Exchange Rate Intervention with Options*

Fernando Zapatero†
and
Luis F. Reverter‡

JEL classification: F31

First draft: February 1997
This draft: September 1997

Abstract

We consider the problem of a Central Bank that has some exchange rate goals. We compare "direct" intervention through sale/purchase of reserves in the currency market with an alternative strategy of intervention with options. The intervention affects the currency markets because as a result of the purchase and sale of derivatives, the investment bank the Central Bank deals with will have to cover its positions and therefore buy or sell the underlying security. In fact this strategy will affect not only the currency market but also the bond market and, therefore, the equilibrium interest rate, since the hedging strategy involves trade in the riskfree security. We simulate different dynamics of the demand and supply of both markets (currency and bonds) involved and compare the loss of reserves of the Central Bank in either type of strategy as well as its effect in the volatility of both equilibrium exchange rate and equilibrium interest rate. Our framework is general enough to allow comparative statics for different parameter values of the model. Central Bank intervention seems to have some advantages but the value of the cross-elasticity exchange rate-interest rate is crucial.

---

*Previous versions of this paper were presented at Banco de México, ITAM and the 1997 Latin-American Meetings of the Econometric Society. We are grateful to Francisco Venegas for comments and to Esteban Rossi for computational assistance and comments.

†Centro de Investigación Económica (ITAM). Av. Sta. Teresa 930. México DF 10700 (México). Ph: (525) 628-4193. Fax: (525) 628-4058. E-mail: zapatero@master.ster.itam.mx

‡Banco de México. Av. 5 de Mayo, 6. México DF 06059 (México). Ph: (525) 227-8843. Fax: (525) 227-8752

1. Introduction

It is clear that fixed exchange rates and target zone regimes require some type of active policy from the Central Bank. Very often it amounts to using reserves of foreign currencies in order to attain the goals of the exchange rate regime. But even in free floating regimes Central Banks often decide to intervene in order to smooth temporary volatility on currency markets or maybe for other long-term currency goals (considerations of an over or undervalued currency). Intervention in the forward market (as opposed as to the spot market) is a refinement that has been argued to be more effective (see Eaton and Turnovsky (1984) for an analysis of this strategy). Alternatively, the Central Bank can use open market operations in order to attain currency objectives (see Hernández and Zapatero (1997) for a comparison of both procedures). The choice between those two alternative procedures amounts to a choice of monetary policy along with currency targets. In this paper we will compare the use of reserves in order to achieve currency goals with an alternative tool that is available today to Central Banks in order to accomplish its currency targets: options on the currency. In order to keep the comparison as "fair" as possible, we will assume that the Central Bank will not buy or sell bonds in either of the intervention strategies. Of course, open market operations could be used additionally. In some cases this amounts to sterilization, but in other instances this implies non-sterilization. We will be more specific as we describe the framework.

Options markets started in the early seventies and have experienced an enormous growth over the last two decades. As a consequence, they have been available to Central Banks as a possible tool only for a few years. The possibility of the use of options in order to attain objectives of monetary policy relies on the functioning of the hedge necessary to cover a position in options. The idea is the following: when a financial institution takes a position in options because of the demand of its clients it is accepting a risk that comes from the changes of the underlying security; if the investment bank takes a long position in an option the risk is the possible loss of value of the option (due to a drop of in price of the underlying security if a call option or an increase in the price of the underlying security if a put option); if the investment bank takes a short position in an option the risk is the possibility that the option will mature in the money and the investment bank will have to pay to the counterpart (the risk increases as the price of the underlying security increases for a short position in a call option and as the price of the underlying security drops in a put option). In order to cover the risk that the position carries, investment banks have to construct synthetically the reverse position so as to keep a zero net exposure to the risk of changes in the price of the underlying security.

A synthetic option is constructed by buying or shortselling the underlying security and shortselling or buying a bond (for calls and puts, respectively). Short positions are replicated in the opposite way. If the Central Bank buys a put option on the currency, for example, the investment bank that sells the option will have to replicate

a long put in order to hedge its position. Replication of a long put involves sale of the underlying security, that is, sale of the currency. The Central Bank can induce the same effect in the currency markets by buying put options than by selling reserves.

Garber and Spencer (1994) describe in detail the effects of the hedges of the investment banks on currency stabilization when the Central Bank uses the interest rate as a tool to sustain a fixed exchange rate regime. In this paper we focus on the use of options as the tool to implement currency stabilization objectives through the hedges of investment banks. We compare different strategies the Central Bank can use in order to induce the same effect as a sale/purchase of reserves. However, the hedging strategy of the investment bank also involves trades in domestic bonds; therefore, this strategy also affects interest rates (and through them exchange rates as a second order effect). In fact, interventions that try to sustain the exchange rate (interventions at the weak-end) will have to induce sales of the underlying security (foreign currency); the counterpart of the hedging strategy is purchases of the domestic bond that will produce a drop of the domestic interest rate (incidentally, that allows for a tightening of the monetary policy as a complementary action). Besides, by taking an appropriate position in options the Central Bank can dispense with the use of reserves. The downside is the fact that the Central Bank undertakes financial responsibilities that might produce a larger loss of reserves when the options expire (or compromise reserves through the functioning of margin requirements).

In order to compare traditional intervention by buying/selling reserves with the use of options as a tool of Central Bank intervention in currency markets we develop an appropriate framework and simulate paths of the economy for given parameters. We first describe the bond market where equilibrium interest rate is determined. As a result of our assumptions about demand and supply we will obtain a one-factor model of interest rates. We then describe the dynamics of the currency market. Demand and supply of the currency market will depend on the exchange rate level and also on the interest rate level; although we do not model explicitly the foreign demand of domestic bonds we assume that part of the demand of domestic bonds will be the result of the differential between foreign and domestic interest rates. As a result of the interaction between equilibrium interest rates and demand and supply in the currency markets, the equilibrium exchange rate will satisfy two-factor dynamics.

We compare paths of both exchange rates and interest rates under both types of interventions and we also perform some comparative static analysis for different parameter values. Overall, Central Bank intervention with options seems to be more efficient from the point of view of the use of reserves. Besides, it clearly gives the Central Bank further maneuvering space by lowering interest rates when the intervention aims at sustaining the currency. However, this effect is larger when the cross-elasticity currency markets-bond markets is lower. The way the two types of intervention affect volatility of exchange rates and interest rates also depends on the values of the parameters.

It is also important to stress that the type of interventions we have in mind in our

model are those that correspond to dirty floating or intra-band interventions. In the case of target zones (or fixed rates) the fact that market participants can anticipate Central Bank interventions will have an effect on the valuation of the options and in the hedging strategies that is not trivial and we do not attempt to measure in this paper. Besides, we will implicitly assume a somehow stable environment. Currency crises would affect the prices of the options and hedging strategies. Currency crises are beyond the scope of this paper.

