Monetary Exchange as a Social Convention: Emergence and Stability of an Intermediating Commodity

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Abstract

This paper investigates the stability of monetary exchange equilibria and the conditions necessary for an equilibrium of this type to emerge endogenously. Previous research on money as a medium of exchange has depended on either initial conditions or steady state conditions to pinpoint equilibria. These approaches are not satisfactory for addressing the issue of emergence, either because the determining element is exogenous, or because there are multiple equilibria without criteria for selecting between them. We construct an N-person non-cooperative anonymous game from an economy with many commodities and bilateral exchange. The environment is restricted so that in addition to direct barter opportunities there is a commodity – a token commodity with no value in consumption – which can be used for intermediate exchanges. We make use of recent advances in evolutionary learning in games pioneered in Young (1993) and Kandori, Mailath, and Rob (1993) to characterize conditions under which monetary trade is typically observed in the long run regardless of initial conditions and discuss how informational assumptions affect transition time.

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1 Introduction

What is required for an economy to switch from barter to monetary exchange? How stable is the institution of monetary exchange? Since early expositions on the difficulty of finding the double coincidence of wants necessary for direct barter, it has been clear that a monetary system of exchange (where some type of good is generally acceptable as a medium of exchange) is of great value to society and the individuals that compose it. People give up valuable goods in exchange for something that they typically cannot consume or use in productive activities. Their intent is to trade at a later date this intermediate item for something they do want. They anticipate that two indirect trades to get from an initial to a final commodity bundle are quicker than one direct trade. In a world of diverse trading goals, where the time available for trading is scarce, it is more efficient (trades occur more quickly) when some commodity (or commodities) serve to expand the number of potential trading partners by reducing the types of trades that are made. However, the value an agent attaches to the intermediate good, and thus his willingness to accept it in exchange, depends entirely on the willingness of other agents to accept it. Given this strategic uncertainty, it is not transparent that a medium of exchange could come into use in the absence of government or some other explicit coordination device.[1]

The emphasis here is on the social coordination aspect of exchange. Models incorporating this were introduced by Jones (1976) and later Kiyotaki and Wright (1989). In these papers agents are endowed with or produce some commodity but prefer to consume a different one. Pairwise meetings occur at random and paired agents choose what types of trades to make (ie. which commodities they would accept in return for their current commodity). Different compositions of trading strategies have different efficiency implications. Especially interesting are equilibria that resemble real world patterns of exchange. The structure of these models emphasizes the relative efficiency of intermediated trade, especially in Kiyotaki and Wright, where no double coincidence opportunities exist.

We focus on the question of what conditions are necessary for monetary exchange to be the unique stable state of the economy (in a manner made precise below). Additionally, it will be eventually be adopted over direct barter regardless initial conditions. In this paper we construct a simple model that contains basic features that make social coordination desirable, as in the earlier models. We show that if an economy is diverse enough in its collection of commodities, regardless of initial conditions, it will eventually develop a social convention that is characterized by a monetary system of exchange. This convention will emerge in finite time from any initial state and can be expected to be maintained for a long time (relative to how

[1] This is the question posed by Menger (1892) of "whether the transition from barter to monetary exchange could be expected to take place spontaneously under the pressure of market forces whenever it promised welfare gains or whether it required an 'invention,' combined with persuasion, convention, or compulsion," Niehans (1978), pp. 2-3.

long it was expected to take to get there).

Kiyotaki and Wright look for steady state equilibria by using stationarity and individual rationality conditions. Many equilibria of this type exist, autarkic equilibria and different patterns of indirect exchange (some of which are monetary exchange patterns) are examples. However, because of their reliance on steady state conditions, they cannot analyze the model outside of steady state equilibria. In particular they cannot eliminate equilibria based on stability. Thus, with no mechanism in their model to select among equilibria, they are not able to make definite statements on the conditions under which a particular pattern of trade would be observed.

Renero (1995) has examined equilibrium dynamics in a specific version of the Kiyotaki and Wright model. He shows that not only do different initial conditions lead to convergence to various steady states, but also that some steady states that appear economically sensible are unstable while there are less sensible ones that are stable. Earlier work by Oh (1989), building on the research by Jones, also had the property that which steady state the economy converged to depended on initial conditions. Thus these models lack an endogenous mechanism to select among multiple stable equilibria.

Our aim is to begin a more comprehensive analysis of social patterns of trade. To do this we use the recent literature on stochastic evolutionary games (Kandori, Mailath, and Rob (1993) and Foster and Young (1990)). This research introduces noise into agents' choice of strategy. While it is reasonable that people make some mistakes, it is not acceptable to assume that people make frequent mistakes or often hold their behavior fixed when they really should correct it – especially when this irrationality is costly. Thus the literature focuses on the effect of the inclusion of arbitrarily small trembles. When combined with a way agents use information about history to form strategies, dynamics can be characterized that are ergodic, eg. the system is not dependent on initial condition for model behavior in the distant future (even though they might be very important in the short and medium term). Thus, this perturbation to the model induces a distribution over states so that some equilibria can be characterized as being observed so infrequently as to be irrelevant and others, or perhaps just one, as having stability properties so that they are most likely to be observed in the long-run. Additionally the stochastic evolutionary approach allows predictions about how long before a particular equilibrium is achieved from initial conditions.

In this paper, as in the earlier models, we construct a simple model that contains basic features that make social coordination desirable. We focus on the question of what conditions are necessary for monetary exchange to be adopted over direct barter from any initial social pattern of exchange. This leads to some differences between our model and the Kiyotaki and Wright economy. Unlike their model, we allow direct barter and we allow only one commodity to be used as an intermediate good. We assume an exchange economy with finitely many commodities, including a "token" commodity that, although unconsumable, has the capability to be used as a medium of exchange. Although exchange is necessary to consume, indirect exchange is not. Agents choose whether to look for direct exchanges or to accept the token commodity as a medium of exchange. Using tokens is risky because although there are always opportunities for direct barter, an agent who accepts the token commodity may find it impossible to locate another agent with whom to trade if other agents do not also use the tokens. Many patterns are equilibria, including both the monetary exchange pattern (where all agents accept tokens) and the direct barter pattern where all agents seek out traders with a double coincidence of wants.

