revenue is much more difficult. There, the answer is ambiguous. It has been believed that the dynamic auction is more profitable than the static auction. We confirm that this is true in the limit. However, we also provide an example in which the static auction produces higher revenue than the dynamic auction. In light of this, it is not so obvious that the dynamic auction is always preferable. In judging the costs and benefits of dynamic auction one should notice that the case when the static auction is more profitable seems pretty rare and unlikely to occur in the real life. Therefore, we sill feel comfortable concluding that dynamic auction is on most occasions the more attractive alternative.

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6 Appendix

Proof of Proposition 1.

Consider strategy profile given by Proposition 1.

Part (i): Suppose $c_i \leq 1 - E[q]$.

Claim 1: If i is the low bidder, $W \neq i$, then, $\tau_i = 1$.

Proof: Suppose $W = j$. In equilibrium j passes and bids 1 in the final round. Then $\tau_{-i} = 0$ and i's payoff from searching is given by (4). When he passes, then $b_{i,l} = 1$ and gets 0, by (5). Since (4) is no less than (5), i.e., $1 - E[q] - c_i \geq 0$, searching is a best response. □

Claim 2: If $W = i$, then, $\tau_i = 0$.

Proof: Since $W = i$, then $p = b_{-i}^{(1)}$ and all other bidders search, $\tau_{-i} = 1$. If i passes, then he gets (3). His payoff from searching is given by (2). The difference between passing and searching is

$$\int_0^1 (1 - \max[b_{-i}^{(1)}, y]) f(y) dy - \left(\int_0^1 (1 - \max[b_{-i}^{(1)}, y]) f(y) dy - c_i \right) = c_i \geq 0,$$

i.e., i optimally passes. □

Claim 3: In the early bidding round, the optimal bid b_i^* is given by

$$\int_0^1 \max[b_i^*, y] f(y) dy - E[q] = c_i.$$

Proof: Bidder i will chose b_i which maximizes his expected payoff. Notice that continuation payoffs (2), (3), (4), and (5) do not depend on the own bid. Bid b_i only determines whether i is the high or the low bidder. Here we show that conditional on the continuation play following the equilibrium, b_i^* weakly dominates any other bid .

By bidding zero, $b_i = 0$, i can guarantee for himself (4),$1 - E[q] - c_i$. If he raises his bid to $b_i > 0$, then, if $W \neq i$, then the behavior in the continuation game is unaffected and i still gets the same payoff. However, if $W = i$, then

$p = b_{-i}^{(1)}$ and in the continuation game i passes while everyone else searches. His payoff changes to (3),

$$\int_0^1 (1 - \max[b_{-i}^{(1)}, y])f(y)dy.$$

Notice that since (3) is decreasing in $b_{-i}^{(1)}$, then whenever i prefers to be the high bidder at $b_{-i}^{(1)} = b_i$ (the worst possible case for i), then he also prefers to be the high bidder for any lower $b_{-i}^{(1)}$, i.e.,

$$\begin{aligned} 1 - E[q] - c_i &\leq \int_0^1 (1 - \max[b_i, y])f(y)dx \\ &\leq \int_0^1 (1 - \max[b_{-i}^{(1)}, y])f(y)dx. \end{aligned} \tag{14}$$

Hence, the optimal bid b_i^* makes bidder i prefer to be the high bidder for all $b_{-i}^{(1)} \leq b_i^*$ that satisfy (14) and makes him prefer to be the low bidder for all $b_{-i}^{(1)} > b_i^*$ that violate (14), i.e.,

$$\begin{aligned} 1 - E[q] - c_i &= \int_0^1 (1 - \max[b_i^*, y])f(y)dy \\ \int_0^1 \max[b_i^*, y]f(y)dy - E[q] &= c_i. \end{aligned}$$

Since F has a full support, the right-hand side is monotone increasing in b_i^* from 0 to $1 - E[q]$. Thus, in the relevant cost range $[0, 1 - E[q]]$, for any c_i we can find a unique b_i^* that satisfies the equality. $\square$

Part (ii): Suppose $c_i > 1 - E[q]$.

Claim 1: In the searching round, i passes, $\tau_i = 0$.

