1

Understanding Why High Income Households Save More Than Low Income Households

Mark Huggett Gustavo Ventura[†‡]

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Abstract

This paper investigates why high income households save on average a higher fraction of income than do low income households in US cross-section data. The three explanations considered are (1) age differences across households, (2) temporary earnings shocks and (3) the structure of social security payments. We use a calibrated life-cycle model to evaluate the quantitative importance of these explanations. We find that age and the structure of social security payments are quantitatively important in replicating the pattern of average savings rates and income found in US cross-section data. Surprisingly, temporary shocks turn out to be of secondary importance.

Key Words: Distribution, Savings, Life Cycle

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[*]Economics Department, University of Illinois, Champaign IL 61820; Centro de Investigacion Economica, ITAM, Mexico DF 10700, Mexico

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[‡]Corresponding Author: Gustavo Ventura, phone: (217) 333- 8153, email: ventura@uiuc.edu, FAX (217) 244- 6678, Address: Department of Economics, University of Illinois, 1206 South Sixth Street, Champaign IL 61820.

1 Introduction

One of the stylized facts of US savings behavior documented in cross-sectional household surveys is that high income households save on average a higher fraction of income than do low income households. The differences in average annual savings rates at different multiples of mean income are striking. Figure 1 shows that households with income levels below one half of mean income in the economy dissave, whereas households with income levels of three or more times mean income save in excess of 20 percent of income.

[Insert Figure 1 Here]

This stylized fact provokes a multitude of questions of which we focus on the following. Are such large differences in average savings rates puzzling relative to standard theory or are they an implication of an empirically specified version of the standard theory? If the latter possibility turns out to be the case, then which features seem to be central in replicating the observations and why is this the case? Answers to these questions would be useful in the development of empirically oriented models of distribution. A first step in the construction of such models is to replicate some of the stylized facts of the distribution of income, wealth and savings.

Our approach to addressing these questions is as follows. First, we adopt the life-cycle / permanent-income theory of household consumption and savings behavior as the "standard theory" to be investigated. Within this framework, we will only allow households to differ at a point in time in their age and in their earnings history. In particular, earnings can differ due to differences at birth and due to random variation in earnings over time. Thus, we abstract from preference heterogeneity, preference shocks and all other possible explanations for saving rate heterogeneity across households. The savings behavior of these households is then examined when some key features of market structure and institutional arrangements are modeled. These key features are the lack of markets for insuring earnings uncertainty and the presence of a social security system with features similar to the current US system.

Within this framework average savings rates at different income multiples can differ for a variety of reasons. First, age could be key as households with high income are more likely to be in the middle of the life cycle and therefore saving at a high rate for retirement. Second, earnings shocks could be important. Temporary earnings shocks will be largely saved if positive and dissaved if negative. Thus, the savings rates of high income households are likely to be higher than low income households in the same age group as the households with high income will have a disproportionate fraction experiencing positive shocks and the opposite pattern for those with low income. Third, social security could be important. One key feature of the US social security system is that benefits are not proportional to social security taxes paid. Thus, households with permanently high earnings levels will save at higher rates before retirement than will households in the same age group with permanently lower earnings levels as benefits are of minor importance to very high earners but a substantial source of retirement income and health benefits to low earners.

The main finding of the paper is that empirically-specified, life-cycle economies with the features described above imply the type of savings behavior that is observed in Figure 1. The key features of the model economies that produce this savings behavior are the demographic structure, the structure of the social security system and the presence of largely permanent differences in earnings across households. We find that neither preference heterogeneity nor a specific pattern of earnings shocks are essential in producing this result. Furthermore, we find that temporary earnings shocks have only a modest contribution to decreasing the savings rate at low income levels and increasing the savings rate at high income levels. We also find that in the absence of a social security system the model economies also imply large differences in average savings rates at different income multiples. The basic reason for this is that without social security the households that are past the retirement age are among the very low income groups. These households are mostly dissaving in the model economies that we examine.

This paper is organized in six sections. Section 2 reviews evidence for the stylized fact discussed in the introduction. Section 3 describes the model economies. Section 4 describes how the parameters of the model economies are selected so that the model economies are realistic descriptions of the US economy along some dimensions. Section 5 answers the question posed in the introduction. Section 6 concludes.

2 Savings Rates and Income in the US

We briefly review some of the evidence documenting the stylized fact that average savings rates increase with the level of income in US cross-section data. This relationship has been documented in numerous studies including Brady and Friedman (1950), Fisher (1952), Kuznets (1953), Friend and Schor (1959), Projector (1968), Avery and Kennickell (1991), Bosworth et al (1991) and Sabelhaus (1993). These studies construct measures of household income and net savings from cross-sectional household surveys.

Consider the results obtained by Kuznets (1953). The data that he considers come from various surveys conducted between 1929 and 1950. For each year, households are separated into different income groups. For a given income group the average savings rate is calculated by dividing total savings in the group by total income in the group. The results of this analysis are presented in Table 1. The averages of these data across the survey years were previously displayed in Figure 1. We observe that the savings rates are typically negative for households with income levels below one-half of mean income. The savings rates increase nearly monotonically as the multiple of mean income increases. For households with income multiples of three or more times mean income the average savings rate exceeds 20 percent. It is interesting to note that these patterns occur in the individual years examined and therefore the pattern in Figure 1 is not solely the result of time averaging the data.

Table 1: Savings Rates at Multiples of Mean Income: US 1929-1950

Income Multiple	1929	1935	1941	1942	1945	1946	1947	1948	1949	1950
.25	-30.4	-32.1	-15.6	-25.1	4.9	-9.3	-14.8	-22.2	-31.1	-15.9
.50	-1.3	-7.4	0.2	-0.1	7.9	1.9	1.4	-1.3	-5.7	-0.8
.75	8.1	-1.5	5.3	8.3	10.7	7.0	4.6	3.2	-0.6	3.9
1.0	11.6	3.5	5.0	10.9	12.9	10.8	7.0	6.4	5.0	7.4
1.5	16.3	9.4	10.7	15.9	15.7	15.9	10.2	10.8	11.2	12.1
2.0	19.5	14.1	13.9	18.2	19.6	19.7	14.0	14.0	15.6	15.4
3.0	23.6	21.9	19.3	22.7	28.6	24.9	21.5	18.5	21.8	20.2
4.0	29.0	27.2	24.8	27.2						
7.0	37.0	37.5								
10.0	38.5	39.8								
25.0	43.1	49.2								

Source: Kuznets (1953, Table 48)

It is interesting to compare the savings rates at different income multiples found by Kuznets to those in more recent data. For example, Projector

(1968, Table 4) presents results from several surveys in the 1960's following the procedure in Kuznets (1953). Projector also finds that savings rates tend to increase with the multiple of mean income. The averages of the different survey years were previously displayed in Figure 1. As Figure 1 shows, the findings are quantitatively similar to those in the Kuznets study. Note that Projector does not have data for income multiples of 4.0 or higher.

More recently, Bosworth et al (1991, Table 5) have analyzed survey data from the 1960's, 1970's and the 1980's. They find that average household savings rates are negative for the lowest income quintile and tend to increase for higher income quintiles in all the survey years they examine. As the income held in the highest income quintile is only about two times average income, their results do not allow a detailed comparison of average saving rates for households at very high income multiples.

We wish to draw attention to one additional feature of the results presented in Bosworth et al (1991). In some surveys the authors calculate savings as the change in wealth across time periods, while in other surveys savings is calculated as income less consumption. They find that both the saving rates of low income quintiles is lower and that the saving rates of high income quintiles is higher when savings is calculated using income less consumption instead of using the change in wealth. Sabelhaus (1993) shows that this same result occurs even when savings are calculated by both methods within the same data set.[1] With a consistent accounting for sources and uses of funds the two definitions must give an identical measure of saving.

