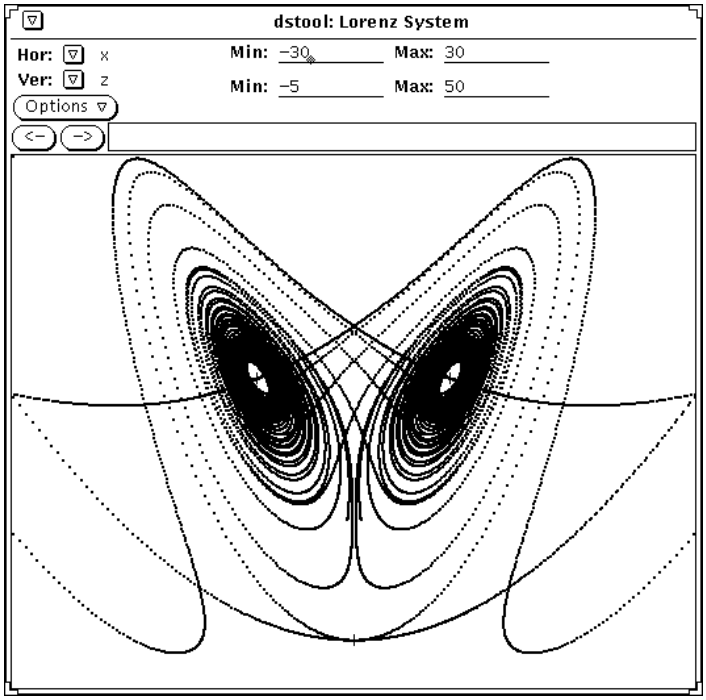

Figure 1.13: Stable and Unstable Manifolds

1.2.15 Multiple Orbits

DsTool supports the convenient display of the evolution of isoriented (edges parallel to coordinate axes) n-cubes or their boundaries under the flow of a dynamical system.

To see this feature, first reload the **Lorenz** system defaults using the procedure described in Section 1.2.12. Now open up the **Multiple Orbits** window by selecting (with MENU) the **Multiple...** menu entry under the **panels** menu button of the **command** window. The lengths of the sides of the cube may be edited in the **Size** column at the bottom of the panel, and the number of division points along each edge in the **Number** column. In the **Size** column, change the **x** and **y** entries to `5`. In the **Number of Points** column, change these two (**x** and **y**) entries to `20`. By default, the **Load:** choice box should be set to **Rectangle** indicating that only the boundary of the specified n-cube will be evolved; selecting **Region** here would also include a mesh of interior points of the specified n-cube. Click SELECT on

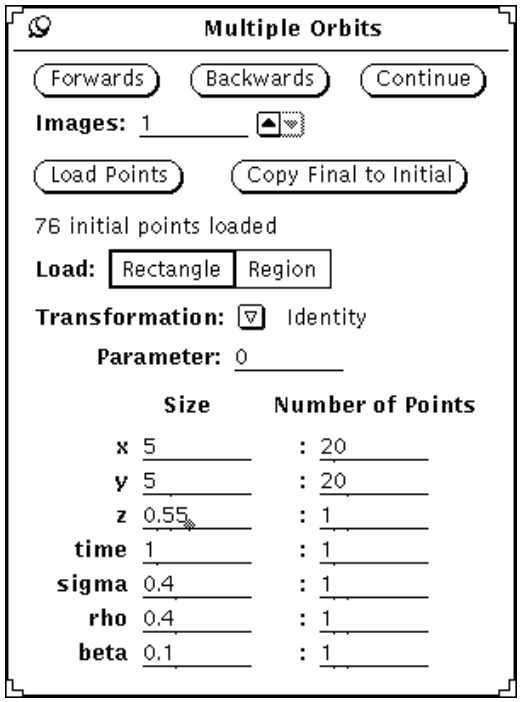

Figure 1.14: The Multiple Orbits Window.

the **Load Points** button near the top of the **Multiple Orbits** panel. The message *76 initial points loaded* will appear. *(See figure 1.14.)*

Before actually evolving the rectangle, make sure you have the **Orbits** panel and a **2-D Image** window open (with **x** and **y** as coordinates displayed.) Recall these are summoned by menu entries of the **Panels** and **View** menu buttons respectively on the main **Command** window. On the **Orbits** panel, edit the **Stop** text field (to the right on the third line) to `1` instead of the default `5000`.