Proof: By passing, bidder i can secure at least zero, while if he searches, then his highest possible payoff is$1 - E[q] - c_i < 0$. Hence, he optimally passes. $\square$

Claim 2: In the early round, $b_i^* = 1$.

Proof: Notice that for $c_i > 1 - E[q]$ inequality (14) is always satisfied since the left-hand side is less than zero and the right-hand side cannot be negative. Hence, bidder i will optimally bid his maximum willingness to pay, $b_i^* = v = 1$. $\square$

Q.E.D.

* * *

Before we give the proof of Proposition 3 we need to restate the continuation payoffs (2), (3), (4) and (5) for the case of independent and private outside prices. When $W = i$ and i searches, then his payoff is

$$1 - \tau_{-i} \int_0^1 \int_0^1 \left(\mathbf{1}_{[y \leq p]} p + \mathbf{1}_{[p < y \leq b_i]} y + \mathbf{1}_{[b_i < y]} \min[y, x] \right) f^{(1,n-1)}(y) dy f(x) dx$$
$$-(1 - \tau_{-i}) \int_0^1 x f(x) dx - c_i \tag{15}$$

and, when he passes, he gets

$$1 - \tau_{-i} \int_0^1 \left(\mathbf{1}_{[y \leq p]} p + \mathbf{1}_{[y > p]} y \right) f^{(1,n-1)}(y) dy. \tag{16}$$

On the other hand, when $W \neq i$ and we suppose that the high bid $b_{-i}^{(1)}$ is known to all bidders, then i gets,

$$1 - \tau_{-i} \int_0^1 \int_0^1 \left(\mathbf{1}_{[x \leq b_{-i}^{(1)}]} x + \mathbf{1}_{[x > b_{-i}^{(1)}]} \min[x, \max[y, b_{-i}^{(1)}]] \right) f^{(1,n-1)}(y) dy f(x) dx$$
$$-(1 - \tau_{-i}) \int_0^1 x f(x) dx - c_i \tag{17}$$

if he searches and gets

$$1 - \tau_{-i} \int_0^1 \left(\mathbf{1}_{[y \leq b_{-i}^{(1)}]} p + \mathbf{1}_{[y > b_{-i}^{(1)}]} y \right) f^{(1,n-1)}(y) dy \tag{18}$$

if he passes.

Proof of Proposition 2.
Consider strategy profile given in Proposition 2.

As a first step we characterize best responses in the searching round. Consider bidder i with cost c_i. We can classify histories as those on-equilibrium path: $\{0, \varnothing\}$, $\{0, k\}$, $\{p, k\}$ and $\{1, k\}$, where $p \in [\beta(\hat{c}), 1)$, and k is an index of any bidder; and those off-equilibrium path: $\{p, k\}$, where $p \in (0, \beta(\hat{c}))$.

After on-path histories $\{0, j\}$ and $\{p, j\}$, bidder i believes that $\Pr(c_j > \hat{c}) = 1$ and since $W = j$ in equilibrium j is passing ($\tau_{-i} = 0$). Similarly, after history $\{1, k\}$ since $p = 1$, i believes that at least for one of his opponents $\Pr(c_j > 1 - E[q]) = 1$ and therefore in equilibrium j is passing ($\tau_{-i} = 0$). Hence, after all of these histories if i passes as well he gets zero. His best response is to search if it's profitable,

$$1 - E[q] \geq c_i. \tag{19}$$

After histories $\{0, \varnothing\}$, $\{0, i\}$ and $\{p, i\}$ bidder i believes that for all of his opponents it is true that $\Pr(c_j \leq c_i) = 1$ and since $W = i$ in equilibrium all are searching ($\tau_{-i} = 1$). His best response is to search if the payoff from searching (15) (left-hand side) is greater than the payoff form passing (16) (right-hand side)

$$1 - \int_0^1 \int_0^1 \left(\mathbf{1}_{[y \leq p]} p + \mathbf{1}_{[p < y \leq b_i]} y + \mathbf{1}_{[b_i < y]} \min[y, x] \right) f^{(1,n-1)}(y) dy f(x) dx - c_i \geq$$

$$1 - \int_0^1 \left(\mathbf{1}_{[y \leq p]} p + \mathbf{1}_{[y > p]} y \right) f^{(1,n-1)}(y) dy,$$

which reduces to

$$\int_0^1 \int_0^1 \mathbf{1}_{[b_i < y]} (y - \min[y, x]) f^{(1,n-1)}(y) dy f(x) dx \geq c_i. \tag{20}$$

After off-path histories $\{p, i\}$ and $\{p, j\}$, where $p \in (0, \beta(\hat{c}))$, i's belief is the same (see the Proposition 2) as after on-path histories $\{p, i\}$ and $\{p, j\}$ respectively. Hence, his best response is given by (20) in the first case and by (19) in the second case.