3 The Economies Investigated

3.1 The Environment

We consider an overlapping generations economy.[2] Each period a continuum of agents are born. Agents live a maximum of N periods and face a probability s_j of surviving up to age j conditional on surviving up to age j-1. The population grows at a constant rate n. These demographic patterns are

[1] The data set is the 1989 Consumer Expenditure Survey

[2] The modeling framework used here is similar to that used by Imrohoroglu et al (1995) and Huggett (1996). For a discussion of related frameworks see Laitner (1992) and Rios-Rull (1995).

stable so that age j agents make up a fraction μ_j of the population at any point in time.[3] All agents have identical preferences over consumption that are given by the following utility function:

$$E\left[\sum_{j=1}^{N}\beta^j\left(\prod_{i=1}^{j}s_i\right)u\left(c_j\right)\right]$$

The period utility function $u(c)$ is of the constant relative risk aversion class, where σ is the coefficient of relative risk aversion and $(1/\sigma)$ is the intertemporal elasticity of substitution.

$$u(c) = c^{(1-\sigma)}/(1-\sigma)$$

An agent's labor endowment in efficiency units is given by a function $e(z,j)$ that depends on the agent's age j and on an idiosyncratic labor productivity shock z. The shock z lies in a set Z and follows a Markov process. Labor productivity shocks are independent across agents. This implies that there is no uncertainty over the aggregate labor endowment even though there is uncertainty at the individual agent level. The function $e(z,j)$ is described in detail in section 4.

At any time period t there is a constant returns to scale production technology that converts capital K and labor L into output Y. The technology improves over time because of labor augmenting technological change. The technology level A_t grows at a constant rate, $A_{t+1} = (1+g)A_t$. Each period capital depreciates at rate δ.

$$Y_t = F(K_t, L_tA_t) = AK_t^{\alpha}(L_tA_t)^{1-\alpha}$$

3.2 The Arrangement

We consider an arrangement where in each period t an age j agent with idiosyncratic shock z chooses consumption c_t and risk-free asset holdings a_{t+1}. The period budget restriction for such an agent is then

$$c_t + a_{t+1} \leq a_t(1 + r_t(1-\tau)) + (1-\theta-\tau)e(z,j)w_t + T_t + b_t(\bar{e},j)$$

[3]The weights μ_j are normalized to sum to 1, where $\mu_{j+1} = (s_{j+1}/(1+n))\mu_j$.

$$c_t \geq 0,\ a_{t+1} \geq \underline{a}_{t+1} \text{ and } a_{t+1} \geq 0 \text{ if } j = N$$

In the above budget constraint resources are derived from asset holdings a_t, labor endowment $e(z,j)$, a lump-sum transfer T_t and a social security benefit $b_t(\bar{e},j)$. Assets pay a risk-free return r_t and labor receives a real wage w_t. Agents are allowed to borrow up to a credit limit $\underline{a}_{t+1}$ in period t. In addition, if an agent survives up to the terminal period $(j = N)$, then asset holdings must be non-negative.

There are income and social security taxes in the model economies. Capital and labor income are taxed at the income tax rate τ. Labor income is also subject to a social security tax θ. The social security benefit $b_t(\bar{e},j)$ is allowed to depend on the agent's age j as well as on an average of past earnings $\bar{e}$. In section 4 this function is specified so as to capture a number of features of the way that benefits are related to both age and earnings history in the US social security system.

For computational purposes we transform variables so as to remove the effects of growth. These transformations are as follows:

$$\hat{a}_t = a_t/A_t, \hat{c}_t = c_t/A_t, \hat{T}_t = T_t/A_t, \hat{b}_t(\bar{e},j) = b_t(\bar{e},j)/A_t, \underline{\hat{a}_t} = \underline{a_t}/A_t$$

$$\hat{K}_t = K_t/L_tA_t, \hat{L}_t = L_t/L_t, \hat{G}_t = G_t/L_tA_t, \hat{w}_t = w_t/A_t, \hat{r}_t = r_t$$

With these transformations in mind we now describe an agent's decision problem in the language of dynamic programming. At a point in time an agent's state is denoted $x = (\hat{a}, z, \bar{e})$, where $\hat{a}$ is (transformed) asset holdings carried into the period, z is the labor endowment shock and $\bar{e}$ is an average of the agents past labor earnings. Optimal decision rules are functions for consumption $c(x,j)$ and asset holdings $a(x,j)$ that solve the following dynamic programming problem, given that after the terminal period N the value function is set to zero, $V(x, N+1) = 0$.

$$V(x,j) = \max_{(\hat{c},\hat{a}')} u(\hat{c}) + \beta(1+g)^{(1-\sigma)} s_{j+1} E[V(\hat{a}', z', \bar{e}', j+1)|x]$$

subject to

$$(1)\hat{c} + \hat{a}'(1+g) \leq \hat{a}(1 + \hat{r}(1-\tau)) + (1-\tau-\theta)e(z,j)\hat{w} + \hat{T} + \hat{b}_j(\bar{e},j)$$
$$(2)\hat{c} \geq 0, \hat{a}' \geq \underline{\hat{a}} \text{ and } \hat{a}' \geq 0 \text{ if } j = N$$

$(3)\bar{e}' = G(\bar{e}, e(z,j)\hat{w}, j)$

Note that the period budget constraint in the dynamic programming problem is essentially the same as the budget constraint written in terms of untransformed variables. The key differences are that time subscripts are dropped and a term $(1+g)$ is added. Time subscripts are dropped as we focus on steady-state equilibria where transformed factor prices are constant over time. The additional term $(1+g)$ appears in the objective of the dynamic programming problem due to the transformation of variables. The credit limit $\underline{\hat{a}}$ appears without a time subscript. This is because we focus on credit limits that are always proportional to the current wage rate. The third restriction in the dynamic programming problem is the law of motion for the average of past earnings $\bar{e}'$. This law of motion will later be specified to approximate how average indexed earnings are calculated in the US social security system.

3.3 Equilibrium

To state the equilibrium concept, some way of describing heterogeneity in the economy at a point in time is needed. At a point in time agents are heterogeneous in their age j and their individual state x. A probability measure ψ_j defined on subsets of the individual state space will describe the distribution of individual states across age j agents. So let $(X,\ B(X),\ \psi_j)$ be a probability space where $X = [\underline{\hat{a}}, \infty) \times Z \times [0, \infty)$ is the state space and $B(X)$ is the Borel σ-algebra on X. Thus, for each set B in $B(X)$, $\psi_j(B)$ is the fraction of age j agents whose individual states lie in B as a proportion of all age j agents. These agents then make up a fraction $\mu_j\psi_j(B)$ of all agents in the economy, where μ_j is the fraction of age j agents in the economy. The distribution of individual states across age 1 agents is determined by the exogenous initial distribution of labor productivity shocks since all agents start out with no assets. The distribution of individual states across age $j = 2, 3, ..., N$ agents is then given recursively as follows:

$$\psi_j(B) = \int_X P(x, j-1, B)\, d\psi_{j-1}$$

The function $P(x, j, B)$ is a transition function which gives the probability that an age j agent transits to the set B next period, given that the agent's current state is x. The transition function is determined by the optimal decision rule on asset holding and by the exogenous transition probabilities on the labor productivity shock z.[4]

We focus on steady-state equilibria. In a steady state the transformed capital and labor inputs, transfers and government consumption are constant over time. Thus, without the transformation these variables all grow at constant rates. In steady state the distribution of states across agents is stationary or unchanged over time when stated in terms of transformed variables.

