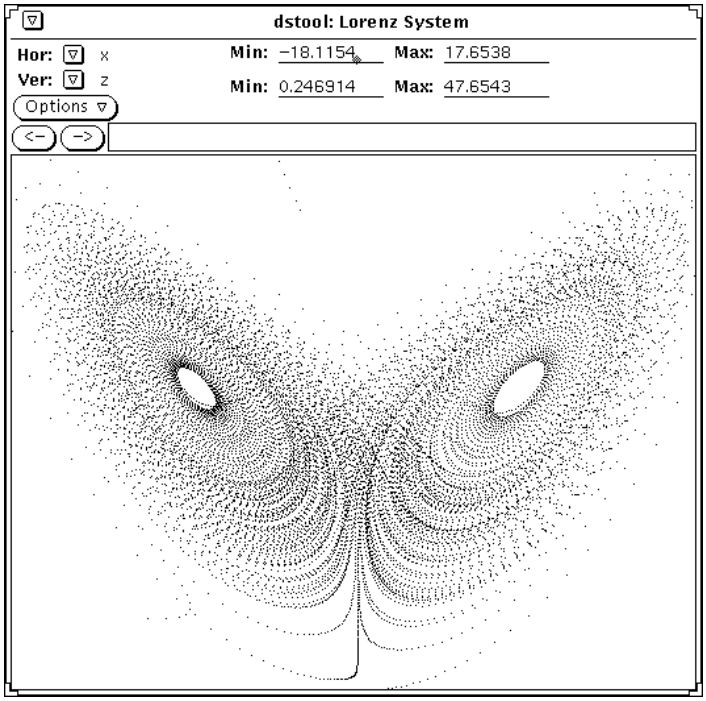

Figure 1.6: Zooming In

1.2.8 Other Mouse Button Functions in View Windows

Since **Refresh** is the default choice of the **Options** menu button, instead of opening the menu, you can simply click SELECT on the **Options** button to refresh the view in the window. Buttons in **DsTool** have their first menu entry as default choice.

You can clear both the view window and the trajectory points in memory by clicking Control-ADJUST in a view window canvas.

By holding and dragging Control-SELECT, you can graphically select a rectangle to evolve under the flow. See Section 1.2.15 below for a complete description.

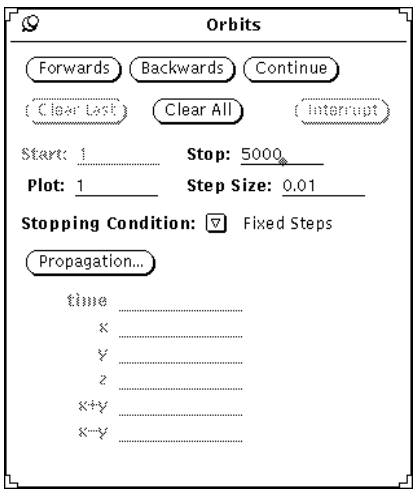

Figure 1.7: The Orbits Window

1.2.9 Quitting and Interrupting

Typing **Control-C** in the original window from which you invoked **DsTool** pops up a notifier box offering the options of either interrupting the current computation, quitting **DsTool**, or canceling. Try this now, and using SELECT, choose the **Cancel** option. You can also quit **DsTool** by choosing the **Quit** entry after clicking MENU in the header of the original **command** window.

1.2.10 Orbits Panel

Additional control of the computation of orbits can be obtained with the **Orbits** panel. With the mouse in the **DsTool command** window and using MENU, select the **Orbits...** entry from the **Panels** menu button. A new panel *(figure 1.7)* will open.

The top two lines of the panel contain buttons **Forwards**, **Backwards**, **Continue**, and **Clear All** which correspond to the mouse actions within the view window already described.

Below these are several text fields which control essential parameters. Edit the **Plot:** value to 100 and the **Step Size:** to `.001`. Using SELECT, click on **Forwards**. `5000` points (the value in the **Stop:** field) will still

be computed but now at time intervals of `.001` with only every hundredth point plotted or stored in memory. You can monitor this by watching the **Final time** entry in the **Selected Point** panel, and noting the number of points in memory displayed to the right of the **Settings** button in the main **command** window.

