Este material fue fotocopiado del libro del profesor Friedman mencionado. Se distribuye gratuitamente para fines didácticos exclusivamente para alumnos inscritos en el curso de Seminario de Economía de México agosto-diciembre 2000 del profesor Enrique Barraza.

Utility Analysis of Savings

Let us turn now to the decision about how much of current receipts from the sale of resource services to spend on current consumption and how much to add to accumulated wealth, or alternatively how much to subtract from wealth to add to current receipts for spending on current consumption. (This analysis will be used and expanded further in some directions in chapter 17.) It is tempting to try to incorporate this decision into utility analysis by the same device as we have just incorporated the decision about how many hours to work, namely, by adding another axis to the indifference diagram on which is measured savings, or the number of dollars per year added to accumulated wealth. Indeed, Leon Walras succumbed to this temptation in the latest edition of his great book, *Éléments d'économie politique pure,* published in English translation under the title, *Elements of Pure Economics,* after having resisted it in earlier editions.[7]

The difficulty with this apparently simple extension of the utility analysis to cover saving can be seen by supposing it to be followed by measuring consumption on one axis and the rate of saving on the other, both measured as number of dollars per year. What is then the price ratio that is relevant? Clearly it is 1: a dollar per year can always be added to savings by subtracting a dollar from consumption. In his desire to include a substitution effect, Walras defined the variable to be measured along the saving axis not as the number of dollars per year devoted to saving but as a commodity E, equal to the permanent income stream purchased with the saving, i.e., the permanent income stream, r, yielded by one dollar of wealth, where r is the rate of interest. The price of one unit of E is then $\frac{1}{r}$ or the reciprocal of the interest rate (if $r = .05$, it costs $20 to buy $1 a year). However, this makes the two axes noncomparable: consumption is a flow, dollars per year; E is a rate of change of a flow, a second derivative, dollars per year per year. With a properly specified utility function, the indifference curves remain the same over time regardless of which point on them is attained, so long as the basic underlying conditions are the same. Not so with indifference curves for consumption and the Walras commodity E. A positive E adds to the stock of wealth so as time passes, the individual becomes wealthier and wealthier. For the same level of consumption, the rate at which the individual will be willing to substitute still further additions to wealth for further additions to consumption will decline. The indifference curves so defined will change.

The difficulty with the simple approach is that saving is not another commodity like food, clothing, etc., which offers utility in accordance with

7. Milton Friedman, "Leon Walras and His Economic System," *American Economic Review,* 45 (December 1955): 900–909.

the rate of saving. Saving is a way of substituting future consumption for present consumption. For a satisfactory analysis of saving, we have to take account of its basic role, not simply add an axis to an indifference diagram. It is essential to consider more than one time period. Accumulated wealth, unlike saving, may have certain characteristics that make it in part a good like other consumption services, insofar as it provides a reserve against emergencies. This service can be measured along an indifference curve axis, and part of income regarded as used to purchase it. The income used to purchase it is the difference between the (anticipated average) maximum return that can be obtained from the wealth and the actual (anticipated average) return from holding the wealth in a form that provides greater utility as a reserve.

If we neglect this role of wealth, the case that it is easiest to present on an indifference diagram is one that Irving Fisher analyzed: the hypothetical case of a finite period, most simply, a two-year period. This case is given in Figure 2.26. The vertical axis measures consumption in year 1, the horizontal axis, consumption in year 2. The diagonal line shows equal consumption in the two years. Let R_1 be receipts in the first year, R_2 receipts in the second, and r the rate of interest, and assume that the individual to whom the figure applies can borrow or lend any sum at the interest rate r that he can repay or make available out of his receipts. The maximum amount he could then spend on consumption in year 1 if he spent nothing in year 2 would be

$$W = R_1 + \frac{R_2}{1 + r}, \tag{3}$$

because $\frac{R_2}{1 + r}$ is the maximum amount he could borrow and repay with his receipts in the second year. W is his initial wealth and defines the intercept A on the vertical axis of the line of attainable combinations. The maximum amount he could spend on consumption in year 2 if he spent nothing in year 1 is