We then examine what the addition of a "best-response" learning rule and a small amount of noise means for the evolution of the economy. This enables us to show that if there are enough commodities in the economy, monetary exchange will be adopted. We also show that the length of time required for this convention to emerge is decreasing in the number of commodities. We also show how the quality of information (degree of detail) that agents have about the population can be crucial in speeding up the transition. This speedup is a somewhat asymmetric implication of the model, information about changes in the behavior of producers (groups of agents who are endowed with the same commodity) can result in large changes in population strategies while information about the behavior of similarly grouped consumers does not. Where the information lacks the detail about producers, convergence is much slower.

2 The Model

In this section we construct a finite economy composed of agents with diverse trading goals and limited opportunities for exchange. In order to get a commodity they can consume, agents must make exchanges (which are based on a trading strategy they choose).

Assumption 1 *There are $l+1$ types of commodities, all indivisible.*

Assumption 2 *Each agent has a type $\theta = (\theta^e, \theta^c) \in \{1, \ldots, l\} \times \{1, \ldots, l\}$, $\theta^e \neq \theta^c$, where θ^e is the type good he is endowed with and θ^c is the type of good he wants to consume.*[2] *There are n agents of each type and $N = n \cdot l(l-1)$ in total.*

Assumption 3 *The $l+1$ commodity, which we call the* token *commodity, is not consumed or produced by anyone. It is also special because it is storeable. An agent can hold one unit of either his endowment commodity or the token commodity without inventory costs. He cannot hold more that one unit and nor can he hold any other commodity.*

[2] Although the assumption that agents have a single good and want a single good appears restrictive, we could consider agents who make repeated trips to the market with different goods until they have a "bundle" that they desire.

Assumption 4 *Agents make only anonymous bilateral voluntary exchanges. These exchanges are restricted to direct barter (double coincidence of wants) or indirect barter with a token as the intermediating commodity.*

Assumption 5 *Agents receive utility from consuming their consumption-type commodity and none from any other commodity. They discount consumption that occurs later at rate β.*

2.1 The Game

Agents choose a strategy s from the set $\{A, D\}$ which specifies the trades they are willing to make conditional on the good they hold when opportunities arise.[3] Those who adopt strategy D will only trade their endowment directly for their consumption good (direct barter). Those who adopt strategy A may also make direct barter trades, but will additionally accept tokens and then try to trade the tokens for their consumption good (indirect barter).

Based on an agent's beliefs about the environment over the relevant horizon he chooses the strategy that maximizes his payoff. Having chosen his strategy; with probability m the agent is given the opportunity to accept a unit of the token commodity in place of his endowment. If he adopted strategy D he retains his endowment, if he adopted strategy A he accepts the token, and his endowment is lost.

The population, divided into types, is now subdivided by the strategy of those agents and by the holding of the agents. As we see in figure 2.1 (where we have for illustration five commodities and consider agents of type $(1, 5)$), the agent of type θ could have adopted: strategy D (and hold his endowment), strategy A and hold his endowment, or strategy A and hold the token commodity. The number of these agents is denoted by d^θ, a^θ, and m^θ (respectively). Define c^i as the set of agents who consume good i (those in column i) and e^j to be those who are endowed with commodity j (those in row j). Referring again to the economy in figure 2.1 a type $\theta = (1, 5)$ can potentially trade only with agents in one or both of the sets e^5 and c^1.

Each agent in the population is randomly paired with another agent. They trade (or not) based upon the strategies they adopted. Agents who successfully trade are re-endowed immediately and return to trade again with the same strategy. The random pairings continue, but for each agent the end of trade arrives randomly. At that time the agent is replaced and his inventory is lost to the economy. For an individual agent the effect of this, combined with other sources of impatience, is a per-period discount factor of β on future consumption.

[3]For the present we consider only pure strategies.

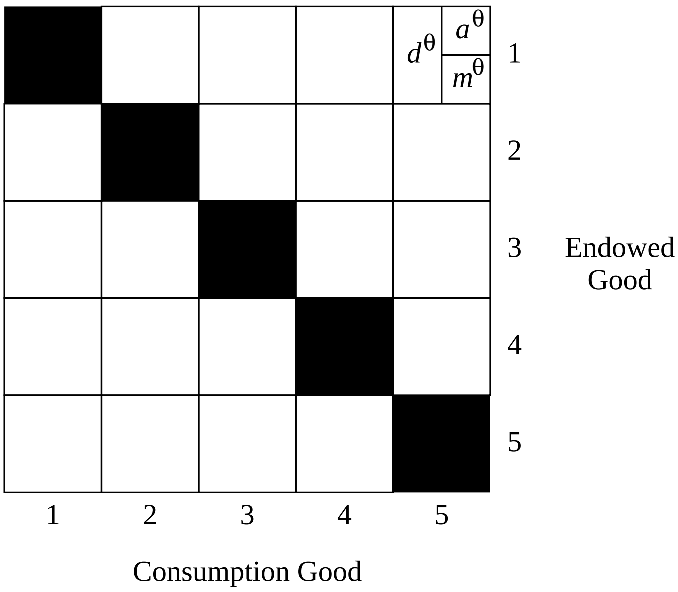

Figure 1: Agents in Type Space (5 commodity illustration)

An agent will always want to make a trade that gives him his consumption good. If a trade would give him a token, he would want to decide based on whether or not he could more easily acquire his consumption good in the future. Thus the agent's entire concern is with present and future trade probabilities. The probability of a θ agent who holds his endowment good acquiring his consumption good at time t is

$$q_t^\theta(c|e) = \frac{1}{N-1}\left[a_t^{\theta'} + d_t^{\theta'}\right],$$

where θ' is the type that has a double coincidence of wants with θ. For an agent who holds his endowment and uses strategy A, the probability of trading for a token is

$$q_t^\theta(m|e) = \frac{1}{N-1}\sum_{\hat{\theta}\in c^\theta} m_t^{\hat{\theta}}.$$

token good, the probability of acquiring his consumption good at time t is

$$q_t^\theta(c|m) = \frac{1}{N-1}\sum_{\hat{\theta}\in e^\theta} a_t^{\hat{\theta}}.$$

The trade decision is more complicated when these probabilities are changing over time, in particular if sometimes tokens are better and sometimes the endowment is better. We can characterize the decision simply if the following condition is satisfied.