The paper is organized as follows: in Section 2 we describe the bond and currency markets; in Section 3 we describe the basic objectives of the traditional sterilized intervention (that we call "direct" intervention); in Section 4 we describe the option markets; in Section 5 we explain the mechanics of Central Bank intervention with options and two different versions of this strategy; Section 6 is devoted to the numerical exercise we perform; in Section 7 we analyze the results; we close the paper with some conclusions.

2. The Setting

We establish the basic framework we will use in order to compare the different Central Bank intervention strategies. We assume that there is a bond market and a currency market. In the bond market the equilibrium interest rate is determined. The bond market can be interpreted as the complementary of a money market: individuals can put their wealth in money or in bonds. Bond demand will be directly related to the interest rate level (money demand would be indirectly related to the interest rate level). Currency demand depends on both the exchange rate (demand from imports/exports) and the interest rate level (foreign demand of domestic bonds). We do not provide microfoundations for the demand and supply that are exogenous. However they are consistent with similar functions in the literature. Besides, the resulting equilibrium interest rate and exchange rate are also consistent with those assumed in the literature.

The setting we describe is the setting that would prevail in case there were no Central Bank interventions. In following sections we will describe the different types of Central Bank intervention and the way they will affect the dynamics of the interest rate and the exchange rate.

2.1. Bond Market

We assume that there is a market of domestic bonds denominated in the local currency. In this setting, bonds are (locally) riskless securities whose price B satisfies the following dynamics,

$$\frac{dB_t}{B_t} = r_t dt \tag{1}$$

where r is the interest rate, to be determined in equilibrium. These securities are free from default risk. Their supply S^B is fixed. Their demand, D^B, however, depends on the nominal interest rate r, determined in equilibrium when demand and supply meet. D^B expresses the nominal amount of domestic bonds demanded. We assume that demand and supply functions are then,

$$D_t^B = \log r_t + z_{1t} \quad (2)$$

$$S_t^B = \bar{B} \quad (3)$$

where $\bar{B}$ is the fixed supply of bonds, and z_1 is a stochastic variable that represents shocks to the demand and satisfies itself,

$$dz_{1t} = \mu_1 dt + \sigma_1 dW_{1t} \quad (4)$$

where μ_1 and σ_1 are constant and W_1 is a standard Brownian motion process. The stochastic shocks to the demand represented by z_1 are, for example, political events or international economic changes (an increase in the level of foreign interest rates) that affect the demand of domestic bonds but we do not model explicitly.

The fact that we assume a constant supply of domestic bonds $\bar{B}$ has implications for the type of Central Bank intervention we consider. We will be explicit about it in Section 3.

In equilibrium $D^B = S^B$. Solving, we derive the equilibrium interest rate that results,

$$r_t = e^{\bar{B} - z_{1t}} \quad (5)$$

We then obtain a one-factor model of the interest rate process; the resulting dynamics of the interest rate are,

$$\frac{dr_t}{r_t} = \left(-\mu_1 + \frac{1}{2}(\sigma_1)^2\right) dt + \sigma_1 dW_{1t} \quad (6)$$

a geometric Brownian motion process. This dynamics are also valid for the case of direct Central Bank intervention (described in Section 3). However, Central Bank intervention with options will modify this dynamics and will transform the equilibrium interest rate in a two-factor stochastic variable.

2.2. Currency Markets

Similarly, we model the demand (D^C) and supply (S^C) in the currency markets. Equilibrium in the currency markets arises when demand and supply meet. We assume that supply and demand depend on both the interest rate r (determined by the equilibrium in the bond market) and the currency level x. The rationale is that part of the demand of bonds will be given by the differential between the domestic and the foreign interest rate (that we implicitly assume constant). That means that investors will shift money in or out of the country depending on the domestic interest

level. With respect to the currency level, imports and exports will depend on it. We denote the currency level by x and it represents the units of domestic currency that can be bought with one unit of the foreign currency (increase in x means devaluation of the domestic currency). We represent by S^C the supply of foreign currency (expressed in units of the foreign currency) and by D^C the demand.

Demand and supply are given by

$$S_t^C = \bar{M} + \frac{1}{2}\log x_t + \frac{1}{2}a_1 \log r_t + \frac{1}{2}z_{2t} \quad (7)$$

$$D_t^C = -\frac{1}{2}\log x_t - \frac{1}{2}a_1 \log r_t - \frac{1}{2}z_{2t} \quad (8)$$

where $\bar{M}$ is constant, a_1 is also a constant coefficient and z_2 a stochastic process that satisfies

$$dz_{2t} = \mu_2 dt + \sigma_2 dW_{2t} \quad (9)$$

with μ_2 and σ_2 constant and, as before, W_2 a standard Brownian motion process independent of W_1. $\bar{M}$ can be interpreted as an inelastic part of the supply (due to transfers from nationals working abroad, for example); it is obvious that the results do not change if we interpret $\bar{M}$ as an inelastic component of the demand, that is, we change its sign. The stochastic process z_2 represents shocks to the supply/demand; the way it is incorporated into both equations implies that both supply and demand are equally affected by these shocks, but does not imply anything related to its sign; it can be the result of, for example, political events that might affect in a negative (positive) way the supply (demand) or viceversa. Similarly, an increase in interest rates will have a positive impact in the demand of domestic bonds and, therefore, a positive impact on the supply of the currency and the opposite effect on the demand of the currency; we also assume symmetry between the effect on supply and on demand.

In equilibrium, $S^C = D^C$ and, therefore,

$$\begin{aligned} x_t &= (r_t)^{-a_1} e^{z_{2t}-\bar{M}} \\ &= e^{-\bar{M}-a_1\bar{B}+(a_1\mu_1+\mu_2)t+a_1\sigma_1 W_{1t}+\sigma_2 W_{2t}} \end{aligned} \quad (10)$$

The dynamics of the exchange rate are then,

$$\frac{dx_t}{x_t} = \left(\mu_1 + \mu_2 + \frac{1}{2}((a_1\sigma_1)^2 + (\sigma_2)^2)\right)dt + a_1\sigma_1 dW_{1t} + \sigma_2 dW_{2t} \quad (11)$$

This is a two-factor geometric Brownian motion process.

3. Direct Central Bank Intervention

The Central Bank of this country is responsible for exchange rate stability. In this section we describe the benchmark type of intervention that we call "Direct Central

Bank Intervention" and is based on sale/purchase of reserves of the foreign currency. The Central Bank will inject or withdraw the foreign money in the currency market. When the exchange rate reaches a given level $\bar{x}$ ("weak" currency) the Central Bank injects an amount of the foreign currency such that the new exchange rate is set at a level $\bar{x} - \bar{\epsilon}$, with $\bar{\epsilon}$ given. Similarly, when the exchange rate reaches a given level $\underline{x}$ ("strong" currency) the Central Bank withdraws an amount of the foreign currency such that the new exchange rate is set at a level $\underline{x} + \underline{\epsilon}$ with $\underline{\epsilon}$ given. In general, in order to simplify the analysis, we will assume that $\bar{\epsilon} = \underline{\epsilon} = \epsilon$.