Next we establish the equilibrium.

Part (i): Suppose $c_i < \hat{c}$.
Claim 1: In the searching round bidder i searches, $\tau_i = 1$.
Proof: The equilibrium history is $\{0, \varnothing\}$. Then since

$$\begin{aligned} c_i &\leq \hat{c} = E[q^{(1,n-1)} - \min[q, q^{(1,n-1)}]] \\ &= \int_0^1 \int_0^1 (y - \min[y, x]) f^{(1,n-1)}(y) dy f(x) dx. \end{aligned}$$

Since $b_i = 0$, then by (20), bidder i searches. □

Claim 2: In the early round bidder i bids zero $b_i^* = 0$.

Proof: Here we show that deviation to $\tilde{b}_i > 0$ is not profitable. First recall that when i bids zero, then it must be that $W \neq i$. Rather than writing down his expected payoff, it is more useful to write down his payoff for the separate cases, i.e., when $c_{-i}^{(1)} < \hat{c}$ then i gets

$$1 - \int_0^1 \int_0^1 \min[y, x] f^{(1,n-1)}(y) dy f(x) dx - c_i \tag{21}$$

and in the opposite case when $c_{-i}^{(1)} \geq \hat{c}$ he gets

$$1 - \int_0^1 x f(x) dx - c_i. \tag{22}$$

Next we show that in both of these cases bidding $\tilde{b}_i > 0$ is no better than bidding zero. If $\tilde{b}_i > 0$, then there are three possible cases: (i) $c_{-i}^{(1)} < \hat{c}$, (ii) $c_{-i}^{(1)} \geq \hat{c}$ and $W \neq i$ and (iii) $c_{-i}^{(1)} \geq \hat{c}$ and $W = i$.

Case (i) $c_{-i}^{(1)} < \hat{c}$: In this case all other bidders bid zero and search. That implies that $W = i$ and $p = 0$. In the continuation game all other bidders search, $\tau_i = 1$. Bidder i's payoff is given either by (15)

$$1 - \int_0^1 \int_0^1 \left(\mathbf{1}_{[y \leq \tilde{b}_i]} y + \mathbf{1}_{[\tilde{b}_i < y]} \min[y, x] \right) f^{(1,n-1)}(y) dy f(x) dx - c_i$$

if he searches or by (16)

$$1 - \int_0^1 y f^{(1,n-1)}(y) dy$$

if he passes. But since $c_i < \hat{c}$ both are less than (21), i.e., deviation is not profitable.

Case (ii) $c_{-i}^{(1)} \geq \hat{c}$ and $W \neq i$: Here at least one of the remaining bidders has bid a positive amount and has outbid bidder i which implies that $p > 0$. Notice that if $p \in (0, \beta(\hat{c}))$ then the play is off the equilibrium path but since the beliefs are set appropriately (see Proposition 2.), the incentives both on- and off-equilibrium path are the same and can be analyzed simultaneously. In this case the high bidder passes, $\tau_{-i} = 1$. If i passes as well he gets zero which is strictly less than (22). The payoff from searching is given by (17) which is equal to (22). Again deviation is not profitable.