Lemma 1 *An agent of type θ prefers strategy A at time s if his belief is that*

$$q_t^\theta(c|m) \geq q_t^\theta(c|e),$$

for all $t \geq s$. Equivalently, if

$$\sum_{\substack{\hat{\theta} \in e^\theta \\ \hat{\theta} \neq \theta'}} a_t^{\hat{\theta}} \geq d_t^{\theta'}.$$

An agent prefers strategy D if the inequality is reversed for all t.

Proof: See appendix.

Even under the Nash equilibrium requirement that these beliefs are correct, we can see that there are still at least two outcomes that might be observed in this economy.

Corollary 2 *In any exchange economy of this type, there are at least the following Nash Equilibria:*

1. *Pure direct barter in which all agents use strategy D.*
2. *Monetary exchange in which agents use strategy A.*

Proof: This follows trivially from Lemma 1.

Typically there will be other Nash equilibria as well. Specifically there may be the following types:

1. All agents in both types of some matched pair, θ and θ', use strategy A while all other types use strategy D.
2. All agents of types in θ_i^e and θ_i^c (all those who either are endowed with or consume i) use strategy A, all others use strategy D.

Suppose that parameters were chosen so that if none of an agent's double coincidence types were using tokens, it then would take k other agents who produce her consumption commodity to use tokens to get her to switch to using tokens also. Thus if all of one of these types (given by a production, consumption good pair) were to use tokens and there is less than k of that type it wouldn't be enough for her to switch. It would be enough for that type's direct-trade type agents to switch (as discussed above). This may be one Nash equilibria. There may be others, for example there may be one "sector" (both producers and consumers of a particular good) which is entirely strategy A and some pairs of types (see item 2 in the corollary) outside of the sector that also use A in a Nash Equilibria.

We will say more below about the problem of multiple equilibria and the method we use to address the issue in section 3. First we consider the two equilibria in Corollary 2 in more detail. This serves the dual purpose of characterizing the value of monetary exchange in different economies and emphasizing why a model that is able select among outcomes with such widely variant welfare properties is important.

2.2 Efficiency of Monetary Exchange vs. Direct Barter

In this section we characterize the relative efficiency of the two Nash equilibria that interest us most ("all A" and "all D"). In any situation such as both of these equilibria, where an equal fraction of each type plays strategy A, it is straightforward to show that the symmetry implies that trade probabilities are constant over rounds. Thus we get the following value functions for agent's in each equilibria using Bellman's equation (refer to the proof of lemma 1).

$$V(\text{All D}) = \frac{\beta}{(1-\beta)(l^2-l)+\beta}$$

$$V(\text{All A}) = \beta(1-m)\cdot\frac{\frac{1}{l-1}+\frac{\beta m}{(1-\beta)l+\beta(1-m)}}{l(1-\beta)+\beta\left(m+\frac{1-m}{l-1}\right)}$$

The comparison of these equilibria is not transparent (especially from the appearance of the second expression). We are mainly interested in the relative value of the two equilibria and with some algebra it is not too difficult to see that although both go to zero as l becomes large, the ratio converges to

$$\frac{V(\text{All A})}{V(\text{All D})} = (1-m)l.$$

Thus it becomes increasingly better to be in a monetary equilibria as the economy becomes more differentiated. Figure 2 illustrates this using the exact expression. It should be emphasized that this represents only relative trading efficiency and not gains to specialization.

Although a thorough analysis of the relative value of the equilibria is not the focus of this paper, for intuition we provide several graphs to illustrate relative efficiency as the parameters β and m vary.

The above approximate ratio for large l, makes it appear that tokens impose an efficiency cost on society. In fact, as Figure 3 illustrates, the effect of the per-capita token stock on efficiency is not so simple. With no tokens the values are equal (as all trades will be direct in either equilibria), larger per-capita token levels increase efficiency until an optimum is reached. However for every token floating around the economy, a unit of a consumable commodity had to be sacrificed. The inability to hold both tokens and endowment imposes a real cost on society that both emphasizes the gains (it is efficient to sacrifice real output, even a lot of it, to trade more quickly) from intermediated trade and understates these gains (unless it is for commodity money, few resources are really sacrificed in carrying or constructing a mediating commodity).

Figure 4 illustrates that although the determination of the efficient equilibria can be sensitive to the discount rate, the monetary equilibria still becomes strictly better as the number of commodities increase.