However, we do not consider explicitly the case of target zones. The literature on target zones assumes that the Central Bank is committed to maintaining the exchange rate within a given band and will intervene whenever the exchange rate reaches the boundaries of the band. Typically, these models assume that the Central Bank only intervenes at the boundaries and no intra-band interventions will take place. In the model we have in mind $\bar{x}$ can be considered as a reservation exchange rate the Central Bank has in mind and triggers intervention. This exchange rate value is not public and can change from intervention to intervention. It might be an intra-band value, for example. The reason why we do not want to consider explicitly a target zone model is twofold. On one hand the existence of a target zone would have an effect on the equilibrium value of the exchange rate (see Krugman (1991)). Besides, it would also have effect in the valuation of options (we will be more specific in the next Section) that would become intractable.

Another issue that has to be discussed is the monetary effects of the intervention. Central Banks can choose between sterilized and non-sterilized intervention (see Hernández and Zapatero (1997) for a comparison of the effects of these alternatives in a target zone regime). Sterilized intervention implies that the Central Bank will keep interest rates constant by providing the necessary liquidity or withdrawing the excess liquidity. In non-sterilized intervention the Central Bank leaves interest rates unaffected. Here we will assume that the Central Bank buys or sells reserves, but they are never exchanged by bonds. That might mean sterilization or not depending on the reason for the pressure on the exchange rate. Suppose, for example, that the exchange rate is "weakening" and the Central Bank decides to intervene by selling reserves. There are two possible reasons for the pressure on the exchange rate: people might be exchanging their holdings of domestic currency for reserves (say an increase in the demand of imported goods); alternatively investors might be exchanging domestic bonds for foreign bonds. In the former case, the Central Bank directly provides the reserves requested by the public; this is sterilized intervention. In the latter case the Central Bank will allow interest rates to go up as sellers of bonds have to attract new buyers. The sellers will exchange the proceeds of their sale for reserves that the Central Bank will provide; this is non-sterilized intervention. The effects of open market operations would be additional to those of the direct intervention with reserves or the intervention with options.

Suppose that at moment τ the exchange rate reaches the level $\bar{x}$, for $\bar{z}_2$ and $\bar{r}$ (or,

equivalently, $\bar{z}_2$ and $\bar{z}_1$). That is,

$$\bar{x} = x_\tau = (\bar{r}_\tau)^{-a_1} e^{\bar{z}_{2\tau} - \bar{M}} \tag{12}$$

The Central Bank intervenes by injecting an amount I (expressed in units of the foreign currency) that is added to the supply S^C or subtracted from the demand D^C. In order to achieve its target, the amount I has to be such that (from (7) and (8))

$$\bar{x} - \epsilon = x_\tau - \epsilon = (\bar{r}_\tau)^{-a_1} e^{\bar{z}_{2\tau} - \bar{M} - I_\tau} \tag{13}$$

From (12) and (13) we get that

$$I_\tau = -\bar{M} - \log(\bar{x} - \epsilon) - a_1 \log \bar{r}_\tau + \bar{z}_{2\tau} \tag{14}$$

Similarly, if the exchange rate reaches the level $\underline{x}$ the Central Bank will intervene by withdrawing an amount J (expressed in units of the foreign currency) that is subtracted from the supply S^C.

Suppose, then, that the Central Bank has to intervene at points $i = (1, ...n)$ (injecting) and at points $j = (1, ...m)$ (withdrawing). The new equilibrium exchange rate is,

$$x_t = r_t^{-a_1} e^{z_{2t} - \bar{M} - \sum_{i=1}^{n} I_i + \sum_{j=1}^{m} J_j} \tag{15}$$

We will assume that the Central Bank has a total balance of reserves denoted by $\bar{R}$ that sets a limit on the total amount of interventions,

$$\bar{R} + \sum_{j=1}^{m} J_j \geq \sum_{i=1}^{n} I_i \tag{16}$$

When (16) becomes a strict equality the Central Bank runs out of reserves and cannot intervene anymore. We move to a full floating rate system.

4. Option Markets

Suppose that, besides the markets described in Section 2, there is an options market where calls and puts on the currency are traded. We will assume that they are european type options with a fixed maturity of T years. Therefore, a call option with strike price K gives the right to buy one unit of the foreign currency at time T for a price K and a put option the opposite. The price of the options is expressed in units of the domestic currency.

These options are traded by investment banks that immediately cover their positions by replicating the opposite position in the option with the underlying security (the currency) and the bond. In order to both replicate the price of the option and to compute the hedging ratios of investment banks ("delta") we need a pricing formula.

We assume that investment banks use an approximation provided by the adjusted Black and Scholes (1973) formula (see Biger and Hull (1983), Garman and Kohlhagen (1982), Grabbe (1983) and Hull and White (1993) for detailed analysis of the technique). This is in fact an exact formula for an exchange rate process that satisfies a Geometric Brownian Motion Process and constant domestic and foreign interest rates. We will denote the foreign interest rate by r_f and will assume that it is constant. However, it is obvious from the model described in Section 2 that neither the domestic interest rate does not satisfy the assumptions required. Besides, although the exchange rate (as explained by equation (11)) satisfies a Geometric Brownian Motion Process, after intervention takes place its volatility will be constant no more. Nevertheless, as an approximation, we will assume that the adjusted Black and Scholes (1973) formula is employed using current interest rate (to be determined in equilibrium in the bond market) and current volatility of the exchange rate (the parameter that explains the dynamics of the exchange rate in equilibrium and that is given by equation (11)) as if they were constant. The justification for this simplification is twofold. First, in practice these formulas are used as no closed form solutions are usually available for these problems. We could certainly do better using numerical pricing techniques (for example, Monte Carlo simulation) but that would come at a huge computational cost. Second, since the model we are using (described in Section 2) is itself a simplification of the true dynamics of the economy, we do not think that this represents a big cost in terms of our conclusions. For a detailed explanation of the functioning of hedging strategies and the role of investment banks in foreign exchange derivatives markets see Garber and Spencer (1994).

We denote the volatility of the exchange rate by σ_e, that from equation (11) is the following two-dimensional vector,

$$\sigma_e = \begin{pmatrix} a_1\sigma_1 \\ \sigma_2 \end{pmatrix} \tag{17}$$

In fact, since we have a two-dimensional factor model we would need three securities to construct a perfect replicating portfolio of the options, but for the same reasons given above we will restrict ourselves to the currency and the domestic bond.