Case (iii): $c_{-i}^{(1)} \geq \hat{c}$ and $W = i$: This case is more difficult. Here, in equilibrium $\beta(\hat{c}) \leq p \leq \tilde{b}_i$. Since $W = i$, then all other bidders are searching, $\tau_{-i} = 1$. Bidder i can either search or pass. Suppose first that he searches. Then, his payoff is given by (15)

$$1 - \int_0^1 \int_0^1 \left(\mathbf{1}_{[y \leq p]} p + \mathbf{1}_{[p < y \leq \tilde{b}_i]} y + \mathbf{1}_{[\tilde{b}_i < y]} \min[y, x] \right) f^{(1,n-1)}(y) dy f(x) dx - c_i.$$

To see that this payoff is indeed lower than (22) it suffices to look at its upper bound, i.e., when $\tilde{b}_i = \beta(\hat{c}) = \hat{p}$ since it is decreasing both in p and b_i. In that case we obtain

$$1 - \int_0^1 \int_0^1 \left(\mathbf{1}_{[y \leq \hat{p}]} \hat{p} + \mathbf{1}_{[\hat{p} < y]} \min[y, x] \right) f^{(1,n-1)}(y) dy f(x) dx - c_i.$$

Bidder i wants to deviate when this is greater than (22), i.e.,

$$\int_0^1 \int_0^1 \left(\mathbf{1}_{[y \leq \hat{p}]} \hat{p} + \mathbf{1}_{[\hat{p} < y]} \min[y, x] \right) f^{(1,n-1)}(y) dy f(x) dx \leq \int_0^1 x f(x) dx = E[q].$$

Now we can substitute for $E[q]$ from (9) and get

$$\begin{aligned}
&\int_0^1 \int_0^1 \left(\mathbf{1}_{[y \leq \hat{p}]} \hat{p} + \mathbf{1}_{[\hat{p} < y]} \min[y, x] \right) f^{(1,n-1)}(y) dy f(x) dx \\
&\quad \leq \int_0^1 \max[\hat{p}, y] f^{(1,n-1)}(y) - \hat{c} \\
&\quad = \int_0^1 \mathbf{1}_{[y \leq \hat{p}]} \hat{p} + \mathbf{1}_{[\hat{p} < y]} y f^{(1,n-1)}(y) dy - \int_0^1 \int_0^1 \left(y - \min[y, x] \right) f^{(1,n-1)}(y) dy f(x) dx.
\end{aligned}$$

After rearranging we obtain

$$\int_0^1 \int_{\hat{p}}^1 \left(y - \min[y, x] \right) f^{(1,n-1)}(y) dy f(x) dx \geq \int_0^1 \int_0^1 \left(y - \min[y, x] \right) f^{(1,n-1)}(y) dy f(x) dx$$

which is a contradiction since $\hat{p} > 0$ and F has a full support. Thus, deviation to $\tilde{b}_i$ followed by searching is not profitable.

Next suppose that after bidding $\tilde{b}_i$, bidder i passes. Then, his payoff is given by (16)

$$1 - \int_0^1 \left(\mathbf{1}_{[y \le p]} p + \mathbf{1}_{[y > p]} y\right) f^{(1,n-1)}(y) dy.$$

As before we look at the upper bound, i.e., when $\tilde{b}_i = \beta(\hat{c}) = \hat{p}$. Bidder i wants to deviate when this payoff is greater than (22),

$$\begin{aligned} \int_0^1 \left(\mathbf{1}_{[y \le p]} p + \mathbf{1}_{[y > p]} y\right) f^{(1,n-1)}(y) dy &\le \int_0^1 x f(x) dx - c_i \\ \int_0^1 \max[\hat{p}, y] f^{(1,n-1)}(y) - E[q] &\le c_i \end{aligned}$$

which is a contradiction by (9) since $c_i < \hat{c}$. Even in this case deviation to $\tilde{b}_i$ is not profitable.

Thus, we have established that deviation to $\tilde{b}_i > 0$ is never strictly better than bidding zero. □

Part (ii): Suppose $\hat{c} \le c_i < 1 - E[q]$.

Claim 1: If $W \ne i$, then $\tau_i = 1$.

Proof: In this case the history is $\{p, j\}$ where $p \in [\beta(\hat{c}), 1)$. Since $c_i < 1 - E[q]$, bidder i optimally searches by (19). □

Claim 2: If $W = i$, then $\tau_i = 0$.