Definition : A steady-state equilibrium is $(c(x,j)$, $a(x,j)$, $\hat{w}$, $\hat{r}$, $\hat{K}$, $\hat{L}$, $\hat{G}$, $\hat{T}$, $\hat{b}(\bar{e},j)$, θ, $\tau)$ and distributions $(\psi_1, \psi_2, ..., \psi_N)$ such that

1. $c(x,j)$ and $a(x,j)$ are optimal decision rules.
2. Competitive Input Markets: $\hat{w} = F_2(\hat{K}, \hat{L})$ and $\hat{r} = F_1(\hat{K}, \hat{L}) - \delta$
3. Markets Clear:
   $(i) \sum_j \mu_j \int_X (c(x,j) + a(x,j)(1+g))d\psi_j + \hat{G} = F(\hat{K}, \hat{L}) + (1-\delta)\hat{K}$
   $(ii) \sum_j \mu_j \int_X a(x,j)d\psi_j = (1+n)\hat{K}$
   $(iii) \sum_j \mu_j \int_X e(z,j)d\psi_j = \hat{L} = 1$
4. Distributions are Consistent with Individual Behavior:
   $\psi_{j+1}(B) = \int_X P(x,j,B)d\psi_j$ for $j = 1, ..., N-1$ and for all $B \in B(X)$.
5. Government Budget Constraint: $\hat{G} = \tau(\hat{r}\hat{K} + \hat{w}\hat{L})$
6. Social Security Benefits Equal Taxes: $\theta\hat{w}\hat{L} = \sum_{j=R}^{N} \mu_j \int_X \hat{b}(\bar{e},j)d\psi_j$
7. Transfers Equal Accidental Bequests:
   $\hat{T} = [\sum_j \mu_j(1-s_{j+1}) \int_X a(x,j)(1+\hat{r}(1-\tau))d\psi_j]/(1+n)$

A brief discussion of the equilibrium concept is in order. Equilibrium condition 1 says that agents optimize. Condition 2 says that factor prices equal marginal products. The first market clearing condition is that aggregate consumption, asset holding and government consumption equals the current output plus the capital stock after depreciation. Note that the term $(1+g)$ appears in this expression so that next period asset holdings are corrected for next periods technology level. The other market clearing conditions are that asset holdings are sufficient to keep the capital stock constant after adjusting for population growth and technological change and that the labor

[4] The transition function is $P(x,j,B) = \text{Prob}(\{z' : (a(x,j), z', \bar{e}') \in B\}|z)$, where the relevant probability is the conditional probability that describes the behavior of the Markov process z.

input per capita is normalized to equal 1. Equilibrium conditions 5 and 6 say that income taxes collected are sufficient to pay for government consumption and that social security taxes are sufficient to cover the benefits paid to agents who are past the retirement age. In this formulation social security is funded on a pay-as-you-go basis. The remaining equilibrium condition is that lump-sum transfers equal accidental bequests. This way of treating accidental bequests, while not a realistic feature of US estate taxation policy, serves to highlight the savings variability that is due to the structure of earnings. More realistic models of the passing of accidental bequests would probably increase the dispersion of savings rates across households in a given age cohort, leading high income households to save more than low income households.

4 Calibration

4.1 Parameters of the Model Economies

Table 2: Model Parameters

β	σ	A	α	δ	g	N	R	n	τ	$\underline{\hat{a}}$
1.011	1.12	.89594	.36	.06	.018	81	46	.012	*	0, $-w$

* See discussion below

The preference parameters (β, σ) are set using a model period of one year. The value of the discount factor β is set equal to the estimate in Hurd (1989) which is greater than 1. Some economists may regard a discount factor greater than 1 with suspicion. However, it should be noted that, in contrast to infinitely-lived agent models, the overlapping generations model does not impose any theoretical restriction on the value of the discount factor. Thus, the value of this parameter is an empirical issue. The value of the parameter σ governing intertemporal substitution and risk aversion is the estimate in Hurd (1989). Hurd's estimate reflects the degree of intertemporal substitution found in household level consumption data [5]. We note that the estimate we use here is in the range of previous estimates from the microeconomics literature that are reviewed by Auerbach and Kotlikoff (1987) and Prescott (1986).

[5] See Rios-Rull (1996) for an analysis of the importance of this parameter in producing realistic capital-output ratios in life-cycle models.

The technology parameters (A, α, δ, g) are set as follows. The technology level A is normalized so that the wage equals 1.0 when the capital-output ratio is 3.0 and the labor input is normalized to equal 1.0. Capital's share of output α is the estimate in Prescott (1986). The depreciation rate δ is set equal to the estimate in Stokey and Rebelo (1995). The rate of technological progress g is set to match the US growth rate of output per capita from 1950-92 as reported in the Economic Report of the President (1994).

The demographic parameters (N, R, s_j, n) are set using a model period of one year. Thus, agents are born at a real-life age of 20 (model period 1) and live up to a maximum real-life age of 100 (model period 81). Agents retire at a real-life age of 65 (model period 46). The survival probabilities s_j are the actual survival probabilities for men in 1990 as reported in Social Security Administration (1992). The growth rate of the population n is set to equal the average population growth rate in the US from 1950-92 as reported in the Economic Report of the President (1994, Table B32).

The income tax rate (τ) is set to match the average share of government consumption in output. The measure of government consumption is federal, state and local government consumption as reported in the Economic Report of the President (1994, Table B1). As the average ratio was .195 from 1959-93 the tax rate is set at $\tau = .195/(1 - \delta(K/Y))$. The tax rate is greater than .195 as capital income is taxed only after subtracting depreciation.

The credit limit $\underline{\hat{a}}$ is set at 0 and for comparison purposes at $-w$. A credit limit of 0 means that agents cannot borrow, whereas a credit limit of $-w$ means that agents can borrow up to one years average earnings in the economy.

4.2 The Structure of Earnings

We consider four models of earnings that differ in stochastic structure. In each of these models earnings are the product of a common real wage per efficiency unit of labor and an agent's labor endowment in efficiency units. We denote y_j and $\bar{y}_j$ the log of an age j agent's labor endowment and the mean log labor endowment of all age j agents. In model 1 all age j agents are identical and receive the mean log endowment of age j agents. In model 2 we allow agents to differ in log labor endowment at birth. As an agent ages his/her log labor endowment at birth is increased or decreased by the change in mean log labor endowment of the agents in the same age cohort. In model 3 agents differ in log labor endowment ($y_1 = \nu_1$) at birth. In addition, each agent receives an idiosyncratic shock ϵ_1 to log labor endowment in each subsequent period. Labor endowments exhibit regression to the age-specific mean at rate γ. In model 4 an agent's log labor endowment is given by the process in model 3 plus a purely temporary shock ϵ_2 each period. Each of models 1-4 are described in Table 3 below. Note that all the random variables (y_1, ϵ_1, ϵ_2) are normally distributed.[6]

Table 3: Labor Endowment Process

Model	$\sigma^2_{y_1}$	γ	$\sigma^2_{\epsilon_1}$	$\sigma^2_{\epsilon_2}$
Model 1: $y_j = \bar{y}_j$				
Model 2: $y_j = y_{j-1} + (\bar{y}_j - \bar{y}_{j-1})$	.45			
Model 3: $y_j = \nu_j = \bar{y}_j + \gamma(\nu_{j-1} - \bar{y}_{j-1}) + \epsilon_{1j}$	.24	.985	.02	
Model 4: $y_j = \nu_j + \epsilon_{2j}$	.24	.985	.02	.01

The parameters of the models are set as follows. First, the values of the mean log labor endowment are selected to match the US cross-sectional age-earnings profile. The values of $\bar{y}_j$ across all four earnings specifications are identical up to a proportional shift. The role of the proportional shift is to make aggregate labor endowment the same across models. The US profile is given in Figure 2.[7]

[Insert Figure 2 Here]

Next, we set all variances and the regression to the mean parameter. In model 2 we set $\sigma^2_{y_1}$ so that the model reproduces the earnings Gini coefficient for the working-age population in the US. Henle and Ryscavage (1980) calculate that the earnings Gini for men averaged .42 in the period 1958-77. When we set $\sigma^2_{y_1} = .45$ the model Gini coefficient within each age group is .36 and the overall Gini for the population under age 65 is .42.