The prices of a call option C and of a put option P given the assumptions made above will be,

$$C_t = e^{-r_f(T-t)}x_t N(d_1) - Ke^{-r(T-t)}N(d_2) \tag{18}$$

$$P_t = e^{-r(T-t)}KN(-d_2) - e^{-r_f(T-t)}xN(-d_1) \tag{19}$$

where $N(\cdot)$ represents the value of the cumulative normal distribution function and

$$d_1 = \frac{\log\frac{x_t}{K} + (r - r_f + \frac{1}{2}(\sigma_e)^\top\sigma_e)(T-t)}{\sqrt{(\sigma_e)^\top\sigma_e(T-t)}} \tag{20}$$

$$d_2 = d_1 - \sqrt{(\sigma_e)\top\sigma_e(T-t)} \tag{21}$$

This formula also provides the basis for the computation of the "delta" of the options,

$$\Delta_C = e^{-r_f(T-t)}N(d_1) \quad (22)$$

$$\Delta_P = -e^{-r_f(T-t)}N(-d_1) \quad (23)$$

where Δ_C represents the "delta" of the call option and gives the number of units of the underlying security that have to be bought in order to replicate the call and the difference between the price of that investment and the price of the call will have to be borrowed (at the domestic interest rate); similarly, Δ_P is the "delta" of the put and represents the number of units of the underlying security that will have to be sold short in order to replicate the put and the amount raised plus the price of the put will have to be invested in the domestic bond.

We will assume that investment banks upgrade their hedging positions continuously taking into account the changes in the underlying x but also the changes in value of the parameters r and σ_e as well as the passage of time that reduces $T-t$.

There is a problem with the previous pricing model and the exchange rate equilibrium that we announced in Section 2. The problem is the following: as the Central Bank intervenes, the volatility of the exchange rate will not be given by the value σ_e expressed in equation (17); the true volatility value will depend on the probability of Central Bank intervention. Dumas, Jennergren and Näslund (1995) provide a valuation model for currency options when there is the risk of realignment. As we stated in Section 3, we will assume that the value $\bar{x}$ at which Central Bank intervention takes place is not public. In fact, agents do not know that Central Bank intervention takes place. A justification would be that the continuous time structure that we present is an approximation and in the truly discrete dynamics of exchange rates the individuals that participate in the currency markets cannot tell whether the change from $\bar{x}$ to $\bar{x}-\epsilon$ was the result of Central Bank intervention or the result of a shock to the markets. In the numerical exercise we perform later we will overcome this problem by using historic volatility instead of σ_e; that is, the agents will compute the volatility of the exchange rate for the previous period; this is more in line with the behavior of investment banks.

4.1. Margin Requirements

In order to guarantee that the party that takes the short position in an option contract can meet its potential financial liability, it is required to satisfy margins. More explicitly, an investor with a short position in options faces the potential of a financial loss if the option finishes "in-the-money", that is, has a positive payoff at maturity. The investor will have to deposit collateral in an account as guarantee. The margin requirement changes day to day with changes in price of the underlying security ("marked to market"). If at a certain point the investor cannot satisfy the margin requirements, her position will be closed using the balance in the account and the

account liquidated. Margin requirements set a limit to the size of the short position an investor can undertake.

We will assume that the amount of collateral the investor has to deposit in the account in order to satisfy the margin is exactly the amount the option is in-the-money, that is, if the investor has a short position in calls, the margin amount m an investor has to keep in the account at a given moment t is,

$$m_t = \max(x_t - K, 0) \tag{24}$$

The opposite if the investor has a short position in puts.

The counterpart of the margin requirements are the market prices of long positions in options. Long positions in options do not generate "positive" margins, but will have a positive market value. This is important because in one of the strategies we consider the Central Bank takes simultaneously long and short positions. Short positions will have to be guaranteed with margins. Long positions have a positive financial value; the Central Bank will be able to post options purchased as collateral in order to satisfy margins of the short positions.

In fact, it can be argued that Central Banks might be exempt from margin requirements or, at least, get a special treatment. That certainly would make more attractive intervention with options.

5. Central Bank Intervention with Options

Suppose that the Central Bank is responsible for achieving the currency policy targets described in Section 3, but has the possibility to deal (buy/sell) in the options market. In Section 3 we described direct intervention in the currency markets. Here we describe alternative strategies trading options that can have the same immediate effects as direct intervention, but also have some second order effects. We assume that reserves are the only asset owned by the Central Bank, in order to abstract from monetary effects. Monetary effects could be analyzed additionally to the effects considered here.

In this approach the Central Bank does not trade in the currency, but in options on the currency. The Central Bank will buy or sell calls or puts from an investment bank. The investment bank has to cover its position and in order to do so will take an equivalent (but with opposite sign) position by constructing a synthetic option with the underlying security and bonds. For example, if the Central Bank sells a call option to the investment bank, the investment bank takes a long position in the call; in order to cover this position, the investment bank will replicate a short position in the call with the underlying security and the bond; in order to replicate a call option it is necessary to buy the underlying (in an amount given by the delta) and sell the bond (up to the value of the underlying purchased minus the cost of the delta); in order to construct the short position the investment bank will sell the underlying security (the currency) and buy the bond.

As we pointed before, by using this strategy the Central Bank will have a direct effect in the currency markets by inducing an increase in the supply of the underlying security. However, there are also limitations to this strategy: the Central Bank will be subject to margin requirements. We will assume that the Central Bank has to satisfy the restriction expressed in (24). In order to do that the Central Bank has to deposit reserves in the account. Those reserves cannot be used for other purposes. However, the Central Bank might also have a long position in options that have a positive value. Therefore, denoting by m^B the total amount of collateral the Central Bank has to deposit in order to satisfy margin requirements and by L the value of the long position in options the Central Bank might have at a given moment, it has to be the case that

$$\bar{R} + \frac{L_t}{x_t} \geq \frac{m_t^B}{x_t} \tag{25}$$

We divide the value of the long position and the margin by the exchange rate because prices of options and, therefore, the value of the long position and the margin in equation (24) are expressed in units of the domestic currency. Whenever the previous equation becomes an equality the Central Bank cannot intervene anymore using options and the regime moves to a full floating exchange system.

Besides the direct effect on the currency markets, trading in options will have an indirect effect on the currency through the bond market. As we just explained, by trading in options the Central Bank will force the investment bank to replicate the option and, in order to do so, sell the underlying security. However, replication of the option always implies a long or short position in the underlying security and the opposite position (short or long) in the bond. Whether the Central Bank sells calls or buys puts, the effect on the currency markets will be the sale of the underlying that always has to be accompanied by a purchase of bonds. More explicitly, if C is the price of a call option and Δ_C the delta of the option, H, the nominal value of bonds the investment bank will have to buy in order to replicate the call option will be,

$$H_t = \Delta_{Ct} x_t - C_t \tag{26}$$

This has an effect on the demand for bonds D^B that now becomes,

$$D_t^B = \log r_t + z_{1t} + H_t \tag{27}$$

and the new equilibrium interest rate will be,

$$r_t = e^{\bar{B} - z_{1t} - H_t} \tag{28}$$

Since the investment bank will increase the demand for bonds, the equilibrium interest rate will go down, as there will be a lower outstanding amount of bonds after the investment bank covers its position. The opposite would happen in an intervention in the other direction.