Proof: In this case the history is $\{p, i\}$ where $p \in [0, \beta(c_i)]$. Since

$$\begin{aligned} c_i &\ge \hat{c} \\ &= E[q^{(1,n-1)} - \min[q, q^{(1,n-1)}]] \\ &> \int_0^1 \int_0^1 \mathbf{1}_{[b_i < y]} (y - \min[y, x]) f^{(1,n-1)}(y) dy f(x) dx, \end{aligned}$$

bidder i optimally passes by (20). □

Claim 3: In the early bidding round bidder i optimally bids b_i^* which is given by

$$\int_0^1 \max[b_i^*, y] f^{(1,n-1)}(y) dy - E[q] = c_i.$$

Proof: This Claim is just a generalization of Claim 3 in the Proposition 1, part (ii). All arguments made there apply here with one exception that here we use value functions (15), (16), (17), (18) in place of (2), (3), (4) and (5). Therefore, we will not repeat the argument here except for stating that i wants to increase his bid whenever

$$1-\int_0^1 xf(x)dx - c_i \leq 1-\int_0^1 \left(\mathbf{1}_{[y\leq b_i]}b_i + \mathbf{1}_{[y>b_i]}y\right) f^{(1,n-1)}(y)dy$$

$$\leq 1-\int_0^1 \left(\mathbf{1}_{[y\leq b_{-i}^{(1)}]}b_{-i}^{(1)} + \mathbf{1}_{[y>b_{-i}^{(1)}]}y\right) f^{(1,n-1)}(y)dy. \qquad (23)$$

The optimal bid b_i^* solves

$$\begin{aligned} 1-\int_0^1 xf(x)dx - c_i &= 1-\int_0^1 \left(\mathbf{1}_{[y\leq b_i^*]}b_i^* + \mathbf{1}_{[y>b_i^*]}y\right) f^{(1,n-1)}(y)dy \\ \int_0^1 \max\left[b_i^* y\right] f^{(1,n-1)}(y)dy - E[q] &= c_i. \end{aligned}$$

Since F has a full support, the right-hand side is monotone increasing in b_i^* from $E[q^{(1,n-1)}]-E[q]$ to $1-E[q]$. Recall that $\hat{c}=E[q^{(1,n-1)}]-E[\min[q,q^{(1,n-1)}]] > E[q^{(1,n-1)}]-E[q]$. Thus, in the cost range $[\hat{c}, 1-E[q]]$, for any c_i we can find a unique b_i^* that satisfies the equality. □

Part (iii): Suppose $c_i \geq 1-E[q]$.

Claim 1: In the searching round, $\tau_i = 1$.

Proof: In this case, $W=k$, where k can be an index of any bidder and $p=1$. Then, by (19) i optimally passes. □

Claim 2: In the early bidding round, $b_i^*=1$.

Proof: Since $c_i > 1-E[q]$ the left-hand side of (23) is less than zero and the right-hand side cannot be negative. This implies that bidder i will optimally bid his maximum willingness to pay which is his value, $b_i^*=1$. □

Q.E.D.

Proof of Proposition 3.

Proof: Consider the strategy profile given by Proposition 3. In the final bidding round, bidders are assumed to bid their best outside alternative as stated in the (1). Therefore, we only have to worry about the equilibrium behavior in the searching round. Each bidder has to decide whether to search or pass. If bidder i searches, then, if all other bidders have costs below the threshold, $c_{-i}^{(1)} \leq \check{c}$, he ends up paying $\min[q_i, q_{-i}^{(1)}]$. However, if at least one of the other bidders has a cost above $\check{c}$, then in the equilibrium that bidder passes and bids 1. Bidder i then ends up paying his outside price, q_i. By using (17) the expected payoff from searching is given by

$$\int_0^{\bar{c}} (\mathbf{1}_{[z\leq\check{c}]}\left(1-\int_0^1\int_0^1 \min[y,x] f^{(1,n-1)}(y)dyf(x)dx\right)+$$
$$\mathbf{1}_{[z>\check{c}]}\left(1-\int_0^1 xf(x)dx\right))g^{(1,n-1)}(z)dz - c_i. \tag{24}$$

On the other hand, if he passes, then he can only get a non-zero payoff if all other bidders search, i.e., $c_{-i}^{(1)} \leq \check{c}$. Hence, his expected payoff is determined by (18),

$$\int_0^{\check{c}} (1-\int_0^1 yf^{(1,n-1)}(y)dy)g^{(1,n-1)}(z)dz. \tag{25}$$

In the equilibrium, there will be a type $\check{c}$ who is exactly indifferent between searching and passing, i.e., for whom (24) is equal to (25),

$$\int_0^{\bar{c}} (\mathbf{1}_{[z\leq\check{c}]}\left(\int_0^1\int_0^1 (y-\min[y,x]) f^{(1,n-1)}(y)dyf(x)dx\right)+$$
$$\mathbf{1}_{[z>\check{c}]}\left(1-\int_0^1 xf(x)dx\right))g^{(1,n-1)}(z)dz = \check{c}. \tag{26}$$