$$(1 + r)\, W = R_1\,(1 + r) + R_2. \tag{4}$$

The line AB thus is the line of attainable combinations. The rate of substitution in the market is such that the individual can add $(1 + r)$ dollars of consumption in year 2 for each dollar reduction in consumption in year 1. As drawn, the equilibrium point P shows a choice involving higher consumption in year 2 than in year 1, but that is of course a result of the particular set of indifference curves and the particular interest rate.

We can use this simple model to illustrate the concept of time preference —the rate at which individuals are willing to substitute future consumption for present consumption. The rate of time preference is thus the slope of the indifference curve and hence varies from point to point in the

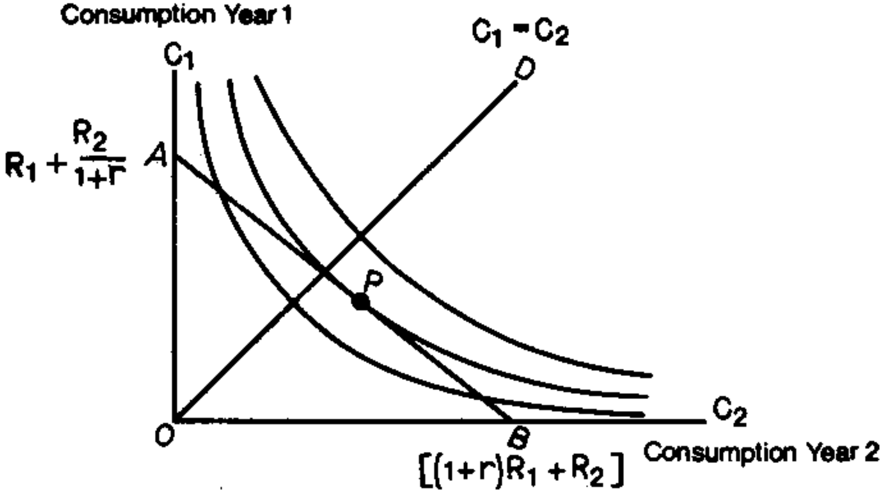

FIGURE 2.26

diagram. At a point corresponding to high consumption in year 1, low consumption in year 2, the individual prefers additional future consumption to present consumption, i.e., he would be willing to give up more than \$1 of current consumption to add \$1 to future consumption. Conversely, at a point corresponding to high future consumption, low present consumption, the individual prefers additional current consumption to future consumption, i.e., it would take more than \$1 of future consumption to compensate him for giving up \$1 of current consumption. The rate of time preference is therefore a variable, depending on the levels of present and future consumption. At point P, the rate of time preference is equal to the market rate of substitution $(1 + r)$ because the individual adjusts his time pattern of consumption to bring about that equality.

It is common to say that individuals "underestimate the future" or have a "preference for the present over the future" or "discount the future." One way to assign a meaning to such expressions is to define them in terms of the rate of time preference on the diagonal line in Figure 2.26. Along this line, future consumption is equal to present consumption. It seems reasonable to say that an individual is neutral between present and future if the slope of the indifference curves for points on this line is unity, or more generally if the indifference curves are symmetrical about this line. An individual underestimates the future if the indifference curves for points on this curve are flatter than the $-45°$ lines and overestimates the future if they are steeper. More generally, we can say he underestimates the future if the indifference curves are asymmetrical about the diagonal line in such a way that a point to the left of the diagonal is on a higher indifference curve than its mirror image to the right of the diagonal.