Therefore, by trading options the Central Bank affects the exchange rate in a double way, directly through the currency market and indirectly through the bond market. Consider a trade in options that results in a sale of the underlying security by the investment bank of size V (units of the foreign currency); given the effect through the bond markets expressed by equation (28) we have that the resulting exchange rate after the intervention will be

$$x_t = e^{a_1(-\bar{B}+z_{1t}+H_t)-\bar{M}+z_{2t}-V_t} \tag{29}$$

Then, in order to get the same impact as an intervention of I units of the foreign currency through direct sale of the currency by the Central Bank we need

$$V - a_1 H = I \tag{30}$$

It is clear that as the price of the underlying security changes, the value of Δ_C and H will change, therefore intervention through options is not a one time effect and it will have consequences all throughout the life of the option. One important effect pointed above after equation (28) is that interventions in the upper bound (weak currency) will yield a lower interest rate level, while interventions in the lower bound (strong currency) will produce higher interest rate levels. As we pointed before, the type of intervention with options we are considering abstracts from all types of monetary policy considerations. It is clear, though, that the Central Bank will have additional possibilities to complement intervention with options. For example, since this type of intervention will lower interest rates the Central Bank could complement intervention in the upper bound with a tightening of the monetary policy (that would further help to stabilize the currency). An additional consequence of this is the fact that the equilibrium interest rate that in the case of direct Central Bank intervention satisfied a one-factor model as explained by equation (6), will now become a two-factor stochastic variable; as the exchange rate changes as a result of changes in z_1 (through the interest rate) and z_2 (directly) the investment bank will adjust its hedge and change its position in bonds and, therefore, change the equilibrium interest rate which now will depend (through the hedging strategy) on changes in z_2. The effect on the volatility of the interest rate, however, is not obvious.

There are several variations of Central Bank intervention with options that can be used. We will focus in two.

5.1. Zero Cost Strategies

The Central Bank constructs a combination selling calls and buying puts on the underlying security in such a way that the market price of the portfolio is zero. In order to compute the strategy, first we choose a strike price $\bar{K}$. For this strike price we compute the price of the call and put, $C(\bar{K})$ and $P(\bar{K})$, from equations (18) and (19). Then we compute the number of short calls and long puts that would yield a portfolio of zero cost; obviously, any portfolio with the same ratio of short calls and

long puts will have a zero cost. Say that for each single call we need $\kappa(\bar{K})$ puts, that is, $\kappa(\bar{K}) = \frac{C(\bar{K})}{P(\bar{K})}$ (clearly, $\kappa > 0$). Constructing this basic zero cost portfolio formed by one call and $\kappa(\bar{K})$ puts will induce a purchase of $\Delta_C(\bar{K}) - \kappa(\bar{K})\Delta_P(\bar{K})$ units of the foreign currency for the hedge of the investment bank where the deltas are computed in equations (22) and (23) and we note that Δ_P is always negative.

Remember that, simultaneously, the investment bank that is trading with the Central Bank will have to buy bonds in order to cover its long position in calls and short position in puts. The nominal amount of bonds to be purchased is given by equation (26) for calls and its equivalent for puts. However, we are considering zero cost positions, which means that the cost of the puts will be equal to the cost of the calls and, therefore, the amount of bonds purchased will correspond to the amount of the underlying security sold; that is, suppose that $\eta(\bar{K})$ represents the number of calls that have to be sold short and we denote by V the number of units of the underlying security (the foreign currency) sold for hedging purposes, that is,

$$V = \eta(\bar{K}) \left(\Delta_C(\bar{K}) - \kappa(\bar{K})\Delta_P(\bar{K}) \right) \tag{31}$$

and by H the value of bonds purchased as given by equation (26),

$$V_t x_t = H_t \tag{32}$$

From equation (30), the total impact in the currency value is

$$I_t = V_t - a_1 H_t = (1 - a_1 x_t) V_t \tag{33}$$

In order to determine the exact number $\eta(\bar{K})$ of calls that have to be shorted we compute,

$$\eta(\bar{K}) = \frac{V}{\Delta_C - \kappa(\bar{K})\Delta_P} = \frac{V}{\Delta_C - \frac{C(\bar{K})}{P(\bar{K})}\Delta_P} = \frac{I}{(1 - a_1 x_t)\left(\Delta_C - \frac{C(\bar{K})}{P(\bar{K})}\Delta_P \right)} \tag{34}$$

where I is determined from equation (14).

Besides, it has to be taken into account that the hedging strategy of the investment bank is dynamic. As the price of the underlying security, the volatility of the underlying security, the interest rate and time left to maturity change, the price of the calls and puts and their deltas will change; that will force the investment bank to update its strategy and buy or sell the currency and bonds. For example, suppose that after the exchange rate is brought down from $\bar{x}$ to $\bar{x} - \epsilon$ it experiences further weakening; before it hits $\bar{x}$ the Central Bank will not do anything, nevertheless the hedge of the investment bank will change: the delta of the call option will go up while the delta of the put option will go down; the effect on the deltas will depend on the closeness of the exchange rate to the strike price, the closer the underlying security to the strike price the more sensitive delta is to changes in the underlying (see Hull (1996)); the increase (in absolute value) of the deltas means that the investment bank

will have to sell more of the currency and buy more bonds; the fall in interest rates will have also a negative impact on the currency; this additional effect will depend on the strike price chosen. Furthermore, the change in the exchange rate and the interest rate will also affect the delta of the hedge. The outcome of this series of interactions is not obvious. It also has to be taken into account that when the option matures the investment bank will close its position buying the currency and selling bonds.

In this case the limits to the Central Bank intervention will be given by the margin requirements of the expression (25); the Central Bank has a long position in calls, therefore L in that expression will have a positive value and will allow the Central Bank to cover larger margin requirements.

5.2. Financing Strategies

In Section 5.1 the Central Bank constructed a combination of zero cost (if we do not take into consideration margin requirements). Here we consider the case when the Central Bank constructs a short position that in fact increases its level of reserves (again, abstracting from margin requirements).

The idea is similar to that in Section 5.1 but here the Central Bank only takes a short position (for a larger amount than before) so as to have the same impact as before through the hedging strategy of the investment bank. Suppose, for example that the exchange rate hits $\bar{x}$ then triggering a Central Bank intervention. In this case the Central Bank will sell call options. The investment bank takes a long position and in order to cover has to replicate a short position in calls which implies selling the underlying security and buying bonds. As we did in Section 5.1 we fix a strike price $\bar{K}$ and compute the corresponding delta $\Delta_C(\bar{K})$. Therefore, each call the Central Bank sells short induces a sale of the currency by an amount $\Delta_C(\bar{K})$; besides each call the Central Bank writes induces a purchase of bonds by the investment bank of $x\Delta_C(\bar{K}) - C(\bar{K})$; but, additionally, in order to be consistent with the approach of previous sections, we have to assume that the Central Bank immediately uses the proceedings of the sale of the options to buy reserves of the foreign currency. This has a negative impact on the currency market of a value $\frac{C(\bar{K})}{x}$ (in units of the foreign currency) per call. Therefore, given a desired intervention of I, from equation (30) the number of calls $\eta(\bar{K})$ the Central Bank will have to sell short is given by the solution to

$$\begin{aligned} I &= \eta(\bar{K})\left(\Delta_C(\bar{K}) - a_1[x\Delta_C(\bar{K}) - C(\bar{K})] - \frac{C(\bar{K})}{x}\right) \\ &= \eta(\bar{K})(1 - a_1 x)\left(\Delta_C(\bar{K}) - \frac{C(\bar{K})}{x}\right) \end{aligned} \tag{35}$$

where I is the value of (14).