Notice that the left-hand side is always greater than zero. Moreover, for all $(x,y)\in[0,1]^2$ it is true that $y-\min[y,x]\leq 1-x$. Since G has a full support (i.e., $y-\min[y,x] < 1-x$ will occur with positive probability) there will always be a type $\check{c}\in(0,1-E[q])$ that satisfies (26). Hence, bidder i best responds by searching when $c_i\leq\check{c}$ and by passing when $c_i>\check{c}$. □

Q.E.D.

Proof of Corollary 2.

Proof: The proof is analogous to that of Proposition 3. The difference is that we use (4) and (5) to construct the equivalents (24) and (25). Then, in the equilibrium, the type who is indifferent, is identified by,

$$\int_{\check{c}}^{\bar{c}} (1-E[q])g^{(1,n-1)}(x)dx = \check{c}. \tag{27}$$

Here it is easy to see that there is always $\check{c} \in (0, 1 - E[q])$ which satisfies (27). $\square$

Q.E.D.

Proof of Proposition 4.

First we look at the common prices environment. We begin with part (i). For any type vector, c, the ex-post social surplus for the dynamic auction is

$$W_{ep}^{D}(c) = \begin{cases} n - \sum_{j=2}^{n} c^{(j)} & \text{if} \quad c^{(1)} \leq 1 - E[q] \\ n - l - \sum_{j=2+l}^{n} c^{(j)} & \text{if} \quad c^{(2+l)} \leq 1 - E[q] < c^{(1+l)} \\ 1 & \text{if} \quad 1 - E[q] < c^{(n)} \end{cases}, \quad (28)$$

where $l \in \{0, n-2\}$. The first case refers to types of all bidders falling in the relevant cost range. In that case everyone but the the bidder with $c^{(1)}$ searches. The middle case covers the intermediate distribution of individual types when some bidders have costs above $1 - E[q]$. All those bidders pass and only one of them wins the auction and trades. Everyone else searches. Hence, $n - l$ bidders trade and $n - l - 1$ search. In the last case all types have costs that are above the threshold $1 - E[q]$. In this case all bidders pass and one of them wins the auction and trades.

Analogously, for the static auction we have

$$W_{ep}^{S}(c) = \begin{cases} n - \sum_{j=1}^{n} c^{(j)} & \text{if} \quad c^{(1)} \leq \check{c} \\ n - l - \sum_{j=2+l}^{n} c^{(j)} & \text{if} \quad c^{(2+l)} \leq \check{c} < c^{(1+l)} \\ 1 & \text{if} \quad \check{c} < c^{(n)} \end{cases}, \quad (29)$$

for $l \in \{0, n-2\}$. There are two differences between the welfare functions. The first difference is in the top case when all bidders have costs bellow the respective threshold. In this case $W_{ep}^{D} > W_{ep}^{S}$. The reason is that in the static auction all bidders search, whereas in the dynamic auction one of the bidders (the bidder with type $c^{(1)}$) passes. In the remaining cases $W_{ep}^{D} = W_{ep}^{S}$.

The second difference is in the thresholds themselves. By Lemma 2, $\check{c} < 1 - E[q]$. Since $n - \sum_{j=2}^{n} c^{(j)} > n - \sum_{j=1}^{n} c^{(j)} \geq n - l - \sum_{j=2+l}^{n} c^{(j)} > 1$ we conclude that for all c, $W_{ep}^{D} \geq W_{ep}^{S}$ which proves part (i) of the Proposition. To show part (ii) it is sufficient to identify a case which occurs in equilibrium with positive probability and for which $W_{ep}^{D} > W_{ep}^{S}$.There are several such cases, but one that is easily identified is when $c^{(n)} \in (\check{c}, 1 - E[q]]$. Because G

has a full support, this case occurs in equilibrium with positive probability, which implies that $W_{ea}^D > W_{ea}^S$ proving part (ii).