In models 3 and 4 we set the parameters according to the following procedure. First, we set $\sigma^2_{\epsilon_1}$ and $\sigma^2_{y_1}$ to match estimates of the variance of persistent shocks to log earnings and to match estimates of the earnings inequality among young people. Second, we set the regression to the mean coefficient

[6] In models 1-3 the labor endowment function is then $e(z, j) = \exp(z + \bar{y}_j)$, where in models 1-3 z is defined as 0, $(y_1 - \bar{y}_1)$ and $(y_j - \bar{y}_j)$ respectively. In model 4, $e(z, j) = \exp(z_1 + z_2 + \bar{y}_j)$ and $z = (z_1, z_2) = ((\nu_j - \bar{y}_j), \epsilon_{2j})$.

[7] We multiply median earnings of men in cross-section data in 1990 (Social Security Bulletin (1995, Table 4.B6) by labor force participation rates in 1990 for each age group. The participation rates are from Fullerton (1992).

γ to match the US earnings Gini. Hubbard et al (1995) estimate that the variance of persistent shocks to log earnings range from .016 for households with a college education to .033 for households with at most a high school education. Our choice of this variance ($\sigma^2_{\epsilon_1} = .02$) implies that a one standard deviation shock increases or decreases earnings by about 15 percent. Our choice of $\sigma^2_{y_1} = .24$ implies that the earnings Gini of the youngest agents in our model economy equals .27. This is in close agreement with the estimates of Lillard (1977) and Shorrocks (1980) who estimate Gini coefficients of .254 and .268 respectively. When we set the regression to the mean parameter $\gamma = .985$ the overall Gini in the model economy equals .42. We note that this value implies slightly less regression towards the mean than the estimates of Hubbard et al (1995) indicate.

The final parameter that we set is the value of the variance of the purely temporary shock to log earnings $\sigma^2_{\epsilon_2}$. We set this variance to .01. This means that a one standard deviation shock increases or decreases earnings by about 10 percent. There are some studies such as that by Carroll (1992) that estimate the magnitude of the stochastic component of log earnings variance that is purely temporary. Carroll's estimate of this variance is .027. As it is difficult to say what part of this figure corresponds to measurement error and what part corresponds to actual temporary variation experienced by households, we arbitrarily consider a substantially smaller variance. In doing so, we follow the practice established by Carroll (1992). Clearly, the results of the importance of temporary shocks would change if this variance were substantially greater than the value we consider.[8]

4.3 Social Security

We set social security benefits and the law of motion of average earnings as follows:

[8]We make a finite approximation to each model. In model 2 the shock z takes on 21 values between $-5\sigma_{y_1}$ and $5\sigma_{y_1}$. In models 3 and 4 the permanent shock takes on 21 values from $-6\sigma_{y_1}$ to $6\sigma_{y_1}$. The temporary shock takes on 3 values between $-2.0\sigma_{\epsilon_2}$ and $2.0\sigma_{\epsilon_2}$. Shocks are evenly spaced over these intervals. Transition probabilities are calculated by integrating the area under the normal distribution, conditional on the value of the state.

$$\hat{b}(\bar{e}, j) = \begin{cases} 0 & \text{for } j < R \\ \\ b + b(\bar{e})/(1+g)^{j-R} & \text{for } j \geq R \end{cases} \tag{1}$$

and

$$\bar{e}' = \begin{cases} (\bar{e}(j-1) + \min\{e(z,j)\hat{w}, e_{\max}\})/j & \text{for } j < R \\ \\ \bar{e} & \text{for } j \geq R \end{cases} \tag{2}$$

In this specification benefits are paid beginning at a 'retirement age' R, which we set to $R = 46$ (a real-life age of 65). At a point in time all agents past the retirement age receive the common benefit b in addition to an earnings-related benefit $b(\bar{e})$. The earnings-related benefit is paid out as a constant real annuity. As we transform variables by dividing by the technology level, the extra term $(1+g)^{j-R}$ appears in the denominator even though this component of benefits is constant in real terms for a given person after retirement.

We calibrate the common benefit b based on the hospital and medical component of social security benefits. These benefits are paid to all qualifying members regardless of earnings history. Over the period 1990-94 the hospital and medical payment per retiree averaged 7.72 and 4.70 percent of US GDP per person over age 20.[9] Thus, we set $b = 0.1242 \times (Y)$, where Y is GDP per capita in the model economy.

We calibrate the earnings-related component of benefits using the rules that determine the old-age component of social security benefits. In the US the monthly old-age benefit payment is determined by applying a progressive benefit formula to a person's average indexed monthly earnings (AIME). The variable AIME is an average of a specified number of the highest indexed earnings years that is converted into a monthly basis.[10] Earnings in a given year are indexed so that mean earnings in all years are equal after indexing. In the calculation of the average only earnings up to some maximum earnings level are used. In the model economies we calculate average indexed earnings

[9]The data for benefit payments comes from the Statistical Supplement of the Social Security Bulletin (1996), Tables 8.A1 and 8.A2, whereas the population and GDP data come from the Economic Report of the President (1996), tables B1 and B30.

[10]The current averaging period is 35 years.

by averaging all years of earnings rather than just the 35 highest years. In the model economies the maximum earnings that goes into the calculation of the average is denoted $e_{\max}$. We set this at 2.47 times average earnings. This number is the maximum creditable earnings in the US social security system 1990-94 as a fraction of average earnings.[11]

In the US, old-age benefits are a progressive function of a person's AIME. In 1994, the old-age benefit equaled 90 percent of the first \$422 of AIME, 32 percent of the next \$2123 of AIME plus 15 percent of AIME over \$2545. The dollar amounts at which the percentages change are called bend points. We set old-age benefits in the model economies by applying the US formula given above to an agent's average indexed earnings $\bar{e}$ at retirement. This requires determining the bend points in the model economy. We set bend points in the model economy equal to the observed bend points in the US economy over the period 1990-94. The two bend points averaged .20 and 1.24 times average earnings.[12] We note that, after amendments to the Social Security legislation in 1977, bendpoints have been automatically increased in proportion to the average wage levels. As we specify how benefits are determined by past earnings, the social security tax rate is endogenously determined within the model to satisfy condition 6 in the definition of equilibrium.

5 Results

This section is organized in three subsections. First, some of the general features of the model economies are documented. Second, the main results of the paper are presented. Third, some additional questions are addressed to provide additional perspective on the model economies as a description of cross-sectional savings behavior. All of the details of how the results reported here are computed are described in the Appendix.

5.1 General Features of the Model Economies

Before discussing the properties of the model economies, we state our measures of wealth and savings. The concept of individual wealth we use in the

[11]Social Security Handbook (1995) and Social Security Bulletin(1993,1994) for average earnings.

[12]Social Security Bulletin (1993, 1994).

model economies is simply net asset holdings, $\hat{a}$. This choice reflects the fact that the concept of wealth typically measured in the US data is one that includes neither social security wealth nor the value of human capital. The notion of individual savings used is then simply the change in net asset holding across a period. Thus, savings for an age j household in state x is $a(x,j)(1+g)-\hat{a}$. This measure of savings is equal at the aggregate level to both economy-wide net savings (S) and private savings. This is due to the fact that public savings are always equal to zero.

