

Aggregate Risk Sharing Across US States and Across European Countries.

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Abstract

The paper analyzes risk sharing across US States and across European countries. The empirical literature on this subject has failed to recognize that, as long as agents can borrow and lend, risk sharing should be measured with respect to shocks to the overall level of wealth, and not to current income. The contribution of this paper is to build an appropriate empirical measure of risk sharing focused on shocks to the expected present value of resources. My main findings are: i) in the US private capital markets allow states to share 40% of shocks to overall domestic wealth; ii) however, risk sharing relative to shocks to the present value of labor income is low (only 5-7%); iii) the amount of risk sharing through the federal system is likely to be less than estimated by Sachs and Sala-i-Martin (20-30% vs 40%); iv) in Europe the extent of risk sharing through private capital markets is almost zero. Since my approach encompasses the approaches of Sachs and Sala-i-Martin and of other contributions in the literature, I am also able to explain the apparent differences in their results.

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1. Introduction.

The paper addresses two issues: the degree of aggregate risk sharing within a country, and the extent to which the federal system of taxes and transfers provides insurance. Recent literature in international macroeconomics (van Wincoop [53], Hess and Shin [34] [35]) has compared the business cycle stylized facts across countries and across regions or states within a country to determine whether the apparent lack of risk sharing observed in *international* data also holds in *intranational* data. At the same time, other literature (Crucini [24], Atkeson and Bayoumi [3], Bayoumi and Klein [8], Asdrubali, Sørensen and Yosha [1], Sørensen and Yosha [47] [48] [49], Athanasoulis and van Wincoop [2]) has attempted to estimate directly the degree of risk sharing both at the international and at the

intranational level. If the lack of risk sharing were as severe within countries as it is across countries,¹ market incompleteness would be the most likely culprit, given that, within regions of the same country, contracts are easier to enforce, transportation costs are lower, there are no official barriers to trade in goods or in assets, and the currency is the same.²

The second issue addressed by the paper is the extent to which the federal system of taxes and transfers provides insurance. Ever since the Delors Report (1989) proposed the idea of the European Monetary Union, the stability of the system to adverse income shocks has been one of the main concerns for European policy-makers.³ A common currency implies the loss of monetary policy independence; furthermore, constraints on fiscal policy impose additional limits to the ability of governments to deal with fluctuations in aggregate income. Existing federations of states, such as the US and Canada, have dealt with this problem by establishing a federal system of taxes and transfers. Since taxes are progressive and transfers are partly related to measures of economic activity, one of the effects of a federal system is to transfer resources from those states that are in a boom to those that are in a recession. It has been argued that an insurance system among members of the European Monetary Union could compensate in part for the loss of monetary policy as an instrument for alleviating adverse shocks to income. This paper, following Sala-i-Martin and Sachs [44] (henceforth, SaSa) and other recent literature (Von Hagen [55], Bayoumi and Masson [9]), attempts to measure the insurance role of the federal system of taxes and transfers. The paper concludes that the federal system of taxes and transfers does not play a large role in terms of sharing either idiosyncratic or systematic risk.

The main contribution of the paper with respect to the empirical literature on this subject consists in recognizing the intertemporal dimension of insurance: as long as countries can borrow and lend, agents' decisions depend on the overall level of wealth, and not on current income.⁴

¹A proper comparison of intranational and international risk sharing needs to take into account that the federal system transfers resources across regions and states. This paper, as well as previous literature, analyzes also the role of the federal system.

²Exchange rate fluctuations may actually allow countries to share risk among each other through across-the-border holdings of nominal bonds (see Svensson [51], and Kim [36]). However, the present paper is not concerned with this issue, as the model involves only real variables.

³The analysis of the insurance role of the US federal system of taxes and transfers is relevant not only for the US and Europe, but also for other countries which are considering the opportunity of forming a currency union (such as the Caribbean countries).

⁴Backus *and al.*[4] show that even in autarky agents still behave in a "permanent income"

The paper finds that the degree of risk sharing through private capital markets among US states is large, about 40% of shocks to domestic wealth, while the degree of risk sharing through private capital markets in Europe is almost zero. The paper also finds that private capital markets in the US play a much smaller role in smoothing shocks to labor income, confirming the fact that the lack of securities explicitly linked to labor income is an important source of market incompleteness.

The paper shows that the different findings on the insurance role of the federal system of SaSa and of Asdrubali, Sørensen and Yosha (henceforth, ASY) are due to differences in the assumptions about the stochastic process and about the nature of the shocks: SaSa focus on shocks to relative income, and assume that relative income follows a deterministic trend, while ASY focus on shocks to gross state product, and assume that the growth rates in gross state product are i.i.d. I argue that focusing on shocks to income may give a more appropriate measure of risk sharing through the federal system, given that the tax base of the federal system is given by income rather than by gross state product, and given that the two differ precisely because of risk sharing through private capital markets. However, I also show that the deterministic trend assumption leads to unrealistic implications in terms of the effects of shocks to relative income on consumption. I argue that the correct estimate of risk sharing through the federal system is between 20 and 30%, which is less than the figure estimated by SaSa, namely 40%. While all the results are obtained constructing and using individual state CPI series in order to deflate nominal variables, they do not appear to be driven by changes in relative prices.

The results of this paper, like the ones from the existing literature, are subject to an important *caveat*: an additional difference between international and intranational data besides the ones already mentioned is that labor is more mobile within countries than across countries.⁵ Whether migration across states is large enough to affect the findings of the paper is an open question, and will be the subject of further empirical research on my part.⁶

fashion, as long as there is a technology that allows them to transfer resources intertemporally.

⁵See Barro and Sala-i-Martin [10], and Blanchard and Katz [11]. Decressin and Fatas [27] show that migration in Europe plays a much smaller role than in the US.

⁶Existing literature (Blanchard and Katz [11]) shows that wages do adjust to state-specific shocks through migration of workers, but also that the adjustment process takes a number of years. This suggests that migration alone may not be able to explain the results of this paper: a good shock to a given state implies that labor income will be higher than the US average for a long period. Therefore the permanent income of workers in that state, and their consumption, should also increase. Section 3 discusses a way to incorporate the effects of migration in the

The outline of the paper is as follows. Section 2 describes the previous literature. Section 3 describes the theoretical framework underlying the empirical exercise and describes the data. Section 4 discusses the econometric methodology. Section 5 discusses the results. Section 6 concludes.

2. The previous literature.

In a seminal paper Backus, Kehoe, and Kydland [4] present evidence that cross country correlations in output far exceed the correlations in consumption, when the series are filtered using the Hodrick-Prescott filter. Such evidence is in sharp contrast with the conclusions of a one-good complete markets general equilibrium model, and is interpreted as suggesting that countries are not actively engaged in risk sharing. This impression was reinforced by the finding of French and Poterba [32] that in the US and Japan foreign assets represent a very small fraction of investors portfolios.

The work by Backus *et al.* was followed by several theoretical papers (Tesar [52], Stockman and Tesar [50], Baxter and Crucini [5] and many others) attempting to reconcile the above "stylized fact" with multi country general equilibrium models. At the same time, a substantial body of empirical literature (Obstfeld [39], Canova and Ravn [20], Lewis [37]) tested the implications of such models.⁷ For the main part, such literature has focused on tests of the Euler equation under the null hypothesis of perfect risk sharing. A limit of this approach is that, in case of rejection of the null, it does not measure how far countries are from perfect risk sharing.

A different strand of literature moved away from tests of the complete markets model, and attempted to measure directly the extent to which countries share risk. Atkeson and Bayoumi [3] estimate the extent to which shocks to labor income are in fact hedged in existing capital markets across both US States and European Countries. They find that factor income plays a much more important role within countries than across countries, suggesting that cross ownership of assets is larger intranationally than internationally, but that labor income shocks are still far from being insured through private capital markets.⁸ Other recent contributions

empirical analysis.

⁷This literature is summarized in Obstfeld[39] and Obstfeld and Rogoff[41].

⁸The last result comes from the regression of changes in income from capital in a given region on the following regressors: changes in income from capital from a broader aggregate of regions, changes in income from labor in that region, and changes in capital product for that region (all

to this strand of literature are ASY and Sørensen and Yosha [47] [48] [49]. ASY measure the extent to which the different "channels of interstate risk sharing", that is, credit markets, capital markets, and the federal system, help to smooth shocks to gross state product across US States. In order to do so, they regress the difference in the growth rates of gross state product and state income, of state income and disposable state income, and of disposable state income and consumption, on the growth rate of gross state product, using a time fixed effects panel data specification.⁹ ASY find that the degree of risk sharing across US states is high, since state consumption ends up bearing only 25% of shocks to GSP, and that the amount of smoothing achieved through the federal system is limited, about 10% of the shocks. Sørensen and Yosha [47] apply the same methodology to OECD countries.

The approach of both ASY and Atkeson and Bayoumi focuses only on current flows, and does not capture the intertemporal dimension of the problem. Let us suppose, for example, that the US stock market crashes because of bad news on the future profitability of firms. If markets are complete, such news represent a decrease in wealth for all agents. However, at least in the short run, they do not necessarily represent any change in net factor payments across states, which is the measure of private capital markets insurance used by ASY, or in current income from capital, which is the measure used by Atkeson and Bayoumi.

The intertemporal dimension of agents' decisions is particularly important when measuring the insurance role of the federal system. The first empirical analysis of the US federal system of taxes and transfers from the perspective of insurance is due to SaSa.¹⁰ They perform a regression of *relative* taxes and transfers on *relative* income, and find that a decline in relative income by one dollar

the variables are in per capita terms). They find that the coefficient on labor income is very small, suggesting that income from capital does not insure labor income shocks.

⁹Since the difference between gross state product and state income is given, among other things (including corporate retained earnings), by factor income from other states, the covariance between such difference and shocks to gross state product should provide an indication of the amount of "smoothing" that occurs through capital markets. Likewise, the covariance between shocks to gross state product and the difference between state income and disposable income should quantify the "smoothing" that occurs through the federal system. Finally, the covariance between gross state product and the difference between disposable income and consumption should give an indication of the amount of "smoothing" that occurs through borrowing and lending. What remains is the amount of the shock that is not smoothed.

¹⁰The concept of a federal system of taxes and transfers as an important ingredient of a currency union, since it helps sharing the consequences of negative income shocks, can be traced to Eichengreen [29].

triggers a combined reaction of taxes and transfers of about forty cents. Their result was interpreted as an indication that the federal system in the US provides a substantial amount of insurance. Their work was followed by several other papers trying to quantify the amount of insurance provided by the federal system using different specifications of the same regression. The general conclusion of these papers is that the amount of insurance provided by the federal system of taxes and transfers is lower than measured by Sala-i-Martin and Sachs.¹¹ Again, this literature focuses on current flows, and does not take into account that, if agents are able to borrow and lend, what matters to them is the present value of their lifetime stream of resources. In this respect, an exception to the previous literature is Crucini [24], who developed a test of consumption risk sharing based on a "present value" model. However, his model and empirical analysis are different from what is done here: Crucini does not present direct results on the role of the federal system and private capital markets in sharing risk across states and countries.¹²

3. A framework for the empirical analysis.

This section describes the theoretical framework which will be used in the empirical exercise. The model adopted here is a simple permanent income model, in which the representative agent for each state receives income from labor and from capital, receives net transfers from the Federal government, and is able to borrow and lend at a given interest rate. More specifically, the assumptions of the model are as follows:

- i) states are able to borrow and lend at given riskless interest rate;
- ii) labor and property income are modeled as an endowment;
- iii) there is a representative agent for each state;

¹¹In Von Hagen [55] the dependent variables are, respectively, the growth rate in the per capita tax burden, and in per capita transfers, while the regressor is the growth rate in Gross State Product. Atkeson and Bayoumi [3] regress the change in net transfers on the change in both labor income, and income from capital. Finally, Bayoumi and Masson [9] regress the change in relative disposable income on the change in relative income. Most studies find that the amount of insurance provided by the federal system is close to ten cents per dollar, except for Bayoumi and Masson, who find a larger value (30 cents).