By adopting this strategy, the Central Bank achieves the desired effect on the exchange rate and, besides, raises its level of reserves by an amount $\frac{\eta(\bar{K})C(\bar{K})}{x}$. As in

Section 5.1, the short position the Central Bank can take is limited by the margin requirements of (25); the difference with that case is that now the Central Bank only has a short position, however, the Central Bank has also a higher level of reserves to start with. Therefore, if intervention takes place at a moment τ, at any moment t, $t > \tau$ it has to be the case that

$$\bar{R} + \frac{\eta(\bar{K})C_\tau(\bar{K})}{x_\tau} \geq \frac{m_t^B}{x_t} \tag{36}$$

Note that reserves are purchased at the moment the options are written (intervention takes place) and their value does not change thereafter. As in the zero cost strategy, increases in the exchange rate after the intervention will increase the delta of the option and, therefore, will have the effect of additional sales of the underlying security and purchases of bonds. The closer to the strike price the larger the effect on the delta. It also has to be considered that the purchase of bonds will lower the interest rate and this will have an additional effect (of the opposite sign) on the exchange rate. This effect will then depend on the choice of the strike price.

6. Numerical Exercise

As we have justified, Central Bank intervention with options gives rise to a situation essentially different from the situation resulting from direct intervention. Even when the immediate effect on exchange rates will be the same in both types of intervention, intervention with options will keep affecting the exchange rate as the investment bank adjusts its hedging strategy. Besides, there is the effect on interest rates. The different implications of both types of intervention are not clear. We try to study the effects through a numerical exercise. In our previous analysis we cast the model in a continuous time setting. In order to perform the numerical exercise we must discretize the setting. As usual, we discretize the continuous time stochastic differential equations of the type of (6) in the following way,

$$\Delta r_t = r_{t+\Delta t} - r_t = r_t \left(-\mu_1 + \frac{1}{2}(\sigma_1)^2 \right) \Delta t + \sigma_1 \omega_t \tag{37}$$

where $\omega \sim \mathcal{N}(0, \sqrt{\Delta t})$ and Δt is an appropriate time interval.

The purpose of the exercise will be to simulate the dynamics of the exchange rate when the Central Bank uses direct intervention first and then the dynamics of the same exchange rate when the Central Bank uses intervention with options. The dynamics of the underlying factors will be common to both types of interventions. However, it is to be expected that the dynamics of the exchange rate and interest rate will be different for each type of intervention. In this exercise we will focus on intervention with options as a zero cost strategy as the one described in Section 5.1.

For our numerical exercise we need to simulate two factors (there are two Brownian motion processes explaining the dynamics of the exchange rate), therefore we need

ω_1 and ω_2, using the notation in (37). We will use standard techniques to generate two series of independent pseudo-random numbers (see Press et al. (1992)) and, from them, the whole exchange rate, interest rate and reserve level series. We set upper and lower intervention rates $\bar{x}$ and $\underline{x}$. For convenience purposes we will keep those rates constant throughout the whole period considered. However, as we pointed in Sections 3 and 5 the type of intervention that we have in mind differs from the one that would take place in a target zone, since the existence of a target zone would affect the valuation of the options and, therefore, the hedging strategies of the investment bank. Nevertheless, we will assume that ϵ (defined in Section 3) is constant in order to keep our setting as simple as possible. With regard to the volatility of the exchange rate, equation (17) gives us an exact solution of the volatility value without intervention. However, in the exercise we perform intervention is likely to arise and that will affect the volatility of the exchange rate. Therefore, instead of using the expression of equation (17) we will compute actual path volatility and assume that the investment bank will use the historic volatility in order to price the option (this would most likely be the case in practice). Since the path we consider represents one year of exchange rate history, we need to generate a previous path in order to compute the starting value (for the time period analyzed) of the volatility of the exchange rate. When the exchange rate surpasses one of the boundaries intervention takes place in the way described in Sections 3 and 5 (we will compare both of them).

The implementation of the exercise for the case of direct Central Bank intervention (Section 3) is straightforward. With respect to the implementation of Central Bank intervention with options, we follow the procedure described in Section 5.1. We first choose a strike price and from it derive the number of calls and puts that will yield the desired effect as explained in Section 5. However, a problem in this case is the fact that if we pick a random strike price, the number of calls may be considerably larger or smaller than the number of puts. Since the intervention changes the price of the options the investment bank has to readjust its hedging portfolio as a result of the price changes; if the number of calls and puts is very unbalanced the effect of the adjustment on the exchange rate can be also important. In order to avoid that problem we select the strike price in such way that the effect on the exchange rate and, therefore, the prices of the options, will not require an adjustment in the hedging portfolio of the investment bank. In order to find that strike price we set a grid of strike prices and compute the second order effect. We stop checking when we find a strike price that gives rise to a second order effect smaller than a given preset level. Then we move to the following time considered and compute the new equilibrium exchange rate and interest rate taking into consideration the new shocks (realizations of ω_1 and ω_2) and the effects of the hedge of the investment bank on the demand/supply of bonds and currency.

This exercise provides a framework to study the effects of the two types of intervention considered on the level of reserves, level of interest rates, volatility of the interest rate and volatility of the exchange rate. In order to study the effects on the

reserves level we study the cost in reserves of each type of intervention. In the case of direct Central Bank intervention this problem is straightforward. In the case of intervention with options, it is not as clear since the margins will have an important effect on the level of reserves. As we explained in Section 4.1 we will assume that the Central Bank has to post reserves in order to guarantee margin requirements, computed as expressed in equation (24). We are assuming that, initially, foreign reserves are the only asset in the balance of the Central Bank. In this way we abstract from other monetary policy considerations. However, since the Central Bank also can take long positions in options we will assume that it can use the options as collateral in order to satisfy the margin requirements of the short positions. In order to simplify the numerical computations we will also assume that, for this purpose, the value of the options is the amount by which they are in the margin, that is the right hand side of equation (24) for calls and the corresponding for puts. This value will have to be transformed to the foreign currency dividing by the current exchange rate. This is a simplification that provides a lower bound on the value of the options as collateral, since the value of the option will be larger than the amount by which it is in the money.

We will repeat this exercise over one hundred paths and will compute the average over those paths. This will be an indication of the relative performance of each type of intervention.