Now restore the **Plot:** field to `1` and the **Step Size:** to `.01`. Clear the view window by clicking **Clear All** inside the **Orbits** panel. Open the abbreviated menu button to the right of **Stopping Condition**. Replace the **Fixed Steps** entry currently chosen by **Event Stopping**. Most of the text fields at the bottom of the **Orbits** panel now darken indicating that their fields are editable. Enter `0` in the lines to the right of $\mathbf{x}+\mathbf{y}$ and $\mathbf{x}-\mathbf{y}$ respectively.

Now click on the **Forwards** button once to make sure the direction is set, and then repeatedly **Continue**. Orbits will be computed until either $x+y$ or $x-y$ reaches `0` or `5000` points are computed. You'll be able to see the orbits wind around near the attractor, as you repeatedly continue them.

You can also select the **Propagation...** panel to control both the choice of integration algorithm and parameters for it.

1.2.11 How to Change Parameters

The Lorenz system contains three parameters, denoted σ, β and ρ. The default values for the parameters are $\sigma = 10$, $\beta = 8/3$, and $\rho = 28$. These values appear in panel text items located on the main panel. The default values correspond to the values originally studied by Lorenz. For the default value of $\rho = 28$, the Lorenz system possesses a global attractor, but for other parameter values different initial conditions may end up in different attractors. This exercise will help you to learn how to alter parameter values and to control the initial conditions with a high degree of accuracy.

One method of changing parameters is by editing text fields in the **Selected Point** panel. If it is not already open, open it with MENU from the menu entry **Selected...** under the **Settings** menu button of the **command** window. *(Figure 1.8)* Change the text item **rho** to the value `19`.

Clear the **view window** by Control-ADJUST and make sure the xz plane is displayed. Restore the range of **x** values to the original `-30` to `30` and the **z** to `-5` to `50`. On the **Selected Point** panel, set the value of the items **x** and **z** to `0.1`, and set the value of **y** to `-0.1`.

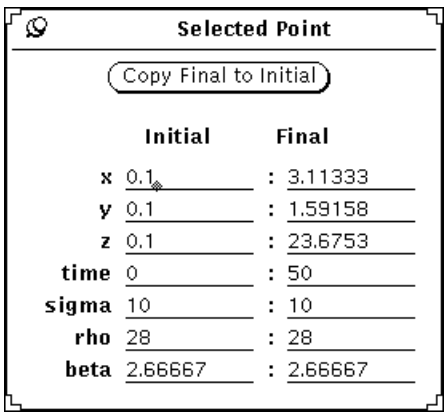

Figure 1.8: The Selected Point Window.

Now select the button item **Forward** in the **Orbits** panel. The orbit starting at the point $(0.1, -0.1, 0.1)$ is drawn in towards an attracting fixed point. Select **Continue** and watch the trajectory spiral in to the sink.

Now change the value of **y** to `-0.099`, and select **Forward** again. The orbit of the point $(0.1, -0.099, 0.1)$ is attracted to a second (different) attracting point! *(Figure 1.9)*

You can also use the mouse to change parameters. By clicking SELECT on the **View** menu button of the **command** window, open a second **2-D Image** window. Change the horizontal and vertical variables being displayed to the parameters σ and ρ. Note that by clicking SELECT on different locations in this window, the values of σ and ρ in the **Selected Point** panel change accordingly. You can mark the selected point on this window by selecting **Display...** from the **Options** menu button on the view window. The popup box allows you to select various symbols and sizes to mark locations in the **view window**. Click SELECT on **Add Point** to actually mark a location with the symbol you have selected.

Being able to control the initial conditions of a trajectory with a high degree of accuracy is often crucial in the exploration of dynamical systems, and in these cases it is often preferable to input initial conditions as text items rather than via a mouse.