¹²Bayoumi and Klein [8] apply a test similar to the one in Crucini to Canadian provinces. In another paper, Crucini [25] evaluates the role of fiscal federalism using a dynamic general equilibrium model, and finds that the observed correlations in consumption growth rates across Canadian provinces could be explained by the insurance role of the federal system.

iv) agents consume only one homogeneous good.

Assumption i) can be partially justified on the ground that state and local governments can, in principle, borrow and lend on behalf of those who do not have access to credit markets.

Assumption ii) is unrealistic, since it implies, among other things, that taxes and transfers are non distortionary. This assumption would be untenable in a theoretical model of insurance, as it assumes away the problem of moral hazard. However, the scope of the present paper, as well as of the previous literature on this topic, is to measure the insurance role of the federal government from the perspective of the net amount of resources received by the agents, without looking at the possible distortions that may arise because of taxes and transfers.

Assumption iii) implicitly rules out migration: when people can move the notion that there is one representative agent for each state is no longer useful.¹³ Previous literature (Blanchard and Katz [11]) suggest that it takes a long time for migration to bring income back to the original level. Consequently, a model that for the time being ignores these effects is perhaps not a bad approximation.

Assumption iv) is also an important restriction of the model, since recent literature in international macroeconomics has pointed out that the presence of non homogeneous goods (non tradable goods and/or several traded goods) is crucial for understanding the patterns of cross country portfolio allocations, and the apparent lack of risk sharing observed internationally.¹⁴ However, from the empirical point of view this paper improves on previous literature by deflating nominal variables using state level CPI indices, which are meant to capture changes in relative prices.

The same model applies to both US States and European countries. However, for the sake of brevity, I will describe the model as if it applied to US states only.

¹³One of the ways migration affects the estimates of the model is the following. When households can migrate a positive permanent shock to productivity in a given state will drive up current labor income for those who live in that state more than it will increase their permanent income, since the inflow of individuals from other states will eventually push income down. If the residents in that state are aware of this mechanism, per capita consumption will rise less than as if there were no migration. Likewise, with a bad shock per capita consumption will not decrease as much as if there were no migration, since some households will move out of the state pushing income up toward the US average. This effect is in part taken into account by allowing for different assumptions about the stochastic process for income.

¹⁴See Lewis [37], Obstfeld and Cole [40], Tesar [52], and Stockman and Tesar [50] among others.

When ambiguities about the definition of the variables arise, I will point them out.¹⁵

Under assumptions i) through iv) the budget constraint of representative household of state i in period t can be written as:

$$B_{i,t} = Y_{i,t}^L + NT_{i,t}^F + NT_{i,t}^S + (1 + r_t)B_{i,t-1} + D_{i,t}^h + NFI_{i,t} - C_{i,t}^{hh}, \quad (3.1)$$

where $B_{i,t}$ is the amount of riskless one period debt held by the agent, $(1 + r_t)$ is the gross interest rate from time $t - 1$ to time t , $C_{i,t}^{hh}$ is private consumption, $Y_{i,t}^L$ is labor income, $D_{i,t}^h$ is income from domestic capital (dividends and rent), $NFI_{i,t}$ is net factor payments from other states,¹⁶ $NT_{i,t}^F$ and $NT_{i,t}^S$ represent net transfers to households from, respectively, federal (for the US) and state and local (henceforth, s&l) governments.¹⁷ All throughout the paper the variables are in per capita terms. The budget constraint of s&l government is:

$$B_{i,t}^S = C_{i,t}^S + G_{i,t}^S + NT_{i,t}^S - \tau_{i,t}^{S,I} + (1 + r_t)B_{i,t-1}^S, \quad (3.2)$$

where $B_{i,t}^S$ is the amount of debt *owed* by the s&l governments, $C_{i,t}^S$ and $G_{i,t}^S$ are respectively consumption and other expenditures by s&l governments, while $\tau_{i,t}^{S,I}$ represent indirect taxes collected by state government. The budget constraint of the federal government is:¹⁸

$$B_t^F = G_t^F + NT_t^F - \tau_t^{F,C} - \tau_t^{F,I} + (1 + r_t)B_{t-1}^F, \quad (3.3)$$

where B_t^F is the amount of debt *owed* by the federal government, G_t^F represents the expenditures (other than transfers to households or grants to s&l governments) of the federal government, $\tau_t^{F,C}$ and $\tau_t^{F,I}$ are respectively corporate taxes and indirect taxes collected by the federal government, and NT_t^F are per capita transfers to households and grants to s&l governments.¹⁹ By definition, $NT_t^F \equiv \sum_{i=1}^n n_i NT_{i,t}^F$ where n_i is the share of population in state i .

¹⁵Section 3.2 will explain in more details the definition of the variables.

¹⁶The treatment of uncertain returns from capital follows Flavin [31]. Interest income, $rb_{i,t-1}^{hh}$, is treated as known at time $t - 1$.

¹⁷For Europe, the variable $NT_{i,t}^F$ represents unilateral transfers, while s&l governments coincide with national governments.

¹⁸This applies for US states only.

¹⁹In the budget constraint we consider for simplicity that grants to s&l governments are made directly to individuals. However, in the intertemporal households budget constraint the distinction does not make any difference.

The intertemporal budget constraints for households, s&l and federal governments are respectively:²⁰

$$\sum_{j=0}^{\infty} \frac{C_{i,t+j}^{hh}}{R_{t,j}} = \sum_{j=0}^{\infty} \frac{Y_{i,t+j}^L + D_{i,t+j}^h + NFI_{i,t+j} + NT_{i,t+j}^F + NT_{i,t+j}^S}{R_{t,j}} + B_{i,t-1}^{hh}, \quad (3.4)$$

$$\sum_{j=0}^{\infty} \frac{C_{i,t+j}^S}{R_{t,j}} + B_{i,t-1}^S = \sum_{j=0}^{\infty} \frac{NT_{i,t+j}^S + G_{i,t+j}^S - \tau_{i,t+j}^{S,I}}{R_{t,j}}, \quad (3.5)$$

and

$$B_{t-1}^F + \sum_{j=0}^{\infty} \frac{NT_{t+j}^F + G_{t+j}^F - \tau_{t+j}^{F,C} - \tau_{t+j}^{F,I}}{R_{t,j}} = 0, \quad (3.6)$$

where $R_{t,j} \equiv \prod_{s=0}^j (1 + r_{t+s})$. Let us define the variable *relative net transfers* as the difference between the net transfers received by agent i from the federal government and the average per capita federal net transfer in the economy in period t , i.e., $\widetilde{NT}_{i,t} \equiv NT_{i,t}^F - NT_t^F$. Using this definition, and plugging expressions (3.5) and (3.6) into equation (3.4), one obtains:

$$\sum_{j=0}^{\infty} \frac{C_{i,t+j}}{R_{t,j}} = \sum_{j=0}^{\infty} \frac{Y_{i,t+j}^L + D_{i,t+j}^h + NFI_{i,t+j} + \widetilde{NT}_{i,t+j}^F - G_{i,t+j}}{R_{t,j}} + B_{i,t-1}, \quad (3.7)$$

where $G_{i,t} \equiv G_{i,t+j}^S - \tau_{i,t+j}^{S,I} + G_{t+j}^F - \tau_{t+j}^{F,C} - \tau_{t+j}^{F,I}$, $C_{i,t} \equiv C_{i,t+j}^{hh} + C_{i,t+j}^S$, and $B_{i,t-1} \equiv B_{i,t-1}^{hh} - B_{i,t-1}^S - B_{t-1}^F$. Under the assumption that the representative household can borrow and lend at the riskless rate, their intertemporal budget constraint is a function of their "permanent disposable income", that is, the discounted present value of their future resources. The latter can be separated into the sum of the net present value of the future stream of labor and property income, the present value of future government expenditures (net of non-personal taxes), and the present value of relative net transfers. It is important to notice that, for a given fiscal policy of the federal government, the insurance role of the federal system, is measured by the present value of *relative* net transfers.

In the terminology of the literature on present value models (see Campbell and Shiller [18], Campbell [14]), the innovations in the annuities are called the "returns". If such innovations compensate each other, that is, if the "returns"

²⁰These budget constraints hold for each single state of nature.

are inversely correlated, agents are able to enjoy a smooth consumption path. In particular, the federal government provides insurance in state i if it compensates shocks to the EPDV of the endowment with changes of opposite sign in the EPDV of relative net transfers.

Since the statistical properties of the variables the paper is dealing with are best described by a specification in logarithms, rather than in levels, I choose to loglinearize the budget constraint (3.7) using the approximation developed by Campbell and Shiller [19], and adopted in several works, among which Campbell and Mankiw [16], and Campbell [24]. First I will define the variable W_t , the present value of future resources, as follows:

$$W_t \equiv Y_t + \sum_{j=1}^{\infty} \frac{Y_{t+j}}{\prod_{s=1}^j R_{t+s}}, \quad (3.8)$$

where $Y_t \equiv Y_{i,t+j}^L + D_{i,t+j} + \widetilde{NT}_{i,t+j}^F - G_{i,t+j}$. From the definition of W_t it follows that:

$$W_{t+1} = R_{t+1}(W_t - Y_t) \quad (3.9)$$

Expression (3.9) can be loglinearized as in Campbell and Shiller [19] and Campbell and Mankiw [16], around the mean "dividend/price" ratio, that is, the difference between the logarithm of the current "dividend" and the logarithm of wealth:

$$y_{i,t} - w_{i,t} = E_t \sum_{j=1}^{\infty} \rho^j \Delta y_{i,t+j} - E_t \sum_{j=1}^{\infty} \rho^j r_{t+j} + c_1, \quad (3.10)$$

where for any variable X_t , $x_t \equiv \log(X_t)$, $\Delta y_{i,t} \equiv y_{i,t} - y_{i,t-1}$, $\rho \equiv 1 - \exp(y - w)$, $y - w$ is the mean log dividend/price ratio, and c_1 is a constant.²¹ This expression means that the unexpected component of the return from wealth can be expressed as a function of shocks to expected future growth rates in income and shocks to expected future discount rates:

$$w_{i,t} - E_{t-1} w_{i,t} = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \rho^j \Delta y_{i,t+j} - (E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j r_{t+j}. \quad (3.11)$$

²¹The value used for ρ is .936 as in Shiller [45]. The parameter ρ can also be expressed as a function of the average discount rate r and of the average growth rate of income g : $\rho = \exp(g-r)$. For this reason, one may think that ρ should be different for different states and for different types of income. However, following the finance literature (Campbell and Mei [21]), I choose to use the same value of ρ for all linearizations. Values for ρ of .9 and .95 were also tried, without affecting the results.

Expression (3.11) is the starting point for the empirical estimation, as it delivers a way to measure empirically shocks to the net present value of future income. The first step consists in decomposing the returns to wealth into a number of different components: returns to human capital, returns to total capital, returns to wealth after redistribution from the federal government, *etcetera*. The second step consists in estimating the extent to which, say, insurance from the federal government helps smoothing shocks to labor income by looking at the correlation between returns to wealth before and after redistribution from the federal system. The next section describes these steps more in depth.

Before delving into the details of the estimation strategy, it is important to explain why I focus on shocks to wealth by considering the intertemporal maximization problem of the agent. Following Campbell and Mankiw and Campbell it can be shown that, if agents maximize a time-additive power utility function over a single homogeneous good, shocks to consumption growth can be written as a function of shocks to the overall wealth:

$$c_{i,t} - E_{t-1}c_{i,t} = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \rho^j \Delta \tilde{y}_{i,t+j} - \sigma(E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j r_{t+j}, \quad (3.12)$$

where σ is the intertemporal elasticity of substitution.²² Therefore, the extent to which shocks to, say, labor income are smoothed through private capital markets and the federal system has an implication in terms of shocks to the growth rates of consumption.