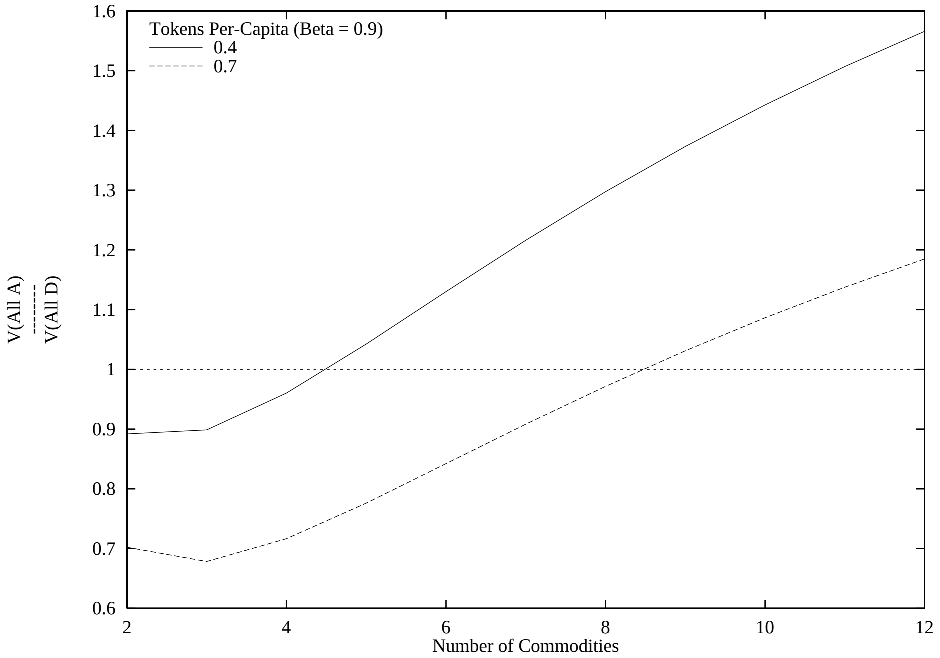

Figure 2: Relative Value of Equilibria vs. Number of Commodities

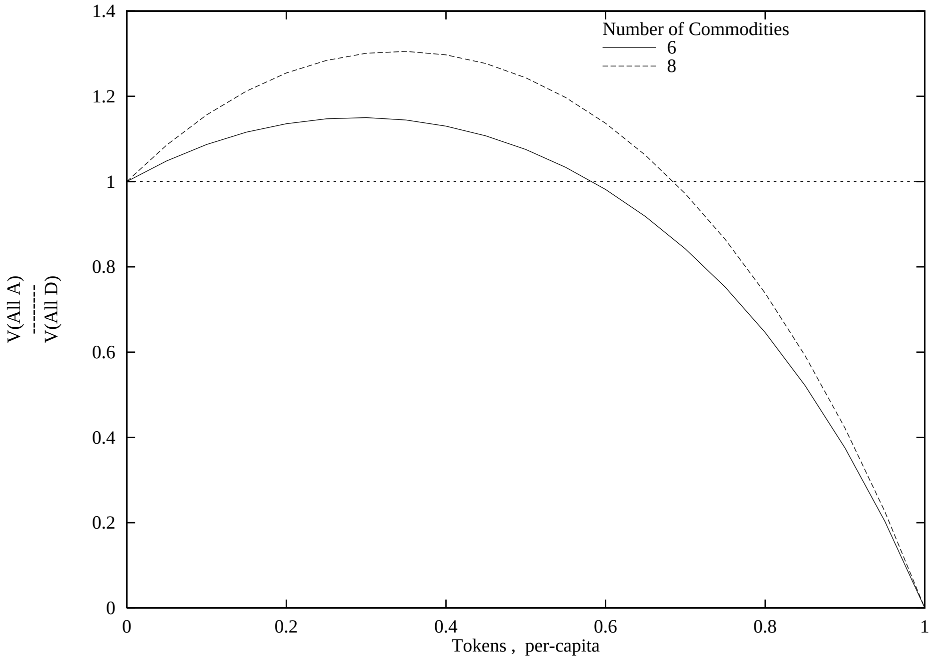

Figure 3: Relative Value of Equilibria vs. Quantity of Tokens

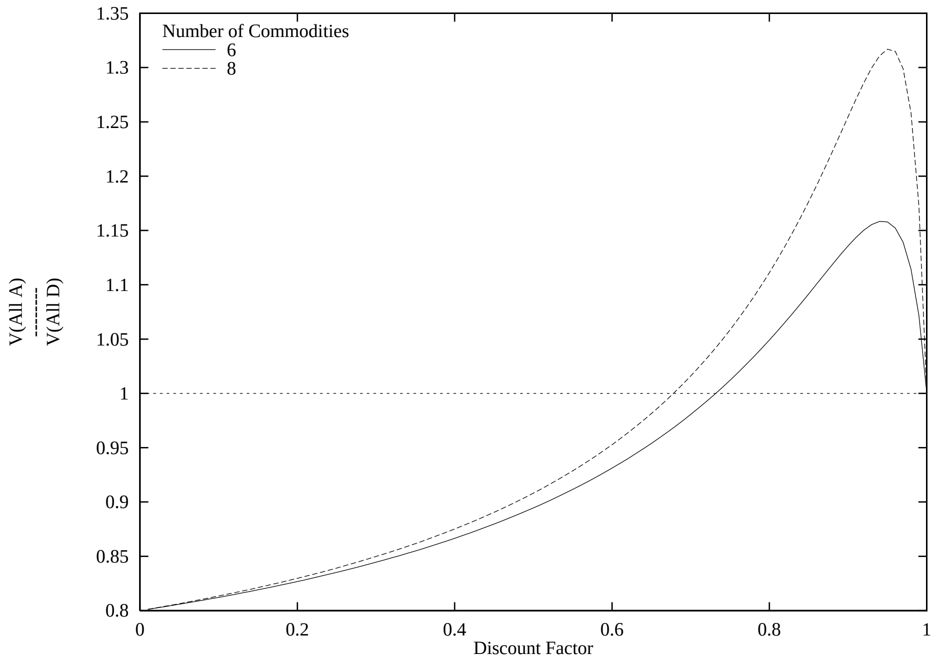

Figure 4: Relative Value of Equilibria vs. Discount Factor

3 Dynamics

We have shown that the relative efficiency of equilibria may differ across economies. Our goal in this section is to see whether society can achieve the better equilibria. We introduce a dynamic environment that will help to evaluate the viability of the equilibria in our model. Search models of monetary exchange have largely accepted multiple equilibria as a necessary result of their steady state analyses. While this permits efficiency comparisons between equilibria it doesn't help enough in evaluating the likely outcome in such a world. Questions about the stability of monetary exchange and the ability of monetary system to emerge without coordination are thus difficult to address with such an analysis.

Work on a Kiyotaki and Wright type of model by Renero (1995) using initial conditions has highlighted the need for caution in accepting equilibria at face value. Furthermore, the existence of multiple equilibria in a model is something that has been viewed as a modeling weakness. This is because of two notions of what makes a good theory; it should be able to eliminate any counterintuitive equilibria and should contain clear predictive content. Models with multiple equilibria often fail on the first test and by definition fail the second.

The game theory literature wrestled with these issues through a long series of refinements. Several of these refinements are based on the idea that "good" equilibria should be stable in the face of small amounts of noise. Kandori, Mailath, and Rob (1993) and Foster and Young (1990) elaborated an explicit stochastic dynamic framework that takes seriously these stories of "mistakes" and evolutionary dynamics that emerged out of the refinement literature. The dynamic structure specifies what *information* agents have about the history of play and state variables and the *learning rule* they use to determine their strategy.[4] A small amount of *noise* (due to unmodeled factors such as experimentation or mistakes) is then added. The presence of these two elements permits the economy to be outside of the equilibria and still have a defined dynamic. As the noise gets small the amount of time spent outside some states (called stochastically stable) becomes small. Not only does this allow the set of equilibria to be reduced, consideration can be given to the manner and frequency with which transitions occur over the long-run. Thus the analysis allows a detailed portrait of social dynamics to be drawn.