7. Analysis of the Results

The numerical framework described in the previous section allows us to do a comparison of the direct intervention with the intervention with options. We can compare the paths of the exchange rate when the Central Bank uses direct intervention (we will denote it by x_d) and when the Central Bank intervenes with options (we will denote it by x_o). Similarly the paths of the equilibrium interest rate under both strategies (we will denote by r_d and r_o respectively). Finally, the cost of reserves (that we denote by c_d and c_o respectively).

The parameters that we will keep constant throughout the exercise are the following, $\bar{x} = 0.30$, $\underline{x} = 0.20$, $\epsilon = 0.01$, $\mu_1 = 0$, $\bar{B} = 1$, $\bar{M} = 1$. We stress the fact that we do not have in mind a target zone regime: as we pointed in Section 5 that would change the value of the options and the hedging strategies. We keep $\bar{x}$ and $\underline{x}$ constant for computational convenience, but they can be interpreted as the reservation prices at which the Central Bank decides to intervene and they could vary from intervention to intervention: our results (assuming that option prices are independent of the intervention level) will not depend on that. We will always consider a positive drift of the exchange rate (some comparative statics are explained later), therefore interventions in the upper boundary are more frequent than in the lower boundary. We believe this is the most interesting exercise.

In the following four tables we study the sensitivity of exchange rate and interest

rate statistics to changes in some of the parameters of the model. We consider four statistics. The first two are the mean over the whole path and the standard deviation over that whole path. Since we generate one hundred paths, the values we report for the mean and standard deviation are the averages of the previous statistics over the hundred paths generated, except for the cost in reserves. Since we compute the final cost of reserves the statistics for the cost of reserves are computed over the hundred paths, i.e., mean of the cost of reserves is the average of final cost of reserves over the one hundred paths generated and similarly for the standard deviation. With respect to the maximum and minimum, they refer to the maximum and minimum over the hundred paths. We certainly do not expect to converge to any values with a small number of paths. Our objective is to make sure that the qualitative results we report are robust, at least for the range of parameters considered.

Some results are consistent for all values of the parameters considered. Since we always choose a positive drift of the exchange rate (which means a tendency towards devaluation) most of the interventions take place at $\bar{x}$, the "weak" boundary. The exchange rate seems to stay, on average, at a higher level when the Central Bank intervenes with options. However, the volatility of the exchange rate is lower under intervention with options; how much lower depends on the parameters. With respect to the interest rate, it stays at a lower level when the Central Bank uses options; besides, it is (in general) less volatile. A result of this is that the Central Bank could also tighten the monetary policy as a complementary action to the intervention in the exchange rate markets. With respect to the cost of reserves, it is always lower when the Central Bank uses options. In fact, it is in all exercises performed negative (which means an accumulation of reserves). Overall, then, our exercise would seem to suggest that Central Bank intervention with options is a better choice than the traditional intervention in currency markets.

In Table 1 we study the sensitivity of the statistics to a_1, the cross exchange rate-interest rate elasticity. The level of the exchange rate is very similar under both types of intervention (but lower under "direct" intervention) and decreases with the elasticity. The volatility of the exchange rate is always lower for the case of intervention with options. It seems to decrease as the elasticity increases. The range of exchange rates increases. Interest rate is lower when the Central Bank intervenes with options, but the spread shrinks as the cross-elasticity increases (it is constant for the case of direct intervention, since in this case it does not depend on the elasticity). The reason is that, as the exchange rate decreases the value of the calls decreases (the argument works in the opposite direction for the puts); as the value of the calls falls, the hedge of the investment bank requires a smaller long position in domestic bonds and that pushes up the interest rate. With respect to the cost in reserves, intervention with options is always more efficient, but the spread in efficiency decreases as the cross-elasticity parameter increases.

Table 2 analyzes changes in the statistics as the "intrinsic" volatility of the interest rate (that we denote σ_1) changes. Also the drift of interest rates goes up. The level

of the exchange rate is almost constant for both types of intervention. Exchange rate volatility goes up for both types of intervention but not very fast. Both interest rates and interest rate volatility go up (as expected). Intervention with options seems to make interest rate volatility less sensitive to changes in σ_1 as a result of the "feedback" effect resulting from the hedging strategy of the investment bank. The spread of efficiency with respect to the cost in reserves increases as the volatility increases.

Table 3 does the same comparative statics but for σ_2, the volatility parameter specific to exchange rates. The exchange rate level seems to drop slightly in both types of intervention (probably as a result of the larger number of interventions). Exchange rate volatility grows under both strategies as expected. The interest rate level (obviously constant for the case of direct intervention) decreases in the case of intervention with options as the volatility increases. The delta of the investment bank is larger and, therefore, its position in bonds is also larger (yielding smaller interest rates). Volatility of interest rates in the case of intervention with options increases as the volatility increases. The spread in the cost in reserves increases very fast as the volatility increases (the cost of reserves increases for the case of direct intervention while the accumulation of reserves increases in the case of intervention with options).

Table 4 considers the effects of changes in μ_2, the expected increase in exchange rates; higher μ_2 makes upper boundary interventions more likely. An increase in μ_2 produces increases in the level of exchange rates (very similar in both cases). The volatility of exchange rates decreases in both cases. The interest rate is obviously not affected by changes in the expected increase of exchange rates in the case of direct intervention; however, it decreases in the case of intervention with options: there will be more interventions in the upper boundary and the investment bank will have to keep larger positions in bonds. The volatility of the interest rate it the latter case hardly changes (it is obviously constant in the former). The spread in the cost efficiency increases.

8. Conclusions

In this paper we have attempted to compare the effects of "direct" Central Bank intervention (with reserves) and intervention with options. We develop a framework that allows us numerical comparisons between both types of interventions. In the economy we consider there is a bond market and a currency market. Central Bank intervention with options affects both markets through the hedge the investment bank that deals with the Central Bank has to undertake. Direct intervention only affects the currency markets; intervention with options also affect bond markets. In order to study the effects of intervention with options we have to assume that the type of interventions we consider are unpredicted by the market (we deviate from the interventions that would take place in a target zone regime). Otherwise the valuation of the options (and computation of the hedging strategies) would become considerably more complex. Our numerical exercise seems to suggest that Central Bank inter-

vention with options would be more efficient from the point of view of the use of reserves. It also has the advantage that an intervention that aims at strengthening the domestic currency would also lower the interest rate level; however this effect depends on the cross-elasticity between the bond market and the currency market. In summary, Central Bank intervention with options seems a good alternative to direct Central Bank intervention but its efficiency depends heavily on the parameters of the economy, in particular in the cross-elasticity currency market-bond market.

References

[1] Biger, N. and J. Hull, 1983, "The Valuation of Currency Options," *Financial Management* 12, 24–28.

[2] Black, F. and M. Scholes, 1973, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* 3, 637–654.

[3] Dumas, B., L.P. Jennergren and B. Näslund, 1995, "Realignment Risk and Currency Option Pricing in Target Zones," *European Economic Review* 39, 1523–1544.

[4] Eaton, J., and S. Turnovsky, 1984, , "The Forward Exchange Market, Speculation, and Exchange Market Intervention," *Quarterly Journal of Economics* 99, 45–69.