3.1. The estimation strategy.

The returns to wealth are decomposed into the following components:

Returns to labor income

$$\Delta W_{i,t}^L \equiv (E_t - E_{t-1}) \sum_{j=0}^{\infty} \rho^j \Delta y_{t+j}^L - (E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j r_{t+j}$$

where $y_{i,t}^L = \ln(Y_{i,t}^L)$.

Returns to domestic (human and non-human) wealth

$$\Delta W_{i,t}^D \equiv (E_t - E_{t-1}) \sum_{j=0}^{\infty} \rho^j \Delta y_{t+j}^D - (E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j r_{t+j}$$

²²The variable $\tilde{y}_{i,t}$ differs from the variable $y_{i,t}$, in that $\tilde{Y}_{i,t}$ includes also "net interest", i.e., $\tilde{Y}_{i,t} = Y_{i,t} + r_t B_{t-1}$. See appendix A.3.

where $y_{i,t}^D = \ln(Y_{i,t}^L + D_{i,t}^h)$. $\Delta W_{i,t}^D$ represents shocks to the wealth of agents if they were not trading on financial markets, and if they had no insurance from the federal system.

Returns to wealth including Net Factor Payments.

$$\Delta W_{i,t}^{NFI} \equiv (E_t - E_{t-1}) \sum_{j=0}^{\infty} \rho^j \Delta y_{t+j}^{NFI} - (E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j r_{t+j}$$

where $y_{i,t}^{NFI} = \ln(Y_{i,t}^L + D_{i,t}^h + NFI_{i,t}^h)$. $\Delta W_{i,t}^C$ represents shocks to the wealth of agents after trading on financial markets, and before the insurance from the federal system.

Returns to private capital after redistribution from the federal government

$$\Delta W_{i,t}^{NT} \equiv (E_t - E_{t-1}) \sum_{j=0}^{\infty} \rho^j \Delta y_{t+j}^{NT} - (E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j r_{t+j}$$

where $y_{i,t}^{NT} = \ln(Y_{i,t}^L + D_{i,t}^h + NFI_{i,t}^h + \widetilde{NT}_{i,t})$. $\Delta W_{i,t}^C$ represents shocks to the wealth of agents after trading on financial markets, and before the insurance from the federal system.

Returns to overall wealth

$$\Delta W_{i,t} \equiv (E_t - E_{t-1}) \sum_{j=0}^{\infty} \rho^j \Delta y_{t+j} - (E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j r_{t+j}$$

where $y_{i,t} = \ln(Y_{i,t}^L + D_{i,t}^h + NFI_{i,t}^h + \widetilde{NT}_{i,t} - G_{i,t} r_t + B_{t-1})$. $\Delta W_{i,t}$ represents shocks to overall wealth, including indirect taxes.

With these definitions, the estimation strategy can be described in two steps.

Step 1. The paper constructs the "returns" $\Delta W_{i,t}^L$, $\Delta W_{i,t}^D$, $\Delta W_{i,t}^{NFI}$, $\Delta W_{i,t}^{NT}$, and $\Delta W_{i,t}$ in accordance with their definition. The estimation of the "returns" depends crucially on the assumptions regarding the stochastic process followed by $y_{i,t}^L$, $y_{i,t}^D$, $y_{i,t}^{NFI}$, $y_{i,t}^{NT}$, and $y_{i,t}$. Section 4 will describe the econometric methodology in more details.

Step 2. The paper will first focus on shocks to domestic (human and non human) capital, that is, $\Delta W_{i,t}^D$. Without trade in assets and insurance from the federal system shocks to domestic wealth would translate into shocks to consumption growth rates. However, both private capital markets and the federal system may help to share shocks to the return on domestic wealth. If private capital markets help to share risk, the impact of a diversifiable shock on the returns to wealth *after* trade in assets is *lower* than the impact on the return to wealth *before* trade on private capital markets. Likewise, the insurance role of the federal system can be gauged from the extent to which the impact of a diversifiable shock

on the returns to wealth *after* net transfers from the federal government is *lower* than the impact on the return to wealth *before* such transfers.

Formally, the degree of risk sharing from private capital markets can be gauged from the panel regression:²³

$$\Delta W_{i,t}^{NFI} - \Delta W_{i,t}^D = \lambda_t^{NFI} + \Delta W_{i,t}^D \gamma^{NFI} + \xi_{i,t}^{NFI}, \quad (3.13)$$

where the coefficient γ^{NFI} measures the percentage of shocks to domestic wealth $\Delta W_{i,t}^D$ that are smoothed through private capital markets: if γ^{NFI} is equal to -0.10 , for instance, it means that the impact of a shock on wealth after trade in assets is 10% lower than the impact on the return to wealth after trade on private capital markets. The error $\xi_{i,t}^{NFI}$ represents a source of randomness to the returns from holding "foreign" assets which is uncorrelated with shocks to domestic wealth. The time fixed effect λ_t^{NFI} is meant to capture the component of risk which cannot be diversified away.

Likewise, the extent to which net transfers from the federal government, or "net government spending" ($G_{i,t}$) smooth or magnify shocks to the net present value of labor income by estimating the following regressions:

$$\Delta W_{i,t}^{NT} - \Delta W_{i,t}^{NFI} = \lambda_t^{NT} + \Delta W_{i,t}^D \gamma^{NT} + \xi_{i,t}^{NT}, \quad (3.14)$$

$$\Delta W_{i,t} - \Delta W_{i,t}^{NT} = \lambda_t^W + \Delta W_{i,t}^D \gamma^W + \xi_{i,t}^W. \quad (3.15)$$

There are no explicit financial instrument which allow agents to trade labor income risk. Given this fact, it may be interesting to ask to what extent private capital markets help sharing shocks to labor income. In order to do so, I estimate the following regression:

$$\Delta W_{i,t}^{NFI} - \Delta W_{i,t}^D = \lambda_t^{NFI,L} + \Delta W_{i,t}^L \gamma^{NFI,L} + \xi_{i,t}^{NFI,L}, \quad (3.16)$$

where $\Delta W_{i,t}^L$ represents shocks to the expected present value of future labor income. Similarly, it may be interesting to see whether federal net transfers play a different role in insuring shocks to total wealth and shocks to human capital only. Therefore I will also estimate the following equations:

$$\Delta W_{i,t}^{NT} - \Delta W_{i,t}^{NFI} = \lambda_t^{NT,L} + \Delta W_{i,t}^L \gamma^{NT,L} + \xi_{i,t}^{NT,L}, \quad (3.17)$$

$$\Delta W_{i,t} - \Delta W_{i,t}^{NT} = \lambda_t^{W,L} + \Delta W_{i,t}^L \gamma^{W,L} + \xi_{i,t}^{W,L}. \quad (3.18)$$

²³This part of the estimation strategy follows Asdrubali *et al.*, although the definition of the variables is different except in the i.i.d. case.

3.2. The data.

3.2.1. US States.

The paper uses annual data from 1969 to 1994. This section provides an initial description of the data. In describing the data, I will refer to the variables appearing in the budget constraints (3.1) through (3.3). Further details, including the sources, can be found in Appendix A.1 and A.2.

$Y_{i,t}^L$ =labor income.

$D_{i,t}$ =income from dividends and rent.

$NT_{i,t}^F$ =net transfers to households from the federal government.

$NFI_{i,t}$ =net factor income (capital income only), measured as follows: gross state product-earnings by place of work-estimated depreciation-&l indirect and corporate taxes-federal indirect and corporate taxes.²⁴

$C_{i,t}^{hh}$ =private consumption (retail sales in state i times the ratio of retail sales to total consumption in the US).

$r_t B_{i,t-1}$ =interest income for households in state i .

$C_{i,t}^S$ =consumption by the s&l governments.

$r_t B_{i,t-1}^S$ =interest payment by the s&l governments.

$G_{i,t}^S$ =investment by the s&l governments.

$\tau_{i,t}^{S,I}$ =indirect taxes by s&l governments.

$r_t B_{t-1}^F$ =per capita interest payment by the federal government.

G_t^F =spending by the federal government, other than transfers to households or s&l governments.

$\tau_t^{F,C}$ and $\tau_t^{F,I}$ =corporate and indirect taxes collected by the federal government.

All the variables indexed by i are converted in real terms using the state CPI series (the details on the construction of these series are given in appendix A.1).

²⁴The measure of net factor income used here includes the portion of corporate profits which is not taxed. The definition adopted here differs from the measure of ASY in that it does not include estimated depreciation (10% of gsp) and "adjustment for residence" (net factor income from labor). Appendix A.2 elaborates in more details on the definition of the variables.

The Bureau of Economic Analysis data on labor income are generally considered quite accurate. However, this does not seem to be the case for South and North Dakota, which reportedly experience an increase in nominal income of about 30% in 1973, when inflation was only 6%. Since these numbers do not appear to be reliable, I decided to exclude South and North Dakota from the data set.²⁵ The data for retail sales are obtained from Sales&Marketing Management and include both durable and non durable goods. The retail sales data from the Bureau of the Census distinguish between durables and non durables, but they are available only for 19 states from 1978 onward.²⁶ All the data are in per capita terms, where the population figures are obtained from the Bureau of Economic Analysis.

3.2.2. European Countries.

The countries included in this study are Belgium, Denmark, France, Germany (West), Greece, Ireland, Italy, Portugal, Spain, and the UK. The period is 1967-1990. All the data are from the OECD, "Main Aggregates", 1967-94.

$Y_{i,t}^L$ ="compensation of employees paid by resident producers" (labor income).

$D_{i,t}^h$ =income from dividends and rent, obtained as follows: "operating surplus" + "consumption of fixed capital" (depreciation) - "gross capital formation".²⁷

$NFI_{i,t}$ ="net factor income from the rest of the world".

$NT_{i,t}^F$ ="net current transfers from the rest of the world" (unilateral transfers).

$C_{i,t}^{hh}$ ="private final consumption expenditures" (private consumption).

$C_{i,t}^S$ ="government final consumption expenditures" (government consumption).

$\tau_{i,t}^{S,I}$ ="indirect taxes".

²⁵The results do not change when South and North Dakota are included, since the econometric methodology takes heteroskedasticity into account.

²⁶The data base from Sales&Marketing Management will allow me to purge the purchases of the most important categories of non durables, such as cars and furniture, from the retail sales data. However, the present version of the paper uses total retail sales.

²⁷In Europe small unincorporated enterprises represent an important part of economic activity. From the accounting perspective, they are part of the household sector.

In order to calculate the aggregate figures for Europe, all the data are transformed in 1985 dollars using a real exchange rate e_t constructed as follows: $e_t = \frac{GDP_{1985\$,t}}{GDP_t}$, where $GDP_{1985\$,t}$ is obtained from the Penn World Tables (Mark 5.6) and GDP_t is the nominal GDP from OECD figures. All the data are in per capita terms, where the population data are from the International Financial Statistics (IMF).

4. The econometric methodology.

Since regressions (3.13) through (3.18) all present similar problems from the perspective of estimation, I will focus on equation (3.13), which I reproduce here for convenience:

$$\Delta W_{i,t}^{NFI} - \Delta W_{i,t}^D = \lambda_t^{NFI} + \Delta W_{i,t}^D \gamma^{NFI} + \xi_{i,t}^{NFI}.$$

where shocks to the expected present value of resources $\Delta W_{i,t}^k$ are defined as:

$$\Delta W_{i,t}^k \equiv (E_t - E_{t-1}) \sum_{j=0}^{\infty} \rho^j \Delta y_{i,t+j}^k - (E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j r_{t+j}, \quad k = D, NFI \quad (4.1)$$

and the error terms $\xi_{i,t}$ are assumed to be uncorrelated both across time and across states and to be heteroskedastic: $E[\xi_{i,t}\xi_{j,s}] = 0$ when either $i \neq j$ or $s \neq t$, and $E[\xi_{i,t}^2] = \sigma_i^2$.²⁸

First of all I notice that the second term in (4.1), $(E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j r_{t+j}$, does not affect the estimate of the parameter γ^{NFI} , since it is wiped out by the time fixed effect λ_t^{NFI} . This is true as long as the discount rate r_t is the same across states, which is the assumption made here. As a consequence, I only need to be concerned with measuring the first term in (4.1), namely $(E_t - E_{t-1}) \sum_{j=0}^{\infty} \rho^j \Delta y_{i,t+j}^k$.