Table 4: Descriptive Statistics

	K/Y	S/Y	r	Income Gini	Percentage of Income Received by the Top 1% 5% 10% 20%
Model 1					
$\underline{\hat{a}}=0$	3.00	0.090	6.0	0.23	1.5 7.7 15.3 30.0
$\underline{\hat{a}}=-\hat{w}$	2.93	0.086	6.3	0.23	1.5 7.8 15.3 30.3
Model 2					
$\underline{\hat{a}}=0$	3.05	0.092	5.8	0.42	5.8 19.3 30.8 47.6
$\underline{\hat{a}}=-\hat{w}$	2.96	0.089	6.1	0.43	5.9 19.6 31.3 48.3
Model 3					
$\underline{\hat{a}}=0$	3.11	0.094	5.5	0.44	6.4 20.7 32.5 49.5
$\underline{\hat{a}}=-\hat{w}$	3.03	0.091	5.9	0.45	6.5 21.0 33.0 50.1
Model 4					
$\underline{\hat{a}}=0$	3.12	0.094	5.5	0.44	6.4 20.7 32.6 49.6
$\underline{\hat{a}}=-\hat{w}$	3.04	0.091	5.8	0.45	6.5 21.0 33.1 50.2

Table 4 summarizes a number of the aggregate properties of the model economies. Table 4 shows that all of the model economies produce similar values of the capital-output ratio (K/Y) and the savings-output ratio (S/Y). Thus, the net return to capital (r) does not vary dramatically across the model economies. Table 4 shows that this before-tax rate is about 6% on an annual basis. As the income tax rate in each of the model economies is slightly in excess of 20%, the after-tax return to capital in the model economies is between 4 and 5 percent.

Table 4 also describes the income distribution properties of the model economies. To take the predictions of a model economy seriously for the cross-section savings fact that is the focus of this paper, a model must have agents with income multiples comparable to those in the data. Model 1 can be criticized in this regard, but the criticism does not extend to models 2-4. It would also be desirable for the model economy to have a similar age structure and income distribution to those in the US economy. Models 1-4 all have a realistic age structure. We will now address how the income distribution properties of the model economies compare to those in the US economy.

In the model economies the concept of income used to compute the properties of the income distribution is labor earnings and capital income before tax plus the value of all transfers received in a model period. A number of recent studies have calculated measures of the concentration of income such as the Gini index for the US economy using this definition of income. Using data from the Current Population Survey, Gramlich et al (1993) find that the income Gini was .42 in 1980, .46 in 1985 and .48 in 1990. Using data from the Survey of Consumer Finances, Avery and Kennickell (1993) find that the household income Gini was .46 in 1983 and .47 in 1986. These measures are similar to those produced in model economies 2-4. In addition, models 2-4 are able to approximate the percentage of income received by the top 20% of US households. Avery and Kennickell (1993) report that the top 20% received 50.1 percent of income in 1983 and 51.2 percent of of income in 1986. The model economies do, however, produce lower levels of income concentration in the extreme upper tail (top 1%) than those reported by Avery and Kennickell (1993). Part of the reason for this is undoubtedly the fact that models of this type have difficulty concentrating sufficient wealth in the upper tail of the wealth distribution. For a discussion of the wealth distribution properties of these model economies see Huggett (1996).

5.2 Savings Rates and Income in Model Economies

Table 5 presents the main findings of the paper. The table presents the savings implications of the model economies and for comparison purposes the averages of the savings rates graphed in Figure 1. Savings rates at different income multiples are calculated by taking a 10 percent band around each income multiple and then dividing total savings of agents in the band by total income of agents in the band. Income is defined as earnings after social security taxes plus interest income and transfers.

Table 5: Savings Rates at Multiples of Mean Income

Income Multiple	US*	Model 1	Model 2		Model 3		Model 4	
		$\underline{\hat{a}}$=0	$\underline{\hat{a}}$=0	$\underline{\hat{a}}$=-w	$\underline{\hat{a}}$=0	$\underline{\hat{a}}$=-w	$\underline{\hat{a}}$=0	$\underline{\hat{a}}$=-w
.25	-19.3	-	-3.8	-16.5	-5.7	-19.3	-6.0	-21.1
.50	-1.3	-11.6	-0.7	-1.0	-0.12	-2.0	0.1	-2.1
.75	4.8	-5.2	4.9	6.4	5.6	6.0	5.4	5.8
1.0	7.9	9.8	9.0	8.8	11.5	11.8	10.9	11.3
1.5	13.0	25.8	17.9	17.3	18.4	18.4	18.0	18.2
2.0	16.5	-	20.8	19.8	22.6	22.6	22.5	22.7
3.0	22.4	-	25.7	26.3	23.4	27.2	27.8	27.8
4.0	27.1	-	27.7	26.7	30.7	30.5	31.3	31.3
7.0	37.3	-	31.8	32.8	34.7	35.2	35.6	35.8
10.0	39.2	-	27.4	28.7	31.7	31.2	32.8	32.4

* Averages from Kuznets (1953) and Projector (1968)

The findings in Table 5 are that a variety of earnings structures imply that in cross-section data the average savings rates are negative for low income multiples and increase as the multiple of mean income increases. Furthermore, model economies 2-4 all produce quantitatively similar results that roughly approximate the magnitudes of the average savings rates observed in US data. This is interesting as the earnings structures differ significantly across these model economies. This suggests that features that are common to all three model economies may be key to generating this stylized fact of savings behavior. The presence of largely permanent differences in earnings abilities across agents together with the demographic structure and the structure of social security payments are common to all of these model economies.

In the remainder of this section we attempt to understand the results in Table 5 at a deeper level. We focus the analysis on models 2-4 as only these models produce enough income heterogeneity to match up to the savings data. Nevertheless, we start out by discussing the individual savings behavior in model 1. This is because it helps in understanding the more complex behavior in model 2. Figure 3 graphs the savings rates for an individual agent over time in model 1. The savings patterns are determined by whether after-tax income lies above or below consumption at a point in time. At the beginning of life the borrowing constraint binds and there are no savings.

Afterwards, after-tax income increases faster than does consumption and therefore agents save. Agents save until just before the retirement age. At this time earnings fall quite sharply while consumption continues to grow. Thus, agents dissave a large fraction of income at this time. At age 65 all agents receive retirement benefits. Agents past the age of 65 dissave until asset holdings are run down to zero.

[Insert Figure 3 Here]

The savings patterns at the individual level in model 2 partly reflect the patterns in model 1. However, there are some important differences as agents within a cohort have different levels of earnings. Figure 4 graphs the average savings rates across age groups for agents with different earnings abilities from the time of birth. Thus, the figure shows that agents in the top 10% of the income distribution for their age group save at higher rates before retirement than agents in the middle $(50 - 60\%)$ of the income distribution or in the bottom 10% of the income distribution. Given that all agents have identical and homothetic preferences and that earnings are identical up to a proportional shift, one might have guessed that savings of any agent would have always been proportional to the savings of any other agent in the same age cohort. Thus, savings rates would be identical. This is not true as social security benefits are not proportional to contributions. In addition, both the borrowing constraint and the transfer due to the taxation of accidental bequests act to break the proportionality of savings.[13] Figure 4(a) shows that these features lead to substantially different savings rates for agents in the same age group, even without any differences in preferences.