Information. We assume that agents are given information directly about the strategies in use.[5] We will analyze the model under two alternative assumptions about how much information is available to agents. That agents know "everything" (the distribution of strategies across types) is one assumption. The other is that agent only have access to an aggregate statistic – the fraction of the whole population using each strategy. Examining the difference the added information makes is one motivation for considering both cases, additionally the simplicity that results in the aggregate statistic case will simplify the analysis there considerably.

Learning Rule. When imposing a learning rule, a right or best method of inference is not self-evident, but two conditions that a reasonable inferences should satisfy are: first, they usually should be correct, and second, avoidable mistakes shouldn't be either too costly or too persistent. A rule that will often satisfy these criteria, is easy to apply, and has been widely used in the literature, is the best-response dynamic.[6] Each agent makes a best response to his information about the strategies in use. Thus every static Nash equilibria (in which this knowledge is both complete and correct) is also potentially an equilibria in the dynamic (which we note is a Markov process).[7]

Finally in defining the intentional population dynamic, we recall that agents, either due to the difficulty of evaluating alternatives or otherwise only occasionally change their strategy (as they are replaced in the population). This gives additional support to the use of best-response learning (the slower the environment changes the "better" is the best-response). This process of replacement is given by the

[4] One effect of the structure we have specified is that the future path inventory holdings is always the same when agents make their strategy choices. This reduces the complexity of the problem by eliminating money and commodity holdings as a state variable. The state then consists solely of the strategy profile of the population.

[5] From a perspective of realism it would be desirable to have agents infer about strategies from observable data such as trades. However it isn't clear that there would be any impact on the results and the cost to the clarity of the analysis would be high.

[6] A different learning rule could be the fictitious play of Fudenberg and Levine (1993).

[7] Cycles are also potential phenomena, this would be troubling because agents don't have to be very smart to take advantage of such phenomena.

probability δ replacement of each agent. However the amount of "inertia" doesn't have any effect on stochastic stability, it only slows the dynamic. Thus although the results hold generally, we can simplify the analysis by ignoring periods where no one is replaced and further reduce notation by analyzing the situation where replacement is perfectly correlated (all agents are replaced at the same time).

The state is $z_t \in Z$, where Z contains all 2^N possible strategy profiles. It is convenient to define z_t as a 2^N-vector, consisting of all zeros except for a one in a position that indicates the state. The information structure $\Omega(Z)$ maps the 2^N states into K observable states (observe that $K = N+1$ in the aggregate information case). Given the information a type θ agent has about the state, he makes a best response. The state that results from the combined best responses of the population is $b(\Omega(z_t)) \in Z$.

We can now define the intentional evolutionary correspondence, $P : Z \to Z$, so that $z_{t+1} = b(\Omega(z_t)) = z_t P$, where P is a $N \times N$ matrix of transition probabilities. When the best-response function is single valued this consists of zeros and ones.

Noise. We impose a small probability, ϵ of each agent making a "mistake" in selecting his strategy. This could be considered to be experimentation (though not optimally chosen). The resulting Markov process is given by $z_{t+1} = z_t P(\epsilon)$ where

$$P_{z_t, z_{t+1}}(\epsilon) = \epsilon^{c(z_t, z_{t+1})}(1-\epsilon)^{N - c(z_t, z_{t+1})},$$

and $c(z_t, z_{t+1})$ is the *transition cost*, the number of agents who have different strategies differently in state z_{t+1} than they would have held given their best-response, $b(\Omega(z_t))$.

The state transition matrix $P(\epsilon)$ has strictly positive elements and hence it has unique ergodic distribution.[8] Let μ^ϵ be the stationary state distribution, then the limit distribution is defined by $\mu^* = \lim_{\epsilon \to 0} \mu^\epsilon$ and *long-run equilibria* elements of μ^* which are greater than zero. In addition to this process on the "true" state we can define another on the "information" state. This is convenient to work with because it is of much lower dimension.

3.1 Case 1: Aggregate Information Only

Suppose that the only information available to agents is the number of agents that used strategy A in the previous period, thus there are $N+1$ possible states. Here the information has the same impact on each agent's decision. Thus there are only three possible equilibria: all agents use strategy A, all agents use strategy D, and agents use strategies such that all are indifferent.

[8]Ergodicity doesn't require all positive elements. There could be zero elements as long as all states communicate (there is a positive probability of moving between them in finite number of steps). Our assumptions allow us to avoid having to work very hard to show ergodicity.

Lemma 3 *For economies with non-trivial stocks of tokens, ie.* $0 < m < 1$*, and a discount factor* $0 < \beta < 1$*, the unique long run equilibria has all agents choosing strategy A if*

$$l > \frac{1}{1-m} + 2$$

and all agents choosing strategy D if the inequality is reversed.

Proof: See Appendix.

As we might expect, the minimum number of commodities where indirect exchange can be supported as the unique long run outcome is three – when the double coincidence problem first arises.

We then have the following result on long run equilibria in this case.

Corollary 4 *Given an sequence of economies* $E(l)$ *with fixed* m *and* β *satisfying lemma 3, there exists an* l^* *such that for all economies with* $l > l^*$ *the unique long-run equilibrium has all agents using strategy A.*

Proof: See Appendix.

Thus with a little noise we are able to characterize the degree of complexity required for a monetary equilibrium to be eventually achieved, and almost always observed. We might wonder if there is anything meaningful in the result, which is a limit result. We would like to know how long before the monetary equilibria is reached if the economy did not begin there. For a reasonably complex economy, this time might be so long as to make the result largely irrelevant. Agents will switch to using tokens from the direct barter equilibria whenever there are $\frac{n(l^2-l)}{(l-2)(1-m)+1}$ or more simultaneous mutations among the $n(l^2 - l)$ members of the population. Thus although the number of mutations needed increases with l, as a fraction of the population it decreases.