The estimates of the terms $\Delta W_{i,t}^D$ and $\Delta W_{i,t}^L$ depend crucially on the assumptions about the stochastic process governing relative income. Since the time series available is too short to obtain any definite answer on the nature of the stochastic process governing income, I will estimate the variables $\Delta W_{i,t}^D$ according to different assumptions on the process followed by relative income. Such assumptions are as follows:

Assumption 1: $\Delta y_{i,t}$ is an i.i.d. random variable.

²⁸The lack of correlation across time is due to the fact that all the variables are innovations.

This is the same assumption used by Fama and Schwert, and ASY. Under this assumption one obtains that:

$$\Delta W_{i,t} = \Delta y_{i,t} \quad (4.2)$$

Assumption 2: $\Delta y_{i,t}$ follow a AR(1) process:

$$\Delta y_{i,t} = \alpha \Delta y_{i,t-1} + \varepsilon_{i,t}, \quad (4.3)$$

Under this assumption, $\Delta W_{i,t}$ can be expressed as follows:

$$\Delta W_{i,t} = (I - \rho\alpha)^{-1} \varepsilon_{i,t}, \quad (4.4)$$

where $e_1 \equiv (1, 0)'$.

Assumption 3: $\Delta y_{i,t}$ follow a VAR(1) process:

$$z_{i,t} = Az_{i,t-1} + \varepsilon_{i,t}, \quad (4.5)$$

where $z_{i,t} \equiv (\Delta y_{i,t}, \Delta y_{us,t})'$. Under this assumption, $\Delta W_{i,t}$ can be expressed as follows:

$$\Delta W_{i,t} = e_1' (I - \rho A)^{-1} \varepsilon_{i,t}, \quad (4.6)$$

where $e_1 \equiv (1, 0)'$.

Assumption 4: relative income follows a trend stationary process:

$$\tilde{y}_{i,t} = \alpha_0 + \alpha_1 t + \eta_{i,t}, \quad (4.7)$$

where

$$\eta_{i,t} = \delta \eta_{i,t-1} + \varepsilon_{i,t},$$

and $\varepsilon_{i,t}$ is an i.i.d. stationary random variable, $\Delta W_{i,t}$ can be rewritten as (see appendix A.4):

$$\Delta W_{i,t} = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \rho^j \Delta y_{us,t+j} + \frac{1 - \rho}{1 - \rho\delta} \varepsilon_{i,t}. \quad (4.8)$$

where the first term in the right hand side of (4.8) does not need to be specified since it is captured by the time fixed effect.

One has also to take into account that estimates of the variables $\Delta W_{i,t}^D$ and $\Delta W_{i,t}^{NFI}$, rather than their actual values, are used in the regression (3.13). In appendix A.5 I show that the panel GLS estimator obtained with the constructed "returns" is consistent.²⁹ The intuition for this results follows closely the result by Pagan [42] on regressions with constructed regressors.

5. The results.

5.1. Risk sharing across US states and across European countries: shocks to domestic wealth.

Without any trade in state contingent assets, and without any redistribution of resources through the federal government, the representative agent in each state (country) would be bound to consume the annuity from domestic wealth, that is, the net present value of their income from labor and from domestic capital. Trading in contingent assets allows agents to smooth shocks to domestic wealth, as part of the dividends from domestic companies accrue to residents in other states, and domestic agents earn income from their investment abroad.

The first column of Table A.1 shows the estimates of the coefficient γ^{NFI} from equation (3.13) for the US, that is, the extent to which net factor payments help smoothing shocks to domestic wealth across US States. The rows of Table A.1 correspond to the different assumptions on the stochastic process for income, as described in section 4.

If income growth rates are i.i.d., they map one to one into shocks to the expected present value, for a given discount rate. For this reason, in spite of the difference in the data,³⁰ the results for the i.i.d. case roughly match the ones found by ASY: private capital markets smooth about 40% of shocks to domestic wealth. Table A.1 show that this result is robust to changes in the assumptions about the underlying stochastic process for income.

The amount of risk sharing through private capital markets across US states is quite large. This result is particularly interesting in comparison with one obtained for Europe, displayed in the first column of Table A.2: the amount of risk

²⁹The asymptotic distribution is not exactly the same as the asymptotic distribution obtained with the actual "returns", because of the estimation error in the VAR (AR) coefficients. The standard errors shown in the tables do not take this fact into account. In the next draft of the paper I intend to deal with this issue.

³⁰See section 3.2.

sharing through private capital markets in Europe is close to zero, regardless of the assumption on the process for income.³¹

As far as the insurance role of the federal system in the US is concerned, in both the i.i.d. and the AR(1) case the results are again very similar to the ones found by ASY: the federal system helps to smooth about 13% of shocks. However, under both the deterministic trend and the VAR(1) assumptions the amount of insurance from the Federal government is noticeably larger, about 20%. In all cases the figure is much smaller than the one found by Sachs and Sala-i-Martin [44], which is 40%.

In Europe the effect of unilateral transfers, which include transfers from the European community, is very small and insignificant.

5.2. Risk sharing across US States and across European countries: shocks to labor income.

There are no financial assets explicitly related to labor income. Given this fact, it is interesting to ask whether US States are sharing shocks to the expected present value of future labor income to the same extent to which they are sharing shocks to overall wealth. The comparison between the first column of Table A.3 and the first column of Table A.1 shows that this is not the case. While 40% of shocks to overall domestic wealth are shared through private capital markets, the corresponding figure for shocks to the present value of labor income is between 4 and 12%, depending on the assumptions about the nature of the stochastic process.³² This result suggests that shocks to labor income are largely uninsured in the US. In Europe, as one might have expected, there is no risk sharing at all across countries through private capital markets: from the second column of Table A.4 one can see in fact that the coefficient for European countries is very small and insignificant in all cases.

³¹Both Sørensen and Yosha [47] and Atkeson and Bayoumi [3] also find that private capital markets do not contribute much to risk sharing across Europe, although their approach, as noted in section 2, differs from the one adopted here. Tests of the null of complete markets usually reject the null hypothesis, with exception of Lewis [37]. However, Lewis' test is *conditional* on the countries having no barriers to capital movements, while most European countries in the period considered by both Lewis and this paper had capital barriers in place.

³²The results are consistent with the ones obtained by the micro literature on risk sharing by households in the US: Cochrane [22] and Mace [38] find that the complete market model is generally rejected by the data. This is not in contrast with what I find here: risk sharing through private capital markets, although important, is far less than perfect.

It is interesting to notice from the second column of Table A.3 that the degree of risk sharing through the federal system is noticeably larger for shocks to labor income than for shocks to overall domestic wealth: the estimated amount of risk sharing through the federal system is close to 20% in most cases, and close to 40% under the assumption of a deterministic time trend. This finding may reflect the fact that transfers from the federal government are more correlated with labor income than with the overall level of wealth. Also, to the extent to which domestic capital is largely owned by residents in other states, as it appears from the results on risk sharing through the federal system, dividends from domestic capital are not part of the tax base of residents in a given state.

The fact that the amount of risk sharing through the federal system is almost 40% under the deterministic trend assumption, which is the assumption adopted by SaSa, reconciles their findings with the findings of ASY. The difference between the two findings can be explained with the different assumption about the stochastic process, and with the fact that while ASY focus on shocks to gross state product, SaSa focus on shocks to income. Given that the tax base of the federal system is given by income rather than by gross state product, and given that the two differ precisely because of risk sharing through private capital markets, focusing on shocks to income may give a more appropriate measure of risk sharing through the federal system.³³ However, the next subsection shows that the assumption of a deterministic trend leads to unrealistic implication in terms of the effects of shocks to consumption.

Table A.5 shows the amount of risk sharing from domestic capital with respect to shock to labor income. In other words, they present the results from the regression:

$$\Delta W_{i,t}^D - \Delta W_{i,t}^L = \lambda_t^{D,L} + \Delta W_{i,t}^L \gamma^{D,L} + \xi_{i,t}^{D,L}. \quad (5.1)$$

Regression (5.1) address the following question: if domestic agents were to hold only domestic capital, which seems to be the case for European countries, how much would they share shocks to labor income? This question is related to the debate on the so called "home bias puzzle", that is, the cross-country evidence that investors tend to hold a large share of domestic securities in their portfolios. If shocks to labor and capital income are both driven by productivity disturbances, as argued in Baxter and Jermann [6] and in Brainard and Tobin [13], "returns" to capital income are not likely to smooth labor income shocks.³⁴ However, if

³³The same result holds when the regressor is shocks to *total* income, rather than shocks to labor income only.

³⁴In order for the coefficients γ^{DL} in equation (5.1) to be negative, it is *not* necessary that

redistributive shocks are also important, as in Bottazzi *et al.* [12], the coefficient $\gamma^{D,L}$ may be negative.

Interestingly, I observe that while for US States the coefficient $\gamma^{D,L}$ is close to zero, for Europe it is very large and negative. One interpretation for this result is that redistributive shocks are a relevant sources of uncertainty in Europe, while this is not the case for the US. Such shocks may arise for instance from unexpected inflation (deflation), which reduces (increases) the real value of labor income when wages are sticky, and raises real profits. In this perspective, the "home bias puzzle" is less surprising, since agents can recover what they lose in terms of real wages by holding shares of domestic firms.³⁵ The results confirm the finding by Bottazzi *et al.* [12].

Tables A.8 shows that in both the US and Europe indirect taxes have an insignificant coefficient in terms of risk sharing with respect to shocks to labor income.

5.3. The final impact of shocks on consumption.

How well does the model predict cross sectional variations in consumption? From equation (3.12), I have that innovations in consumption growth rates are a function of shocks to the overall wealth and of shocks to the expectations about future discount rates:

$$c_{i,t} - E_{t-1}c_{i,t} = \Delta W_{i,t} + (1 - \sigma)(E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j r_{t+j}, \quad (5.2)$$

where I use the definition $\Delta W_{i,t} \equiv (E_t - E_{t-1}) \sum_{j=0}^{\infty} \rho^j \Delta y_{t+j} - (E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j r_{t+j}$ (see section 3.1). Expression (5.2) implies that the coefficients γ^C and $\gamma^{C,L}$ in the

income from capital is *negatively* correlated with labor income. All that is required is that the former is *less* affected by shocks than the latter. In fact, since $d \ln(y(t))$ is approximately $w d \ln(y(t)) + (1 - w) d \ln(\tilde{s}(t))$, we have that

$$Cov(d \ln(y(t)), x(t)) - Cov(d \ln(yl(t)), x(t)) \simeq (1 - w)(Cov(d \ln(\tilde{s}(t)), x(t)) - Cov(d \ln(yl(t)), x(t)))$$

where w is the weight of human capital in the wealth of agents, and $x(t)$ represents either systematic or idiosyncratic shocks. This is the well known result that the beta of the overall portfolio is equal to the value-weighted sum of the betas of each asset. However, if productivity disturbances were the only driving force, income from capital and from labor would be perfectly correlated: the coefficients in equation (5.1) would be zero.

³⁵This explanation holds only if the redistributive effects last for long enough to affect the expected present values of variables.

following regressions should be zero:

$$c_{i,t} - E_{t-1}c_{i,t} - \Delta W_{i,t} = \lambda_t^C + \Delta W_{i,t}^D \gamma^C + \xi_{i,t}^C, \quad (5.3)$$

$$c_{i,t} - E_{t-1}c_{i,t} - \Delta W_{i,t} = \lambda_t^{C,L} + \Delta W_{i,t}^L \gamma^{C,L} + \xi_{i,t}^{C,L}, \quad (5.4)$$

where the second term on the right hand side of equation (5.2) is incorporated by the time fixed effect, since it is the same for all states (countries).³⁶

The first and third columns in Tables A.6 report, respectively, estimates of the coefficients γ^C and $\gamma^{C,L}$ for the US. One notices that all the coefficients, except under the deterministic trend assumption, are negative and significant, and are in the range between -10% and -20% for shocks to domestic wealth, and between -35% and -45% for shocks to the expected present value of future labor income. These coefficients have a number of possible interpretations. A first interpretation is related to migration: since individuals can move across states, consumption does not necessarily move one to one with state-specific shocks to labor income, or to domestic wealth. However, other interpretations are also possible. In the first place, it is unlikely that measured changes in the expected present value of income from capital fully captures changes in the market value of assets. In the second place, the measure of consumption used here includes durable as well as non durable goods.