To return to the determinants of consumption and saving, we are back in a familiar situation. It appears that the pattern of consumption depends on three variables: R_1, R_2, r, yet it is clear from Figure 2.26 that only two variables are important: $W = R_1 + \frac{R_2}{1+r}$, and r, namely wealth and the interest rate:

$$C = f(r, W). \tag{5}$$

If we interpret R_1 and R_2 as measured incomes in the two years, consumption in each year depends not on income but on wealth (or "permanent income"). On the other hand, if we define savings as the difference between measured income and consumption, savings does depend on income, because

$$S_1 = R_1 - C = R_1 - f(r, W). \tag{6}$$

In this model, there are two motives for saving: to "straighten out the income stream," that is, to make consumption steadier over time than receipts—this motive causes R_1 to enter into equation 6; and to earn a return on savings, this motive causes r to enter into equation 5. W in equation 5 can be regarded as playing a dual role as a measure both of available opportunities and of the consumption service of a reserve against emergencies.

A special case of equation 5 arises if the indifference curves in Figure 2.26 are similar in the sense that all indifference curves have the same slope along any ray from the origin. Equation 5 then reduces to

$$C = k(r) \cdot W \tag{7}$$

or, to include other factors that might affect consumption not included in our simple representation:

$$C = k(r, u) \cdot W, \tag{8}$$

where u stands for these other factors. In this special case, we could define the consumer's numerical rate of time preference by the common slope along the diagonal. If he has neutral time preference in this sense, then for any positive rate of interest, future consumption will exceed present consumption. If he discounts the future, then for some positive rates of interest current consumption will exceed future consumption.

The simple time period model can also be used to illustrate the effect of a difference between the rate of interest at which the individual can borrow and the rate at which he can lend. This difference may arise simply from the costs of financial intermediation between borrowers and lenders or from the difference between human and nonhuman capital that makes human capital generally less satisfactory as collateral for a loan. Let r_B be the rate of interest at which he can borrow and r_L at which he can lend, with $r_B > r_L$. Then the budget line will have a bend as in Figure 2.27 at the point corresponding to receipts (R_1, R_2) in the two years. There is then no unambiguous measure of wealth, and the final outcome may depend on the initial position, depending on where it is and the shape of the indifference curves.

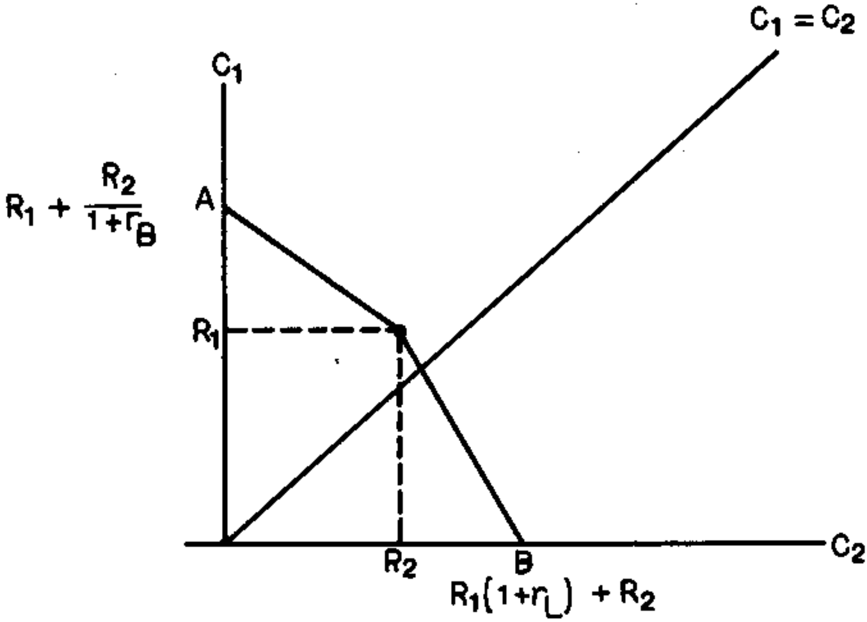

FIGURE 2.27

Generalizing this analysis to an indefinite time period is easy to do formally, hard to do in two-dimensional graphs. The formal generalization is that the economic agent is regarded as having a utility function that is a function of the whole future pattern of consumption:

$$U = F[C(t)], \tag{9}$$

where C(t) represents the flow of consumption at time t, and t extends from the time period in question to the indefinite future, say t_0 to ∞. He also is regarded as having an opportunity set

$$G[C(t)], \tag{10}$$

that summarizes the alternative time patterns of consumption that are available to him. He is then regarded as maximizing the utility function in equation 9 subject to the opportunity set of equation 10.