[5] Garber, P. and M. Spencer, 1994, "Foreign Exchange Hedging with Synthetic Options and the Interest Rate Defense of a Fixed Exchange Rate Regime," working paper, Intenational Monetary Fund.

[6] Garman, M. and S. Kohlhagen, 1983, "Foreign Currency Option Values," *Journal of International Money and Finance* 2, 231–237.

[7] Grabbe, J., 1983, "The Pricing of Call and Put Options on Foreign Exchange," *Journal of International Money and Finance* 2, 239–253.

[8] Hernández D., A. and F. Zapatero, 1997, "Monetary Policy in a Target Zone with Stochastic Capital Flows," working paper, ITAM.

[9] Hull, J., 1993, *Options, Futures and other Derivative Securities*, Prentice-Hall, New York.

[10] Krugman, P., 1991, "Target Zones and Exchange Rate Dynamics," *Quarterly Journal of Economics* 106, 311–325.

[11] Press, W., S. Teukolsky, W. Vetterling and B. Flannery, 1992, *Numerical Recipes in C: the Art of Scientific Computing,* Cambridge University Press, New York.

Table 1

$\sigma_1 = 0.02, \sigma_2 = 0.02, \mu_2 = 0.2$

	$a_1 = 0.2$		$a_1 = 0.3$		$a_1 = 0.4$	
	x_o	x_d	x_o	x_d	x_o	x_d
Mean	0.28104	0.27920	0.27556	0.27466	0.26206	0.26159
SD	0.01484	0.01511	0.01440	0.01478	0.01269	0.01294
Maximum	0.28849	0.29037	0.28715	0.28398	0.28040	0.28088
Minimum	0.26226	0.25476	0.24603	0.24603	0.22873	0.22697
	r_o	r_d	r_o	r_d	r_o	r_d
Mean	2.59084	2.74656	2.65956	2.74656	2.70205	2.74656
SD	0.25476	0.33763	0.26080	0.33763	0.26193	0.33763
Maximum	3.33363	3.41823	3.42240	3.41823	3.47204	3.41823
Minimum	2.09392	2.18359	2.13273	2.18359	2.15765	2.18359
	c_o	c_d	c_o	c_d	c_o	c_d
Mean	-0.12486	0.36537	-0.07592	0.27700	-0.04483	0.21088
SD	0.11715	0.19402	0.09271	0.19319	0.06405	0.17464
Maximum	0.00637	0.89779	0.00637	0.82002	0.02442	0.72186
Minimum	-0.41694	0.04005	-0.41694	0.00000	-0.42241	0.00000

Table 2

$\sigma_2 = 0.02, \mu_2 = 0.2, a_1 = 0.3$

	$\sigma_1 = 0.01$		$\sigma_1 = 0.02$		$\sigma_1 = 0.03$	
	x_o	x_d	x_o	x_d	x_o	x_d
Mean	0.27624	0.27531	0.27556	0.27466	0.27488	0.27368
SD	0.01388	0.01432	0.01440	0.01478	0.01506	0.01552
Maximum	0.28709	0.28710	0.28715	0.28398	0.28563	0.28691
Minimum	0.24928	0.24928	0.24603	0.24603	0.24352	0.24349
	r_o	r_d	r_o	r_d	r_o	r_d
Mean	2.64047	2.72667	2.65956	2.74656	2.69133	2.77828
SD	0.15104	0.16719	0.26080	0.33763	0.38297	0.51495
Maximum	3.03386	3.04122	3.42240	3.41823	3.89227	3.87050
Minimum	2.34286	2.42786	2.13273	2.18359	1.94104	1.97731
	c_o	c_d	c_o	c_d	c_o	c_d
Mean	-0.06249	0.26879	-0.07592	0.27700	-0.07734	0.29021
SD	0.06904	0.19007	0.09271	0.19319	0.09083	0.19646
Maximum	0.01778	0.80559	0.00637	0.82002	0.03005	0.82787
Minimum	-0.34215	0.00000	-0.41694	0.00000	-0.47598	0.00000

Table 3

$\sigma_1 = 0.02, \mu_2 = 0.2, a_1 = 0.3$

	$\sigma_2 = 0.01$		$\sigma_2 = 0.02$		$\sigma_2 = 0.03$	
	x_o	x_d	x_o	x_d	x_o	x_d
Mean	0.28305	0.28311	0.27556	0.27469	0.26978	0.26790
SD	0.00910	0.00890	0.01440	0.01472	0.01923	0.02114
Maximum	0.28990	0.29006	0.28715	0.28704	0.28424	0.28463
Minimum	0.26627	0.26627	0.24603	0.24603	0.23844	0.23870
	r_o	r_d	r_o	r_d	r_o	r_d
Mean	2.68842	2.74656	2.65956	2.74656	2.63544	2.74656
SD	0.25605	0.33763	0.26080	0.33763	0.27167	0.33763
Maximum	3.41823	3.41823	3.42240	3.41823	3.46765	3.41823
Minimum	2.14471	2.18359	2.13273	2.18359	2.11690	2.18359
	c_o	c_d	c_o	c_d	c_o	c_d
Mean	-0.03325	0.18504	-0.07592	0.27700	-0.13183	0.42276
SD	0.03922	0.11812	0.09271	0.19319	0.14285	0.24867
Maximum	0.00542	0.49113	0.00637	0.82002	0.05101	1.12851
Minimum	-0.22452	0.00000	-0.41694	0.00000	-0.71117	0.00000

Table 4

$\sigma_1 = 0.02, \sigma_2 = 0.02, a_1 = 0.3$

	$\mu_2 = 0.1$		$\mu_2 = 0.2$		$\mu_2 = 0.3$	
	x_o	x_d	x_o	x_d	x_o	x_d
Mean	0.27153	0.27008	0.27556	0.27466	0.27872	0.27810
SD	0.01502	0.01568	0.01440	0.01478	0.01435	0.01386
Maximum	0.28561	0.28611	0.28715	0.28398	0.28623	0.28822
Minimum	0.23530	0.23521	0.24603	0.24603	0.25542	0.25487
	r_o	r_d	r_o	r_d	r_o	r_d
Mean	2.69050	2.74656	2.65956	2.74656	2.62519	2.74656
SD	0.25683	0.33763	0.26080	0.33763	0.26509	0.33763
Maximum	3.45710	3.41823	3.42240	3.41823	3.39684	3.41823
Minimum	2.16306	2.18359	2.13273	2.18359	2.11400	2.18359
	c_o	c_d	c_o	c_d	c_o	c_d
Mean	-0.04997	0.21366	-0.07592	0.27700	-0.10142	0.35130
SD	0.06278	0.17169	0.09271	0.19319	0.11453	0.20845
Maximum	0.00732	0.71059	0.00637	0.82002	0.00087	0.91964
Minimum	-0.38137	0.00000	-0.41694	0.00000	-0.67793	0.00000