Under the deterministic trend assumption the coefficients are positive and very large, especially for shocks to labor income. This may suggest that the deterministic trend assumption tends to underestimate the persistence of shocks to relative income.

The second and fourth column in Tables A.6 report, respectively, estimates of the coefficients γ^F and $\gamma^{F,L}$ for the US, from the following equations:

$$c_{i,t} - E_t c_{i,t} - \Delta W_{i,t} = \lambda_t^C + \Delta W_{i,t}^D \gamma^C + \xi_{i,t}^C, \quad (5.5)$$

³⁶In view of the empirical implementation, I will start by assuming that the expected return on wealth is constant and equal to the reciprocal of the intertemporal discount rate β . This assumption implies that:

$$E_{t-1} \Delta c_t = 0$$

which is the logarithmic version of Hall's condition, so that A.10 becomes:

$$\Delta c_t = c_t - E_{t-1} c_t.$$

The results do not change when $c_t - E_{t-1} c_t$ is estimated assuming that consumption growth rates follow an AR(1) process.

$$c_{i,t} - E_t c_{i,t} - \Delta W_{i,t} = \lambda_t^{C,L} + \Delta W_{i,t}^L \gamma^{C,L} + \xi_{i,t}^{C,L}. \quad (5.6)$$

The coefficients γ^F and $\gamma^{F,L}$ represent the final impact of the shocks on consumption.

The first and third columns in Table A.7 report, respectively, estimates of the coefficients γ^C and $\gamma^{C,L}$ for European countries. The results depend very much on the assumptions about the stochastic process. Under the deterministic trend assumption the coefficients are again positive and very large. Under other assumptions the coefficients are not significantly different from zero, or negative. Further work using measures of consumption that take into account the difference between durable and nondurable, and tradable and nontradable goods, is needed to get a definite answer on the extent to which this permanent income model explains cross sectional variations in consumption growth rates.

6. Conclusions and extensions.

The paper finds that the degree of risk sharing through private capital markets among US states is large, about 40% of shocks to domestic wealth, while the degree of risk sharing through private capital markets in Europe is almost zero. The paper also finds that private capital markets in the US play a much smaller role in smoothing shocks to labor income, confirming the fact that the lack of securities explicitly linked to labor income is an important source of market incompleteness.

As far as risk sharing through the federal system is concerned, the paper shows that the differences in the findings of SaSa, namely that the federal system helps smoothing about 40% of shocks, with the findings of other literature (ASY) that the insurance role of the federal system is much more limited, around 10%, are due to the fact that: i) SaSa look at shocks to income while ASY look at shocks to gross state product, ii) SaSa assume a deterministic trend in relative income while ASY look at shocks to gross state product. The paper finds that the assumption of a deterministic trend in relative income delivers implausible predictions in terms of consumption responses to income shocks, and that according to all other assumptions about the stochastic process the amount of risk sharing from the federal system is between 20 and 30%, that is, less than estimated by SaSa. While the results are obtained constructing and using individual state CPI series in order to deflate nominal variables, they do not appear to be driven by changes in relative prices.

In terms of policy implications, the results suggest that the different degree of risk sharing through private financial markets is the most important difference

between the US and Europe, while the role of the federal system of taxes and transfers is perhaps less important than suggested by SaSa. One may think that the integration in financial markets, which is likely to follow the process of monetary unification, may enhance in the long run the degree of risk sharing across European countries. This process may be accelerated by national governments in a number of ways. A possible option consists in privatizing the pension system and allow pension funds to diversify their portfolio internationally. In fact, a likely explanation for the high degree of risk sharing in the US is that US pension funds invest on the national market, rather than on the local one.³⁷ Another option for governments would be to help establish a "macro market" for futures on national incomes, following the idea of Shiller [45]. Fiscal federalism among European countries, on the contrary, appears to be a less attractive option from the perspective of risk sharing than the figures by SaSa had suggested.³⁸

The paper can be extended in several directions. From the perspective of the data, it would be important to purge purchases of durable goods from the measures of consumption, and to use direct measures of changes in the market value of wealth, instead of using changes in the present discounted value of income from wealth as a proxy. For some components of wealth, such as real estate, this should be possible. From the perspective of the model, it would be important to incorporate relevant features of reality, such as migration, liquidity constraints, and the distinction between tradable and non tradable goods into the analysis.³⁹ With regard to the last extension, namely incorporating nontradables in the analysis, lack of availability of data for the US may undermine the feasibility of the project. However for other countries, such as Canada, data on consumption of tradables and non tradables by provinces are available.

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³⁷Coval and Moskowitz [23] show however that *some* bias in favor of local companies may be present even in the US.

³⁸Persson and Tabellini [43] also point out at the moral hazard problems that would arise when risk sharing is achieved by means of a federal system.

³⁹So far the effect of non tradable goods is indirectly taken into account by using individual CPI indices for all states.

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A. Appendix

A.1. The construction of the state CPI indexes.

There are no existing available series for CPI by state. The gross state product data are converted in real terms, since 1978, using a deflator that takes into account the differences in the product mix of the states, but uses national price data (see Friedenber and Beemiller, *The Survey of Current Business*, June 1997).⁴⁰ For this reason, the gross state product deflator may be a misleading measure when used as a consumption deflator. The paper constructs state CPI indexes using the following sources:

i) The ACCRA (American Chamber of Commerce Realtors Association) publishes since 1978⁴¹ a quarterly City Cost of Living Index.⁴² The index contains the cost of living relative to the US average for a number of cities that varies from year to year (from above 200 in 1980, to above 300 in 1995).⁴³ Most recent data are published in the Statistical Abstract.

ii) The Bureau of Labor publishes data on CPI for Urban Consumers (CPI-U) for about 30 metropolitan areas. For most of them the data are available since 1969. The data are published in the Statistical Abstract and in the Economics Indicators Handbook.

iii) The Bureau of Labor developed until 1980 a quarterly "cost of living for intermediate level budget" for about 40 metropolitan areas,⁴⁴ as well as for rural areas in the Northeast, Northcentral, South and West regions. The data are published partly in the Statistical Abstract, and partly on various issues of the Monthly Labor Review. I used the data on "Total. Cost of family consumption", which do not include personal income taxes in the computation of the budget.⁴⁵

iv) The Bureau of Labor publishes since 1978 on the Monthly Labor Review data on CPI by regions (Northeast, Northcentral, South and West) and by urban population (A: areas with population larger than 1250000, B: areas

⁴⁰These are the data used by Hess and Shin [34].

⁴¹Current CPI data include ACCRA data only since 1980.

⁴²I construct annual data, and therefore use only the Fall issue for each year.

⁴³See explanations in their publication.

⁴⁴The data are quarterly. I used the fall data for each year (except 1969, for which I use the spring data).

⁴⁵This is because CPI data, as well as the ACCRA data, do not include such taxes.

with population between 385000 and 1250000, C: areas with population between 75000 and 385000, D: areas with population less than 75000).⁴⁶

v) The Bureau of Economic Analysis site at the University of Virginia has data on population for more than 200 Metropolitan Areas.

vi) The Statistical Abstract publishes data on the percentage of rural population by state.

vii) The Statistical Abstract and The US Atlas contain data on metropolitan areas not included in the BEA data set. When large metropolitan areas extend to more than one state (such as New York), those sources contain data on the portion of population belonging to each state (New York, New Jersey, etc.).

The inflation rate for state i is constructed as follows:

$$\pi_{i,t} = w_i^U \pi_{i,t}^U + (1 - w_i^U) \pi_{i,t}^R$$

where $\pi_{i,t}^R$ (inflation in rural areas of state i) is obtained from source iv)⁴⁷ after 1978 and from source iii) before, and w_i^U (fraction of population living in rural areas of state i) is obtained from source vi). The data on $\pi_{i,t}^U$ are constructed as follows:

$$\pi_{i,t}^U = \sum_{k=1}^K w_i^k \pi_{i,t}^k + (1 - \sum_{k=1}^K w_i^k) \pi_{i,t}^B$$

where $\pi_{i,t}^k$ is the inflation in metropolitan area k ,⁴⁸ obtained from sources i), ii), and iii), w_i^k is the percentage of urban population living in metropolitan area k (obtained from v), vi) and vii)), and $\pi_{i,t}^B$ is the inflation in other urban areas (obtained from iv)).⁴⁹ Finally, I normalize the state CPI data so that in each year their population average coincides with the US CPI.⁵⁰

⁴⁶So I have data on Notheast/A, Northeast/B, etc.

⁴⁷The paper uses D data, when available, C otherwise.

⁴⁸Metropolitan area k is included when we have data on its CPI on year t as well as $t + 1$. When we have cost of living data, which are *relative* to the US cost of living, we obtain the CPI for that metropolitan area by multiplying the US CPI for that year by the corresponding relative cost of living number.

⁴⁹Before 1978, $\pi_i^B(t)$ is the average inflation in all the metropolitan areas available in the same region.

⁵⁰The paper finds that the US inflation obtained by taking the population averages of this data differs from the actual US inflation less than .007% for all years, and most of the time much less.

A.2. The other data for the US.

The source of most of the data is the Bureau of Economic Analysis (BEA), and the related data set Regional Economic Information System (REIS).⁵¹ This is the same source of the data of Sachs and Sala-i-Martin, and Asdrubali et al. The paper uses annual data from 1969 to 1994.

Pre tax and transfers labor income ($yl_{i,t}$): The REIS data set contains data on Personal Income by state, by Region, and for the US as an aggregate. The data on Personal Income published by the BEA is constructed as follows:

$$\begin{aligned} \text{Personal Income} = & \text{Earnings by place of work} \\ & - \text{Personal contributions for Social Insurance} \\ & + \text{Adjustment for residence} \\ & + \text{Dividends, Interest, and Rent} \\ & + \text{Transfer payments} \end{aligned}$$

The definition of state labor income adopted here is the following:

$$\begin{aligned} \text{state labor income} = & \text{Earnings by place of work} \\ & + \text{Adjustment for residence} \\ & + \text{Business transfers} \end{aligned}$$

The definition of state labor income adopted here is therefore equal to the BEA definition of personal income, minus transfers from federal, s&l governments, plus Social Insurance contributions to federal, s&l governments, and minus property income (Dividends, Interest, and Rent).⁵²

Property income(Dividends and Rent)($d_{i,t}$): the BEA makes these data available on request.

Interest income for households($rb_{i,t}$): the BEA makes these data available on request.

Net transfers from federal government ($nt_{i,t}^F$):The REIS data set contains data on personal income tax payments to the federal government, and on other non tax payments. The BEA makes the data on contributions for Social Insurance by State available on request. This is an improvement over the data set constructed by Asdrubali et al., who allocate the total amount of social insurance contributions for the US to states

⁵¹<http://www.lib.virginia.edu/socsi/reis>, and <http://leap.nlu.edu>.