This generalization is perfectly general and perfectly empty. To give it content, it is necessary to specialize equations 9 and 10. For example, equation 9 can be specialized by supposing that there exists some internal rate of discount, say ρ, such that a particular form of equation 9 can be written

$$U(t_0) = \int_{t_0}^{\infty} f[C(t)]e^{-\rho t}dt \tag{11}$$

in which case, of course, any monotonic transformation of equation 11, say

$$U^* = F(U), \tag{12}$$

will also do provided $F'(U) > 0$. Equation 10 can be specialized by supposing that there exists some market rate of interest r such that any pattern of consumption is available for which

$$W(t_0) \geq \int_{t_0}^{\infty} C(t)e^{-rt}dt, \tag{13}$$

where W is the similarly discounted value of the individual's anticipated stream of receipts in the future. There has been ~~much analysis, especia~~lly in the literature on growth models, ~~using such~~ specializations but no such specialization as yet has reason to be singled out as deserving particular confidence.

One way to present an indefinite time period in a two-dimensional graph is to specialize the opportunity set in equation 10 by supposing that the only alternatives available to the individual are two-dimensional: a rate of consumption of C_1 for one time unit, say a year; a rate of consumption of C_2 for the indefinite future thereafter. For this to be at all reasonable, we must suppose the individual to have an infinite life with unchanging tastes. This may seem absurd but in fact is not. It simply is a way of representing the observed phenomenon that the family, not the individual, is the basic consumption unit, and that in deciding on current consumption versus future consumption, the person making the decision takes into account the utility that his descendants will derive from consumption as well as his own. The infinitely lived and unchanging individual thus represents the long-lived family line. Though highly special, the two-dimensional representation brings out one important feature of the saving-spending process concealed by the two-period example.

Let R_1 be the rate of flow of receipts in the first year, R_2 the assumed steady rate of flow indefinitely thereafter, and r an assumed constant rate of interest over time at which the individual can borrow or lend. Then his initial wealth is

$$W = R_1 + \frac{R_2}{r}, \tag{14}$$

where r enters into the denominator of the final term rather than $1 + r$ as in equation 3, because R_2 is here a perpetual income stream rather than simply a one-period receipt. This initial wealth defines the point A, the maximum consumption in the first period if consumption thereafter is zero. The maximum consumption after the end of the first year is R_2, the perpetual receipt thereafter plus interest on the first year's receipt if consumption in the first year is zero, or rR_1, so $rW = rR_1 + R_2$ defines point B, and the line connecting them is the line of attainable combinations. Its slope with respect to the C_2 axis is $\frac{1}{r}$, or the number of dollars of current consumption that must be given up to add \$1 per year to all future consumption; with respect to the C_1 axis, the slope is r, or the number of dollars that can be added to future consumption by giving up \$1 of current consumption. Figure 2.28 is drawn for an interest rate of .20 in order to make it possible to distinguish the different points.

P_1 is the equilibrium position, involving as the figure is drawn, lower consumption in the first year than indefinitely in order to raise future consumption. Let us now move one year ahead and look at the situation again, again assuming that the only alternatives are a rate of consumption of C_1 for one year and of C_2 thereafter (this is the unsatisfactory element of the analysis because, of course, we would expect the individual at time 0 to choose a whole future pattern of consumption and not proceed in this step-at-a-time fashion). The indifference curves are the same, since we have assumed the individual to have unchanging tastes, but the opportunity line is different because saving in year 1 has added to his wealth. The new opportunity line (A'B') will go through the point on the diagonal corresponding to the abscissa of P_1. The new equilibrium is P_2.