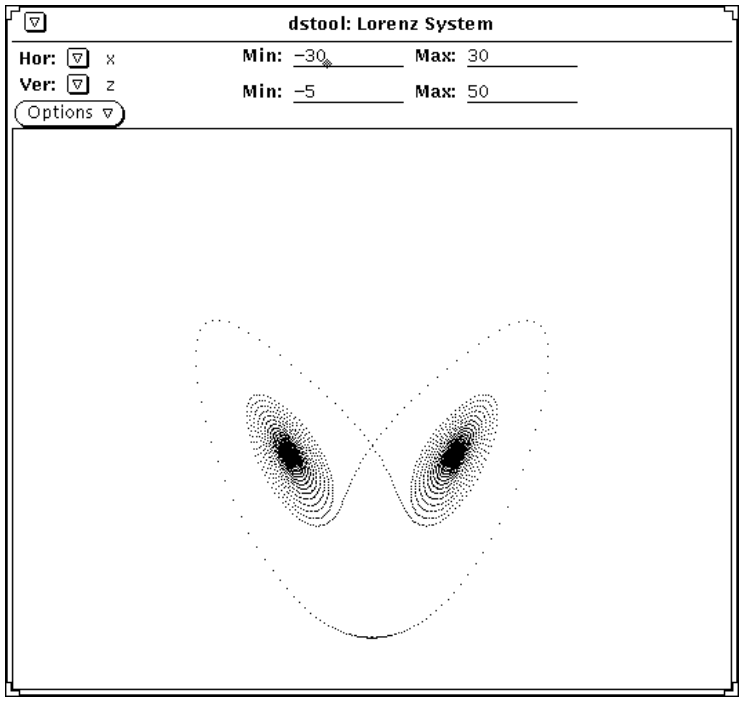

Figure 1.9: Similar initial conditions with different fates

1.2.12 Loading Other Dynamical Systems and Restoring Defaults

The abbreviated menu button to the right of **Model:** in the **command** window can be used to load a new dynamical system or reload the defaults of an old one. This button holds a **pullright menu** where each of the entries **Mapping Examples**, **Vector Field Examples**, and **Special Dynamical Systems** contains a submenu which may be displayed by moving right while continuing to hold the MENU button. This can also be used to conveniently restore defaults for a system. Follow the **Vector Field Examples** entry to the right while holding MENU and select **Lorenz System** again. Now the values of parameters and variable ranges for graphing are as they originally were.

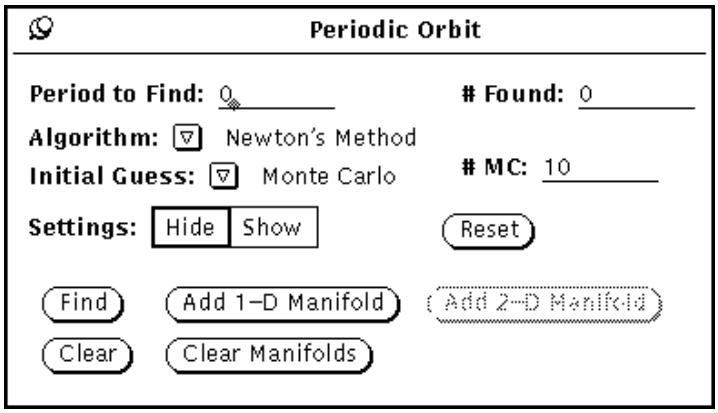

Figure 1.10: The Fixed Point Window.

1.2.13 Computing Equilibria

If you have not already done so, use the procedure of Section 1.2.12 to restore the default **Lorenz System**. Adjust your view window to the **xz** plane.

We shall numerically compute the equilibria for the Lorenz system. First using the **Panels** menu button of the **DsTool command** window, click with MENU on the **Fixed Point...** entry. The **Fixed Point** panel appears. *(Figure 1.10)* The entry 0 in the **Period to Find:** text field in the upper left corner indicates to **DsTool** a wish to find equilibria. Using SELECT, press the **Find** button in the fifth line of the panel, and **DsTool** will use Newton's method starting at a random point to search for equilibria. The number of tries (by default 10) is specified in the **# MC:** field, and the number of equilibria found is reported in the **# Found:** field. **DsTool** marks saddles in view windows with green plus signs, sinks as blue triangles, and sources as red squares. After one or two presses of **Find**, you'll almost certainly have located the three saddles of the **Lorenz system**. *(Figure 1.11)*

To the right of **Settings:** on the fourth line of the **Fixed Point** panel is the **Hide/Show** choice box. Click SELECT on **Show** and the panel will expand, with a number of parameters accessible for editing. *(Figure 1.12)* You can also change the **Algorithm:** and **Initial Guess:** methods via the abbreviated menu buttons to the right of these labels.