[Insert Figure 4 Here]

We are now ready to describe how the behavior in model 2 aggregates to generate the results in Table 5. First, the agents at low multiples of mean income are largely the very youngest agents and also the agents just before the retirement age. This fact can be read off of the cross-sectional age-income distribution described in Figure 4(b). These agents have zero or negative savings rates as Figure 4(a) shows. When the credit limit is set to allow borrowing these savings rates are even smaller as the young can dissave. As the multiple of mean income increases the composition of the agents changes. There are more agents above age 25 when savings rates start

[13]The structure of earnings in earnings models 3-4 provide additional reasons for savings rates to differ within age groups.

to increase and there are fewer agents just before the retirement age. Both of these considerations dictate that average savings rates should increase. At higher income multiples the composition changes to include higher fractions of middle-age agents and agents in the upper tail of the income distribution for their age group. Figure 4(a) illustrates that both of these effects lead to very high savings rates.

We now investigate the savings behavior in the models with temporary and permanent shocks. One way to understand why the results for model 4 are so similar to those for model 2 is to produce the analog of Figures 4(a)-(b) for model 4. This is done in Figure 5(a)-(b). A quick look at Figure 5 shows that both the age-savings rate distribution (Figure 5(a)) and the age-income distribution (Figure 5(b)) are quite similar to the distributions in Figure 4. Thus, it is not surprising that the average savings rates at different multiples of mean income are also similar.

[Insert Figure 5 Here]

One surprising feature of Table 5 is that the results of models 3 and 4 are so similar. A natural conjecture is that temporary shocks could be a quantitatively important source of differences in savings rates at different income multiples. The conjecture is based on the assumption that positive temporary shocks will be largely saved, whereas negative temporary shocks will be largely dissaved. It is also based on the assumption that high income groups will have a high fraction of agents receiving positive temporary shocks and low income groups will have a high fraction receiving negative temporary shocks.

It turns out that both of these assumptions are true. It is possible to show theoretically that off corners part of a positive temporary shock is saved and part of a negative temporary shock is dissaved. Furthermore, we have calculated for model 4 some properties of the distribution of marginal savings rates out of temporary earnings. These marginal savings rates are defined as the ratio of additional savings to the change in before-tax income produced by the shock. We find that in model 4 the median marginal savings rate for the working-age population is to save approximately 64 percent of a temporary shock. This occurs at both of the credit limits considered.[14]

[14] Recall that there are three values of the temporary shock. The result occurs both when comparing the savings change in moving from the middle to the high and from the middle to the low shock.

These distributions are quite concentrated around the median except in the economy where the borrowing constraint is set at $\underline{\hat{a}} = 0$ [15], and are in all cases skewed to the left [16]. Furthermore, it is true that positive shocks are concentrated in high income groups and negative shocks are concentrated in low income groups.

There could be a number of reasons as to why temporary shocks don't matter too much in Table 5. We list one of these. Part of the story may be that other features of the model economy are already capable of producing large differences in average saving rates at different income levels. Thus, savings out of temporary shocks will not alter dramatically these already large differences in average saving rates, unless the temporary shocks are substantially larger in magnitude than those considered here.[17]

To close this section we ask the following question. To what degree do the results in Table 5 depend on differences in savings rates within age groups versus simply differences in average savings rates across age groups? To answer this question we set savings rates of all agents within an age group equal to the average for the age group, while holding the age-income distribution unchanged. This is accomplished by altering the decision rule $a(x, j)$ in an obvious way. The results are shown in Table A1 in the Appendix. The findings are that average savings rates now tend to be much higher and even positive at the lowest income multiples and that average savings rates at high income multiples now tend to be much smaller. Thus, it can be concluded that the features of the model economies that lead to differences of savings rates within an age group are quantitatively quite important in producing the results in Table 5.

[15]When the borrowing constraint is set at zero, there are many more agents that are exactly at the corner of their borrowing constraint and thus have zero marginal savings rates out of a temporary shock.

[16]Average marginal savings rates range from 56.5 percent in the case of $\underline{\hat{a}} = 0$ and moving from the medium to the low shock, to 64.2 percent in the case of $\underline{\hat{a}} = -w$ and moving from the high to the medium shock.

[17]We calculated the results for model 4 when the temporary shock is approximated by 5 points rather than the 3 point approximation listed in Table 5. This allows us to consider more extreme values (ranging from $-3\sigma_{\epsilon_2}$ to $3\sigma_{\epsilon_2}$) of the temporary shock which could potentially be important. The results were almost identical to those previously reported in Table 5.

5.3 Additional Questions

The previous section demonstrated that calibrated life-cycle economies which capture only a few ways in which households differ at a point in time are able to approximate the pattern of average savings rates and income observed in US cross-section data. Thus, the results of the previous section provide an answer to the question posed in the beginning of the paper. Below, we pose a series of additional questions. Answers to these questions should shine some light on how seriously the models previously analyzed should be taken as a description of why average savings rates increase with income in US cross-section data.

Questions:

1. Would the results still hold in the absence of social security?
2. Would the results still hold with other ways of distributing accidental bequests?
3. Do the model economies match other facts of the distribution of savings such as the distribution of savings rates within age groups?

5.3.1 Sensitivity to Social Security

In this section we address question 1 stated above. The motivation comes from three main sources. First, there have been large changes over time in US social security payments. In particular, before 1935 the US social security system did not exist. Since then the magnitude of government supported transfers has increased substantially. Second, the results in Table 5 could be quite sensitive to changes in the importance of social security payments. This is because savings rate differences within age groups are quantitatively important in producing these results (recall Table A1 in the Appendix) and because social security is one of the key features causing savings rate differences within age groups. Third, the data from Kuznets (1953, Table 48) for 1929 show that high income households saved on average a substantially higher fraction of income than low income households even before the US social security system was established.[18] Thus, if the results in Table 5 were particularly sensitive to changes in social security payments, then the very parsimonious explanation of the US cross-section savings fact offered here would be much less convincing.

In this section we eliminate all social security payments from the model economies. Thus, the social security tax is set at zero. This is an extreme way of examining model sensitivity. A social security system that is intermediate between no social security system and the system analyzed in Table 5 is likely to produce properties between these extremes.

Table 6
Savings Rates at Multiples of Mean Income: No Social Security

Income Multiple	US*	Model 2		Model 3		Model 4	
		$\underline{\hat{a}}$=0	$\underline{\hat{a}}$=-w	$\underline{\hat{a}}$=0	$\underline{\hat{a}}$=-w	$\underline{\hat{a}}$=0	$\underline{\hat{a}}$=-w
.25	-19.3	-62.6	-68.0	-57.9	-66.4	-58.0	-65.0
.50	-1.3	-2.8	-5.1	-7.0	-6.5	-7.1	-7.5
.75	4.8	10.7	9.8	10.2	10.0	10.0	10.0
1.0	7.9	20.4	19.3	18.5	18.4	18.5	18.6
1.5	13.0	22.0	22.8	27.2	27.3	26.9	27.3
2.0	16.5	26.1	27.7	30.8	31.1	30.6	31.1
3.0	22.4	29.5	30.4	33.3	34.0	33.5	34.3
4.0	27.1	29.1	30.0	35.2	35.9	35.5	36.2
7.0	37.3	33.5	34.0	37.7	38.0	37.9	38.3
10.0	39.2	26.2	27.5	30.7	30.9	33.2	33.1

* Averages from Kuznets (1953) and Projector (1968)

The main findings of the sensitivity analysis are presented in Table 6. Table 6 shows that, without social security, savings rates still increase with the multiple of mean income. In fact, the differences in savings rates at different income multiples are even greater than the results in Table 5. Why is this so? To answer this question consider Figure 6. Figure 6(b) shows that the agents at the lowest income multiples are largely well into retirement. It is clear from Figure 6(a) that these agents are dissaving. Thus, it is clear why these models produce lower savings rates at low income multiples than in

[18] Kuznets provides two separate estimates in 1929. We list in Table 1 Kuznets' preferred estimate. Both estimates reveal large differences in average saving rates at different income multiples.

the same models with social security. It is simply that with social security the lowest income households are mainly the youngest agents, whereas in the absence of social security these agents are mainly the oldest agents. It is interesting to note that Boskin, Kotlikoff and Knetter (1985) have documented that exactly this type of change in the age-income distribution occurred in the US economy in the period 1968-84. The explanation that they give is that the growing importance of social security was responsible for the shift in the age-income distribution.