Now back on the **Multiple Orbits** panel, click SELECT on **Forwards**. In the **view** window, you will see the original rectangle and its evolution one time step later. Click on **Continue** several times and you will see the rectangle evolve towards the line $x = y$. *(Figure 1.15)*

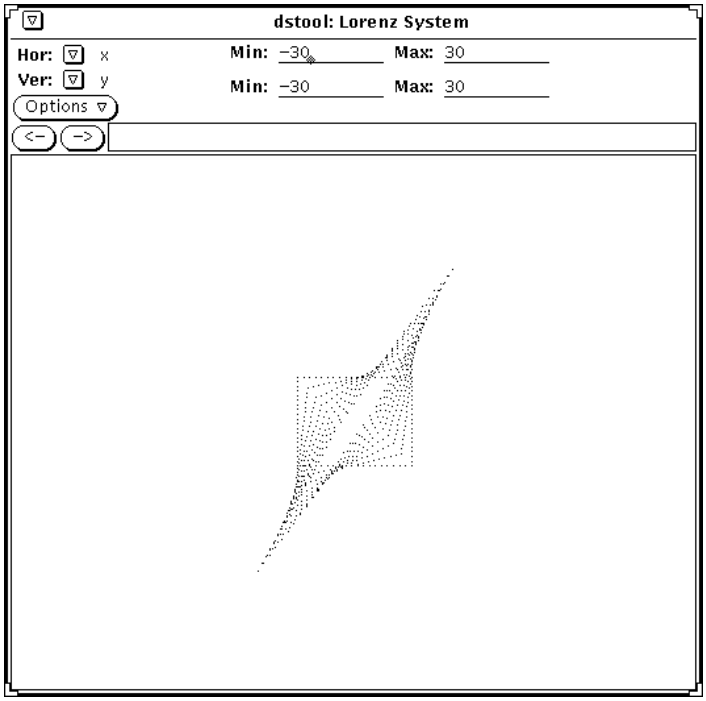

Figure 1.15: Evolution of an xy plane rectangle centered near the origin.

1.2.16 How to Print

The phase portraits and bifurcation diagrams produced in **DsTool** may be printed on a laser printer. Choose the menu entry **Print...** from the menu under the **Options** menu button of a **2-D Image** window. A pop-up panel labeled **Print** will appear. Near the bottom of this panel is the **Destination:** choice box. Using SELECT, choose either **Printer** or **File**. If **Printer** is chosen, edit the **Printer:** name to an appropriate local printer. If **File** is chosen, edit the **Directory** and **File** text fields below in this panel to specify the file you wish to create. Now clicking on the button **Print** will send a PostScript file to either the printer or file.

1.2.17 Use of Color

The use of the color palette in **2-D Image** windows is programmable. To adjust this, use MENU to select the **Display...** menu entry from the **Op-**