⁵²This definition of state income differs slightly from the one used by Asdrubali et al.??, since they add to it federal non personal taxes, s&l non personal taxes, and interest on state funds, and they subtract s&l contributions for Social Insurance. Data on federal non personal taxes, interest on s&l funds are not available by state: Asdrubali et al. allocate the aggregate figures for the US according to the weights suggested by the Tax Foundation. Figures for s&l non personal taxes are calculated by Asdrubali et al. as the difference between total s&l tax revenues, which are available by fiscal year, and s&l personal taxes, which are available by calendar year.

using weights proportional to the states' income and population. In fact, social security contributions represent a large fraction of transfers from households to the federal government. The REIS data set contains data on personal transfers to individuals by program (i.e. social insurance, medicare, etc.), and it is possible to identify the programs that are financed by the federal government. I follow Sachs and Sala-i-Martin in considering State Unemployment Insurance as a State program. Grants from the federal governments to the states are also included in the definition of this variable. Unfortunately, these data are on a fiscal year basis.⁵³

Net factor income: net factor income is measured as the difference between gross state product and the sum of the following items: earnings by place of residence, estimated depreciation, s&l corporate and indirect taxes, federal corporate and indirect taxes by state. Gross state product is obtained from the BEA. Depreciation is estimated as 10% of gross state product. State and Local corporate and indirect taxes are measured as the difference between total taxes collected by s&l governments and total personal taxes collected by s&l governments, where the first figure is obtained from the "Statistical Abstract of the United States" and from "Governmental Finances", a publication of the US Bureau of the Census, and the second figure is obtained from the BEA. Federal corporate and indirect taxes by state are measured by allocating total federal corporate and indirect taxes (obtained from The Budget of the US Government, Historical Tables) by state according to the weights suggested by the Tax Foundation.⁵⁴ Even assuming that depreciation is estimated correctly, the measure of net factor payments used exceeds the true measure by as it includes the component of profits which is not taxed. The measure used here is different from the measure adopted by ASY, in that their measure includes also depreciation and net factor income from labor ("adjustment for residence").

Private consumption ($c_{i,t}^{hh}$): this is equal to retail sales in state i times the ratio of retail sales to total consumption in the US.⁵⁵ The data on retail sales are proprietary. I am grateful to Sales & Marketing Management for allowing me to use them. In doing this I follow Asdrubali et al.⁵⁶

⁵³The source of these data is the "Statistical Abstract of the United States", and the publications of the US Bureau of the Census, "Federal Expenditures by State for Fiscal Year", and "Governmental Finances".

⁵⁴The weights for corporate taxes are the (half-half) average of the share of the state in US personal income and its share in property income (dividends, interest, and rent). The weights for indirect taxes are the (half-half) average of the share of the state in US personal income.

⁵⁵The data on total private consumption in the US were obtained from DataStream.

⁵⁶Hess and Shin[34] show that the data on retail sales are a very good proxy for total

Government consumption ($c_{i,t}^S$): consumption by the s&l governments, obtained as the difference between total expenditures on the one side, and the sum of transfers to households, capital outlays, and interest payments by s&l governments on the other side. The data on transfers to households are obtained from the REIS. The other data are obtained from the publications of the US Bureau of the Census, "Federal Expenditures by State for Fiscal Year", and "Governmental Finances".

Interest payment by the s&l governments ($rb_{i,t}^S$); capital outlays by s&l governments ($g_{i,t}^S$): these data are obtained from the publications of the US Bureau of the Census, "Federal Expenditures by State for Fiscal Year", and "Governmental Finances".

Spending by the federal government, other than transfers to households or s&l governments (g_t^F), corporate and indirect taxes collected by the federal government ($\tau_t^{F,C}$, $\tau_t^{F,I}$): these data are obtained from The Budget of the US Government, Historical Tables.

All data are in per capita terms. Population data are obtained from the REIS data set.

The data on the instrumental variables besides the rate of inflation, that is, the price of oil, an index of the price of raw materials, a trade-weighted index of the US nominal exchange rate, are obtained from the International Financial Statistics, published by the IMF.

A.3. Loglinearization of the budget constraint.

In this appendix I want to show that the loglinearized solution used in Campbell and Mankiw is also a valid first order approximation of the solution of the model used in this paper. I will first describe briefly the approach by Campbell and Mankiw.

The agent maximizes a time-additive power utility function over a single homogeneous good. More specifically, the maximization problem of the agent can be described as:

$$\max \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \tag{A.1}$$

where C_t represents per capita consumption in period t .

Assuming all wealth is tradable, the dynamic budget constraint of the agent can be written as:

$$W_t = R_t(W_{t-1} - C_{t-1}) \tag{A.2}$$

private consumption, at least for the US as a whole.

The first-order Taylor approximation of the Euler equation is:⁵⁷

$$E_{t-1}\Delta c_t = \sigma(E_{t-1}r_t - \beta), \quad (\text{A.3})$$

where $r_t \equiv \ln R_t$ (as in the paper, I adopt the convention $x_t \equiv \ln X_t$). The intertemporal budget constraint of the agent (A.2) can be loglinearized around the mean log consumption/wealth ratio, assuming the latter is stationary, obtaining:

$$c_t - w_t = \sum_{j=1}^{\infty} \rho^j r_{t+j} - \sum_{j=1}^{\infty} \rho^j \Delta c_{t+j} + c_2, \quad (\text{A.4})$$

where $\rho \equiv 1 - \exp(c - w)$, and $c - w$ is the mean log consumption/wealth ratio, and c_2 is a constant.⁵⁸ Taking expectations of expression (A.4) conditional on the information available at time t , and plugging (A.3) into (A.4) one obtains:

$$c_t - w_t = (1 - \sigma)E_t \sum_{j=1}^{\infty} \rho^j r_{t+j} + c_3. \quad (\text{A.5})$$

If one thinks of the wealth of the agent as consisting of N_t shares, each with price P_t and dividend Y_t , that is, $W_t = N_t(D_t + P_t)$, one can write the returns to wealth R_t as:

$$R_t = \frac{(Y_t + P_t)}{P_{t-1}}. \quad (\text{A.6})$$

From (A.6) and from the definition of wealth one obtains:

$$\frac{W_t}{N_t} = R_t \left(\frac{W_{t-1}}{N_{t-1}} - Y_{t-1} \right), \quad (\text{A.7})$$

which has the same structure as (A.2) and can be loglinearized in a similar fashion around some level of the log-dividend/price ratio $y - w + n$, yielding:

$$y_t - w_t = -n_t + \sum_{j=1}^{\infty} \rho^j r_{t+j} - \sum_{j=1}^{\infty} \rho^j \Delta y_{t+j} + c_3. \quad (\text{A.8})$$

⁵⁷Campbell [24] uses the second-order Taylor approximation of the Euler equation. As this paper is not concerned with pricing consideration, I use only the first-order Taylor approximation.

⁵⁸Like Campbell and Mankiw [16] I am implicitly assuming that the mean value of $c - w$ is the same as the mean value for $y - w$, as I am using the same ρ for equation ?? and for equation 3.10.

Normalizing $N_t = 1$, taking the time t conditional expectation one obtains:

$$y_t - w_t = E_t \sum_{j=1}^{\infty} \rho^j r_{t+j} - E_t \sum_{j=1}^{\infty} \rho^j \Delta y_{t+j} + c_3. \quad (\text{A.9})$$

Subtracting (A.5) from (A.9), and subtracting again the time t conditional expectation of the resulting expression one obtains:

$$c_t - E_{t-1} c_t = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \rho^j \Delta y_{t+j} - \sigma (E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j r_{t+j}, \quad (\text{A.10})$$

which is the same expression obtained in Campbell and Mankiw [16].

The budget constraint of the agent in the model presented in section 3 of this paper is of the kind:

$$B_t = R_t B_{t-1} + Y_t - C_t, \quad (\text{A.11})$$

where for simplicity I am assuming that the agent is earning only one type of income, Y_t . If I define the variable W_t^h as:

$$W_t^h \equiv Y_t + \sum_{j=1}^{\infty} \frac{Y_{t+j}}{\prod_{s=1}^j R_{t+s}}, \quad (\text{A.12})$$

and total wealth as:

$$W_t \equiv R_t B_{t-1} + W_t^h, \quad (\text{A.13})$$

and substitute (A.12) and (A.13) into (A.11), I obtain (A.2), which in turn can be loglinearized to yield (A.4). From the definitions of W_t^h and W_t it also follows that:

$$W_t = R_t (W_{t-1} - \tilde{Y}_{t-1}), \quad (\text{A.14})$$

where $\tilde{Y}_t \equiv Y_t + R_t B_{t-1} - B_t = Y_t + (R_t - 1) B_{t-1} - (B_t - B_{t-1})$. Loglinearization of (A.14) delivers the following expression, similar to (A.9):

$$\tilde{y}_t - w_t = E_t \sum_{j=1}^{\infty} \rho^j r_{t+j} - E_t \sum_{j=1}^{\infty} \rho^j \Delta \tilde{y}_{t+j} + c_3, \quad (\text{A.15})$$

which, combined with (A.5), delivers expression (3.12):

$$c_t - E_{t-1} c_t = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \rho^j \Delta \tilde{y}_{t+j} - \sigma (E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j r_{t+j}.$$

In the empirical implementation I will use as a proxy for \tilde{y}_t the variable $\ln(Y_t + (R_t - 1)B_{t-1})$. This is admissible if:

$$\begin{aligned} (E_t - E_{t-1}) \left(\ln \left(1 - \frac{B_t - B_{t-1}}{Y_t + (R_t - 1)B_{t-1}} \right) - \ln \left(1 - \frac{B_{t-1} - B_{t-2}}{Y_{t-1} + (R_{t-1} - 1)B_{t-2}} \right) \right) &\simeq \\ (E_t - E_{t-1}) \left(\frac{B_t - B_{t-1}}{Y_t + (R_t - 1)B_t} - \frac{B_{t-1} - B_{t-2}}{Y_{t-1} + (R_{t-1} - 1)B_{t-2}} \right) &\simeq 0, \end{aligned}$$

that is, if unexpected changes in bond holdings are small enough as fraction of income.

A.4. Construction of the "returns" in the deterministic trend case.

I want to show that $(E_t - E_{t-1}) \sum_{j=0}^{\infty} \rho^j \Delta \tilde{y}_{i,t+j}$ is equal to $\frac{1-\rho}{1-\rho\delta} \varepsilon_{i,t}$ when the $\tilde{y}_{i,t}$ behaves according to the process:

$$\tilde{y}_{i,t} = \alpha_0 + \alpha_1 t + \eta_{i,t}$$

where

$$\eta_{i,t} = \delta \eta_{i,t-1} + \varepsilon_{i,t}.$$

Since $d\tilde{y}_{i,t+j} = \alpha_1 + \eta_{i,t+j} - \eta_{i,t-1+j}$ I have that for $j = 0$:

$$(E_t - E_{t-1}) \Delta \tilde{y}_{i,t} = \varepsilon_{i,t}$$

while for $j = 1, \dots$:

$$(E_t - E_{t-1}) \Delta \tilde{y}_{i,t} = \frac{\delta - 1}{\delta} \delta^j \varepsilon_{i,t}$$

so that I can write:

$$\begin{aligned} (E_t - E_{t-1}) \sum_{j=0}^{\infty} \rho^j \Delta \tilde{y}_{i,t+j} &= \varepsilon_{i,t} + \frac{\delta - 1}{\delta} \frac{\delta \rho}{1 - \delta \rho} \varepsilon_{i,t} \\ &= \frac{1 - \rho}{1 - \rho \delta} \varepsilon_{i,t} \end{aligned}$$

A.5. Consistency of the estimator.