The argument for the case of independent outside prices is exactly the same with the exception the welfare function for the dynamic auction is a bit more complicated because there is an additional threshold[14], $\hat{c}$, that separates the late from the early bidders. For the dynamic auction we have

$$W_{ep}^D(c) = \begin{cases} n - \sum_{j=1}^n c^{(j)} & \text{if} & c^{(1)} \le \hat{c} \\ n - \sum_{j=2}^n c^{(j)} & \text{if} & \hat{c} < c^{(1)} \le 1 - E[q] \\ n - l - \sum_{j=2+l}^n c^{(j)} & \text{if} & c^{(2+l)} \le 1 - E[q] < c^{(1+l)} \\ 1 & \text{if} & 1 - E[q] < c^{(n)} \end{cases}, \quad (30)$$

for $l \in \{0, n-2\}$. For the static auction, W_{ep}^S is identical to (29).

Notice that (28) is a lower bound on $W_{ep}^D(c)$ since it combines the top two cases of (30) into a single case, $n - \sum_{j=2}^n c^{(j)}$ if $c^{(1)} \le 1 - E[q]$ in (28). This observation allows us to apply the same argument as in the case of common outside prices to conclude that $W_{ep}^D \ge W_{ep}^S$ and $W_{ea}^D > W_{ea}^S$. □

Q.E.D.

Proof of Proposition 5.

This is straight forward. Notice that for $k \in [0, \bar{c}]$ since $E[q] < 1$,

$$\pi(k) = \int_0^k E[q] g^{(2,n)}(z) dz + \int_k^{\bar{c}} 1 g^{(2,n)}(z) dz$$

is strictly decreasing in k. Hence for $k' < k''$ we have $\pi(k') > \pi(k'')$. □

Q.E.D.

Example 4a: Proof of the Asymptotic Result.

The Figure 5 is helpful to understand the proof.

[14] as defined in Proposition 2

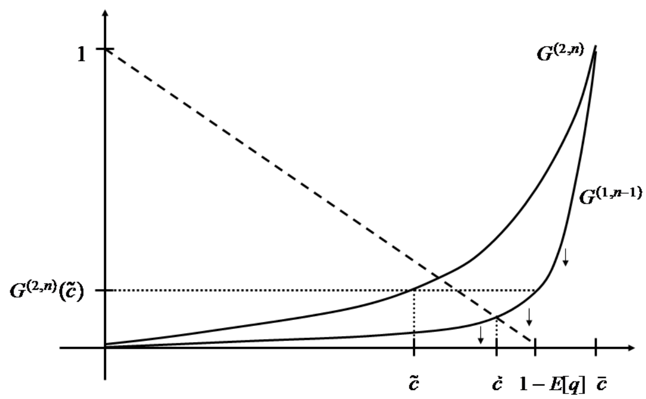

Figure 5

Consider arbitrary number of bidders n. By (12) define $\check{c}_n$ as a point corresponding to the intersection of the straight line $1-\frac{x}{1-E[q]}$ and $G^{(1,n-1)}(x) = G(x)^{n-1}$ on $[0, 1-E[q]]$. Since $\bar{c} > 1-E[q]$ it is easy to see that on the partial domain $[0, 1-E[q]]$ the extreme value distribution converges uniformly $G(x)^{n-1} \to 0$ as $n \to \infty$. Hence, in the limit we have $1 - \frac{\check{c}_n}{1-E[q]} = 0$, i.e., $\check{c}_n \to 1-E[q]$.

Next, define $\tilde{c}_n$ as the point for which (13) holds. Since $G^{(2,n)}(x)$ is strictly increasing on $[0, 1-E[q]]$ and $G^{(2,n)}(0) = 0$, then $\int_0^{1-E[q]} G^{(2,n)} < (1-E[q])G^{(2,n)}(1-E[q])$. Then, by Intermediate Value Theorem there exists $\tilde{c} \in (0,1)$ such that $\int_0^{1-E[q]} G^{(2,n)} = (1-E[q])G^{(2,n)}(\tilde{c})$. Hence, for an arbitrary $m \in \mathbb{N}$, $\tilde{c}_m < \lim\limits_{n\to\infty} \check{c}_n$. $\square$