[Insert Figure 6 Here]

To what degree do the results in Table 6 depend on differences in savings rates within age groups versus differences in average savings rates across age groups? The answer to this question is given in Table A2 in the appendix. In this table all savings rates within an age group are set equal to the average of the group while holding the age-income distribution constant. The findings are that savings rate differences across age groups are the dominant cause of why average savings rates increase with income in cross section in the absence of social security.[19]

5.3.2 Sensitivity to the Distribution of Accidental Bequests

In this section we address the question whether the main results of the paper would hold under alternative ways of distributing accidental bequests. The particular alternative that we focus on is specifically designed so as not to provide additional reasons for high income households to save more than low income households within an age group. Thus, we allow the government to tax all accidental bequests and use the proceeds to finance government consumption. The answer to this question is displayed in Table 7 below.

Table 7 shows that the results are quite similar to those previously reported in Table 5. Thus, the previous results hold even under unrealistic ways of treating accidental bequests that are designed to make accidental bequests largely irrelevant for savings purposes. We conjecture that more realistic ways of handling accidental bequests than have been considered in this paper could very well be capable of further increasing the savings rates of

[19] Computations of the importance of savings rate differences within age groups versus across age groups could also be done with US data. It would be interesting to see if the within-age-group effect for the cross-section facts has become more important as social security payments have become more important.

high income households within a cohort. This could occur when the receipt of an accidental bequest is modeled as a largely temporary event.

Table 7
Savings Rates at Multiples of Mean Income:
Government Taxation of Accidental Bequests

Income Multiple	US*	Model 2		Model 3		Model 4	
		$\underline{\hat{a}}$=0	$\underline{\hat{a}}$=-w	$\underline{\hat{a}}$=0	$\underline{\hat{a}}$=-w	$\underline{\hat{a}}$=0	$\underline{\hat{a}}$=-w
.25	-19.3	-0.2	-11.2	-4.6	-17.6	-5.2	-19.0
.50	-1.3	-1.0	-0.8	-0.6	0.0	-0.4	-0.8
.75	4.8	6.9	8.6	6.7	7.2	6.3	6.7
1.0	7.9	8.9	11.3	12.3	13.1	11.1	12.5
1.5	13.0	18.9	19.4	19.8	20.0	19.3	19.6
2.0	16.5	19.5	20.2	24.2	24.3	24.0	24.2
3.0	22.4	26.6	25.6	28.5	28.5	29.3	29.3
4.0	27.1	27.7	28.8	32.2	32.3	33.0	32.9
7.0	37.3	34.2	35.0	37.3	37.5	37.7	38.1
10.0	39.2	29.2	28.1	32.7	32.7	33.2	33.0

* Averages from Kuznets (1953) and Projector (1968)

5.3.3 Other Savings Facts

In this section we address whether the model economies considered in this paper are able to match features of the cross-section distribution of savings rates within and across age groups. The first issue to discuss is what the existing evidence indicates about the distribution of savings rates in cross section. Attanasio (1994) and Dynan et al (1996) both provide evidence of the distribution of savings rates. Attanasio examines the 1989 Consumer Expenditure Survey (CES) and finds that within age groups low income households always dissave and high income households save at high rates. Dynan et al find the same pattern of saving rates when the CES from 1982-89 is pooled. They also present results when savings is calculated as the change in wealth using data from the Survey of Consumer Finances (SCF) 1983-89 and from the Panel Study of Income Dynamics (PSID) 1984-89. Their results are listed in Table 8.

Table 8
Median Saving Rates in Quintiles of the Age-Income Distribution *

Quintile	Data Set	30-39 years	40-49 years	50-59 years	70-79 years
1	CES	-20	-24	-38	-59
	PSID	0	0	0.7	3
	SCF	3	-1	-5	1
2	CES	18	15	9	-35
	PSID	3.3	2.5	0.7	0
	SCF	6	9	5	-3
3	CES	27	27	27	-17
	PSID	5.4	3.8	5.6	-3
	SCF	4	12	2	-15
4	CES	34	35	37	4
	PSID	3.4	5.5	8.3	5
	SCF	11	18	7	1
5	CES	45	46	49	32
	PSID	10.8	11.4	10.9	6
	SCF	7	23	20	19

(*) Dynan et al (1996, Tables 2-4 and 12)

In Table 8 we see that households in the lowest income quintile (quintile 1) have a lower median savings rate than do households in the highest income quintile (quintile 5). This holds within all age groups for each of the three surveys considered in Table 8. This cross-section savings pattern is qualitatively consistent with the pattern from the model economies previously presented in Figure 5 at least up until the retirement age. At a quantitative level the savings rates of low income households in the CES that are short of the retirement age are considerably lower than those in Figure 5. In this respect the CES results also differ from those in the SCF and the PSID. There could be a number of reasons for this discrepancy. One of these is that savings are measured in the CES as income less consumption rather than as the change in wealth. Thus, measurement error in income would act to overstate the savings rates of high income and to understate the savings rates of low income households. In fact, Sabelhaus (1993) has shown that when savings are measured as the change in wealth in the 1989 CES the savings rates of low income households increase significantly and the savings rates of high income households decrease relative to when savings are measured as income less consumption.[20]

The key difference between Figure 5 and the savings patterns in Table 8 is the behavior of households aged 70-79. In Table 8 the highest income households (quintile 5) in this age group save. This is true of all three surveys. Figure 5 shows that in the model economies the high income households in this age group are dissaving rather than saving. If this proves to be a robust empirical finding, then it seems clear to us that the model economies considered here must be missing some key feature of reality. Thus, it is quite important to investigate this finding in more detail.

There are several issues to consider of which we will briefly mention two. First, the sample sizes of households in the fifth quintile in this age group are relatively small for both the PSID (78 households) and the SCF (154 households). These are the data sets for which measurement error in measuring income does not act to overstate savings of high income groups. Given the small sample sizes the natural question is what inferences can be made from the data. Currently, the answer to this question is not known as the authors have yet to report information that would allow inferences to be made for larger populations of households as to, for example, whether high income households past the retirement age save at higher rates than do low income households in this age group. At this moment, one conclusion that could be made is that these results provide no evidence that high income households that are past the retirement age save at lower rates than do low income households in the same age group as Figure 5 predicts.

The second point we wish to mention is the treatment of pensions. The

[20]Dynan et al (1996) discuss other reasons for differences in results across the three surveys.

data in Table 8 do not account for the saving and dissaving in private pensions. Previous studies have found that attempting to incorporate private pensions into the calculation of household savings rates tends to increase savings rates before the retirement age and to decrease savings rates after retirement (see Ando and Kennickell (1986) and Bosworth et al (1991)). Therefore, an important issue is whether dissaving in pensions is sufficiently large and sufficiently concentrated in high income households so as to imply that high income households aged 70-79 dissave.

6 Conclusion

The paper asks whether the large differences in average savings rates across different income groups observed in US cross-section data are puzzling relative to standard theory. The main finding of this paper is that the calibrated model economies that we consider imply the type of behavior that is observed in Figure 1. The key features of the model economies that produce this savings behavior are the demographic structure, the structure of social security transfers and the presence of largely permanent differences in earnings across households. We also find that neither preference heterogeneity nor a specific pattern of earnings shocks are essential to produce this result. In fact, we find that temporary earnings shocks have only a very modest contribution to decreasing the savings rate at low income levels and increasing the savings rate at high income levels. Clearly, these findings do not imply that features that we have abstracted from, such as preference heterogeneity, differences in household composition, differences in earnings profiles and so forth, are not important features of actual economies. The findings only imply that such heterogeneity is not essential to produce the large differences in average savings rates that are observed.