Lemma 5 *For an economy with* $0 < m < 1$*, and discount factor* $0 < \beta < 1$*, the expected wait to reach the "All A" equilibrium decreases as the number of commodities becomes large.*

$$\lim_{l \to \infty} W(\epsilon) = 0$$

Proof: Using the normal approximation to the binomial,

$$W = \left[1 - Z\left(n(l^2-l)\frac{1-\epsilon((l-2)(1-m)+1)}{((l-2)(1-m)+1)\sqrt{n(l^2-l)\epsilon(1-\epsilon)}}\right)\right]^{-1}.$$

The argument of Z goes to $-\infty$ as l gets large. ■

We have already warned about taking the efficiency comparisons too literally given the way tokens displace consumable commodities in the model economy. It

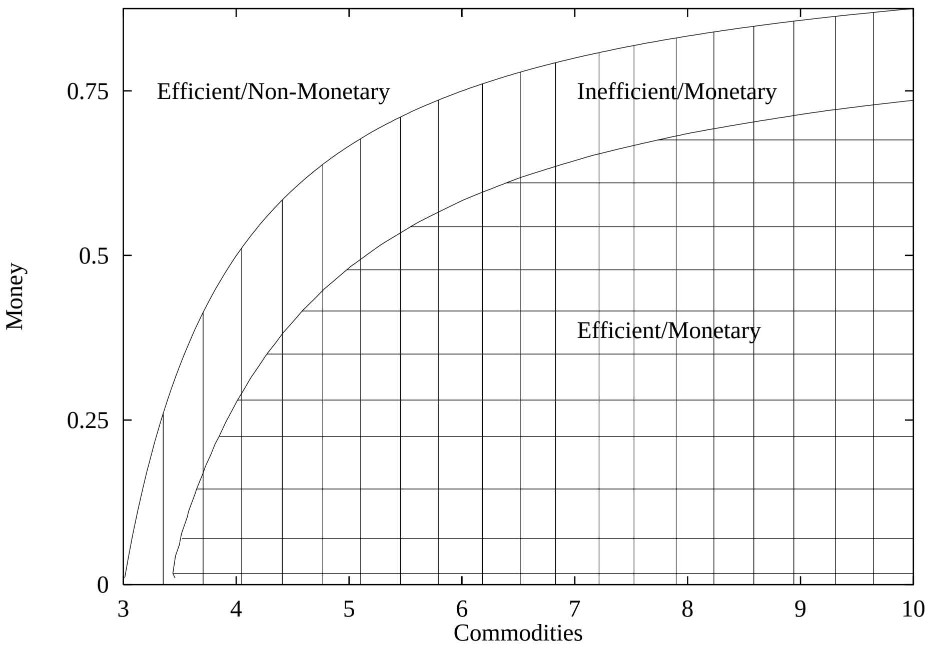

Figure 5: Long-Run Equilibrium vs. Efficiency (Agg. info)

is still worth noting that efficiency and and convergence do not coincide in this model. In fact, as figure 5 shows, the stochastically stable (long-run) equilibria is the monetary equilibria before there are enough commodities for it to be efficient. [9]

3.2 Case 2: Disaggregated Information

The ability to provide a clear analytical result in the example above hinged on the the assumption that agents have very limited information. We would like to be able to examine situations where agents have better information because it might not be clear to what degree our results rely on the information structure.

We now consider the situation where agents learn the exact distribution across types of the strategies used. With the more detailed information agents are given, we can no longer exploit symmetry to simplify the complicated dynamic system. The value functions are complicated.[10]

Although we are unable to prove a theorem analogous the Theorem 3 for this case, we can compare the results of simulations. We include for comparison in the

[9] This may be related to risk dominance, but since in this model expected payoff is not a linear function of population strategy, the right notion of risk dominance is not the usual one.

[10] Given a expectation of population strategies S, they are

$$V(D|S) = \sum_{r=1}^{\infty} \beta^r q_r^\theta(c|e) \prod_{s=1}^{r-1} \left(1 - q_s^\theta(c|e)\right),$$

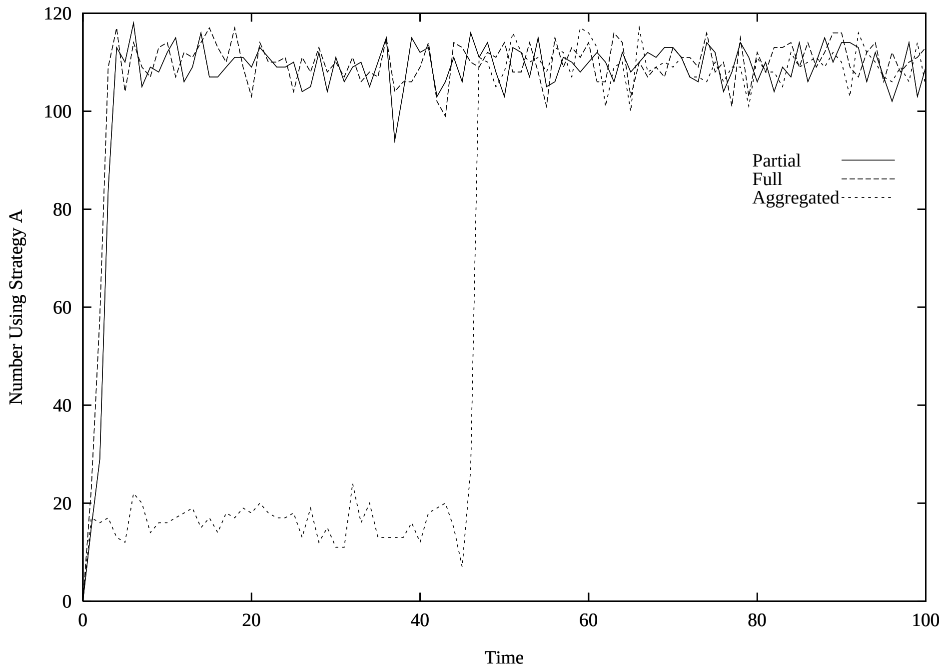

Figure 6: Eventual Convergence of Three Information Structures

simulations a third information structure. We were able to prove an analogous result for this structure, where the information given agents about the strategy choices is aggregated only by endowment type (ie what fraction of each "endowment sector" accepted tokens) in the previous round. We do not include the analysis of this case because the algebra doesn't doesn't provide much intuition.[11] Figures 6 and 7 are representative, they shows that convergence not only still occurs – it is considerably faster. We summarize in Table 3.2 the effect of the number of commodities on convergence time in simulations.[12] This leads to the question of why convergence is faster when there is more information.