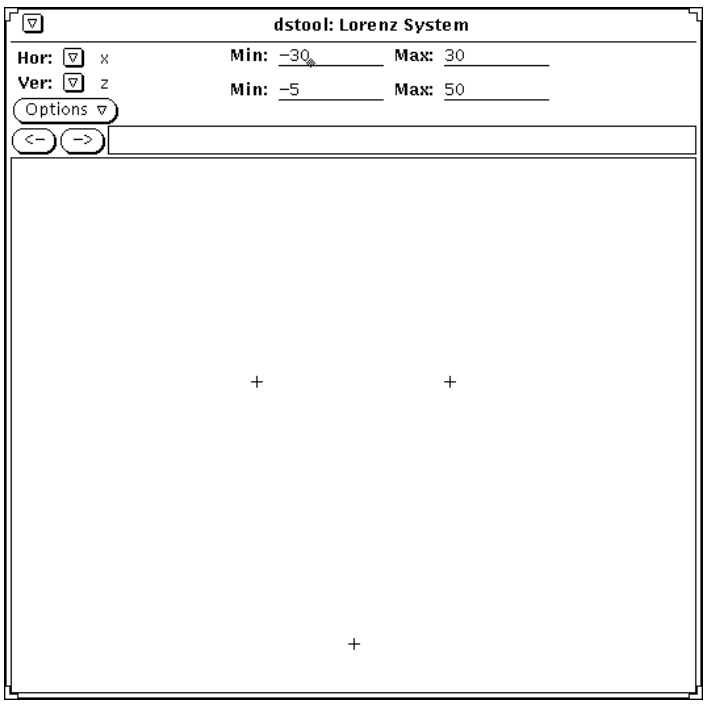

Figure 1.11: Equilibria Found

1.2.14 Computing Manifolds

Before beginning this section of the tutorial, Section 1.2.13 must be completed. If the three equilibria have been found and marked, then we may compute the stable and unstable manifolds of the hyperbolic saddle located at the origin. At the moment, when these are of dimension greater than `1`, **DsTool** computes one dimensional pieces of these tangent at the equilibrium to the eigen-directions of the linearization, for those eigen-directions with real eigen-values.

First, in the **Fixed Point** panel, click SELECT on the **Show** choice box to the right of the **Settings:** entry. We will edit the **1-D Manifold** entries to produce longer, more detailed stable and unstable manifolds. Select the input field to the panel text item **Unstable Manifold Divisions per Step:**, and change the default value of `1` to `3`. This determines the number of interpolating points which we will use to "fill in" the space between two calculated points on the unstable manifold. Below this field, change the

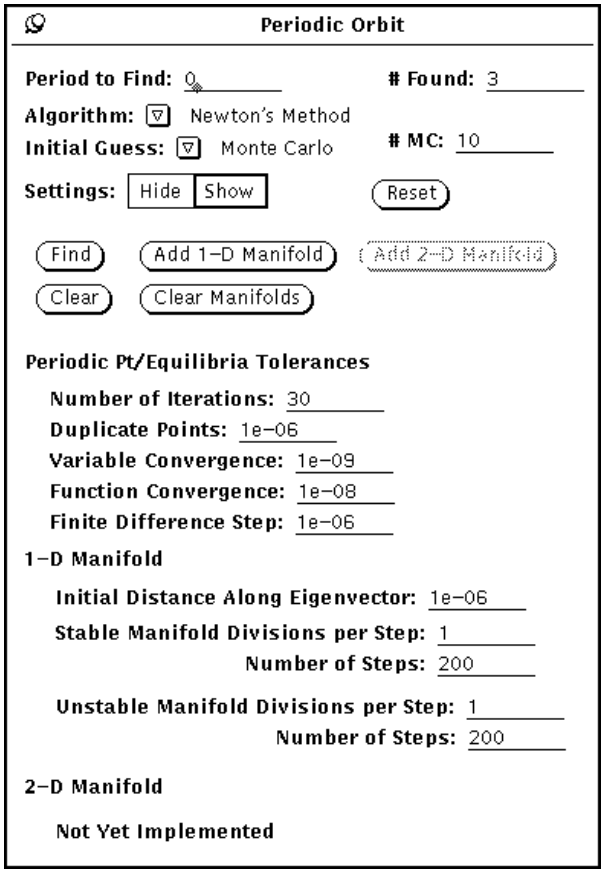

Figure 1.12: The Full Fixed Point Panel.

Number of Steps: default entry of `200` to `2500`. This basically determines the length of the unstable manifold: the bigger this value is, the longer the manifold will be.

Above the entries for unstable manifolds are corresponding ones for stable manifolds. Edit the **Stable Manifold Divisions per Step:** text value to `10`.

After setting the above values, select the button item **Add 1-D Manifold** in the **Fixed Point** panel. *(Figure 1.13)* If you are on a color monitor, **DsTool** draws the stable manifolds in red and the unstable manifold in blue.