One question to ask is whether the findings described above were due mainly to features of the model economies that lead to differences in average savings rates across age groups or to differences in savings rates within age groups. We find that the features of the model economies that lead to savings rate heterogeneity within an age group are quite important. One of these features is a social security system in which households with low earnings realizations receive relatively generous benefits as compared to households with high earnings realizations over a lifetime. In light of this result, it is interesting to recall that even without any social security system the model economies also predict that high income households save at substantially higher rates than low income households. In this case, the role played by the fact that average savings rates differ across age groups is very important.

This paper has focused very narrowly on a single observation. In the future it would be interesting to investigate in more detail a number of other aspects of savings behavior. In particular, the saving behavior of the aged discussed in section 5.3.3 is a key issue. We believe that it would be valuable to investigate the savings behavior of households with pensions both theoretically and especially empirically. A careful treatment of pensions would allow researchers to determine with more precision whether life-cycle models (absent motivations for intentional bequests) are at odds with the savings behavior of older households.

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7 Appendix 1

Equilibria are computed using the following algorithm:

1. Guess the value for capital $\hat{K}$, transfers of accidental bequests $\hat{T}$ and the social security tax θ.

2. Compute factor prices: $\hat{w} = F_2(\hat{K}, 1)$ and $\hat{r} = F_1(\hat{K}, 1) - \delta$. Obtain the income tax rate from conditions 5 in the definition of equilibrium. Obtain social security tax collections implied by the guess of $\hat{K}$ and θ. Parameterize the social security scheme by calculating the first component of the social security payments and the bend points for the history dependent component.

3. Calculate optimal decision rules: $a(x, j)$ and $c(x, j)$.

4. Calculate values of $\hat{K}$ and $\hat{T}$ that are implied by $a(x, j)$. Calculate aggregate social security payments.

5. If the values guessed for $\hat{K}$ and $\hat{T}$ in step 1 and the value in step 2 for social security tax collections equal the implied values in step 4 , then this is a steady-state equilibrium. Otherwise, try new values and repeat these steps.

Steps 3 and 4 above need to be explained. To carry out step 3, we work on the first order conditions of the household's decision problem:

$$U'(\hat{c}_j)(1+g) \geq \beta s_{j+1}(1+g)^{(1-\sigma)} E\left[V_1(\hat{a}', z', \bar{e}', j+1)|x\right]$$

$$\hat{c}_j = \hat{a}(1+\hat{r}(1-\tau)) + \hat{w}e(z,j)(1-\theta-\tau) + \hat{b}(\bar{e}, j) + \hat{T} - (1+g)\hat{a}'$$

$$\begin{aligned} V_1(\hat{a}', z', \bar{e}', j+1) \;&=\; U'[\hat{a}'(1+\hat{r}(1-\tau)) + \hat{w}e(z', j+1)(1-\theta-\tau) \\ &\quad + \hat{b}(\bar{e}', j+1) + \hat{T} - (1+g)a(\hat{a}', z', \bar{e}', j+1)](1+r(1-\tau)) \end{aligned}$$

The above conditions, together with the requirement that $a(x, N) = 0$ define a recursive algorithm for the computation of optimal decision rules for asset holdings at every age and state $x \in X$. With $a(x, j)$ at hand, the decision rule $c(x, j)$ is determined from the budget constraint.

To implement this algorithm on a computer, we require the first order condition to hold exactly on gridpoints defined over the space of asset holdings.

The solution to the first order condition at a particular grid point x is our approximation of $a(x,j)$. The approximation is obtained using a simple bisection procedure to solve the Euler equation.[21] As in Huggett (1996), we choose a uniform grid of 301 points. The distance between gridpoints is 0.25 units of output. Between gridpoints the decision rules are given by a linear interpolation. Thus, decision rules are piecewise-linear functions.

Step 4 requires for aggregation purposes the computation of the probability measures ψ_1, ψ_2,..,ψ_N. Instead, we perform the equivalent aggregation procedure through simulation. We simulate decision rules and law of motions for the social security scheme for a large number of agents agents per cohort in stationary equilibrium [22]. Since decision rules for assets and values for average past earnings are not restricted to fall on gridpoints, simulated decision rules and values of $\bar{e}$ off-gridpoints are obtained by linear interpolation of values at gridpoints. Once the corresponding equilibrium is computed, we use the simulated decision rules to construct a sample of savings and income in order to compute the savings facts and distributional properties of the model economies reported in the paper.

[21]See Huggett (1993) for a more detailed exposition on this algorithm applied to economies with infinitely-lived agents. Coleman (1990) describes a similar version of this algorithm.

[22]We simulate 20,000 agents in the cases of Models 2 and 3, and 40,000 agents in the case of model 4. Higher number of agents turned out to be irrelevant in the sense that they affect neither the aggregate statistics of the model economies nor the implied saving facts.

8 Appendix 2

Table A1
Savings Rates at Multiples of Mean Income
[Equal Savings Rates within Age Groups]

Income Multiple	US*	Model 2		Model 3		Model 4	
		$\underline{\hat{a}}$=0	$\underline{\hat{a}}$=-w	$\underline{\hat{a}}$=0	$\underline{\hat{a}}$=-w	$\underline{\hat{a}}$=0	$\underline{\hat{a}}$=-w
.25	-19.3	5.2	-7.8	10.0	-2.6	10.7	-2.5
.50	-1.3	0.5	1.9	4.7	4.6	5.3	5.1
.75	4.8	8.8	9.7	11.0	11.4	11.3	11.3
1.0	7.9	11.6	10.8	15.3	15.3	15.0	14.8
1.5	13.0	18.3	17.2	19.0	18.8	18.8	18.5
2.0	16.5	19.0	17.5	20.6	20.3	20.4	20.3
3.0	22.4	21.4	21.3	21.4	21.3	21.6	21.6
4.0	27.1	21.2	20.2	21.8	22.0	22.4	22.4
7.0	37.3	23.9	24.8	22.8	23.1	23.2	23.6
10.0	39.2	25.9	27.1	24.5	23.8	24.7	25.0

* Averages from Kuznets (1953) and Projector (1968)

Table A2
Savings Rates at Multiples of Mean Income: No Social Security
[Equal Savings Rates within Age Groups]

Income Multiple	US*	Model 2		Model 3		Model 4	
		$\underline{\hat{a}}$=0	$\underline{\hat{a}}$=-w	$\underline{\hat{a}}$=0	$\underline{\hat{a}}$=-w	$\underline{\hat{a}}$=0	$\underline{\hat{a}}$=-w
.25	-19.3	-59.2	-61.7	-50.5	-55.0	-50.2	-53.2
.50	-1.3	0.3	-2.1	1.0	0.2	1.3	-0.8
.75	4.8	12.4	10.9	14.8	13.8	14.8	13.9
1.0	7.9	21.4	20.0	21.0	20.3	21.0	20.5
1.5	13.0	21.5	21.8	26.5	26.5	26.3	26.4
2.0	16.5	25.2	26.4	28.5	28.7	28.3	28.7
3.0	22.4	28.0	28.4	30.0	30.3	29.7	30.4
4.0	27.1	27.3	27.8	30.6	31.1	30.4	31.2
7.0	37.3	32.5	33.0	32.0	32.5	31.7	32.5
10.0	39.2	32.5	33.7	31.4	32.4	31.7	32.7

* Averages from Kuznets (1953) and Projector (1968)