I assume that the "true model" is:

$$y_{i,t} = \lambda_t + x_{i,t} \gamma + \xi_{i,t}, \quad i = 1..N, \quad t = 1..T \quad (\text{A.16})$$

where $y_{i,t}$ and $x_{i,t}$ are the actual shocks to the present values, or "returns", λ_t is a fixed time effect, and $\xi_{i,t}$ is the error term, uncorrelated with $x_{i,t}$ for all i, t .⁵⁹ Since shocks to the present values are innovations, the error terms are uncorrelated across time and, by assumption, across i : $E[\xi_{i,t}, \xi_{j,s}] = 0$ for all $s \neq t$ and $i \neq j$, and $E[\xi_{i,t}^2] = \sigma_i^2$. Under Assumption 2 (VAR) in section 4. The shocks to the present values $y_{i,t}$ and $x_{i,t}$ are defined according to the following formulas:

$$x_{i,t+1} \equiv e_1'(I - \rho A_i)^{-1} \varepsilon_{i,t+1}, \quad (\text{A.17})$$

$$y_{i,t+1} \equiv e_1'(I - \rho B_i)^{-1} \eta_{i,t+1}, \quad (\text{A.18})$$

where $\varepsilon_{i,t+1}$, $\eta_{i,t+1}$, A_i , and B_i are respectively $k \times 1$ vectors of innovations and a $k \times k$ matrices of coefficients from the following VAR models:

$$z_{i,t+1} = A_i z_{i,t} + \varepsilon_{i,t+1}. \quad (\text{A.19})$$

$$w_{i,t+1} = B_i w_{i,t} + \eta_{i,t+1}. \quad (\text{A.20})$$

However, the shocks to the present values are not observed, except in the i.i.d. case. Rather, they are estimated as follows:

$$\hat{x}_{i,t+1} \equiv e_1'(I - \rho \hat{A}_i)^{-1} \hat{\varepsilon}_{i,t+1}, \quad (\text{A.21})$$

$$\hat{y}_{i,t+1} \equiv e_1'(I - \rho \hat{B}_i)^{-1} \hat{\eta}_{i,t+1}, \quad (\text{A.22})$$

where $\hat{\varepsilon}_{i,t+1}$, $\hat{\eta}_{i,t+1}$, \hat{A}_i , and \hat{B}_i are respectively the (OLS) estimated vectors of innovations and a $k \times k$ matrices of (OLS) estimated coefficients from (A.19) and (A.20).

A little investment in notation is needed. For any scalar $q_{i,t}$ I adopt the following definitions: $q_i \equiv (q_{i,1} \dots q_{i,T})'$ is a $T \times 1$ vector; $q \equiv (q_1' \dots q_N')'$ is a $NT \times 1$ vector; Q is the $T \times N$ matrix whose $(t, i)^{th}$ element is $q_{i,t}$; $\bar{q} \equiv \frac{\sum_i q_i}{N}$ is the $T \times 1$ vector of cross sectional averages; $\tilde{q}_i \equiv q_i - \bar{q}$ is the vector of deviations from the averages. Also, for a matrix Q , M_Q is defined as $M_Q \equiv I - Q(Q'Q)^{-1}Q'$. Using these definitions, I can rewrite (A.16) as:

$$\tilde{y} = \tilde{x}\gamma + \tilde{\xi}, \quad (\text{A.23})$$

⁵⁹In the regressions $y_{i,t}$ is actually the difference between shocks to present values. All the following arguments do not change.

where I recall that $\tilde{y} \equiv (\tilde{y}'_1 \dots \tilde{y}'_N)'$, etc. Expressions (A.17) through (A.22) can also be rewritten in stacked form as follows:

$$\begin{aligned} y_i &\equiv \varepsilon_i f(A_i)', \\ x_i &\equiv \eta_i f(B_i)', \\ \hat{y}_i &\equiv \hat{\varepsilon}_i f(\hat{A}_i)', \\ \hat{x}_i &\equiv \hat{\eta}_i f(\hat{B}_i)', \\ Z &= Z_{-1}A_i + \varepsilon_i, \\ W &= W_{-1}B_i + \eta_i, \end{aligned}$$

where $f(A_i) \equiv e'_1(I - \rho A_i)^{-1}$, $f(B_i) \equiv e'_1(I - \rho B_i)^{-1}$.⁶⁰ Notice that the covariance matrix of the errors $\tilde{\xi}$, defined as $\tilde{\Delta} = \tilde{\Sigma} \otimes I_T$, where $\tilde{\Sigma}$ is given by:

$$\tilde{\Sigma} = (I_N - \frac{1_N 1'_N}{N}) \Sigma (I_N - \frac{1_N 1'_N}{N}), \quad (\text{A.24})$$

and Σ is a diagonal matrix with the elements σ_i^2 , $i = 1..N$, on the diagonal, I_N is the $N \times N$ identity matrix, and 1_N is a $N \times 1$ vector of ones.

The first question I want to address is whether the GLS fixed time effect panel estimator obtained using the constructed "returns" is a consistent estimator of the true γ , for T large and N fixed. In other terms, I want to show that:

Theorem A.1. *The estimator $\hat{\gamma}$, defined as:*

$$\hat{\gamma} \equiv (\tilde{x}' \tilde{\Delta}^{-1} \tilde{x})^{-1} \tilde{x}' \tilde{\Delta}^{-1} \tilde{y},$$

is a consistent estimator of γ for $T \rightarrow \infty$ and N fixed:

$$Plim_{T \rightarrow \infty} \hat{\gamma} = \gamma.$$

under the following assumptions:

- A1) $\frac{Z'_{-1i} \varepsilon_j}{T} \xrightarrow{a.s.} 0$, $\frac{W'_{-1i} \eta_j}{T} \xrightarrow{a.s.} 0$, $\frac{Z'_{-1i} \eta_j}{T} \xrightarrow{a.s.} 0$, $\frac{W'_{-1i} \varepsilon_j}{T} \xrightarrow{a.s.} 0$, all i, j .
A2) $\frac{Z'_{-1i} Z'_{-1j}}{T} \xrightarrow{a.s.} R^1_{ij}$, $\frac{W'_{-1i} W_{-1j}}{T} \xrightarrow{a.s.} R^2_{ij}$, $\frac{Z'_{-1i} W_{-1j}}{T} \xrightarrow{a.s.} R^3_{ij}$, R^1_{ij} , R^2_{ij} , R^3_{ij} positive definite and non singular all i, j .

⁶⁰Notice that by construction $\hat{\varepsilon}_i = M_{Z_{-1}} Z$, $M_{Z_{-1}} = I - Z_{-1}(Z'_{-1} Z_{-1})^{-1} Z'_{-1}$, $\hat{A}_i = (Z'_{-1} Z_{-1})^{-1} Z'_{-1} Z$, $\hat{\eta}_i = M_{W_{-1}} Z$, $M_{W_{-1}} = I - W_{-1}(W'_{-1} W_{-1})^{-1} W'_{-1}$, $\hat{B}_i = (W'_{-1} W_{-1})^{-1} W'_{-1} W$.

- A3) $f(\cdot)$ is continuous at A_i , all i .
A4) For all t, i, j , $E_{t-1}[Z_{i,t-1}\varepsilon_{j,t}] = 0$, $E_{t-1}[W_{i,t-1}\eta_{j,t}] = 0$.
A5) $\frac{\varepsilon_i\varepsilon_j}{T} \xrightarrow{a.s.} S_{ij}^1$, $\frac{\eta_i\eta_j}{T} \xrightarrow{a.s.} S_{ij}^2$, $\frac{\varepsilon_i\eta_j}{T} \xrightarrow{a.s.} S_{ij}^3$, $S_{ij}^1, S_{ij}^2, S_{ij}^3$ positive definite and non singular all i, j .
A6) The elements of $\tilde{\Sigma}$ are known.

Remark 1. Assumptions A1 and A2 guarantee that $\hat{A}_j \xrightarrow{a.s.} A_j$. Since by assumption A3 $f(\cdot)$ is continuous at A_j , I have that:

$$f(\hat{A}_j) \xrightarrow{a.s.} f(A_j).$$

Assumptions A2 and A4 imply that:

$$\frac{Z'_{-1i}\varepsilon_j}{\sqrt{T}} \xrightarrow{D} N(0, R_{ij}^1),$$

$$\frac{W'_{-1i}\eta_j}{\sqrt{T}} \xrightarrow{D} N(0, R_{ij}^2).$$

Before proving theorem A.1, I will prove the following lemma:

Lemma A.2. Given the same set of assumptions as in theorem A.1, and defining

$$g_{i,j} \equiv \tilde{x}'_i\hat{x}_j - x'_i x_j,$$

$$h_{i,j} \equiv \tilde{x}'_i\hat{y}_j - x'_i y_j,$$

the following statements hold true:

$$\frac{g_{i,j}}{T} \xrightarrow{a.s.} 0, \text{ for } T \rightarrow \infty, \text{ all } i, j, \quad (\text{A.25})$$

$$\frac{h_{i,j}}{T} \xrightarrow{a.s.} 0, \text{ for } T \rightarrow \infty, \text{ all } i, j, \quad (\text{A.26})$$

Proof: Statement (A.25) can be proven by showing that:

$$\begin{aligned} & \frac{1}{T}\tilde{x}'_i\hat{x}_j - \frac{1}{T}x'_i x_j \\ &= \frac{1}{T}f(\hat{A}_i)\tilde{\varepsilon}'_i\hat{\varepsilon}_j f(\hat{A}_j)' - \frac{1}{T}x'_i x_j \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{T}f(\widehat{A}_i)Z_i'M_{Z_{-1i}}M_{Z_{-1j}}Z_jf(\widehat{A}_j)' - \frac{1}{T}x_i'x_j \\
&= \frac{1}{T}f(\widehat{A}_i)(Z_{-1i}A_i + \varepsilon_i)'M_{Z_{-1i}}M_{Z_{-1j}}(W_{-1j}B_j + \varepsilon_j)f(\widehat{A}_j)' - \frac{1}{T}x_i'x_j \\
&= (f(\widehat{A}_i) - f(A_i))\frac{\varepsilon_i'\varepsilon_j}{T}f(\widehat{A}_j)' + f(A_i)\frac{\varepsilon_i'\varepsilon_j}{T}(f(\widehat{A}_j) - f(A_j))' \\
&\quad + (f(\widehat{A}_i) - f(A_i))\frac{\varepsilon_i'\varepsilon_j}{T}(f(\widehat{A}_j) - f(A_j))' \\
&\quad - f(\widehat{A}_i)\frac{\varepsilon_i'Z_{-1i}}{T}\left(\frac{Z_{-1i}'Z_{-1i}}{T}\right)^{-1}\frac{Z_{-1i}'\varepsilon_j}{T}f(\widehat{A}_j)' \\
&\quad - f(\widehat{A}_i)\frac{\varepsilon_i'Z_{-1j}}{T}\left(\frac{Z_{-1j}'Z_{-1j}}{T}\right)^{-1}\frac{Z_{-1j}'\varepsilon_j}{T}f(\widehat{A}_j)' \\
&\quad + f(\widehat{A}_i)\frac{\varepsilon_i'Z_{-1i}}{T}\left(\frac{Z_{-1i}'Z_{-1i}}{T}\right)^{-1}\frac{Z_{-1i}'Z_{-1j}}{T}\left(\frac{Z_{-1j}'Z_{-1j}}{T}\right)^{-1}\frac{Z_{-1j}'\varepsilon_j}{T}f(\widehat{A}_j)' \\
&\xrightarrow{a.s.} 0,
\end{aligned}$$

where in the last step the first three terms converge a.s. to zero from Remark 1, while the other terms converge to zero from assumption A1. By similar arguments, statement (A.26) can be proven by showing that:⁶¹

$$\begin{aligned}
&\frac{1}{T}\widehat{x}_i'\widehat{y}_j - \frac{1}{T}x_i'y_j \\
&= (f(\widehat{A}_i) - f(A_i))\frac{\varepsilon_i'\eta_j}{T}f(\widehat{B}_j)' + f(A_i)\frac{\varepsilon_i'\eta_j}{T}(f(\widehat{B}_j) - f(B_j))' \\
&\quad + (f(\widehat{A}_i) - f(A_i))\frac{\varepsilon_i'\eta_j}{T}(f(\widehat{B}_i) - f(B_i))' \\
&\quad - f(\widehat{A}_i)\frac{\varepsilon_i'Z_{-1i}}{T}\left(\frac{Z_{-1i}'Z_{-1i}}{T}\right)^{-1}\frac{Z_{-1i}'\eta_j}{T}f(\widehat{B}_j)' \\
&\quad - f(\widehat{A}_i)\frac{\varepsilon_i'W_{-1j}}{T}\left(\frac{W_{-1j}'W_{-1j}}{T}\right)^{-1}\frac{W_{-1j}'\eta_j}{T}f(\widehat{B}_j)' \\
&\quad + f(\widehat{A}_i)\frac{\varepsilon_i'Z_{-1i}}{T}\left(\frac{Z_{-1i}'Z_{-1i}}{T}\right)^{-1}\frac{Z_{-1i}'W_{-1j}}{T}\left(\frac{W_{-1j}'W_{-1j}}{T}\right)^{-1}\frac{W_{-1j}'\eta_j}{T}f(\widehat{B}_j)' \\
&\xrightarrow{a.s.} 0,
\end{aligned}$$

where in the last step the first three terms converge a.s. to zero from Remark 1, while the other terms converge to zero from assumption A1 and Remark 1.⁶²

⁶¹The intermediate steps are similar to the proof of (A.25).