The agents' payoffs are principally affected by the strategies of agents with whom

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and

$$V(A|S) = \sum_{r=1}^{\infty} \beta^r \left[q_r^\theta(c|e) + q_r^\theta(m|e)v_r^m\right] \prod_{s=1}^{r-1} \left(1 - q_s^\theta(c|e) - q_s^\theta(m|e)\right),$$

given that

$$v_r^m = \sum_{s=r+1}^{\infty} \beta^{s-r} q_s^\theta(c|m) \prod_{w=r+1}^{s-1} \left(1 - q_w^\theta(c|m)\right).$$

[11]A fourth case, where information is aggregate by consumption type results in similar properties to the completely aggregated case.

[12]This is especially rough for cases where the passage time was long. We ran simulations with 4000 switch opportunities in each case. Thus there were few or no switches towards the top of the table. The parameters used in Figure 7 were: $l = 6$, $m = .15$, $\epsilon = .13$, and $n = 3$. The only change in Figure 7 is that $\epsilon = .03$ and $l = 7$. For Table the parameters were 3.2 $m = .25$ and $\epsilon = .07$.

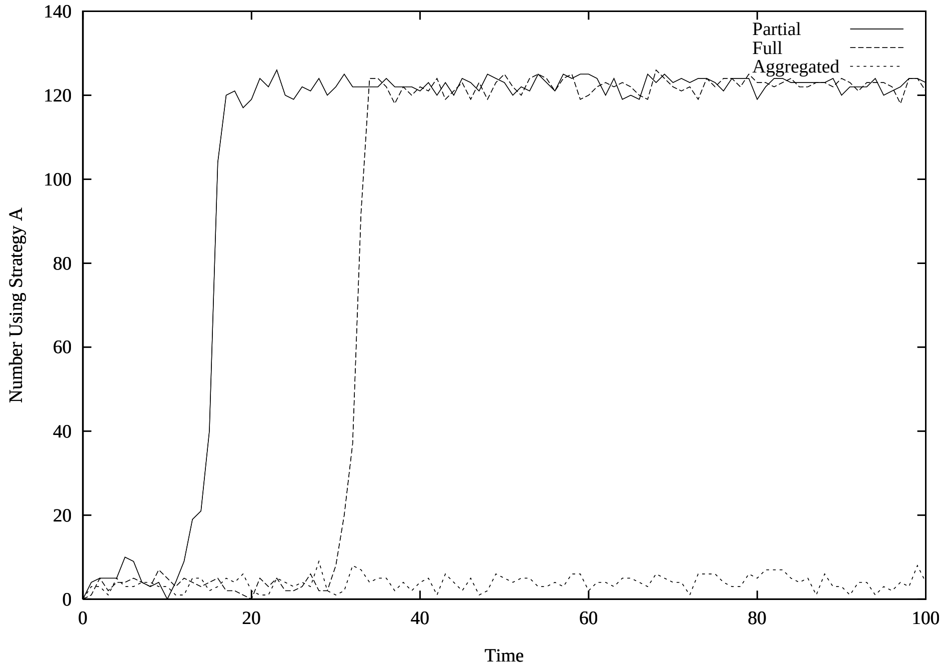

Figure 7: Rapid Convergence of Finer Information Models

Information Structure			
l	Complete	Partial	Aggregate
3	?	?	?
4	?	?	?
5	136.3	2000.5	?
6	11.8	8.8	?
7	6.7	8.9	?
8	4.3	5.3	?
9	3.6	4.5	?
10	2.9	3.5	1000.3
11	2.6	2.9	232.9
12	2.5	2.5	30.3
13	2.3	2.3	14.1
14	2.1	2.3	6.3
15	2.2	2.3	2.3

Table 1: Average First Passage Time (Barter to Monetary)

they could trade (and only secondarily by those who trade with those agents). Specifically the agents who matter most to an agent of type θ are those who affect $q_r^\theta(c|e)$ or $q_r^\theta(c|m)$. Better information about producers' trade strategy gives agents the ability to react to those agents whose strategy matters. The model has a particular "location" structure (in terms of commodities) and only with the detailed information can agents exploit it. In the static context the extra information gives the model more equilibria (ones like those discussed in section two).

These additional equilibria are the source of the increase in convergence speed. A mutation moves the state of the system, but if it was insufficient to escape the basin of attraction of the equilibria, the system returns to the initial equilibria. With only two equilibria, as in the first example, each has a large basin of attraction. For a mutation to cause a response it must be large enough to convince the entire population to immediately switch and this occurs only with a low probability.

The dynamic system is like a hill, the two extreme equilibria are its two sides. Jump to the top and you can roll to the bottom of the other side, but if you jump short you roll back to where you began and have to try again. As the number of commodities increases the hill gets higher. In the simple example, the theorem the says that the two sides don't grow at the same rate[13] and it gets relatively harder to make the switch from the monetary side compared to the return trip. But it may be a long time before a successful jump is made.

The increased number of equilibria is like the addition of ledges to the hill, where one can rest between short jumps. Not only that, but the steps are only the non-monetary side, making it relatively easy to get to the monetary equilibria and very hard to get out.[14]

We can make a perturbation to the model that helps show why this is so. So that the decision for an agent would be much simplified, suppose that each agent is only given one trading opportunity after his choice of strategy and the random token opportunity. All he would have to look at is whether the number of agents he expects to be holding his consumption good and using strategy A is greater than the number of agents with whom he can make a double coincidence trade. Thus any agent's best response is to use A if previously

$$(1-m)\sum_{\hat\theta\in e^\theta} a_{\hat\theta} > (1-m)a^{\theta'} + d^{\theta'}.$$

[13]Stretching the metaphor, on the sides of the hill the relative elevations of the valleys are changing.

[14]The intuition for this is already available from research on local interaction. From the switch of some locale (which is of relatively high probability), the effect quickly spreads to the rest of the economy. Part of this intuition has been formalized in Ellison. He defines a modified transition cost that takes account of the lower cost of a path that passes through stable sets. His bound would still not capture fully the increased speed of convergence here because his bound is unable to account for the possibility that mutations out of intermediate stable sets may be more likely to move us closer rather than further away.

A mutation necessary for this to be satisfied for some agent (and thus column of agents) becomes more likely as the number of commodities increase. Furthermore the number of columns that must switch for the entire population to follow and to switch does not increase – so this also becomes increasingly likely. Finally there are a number of smaller mutations of agents switching from D to A (such as to equilibria mentioned in section 2), that although leading directly to all monetary exchange, make the mutation necessary to complete the transition much more likely.