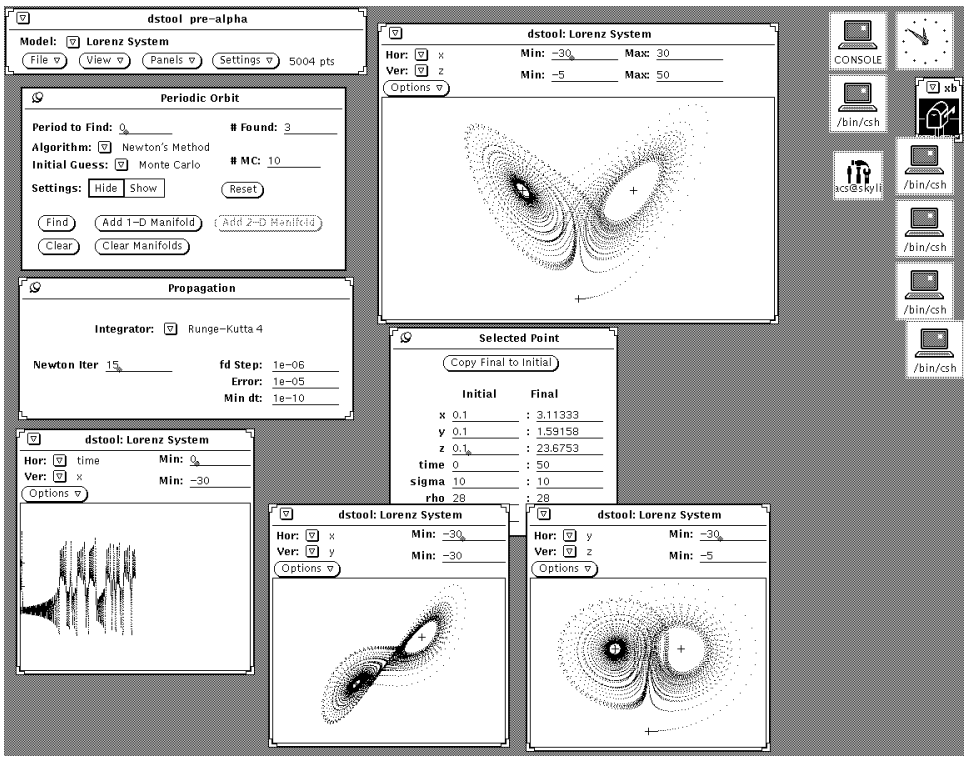

Figure 1.16: Using Multiple Windows

tions menu button of this window. The **2-D View Options** panel will appear. Options supported include using the menu button to the right of **Colormap Type:** to change to the entry **Pick Colors**. When this option is selected, the user can click SELECT on a color in the palette directly above the drawing area. This will then set the chosen color as the color for drawing orbits.

1.3 A Torus Diffeomorphism

Despite the richness of the Lorenz system, there are aspects of **DsTool** that we did not explore in the previous example. This example is designed to explore some of the remaining elementary features of **DsTool**.

This example is a diffeomorphism of the two dimensional torus $\mathcal{T}^2 = S^1 \times$

S^1 onto itself. The map is given by a non-linear perturbation of translations of the torus:

$$\mathbf{f}(\mathbf{x}) = \mathbf{x} + \mathbf{\Omega} + \mathbf{g}(\mathbf{x})$$

where $\mathbf{x} = (x, y) \in \mathcal{T}^2$, $\mathbf{\Omega} = (\omega_x, \omega_y) \in \mathbf{R}^2$, and

$$\mathbf{g}(\mathbf{x}) = \left(\frac{-ab}{2\pi} \sin 2\pi y, \frac{-a}{2\pi b} \sin 2\pi x \right).$$

Because the map is from the torus to itself, the phase space in this example is the unit square with opposite edges identified.

1.3.1 Choosing an Installed Dynamical System

The first task is to load the torus diffeomorphism into **DsTool**. To do this, hold down MENU on the abbreviated menu button to the right of **Model:** in the **command** window and follow the pullright menu entry **Mapping Examples** to the entry **Kim-Ostlund Torus Map**.

The parameter **b** above is labeled in the **2-D Image window** as **asymm**.

1.3.2 Orbits of Maps

Forward orbits of maps are generated in the same manner as generating the forward orbit of a flow. Position the pointer within the **2-D Image** window; click SELECT to set an initial point and ADJUST to iterate the map forward. Continue the iteration with ADJUST. You can also use the **Orbits** and **Selected Point** panels as with vector fields. You will see that for the default parameter values almost every initial condition is mapped asymptotically towards an invariant curve which winds around the torus.

Since this map is a diffeomorphism (for small values of the nonlinearity a), the inverse of the map exists, and therefore we can compute backward orbits. Click MENU followed by ADJUST a few times and convince yourself that almost every initial condition is backwards asymptotic to a period five orbit for the default parameter values. *(Figure 1.17)*

1.3.3 More on Fixed Points

For the default parameter values, the torus map has two distinct sets of periodic points of period five. Open the **Fixed Point** panel as described