⁶²Remark 1 guarantees that the terms $\frac{\varepsilon_i'Z_{-1i}}{\sqrt{T}}$ and $\frac{\eta_i'W_{-1i}}{\sqrt{T}}$ converge in distribution to an $O_p(1)$

Proof of theorem A.1: Defining $\bar{\sigma}_{ij}^2$ as the (i, j) element of $\tilde{\Sigma}^{-1}$, Theorem A.1 can be proven as follows:

$$\begin{aligned}
\hat{\gamma} &\equiv (\tilde{x}'\tilde{\Delta}^{-1}\tilde{x})^{-1}\tilde{x}'\tilde{\Delta}^{-1}\tilde{y} \\
&= \left(\sum_i \sum_j \frac{1}{T} \bar{\sigma}_{ij}^2 \tilde{x}'_i \tilde{x}_j\right)^{-1} \left(\sum_i \sum_j \frac{1}{T} \bar{\sigma}_{ij}^2 \tilde{x}'_i \tilde{y}_j\right) \\
&= \left(\sum_i \sum_j \frac{1}{T} \bar{\sigma}_{ij}^2 \tilde{x}'_i \tilde{x}_j\right)^{-1} \left(\sum_i \sum_j \frac{1}{T} \bar{\sigma}_{ij}^2 \tilde{x}'_i \tilde{y}_j\right) + o_p(1) \\
&= (\tilde{x}'\tilde{\Delta}^{-1}\tilde{x})^{-1}\tilde{x}'\tilde{\Delta}^{-1}\tilde{y} + o_p(1) \\
&= \gamma + o_p(1)
\end{aligned}$$

where the last step follows from assumption A1 by standard asymptotic arguments, and the second step follows immediately from statement A.25 in lemma A.2. In fact, for any element $\tilde{x}'_i \tilde{x}_j$ I have that:

$$\frac{\tilde{x}'_i \tilde{x}_j}{T} - \frac{\tilde{x}'_i \tilde{x}_j}{T} \xrightarrow{a.s.} 0,$$

given that:⁶³

$$\begin{aligned}
\frac{\tilde{x}'_i \tilde{x}_j}{T} - \frac{\tilde{x}'_i \tilde{x}_j}{T} &= \frac{\hat{x}'_i \hat{x}_j}{T} - \frac{1}{N} \sum_k \frac{\hat{x}'_i \hat{x}_k}{T} - \frac{1}{N} \sum_h \frac{\hat{x}'_i \hat{x}_k}{T} + \frac{1}{N^2} \sum_k \sum_h \frac{\hat{x}'_h \hat{x}_k}{T} \\
&\quad - \left(\frac{x'_i x_j}{T} - \frac{1}{N} \sum_k \frac{x'_i x_k}{T} - \frac{1}{N} \sum_h \frac{x'_h x_j}{T} + \frac{1}{N^2} \sum_k \sum_h \frac{x'_h x_k}{T} \right) \\
&= \frac{g_{ij}}{T} - \frac{1}{N} \sum_k \frac{g_{ik}}{T} - \frac{1}{N} \sum_h \frac{g_{hj}}{T} + \frac{1}{N^2} \sum_k \sum_h \frac{g_{hk}}{T} \\
&\xrightarrow{a.s.} 0,
\end{aligned}$$

given that a finite sum of elements converging a.s. to zero also converges a.s., and hence in probability, to zero. By similar arguments, from statement ?? in

random variable, and the product of an $O_p(1)$ random variables and of a random variable that converges a.s. to zero also converges to zero.

⁶³Remember that by definition:

$$\tilde{x}_i = \hat{x}_i - \frac{1}{N} \sum_k \hat{x}_k$$

lemma A.2 it follows that:

$$\frac{\tilde{x}'_i \tilde{y}_j}{T} - \frac{\tilde{x}'_i \tilde{y}_j}{T} \xrightarrow{a.s.} 0.$$

In order to have a feasible GLS estimator one needs a consistent estimate of the covariance matrix $\tilde{\Delta}$. This can be obtained as follows:

i) Estimate γ by OLS (i.e., use the estimator $\hat{\gamma}_{OLS} \equiv (\tilde{x}'\tilde{x})^{-1}\tilde{x}'\tilde{y}$, which is also a consistent estimator of γ).⁶⁴

ii) Obtain a consistent estimate the residuals from:

$$\tilde{\xi} = \tilde{y} - \tilde{x}\hat{\gamma}_{OLS}$$

and hence a consistent estimate of the diagonal elements of $\tilde{\Sigma}$.

iii) Obtain a consistent estimate of the diagonal elements of Σ using the relationship:

$$diag(\Sigma) = \left(\left(1 - \frac{2}{N}\right)I_N + \frac{1}{N^2}1_N \otimes 1'_N \right)^{-1} diag(\tilde{\Sigma})$$

iv) Use (A.24) to obtain $\tilde{\Delta}$ from Σ .

⁶⁴The proof is identical to the one of theorem A.1.

A.6. Tables

Table A.1: RISK SHARING ACROSS US STATES: SHOCKS TO DOMESTIC WEALTH

	Capital Markets	Fed. Syst.
I.I.D.	-0.4014 (0.0119)	-0.1355 (0.0062)
AR(1)	-0.4129 (0.0132)	-0.1449 (0.0068)
VAR(1)	-0.4488 (0.0137)	-0.1815 (0.0076)
TREND	-0.4110 (0.0111)	-0.2313 (0.0058)

Table A.2: RISK SHARING ACROSS EUROPEAN COUNTRIES: SHOCKS TO DOMESTIC WEALTH

	Capital Markets	Unilateral Transfers
I.I.D.	0.0029 (0.0134)	-0.0239 (0.0115)
AR(1)	0.0517 (0.0158)	-0.0403 (0.0146)
VAR(1)	0.0456 (0.0173)	-0.0755 (0.0157)
TREND	-0.0340 (0.0151)	-0.0660 (0.0140)

Estimates from the regressions:

$$\Delta W_{i,t}^{NFI} - \Delta W_{i,t}^D = \lambda_t^{NFI} + \Delta W_{i,t}^D \gamma^{NFI} + \xi_{i,t}^{NFI}$$

$$\Delta W_{i,t}^{NT} - \Delta W_{i,t}^{NFI} = \lambda_t^{NT} + \Delta W_{i,t}^D \gamma^{NT} + \xi_{i,t}^{NT}$$

Table A.3: RISK SHARING ACROSS US STATES: SHOCKS TO LABOR INCOME

	Capital Markets	Fed. Syst.
I.I.D.	-0.0796 (0.0190)	-0.1929 (0.0084)
AR(1)	-0.0599 (0.0187)	-0.1736 (0.0081)
VAR(1)	-0.0440 (0.0226)	-0.2576 (0.0095)
TREND	-0.1253 (0.0201)	-0.3672 (0.0067)

Table A.4: RISK SHARING ACROSS EUROPEAN COUNTRIES: SHOCKS TO LABOR INCOME

	Capital Markets	Unilateral Transfers
I.I.D.	-0.0119 (0.0108)	0.0019 (0.0098)
AR(1)	0.0001 (0.0079)	0.0016 (0.0073)
VAR(1)	-0.0016 (0.0095)	-0.0025 (0.0089)
TREND	-0.0029 (0.0093)	-0.0142 (0.0090)

Estimates from the regressions:

$$\Delta W_{i,t}^{NFI} - \Delta W_{i,t}^D = \lambda_t^{NFI,L} + \Delta W_{i,t}^L \gamma^{NFI,L} + \xi_{i,t}^{NFI,L}$$

$$\Delta W_{i,t}^{NT} - \Delta W_{i,t}^{NFI} = \lambda_t^{NT,L} + \Delta W_{i,t}^L \gamma^{NT} + \xi_{i,t}^{NT,L}$$

Table A.5: DOES DOMESTIC CAPITAL HELP TO SMOOTH SHOCKS TO LABOR INCOME?

	US	EUROPE
I.I.D.	0.0057 (0.0192)	-0.7399 (0.0480)
AR(1)	-0.1109 (0.0187)	-0.8881 (0.0309)
VAR(1)	-0.0476 (0.0230)	-0.9095 (0.2047)
TREND	0.0944 (0.0198)	-0.7790 (0.0332)

Estimates from the regressions:

$$\Delta W_{i,t}^D - \Delta W_{i,t}^L = \lambda_t^{D,L} + \Delta W_{i,t}^L \gamma^{D,L} + \xi_{i,t}^{D,L}$$

Table A.6: THE IMPACT OF SHOCKS ON CONSUMPTION: US STATES

	Residual	Final	Residual	Final
I.I.D.	-0.1428 (0.0321)	0.2485 (0.0292)	-0.3792 (0.0434)	0.4016 (0.0407)
AR(1)	-0.2047 (0.0294)	0.1770 (0.0251)	-0.4441 (0.0348)	0.2469 (0.0319)
VAR(1)	-0.1092 (0.0267)	0.1636 (0.0234)	-0.3397 (0.0351)	0.2361 (0.0315)
TREND	0.5265 (0.1034)	0.8439 (0.1052)	0.8508 (0.1542)	1.4943 (0.1549)

Table A.7: THE IMPACT OF SHOCKS ON CONSUMPTION: EUROPEAN COUNTRIES

	Residual	Final	Residual	Final
I.I.D.	-0.4514 (0.0581)	0.2353 (0.0609)	0.3680 (0.0508)	0.5944 (0.0414)
AR(1)	-0.4967 (0.0582)	0.2276 (0.0540)	0.1489 (0.0334)	0.2590 (0.0237)
VAR(1)	-0.4562 (0.0598)	0.2372 (0.0580)	0.0991 (0.0423)	0.2210 (0.0294)
TREND	0.8055 (0.3595)	1.6300 (0.3699)	1.6601 (0.1780)	1.9224 (0.1820)

Estimates from the regressions:

$$c_{i,t} - E_{t-1}c_{i,t} - \Delta W_{i,t} = \lambda_t^C + \Delta W_{i,t}^D \gamma^C + \xi_{i,t}^C,$$

$$c_{i,t} - E_{t-1}c_{i,t} - \Delta W_{i,t} = \lambda_t^{C,L} + \Delta W_{i,t}^L \gamma^{C,L} + \xi_{i,t}^{C,L},$$

Table A.8: INDIRECT TAXES

	US: dom.wealth	US: labor	EUROPE: dom.wealth	EUROPE: labor
I.I.D.	0.0184 (0.0059)	0.0422 (0.0074)	-0.3118 (0.0272)	0.0075 (0.0272)
AR(1)	0.0183 (0.0071)	0.0522 (0.0080)	-0.2790 (0.0334)	0.0111 (0.0189)
VAR(1)	-0.0197 (0.0066)	-0.0074 (0.0086)	-0.2821 (0.0308)	-0.0106 (0.0212)
TREND	-0.0022 (0.0043)	0.0132 (0.0063)	-0.1495 (0.0329)	0.0213 (0.0195)

Estimates from the regressions:

$$\Delta W_{i,t} - \Delta W_{i,t}^{NT} = \lambda_t^W + \Delta W_{i,t}^D \gamma^W + \xi_{i,t}^W$$

$$\Delta W_{i,t} - \Delta W_{i,t}^{NT} = \lambda_t^{W,L} + \Delta W_{i,t}^L \gamma^{W,L} + \xi_{i,t}^{W,L}$$