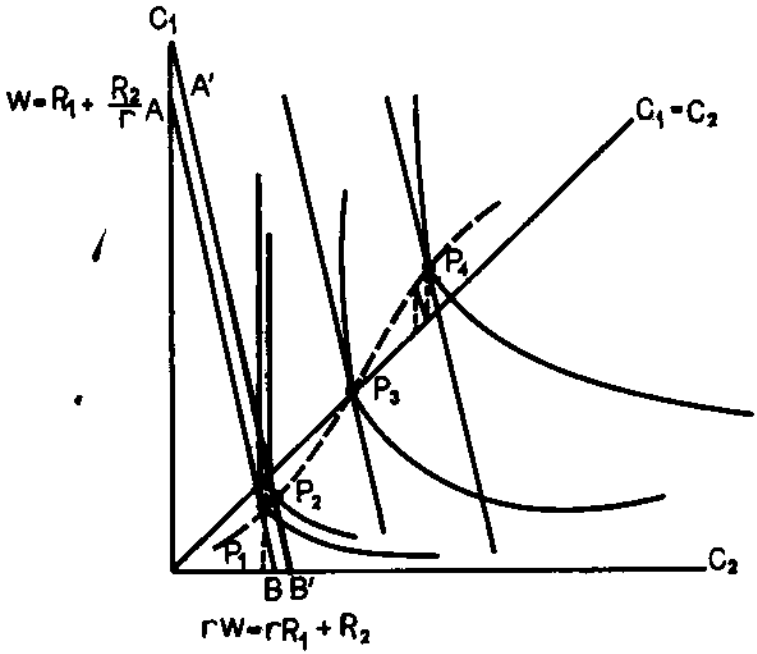

FIGURE 2.28

The dashed line is the locus of such points of equilibrium in successive years and defines the individual's future consumption path. As drawn, the dashed line cuts the diagonal at P_3. At this point the rate of time preference of the individual as defined earlier (for a constant level of consumption) equals the rate at which he can substitute current for future income. This is a point which if attained will be maintained.

Suppose we had started the individual with a wealth such that P_4 is the equilibrium. Then the individual would have dissaved in the sense of reducing wealth to add to current consumption. He would have followed the path suggested by the zigzag line down the dashed line until again he arrived at P_3.

The advantage of this construction is that it brings out the difference between the equilibrium stock of wealth (desired wealth) and the equilibrium rate of approach to that stock of wealth. The wealth corresponding to point P_3 is the equilibrium stock of wealth. If the individual does not have that stock of wealth, he will move toward it. There will be an equilibrium rate at which he will want to move toward it that will depend both on how far he is from his desired wealth and on what his current wealth is. The considerations determining the desired stock of wealth are different from those determining how fast he wants to move toward it, though this distinction is blurred by the two-dimensional representation in Figure 2.28.

In that figure, in order for there to be an equilibrium stock of wealth, it is necessary that the slope of the indifference curves become flatter along the diagonal line as wealth increases; that is, that it require larger and larger increments in future consumption to compensate for giving up $1 of current consumption, or, alternatively, that the preference for present over future consumption increase with wealth. This seems intuitively perverse. It seems more plausible that if anything the reverse would occur.

If the indifference curves were similar in the sense that they all have the same slope along any ray from the origin, the dashed line would, unlike the dashed line in Figure 2.28, never cut the diagonal. It would be rather such a ray. If below the diagonal, it would imply indefinite accumulation of wealth; if above, indefinite decumulation. But in both cases there would be an equilibrium rate of accumulation or decumulation. For modern progressive societies, there is no inconsistency between observable phenomena and a representation implying indefinite accumulation.

This is a very incomplete treatment of a very complex problem. Its purpose is to illustrate how the apparatus we have developed can illuminate such problems.