On the other hand for the switch from all A to all D the number of mutations required is larger in economies with more commodities. This type of mutation because an very low probability occurrence.

4 Conclusion

We have shown that regardless of initial conditions, in an economy with a diverse enough collection of commodities, a population of agents will develop a social convention that is characterized by a monetary system of exchange. Furthermore this convention will emerge in finite time, unaided by any explicit coordination mechanism such as a government.

We have seen in this model that the level of detail in the information that agents have about others' trade strategies is critical in speeding up the transition. In particular, this allows relatively small behavioral changes within a group of agents, who are all endowed with (or produce) the same commodity, to have a large affect on the population. This is interesting both because it speeds the transition to monetary equilibria and because the effect is asymmetric, similarly sized behavioral changes in a group of agents who consume the same commodity will not spread. Further work with a model that allows sectoral asymmetries in information may show that visible trade, such as takes place in markets, can not only assist a monetary system, but also result in particular market structures.

Our model points to the importance of the relationship between specialization and exchange. Diverse commodity demands increase the the double coincidence of wants problem and this impels the economy to develop a monetary system. Thus it is clear that the presence of money, or more precisely the degree of difficulty in finding trade opportunities, should be closely related to the degree of specialization. Further work, in a model where agents choose their consumption and production goods may shed light on the manner in which a money encourages specialization and specialization encourages monetary exchange.

The main lesson to be drawn is that intermediated exchange is a robust institution. Despite difficulties imposed by the model – inability to explicitly coordinate and the displacement of real resources by the intermediate good – in an economy with a reasonably diverse set of commodities, intermediated exchange is the unique stochastically stable state over direct barter. It remains for future research to examine the relative robustness of particular forms of intermediated exchange.

Proof of Lemma 1: Denote the conditional values of agents by $v_r(a)$, $v_r(m)$, and $v_r(d)$ (we suppress the type temporarily for notational simplicity). Thus whether an agent chooses strategy A or not depends on whether or not

$$v_0(a) \geq v_0(d).$$

The value are given recursively by

$$v_r(a) = \beta\left[q_r(c|e) + q_r(m|e)v_{r+1}(m) + (1 - q_r(m|e) - q_r(c|e))v_{r+1}(a)\right]$$

$$v_r(d) = \beta\left[q_r(c|e) + (1 - q_r(c|e))v_{r+1}(d)\right]$$
$$v_r(m) = \beta\left[q_r(c|m) + (1 - q_r(c|m))v_r(m)\right]$$

Generally, we see that

$$\begin{aligned} v_r(a) - v_r(d) \quad = \quad & \beta\{q_r(m|e)\left[v_{r+1}(m) - v_{r+1}(a)\right] + \\ & \qquad (1 - q_r(c|e))\left[v_{r+1}(a) - v_{r+1}(d)\right]\}. \end{aligned}$$

Thus A has a higher payoff than D if

$$q_r(m|e)\left\{v_{r+1}(m) - v_{r+1}(a)\right\} \geq 0 \text{ for all } r,$$

and is strictly greater for at least one r, (and D has the higher payoff if the inequality is reversed). Since $q_r(m|e) \geq 0$ for all r, we need for all r,

$$v_r(m) - v_r(a) \geq 0.$$

This is given by the following, and if for all r, $q_r(c|m) \geq q_r(c|e)$ we have the inequality below.

$$\begin{aligned} = \quad & \beta\{q_r(c|m)(1 - v_{r+1}(m)) - q_r(c|e)(1 - v_{r+1}(a)) \\ & \qquad +(1 - q_r(m|e))\left[v_{r+1}(m) - v_{r+1}(a)\right]\} \\ \geq \quad & \beta\left[(1 - q_r(m|e) - q_r(c|e))(v_{r+1}(m) - v_{r+1}(a))\right] \\ \overset{r\to\infty}{\longrightarrow} \quad & 0 \end{aligned}$$

Finally, $q_r(c|m) \geq q_r(c|e)$ if and only if

$$\sum_{k=1}^{l} a_r^{(j,k)} \geq a_r^{(j,i)} + d_r^{(j,i)},$$

equivalently if

$$\sum_{\substack{k=1 \\ k\neq i}}^{l} a_r^{(j,k)} \geq d_r^{(j,i)}.$$

The second half of the proof follows a parallel argument, basically only reversing the inequalities. ■

Proof of Lemma 3: Let K be the number of agents that used strategy A in the previous period. Then agents believe that $\frac{K}{l^2-l}$ agents of each type will use A. Thus,

$$d_0 = n - \frac{K}{l^2 - l},$$

$$a_0 = (1-m)\,\frac{K}{l^2 - l},$$

and

$$m_0 = m\frac{K}{l^2 - l}.$$

It is then trivial (using the definitions of $q(c|e)$, $q(m|e)$, and $q(c|m)$) that these are constant for all rounds. Using Lemma 1, the condition under which an agent (and thus all agents) are willing to choose strategy A is

$$(l-2)\,(1-m)\,\frac{K}{(l^2-l)} \geq n - \frac{K}{l^2-l}.$$

or, rearranging to solve for K,

$$K \geq \frac{n(l^2-l)}{(l-2)(1-m)+1}.$$

Agents choose D if the inequality is reversed. All states other than "All A" or "All D" are transient. The dynamic path will consist of mutations that either carry us into the basin of the other stable equilibria, or just return back to the initial equilibria. A state is stochastically stable, or a long run equilibria if the limit of the unconditional probability of being at that state goes to one as $\epsilon \to \infty$. An equivalent condition is

$$\lim_{\epsilon\to\infty} \frac{P(\text{Switch to } D|\text{Basin of } A)}{P(\text{Switch to } A|\text{Basin of } D)} = 0$$

The switch to A and the switch to D take K and $N-K$ simultanous mutations respectively, so this is equivalent to

$$\lim_{\epsilon\to\infty} \frac{\epsilon^{N-K}}{\epsilon^{K}} = 0.$$

The condition for this is $N - 2K > 0$, and so "All A" is the long run equilibria if

$$l > \frac{1}{1-m} + 2.$$

■

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