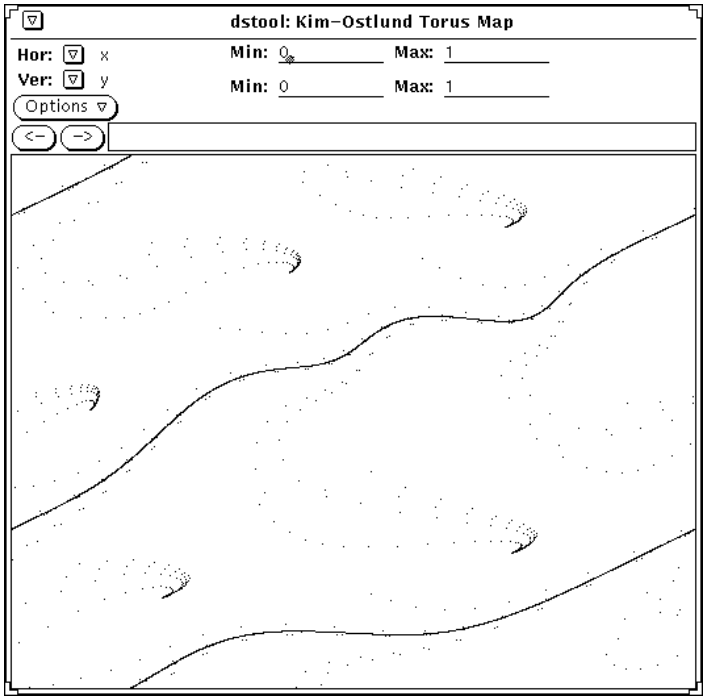

Figure 1.17: Orbits of a torus diffeomorphism

in Section 1.2.13. We want to find the periodic points of period five, so we select the input field for the text item **Period To Find:**, and set its value to 5. Now click SELECT on the **Find** panel button item. There are two orbits of period five. One orbit is attracting; we saw this orbit in the previous section. The other orbit consists of five saddle points, so we did not observe this orbit when we computed random trajectories. It is almost always the case that **DsTool** finds both sets of period five points on the first try. If the text item **# Found:** does *not* read 2 after selecting **Find**, then try selecting **Find** until it does.

1.3.4 More on Manifolds

Once the periodic points of period 5 are found, we can compute the stable and unstable manifolds of these points. In the periodic orbit panel, first click SELECT on the **Show** choice box next to the label **Settings**. This will

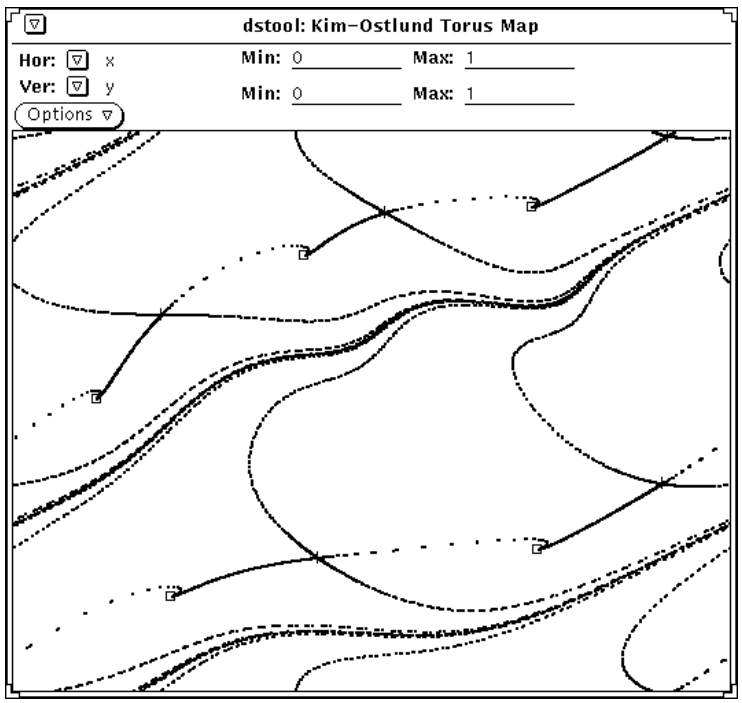

Figure 1.18: Stable and Unstable Manifolds on the Torus

reveal a large collection of parameters which can be adjusted. Under **1-D Manifold**, change the value of **Stable Manifold Divisions per Step** to `3`. This determines the number of interpolating points which we will use to "fill in" the space between two calculated points on the unstable manifold. Immediately below the **Stable Manifold Divisions per Step** entry, edit the **Number of Steps** text field to the value `300`. This basically determines the length of the unstable manifold: the bigger this value is, the longer the manifold will be. Change the corresponding **Unstable Manifold** entries similarly. Then select the button item **Add 1-D Manifold**. As we saw in the Lorenz system, on a color monitor **DsTool** displays the stable manifolds in red and the unstable manifolds in blue. *(Figure 1.18)*

An often discussed phenomenon in the study of nonlinear dynamics is the transverse intersection of the stable and unstable manifolds of a non-integrable system. These intersections imply the existence of infinitely many saddle points and the presence of Smale Horseshoes. The torus map has a

homoclinic orbit which intersects itself transversally for a certain parameter range, so our goal in the next few paragraphs is to view this transverse intersection. As a word of warning, the computation involved is rather extensive, and may take a few minutes.

First we'll adjust the number of significant digits possible in panel items. Going back to the **DsTool command** window, open the **Settings** menu button to select the entry **Defaults....** On the window which pops up, change the **Window Precision:** entry from `6` to `12`.

Now if it is not already open, use the **Settings** menu button to open the **Selected Point** window. Select the input field of the text item **wx:** in this panel, and replace the old parameter value with the value `0.59547797038`. Now clear the **2-D Image** window via Control-MIDDLE and then create the periodic orbit panel (if it is not already created) as is detailed in section 1.2.13.

First, in the **Fixed Point** panel, click SELECT on the **Show** choice box to the right of the **Settings:** entry. We will edit the **1-D Manifold** entries to produce more complete stable and unstable manifolds. As before, we want to modify how "long" the manifolds are and how "fine" they are. Change the input fields of the following text items on the **Fixed Point** panel:

- Change **Unstable Manifold Divisions per Step:** to `20`.
- Change **Number of Steps:** immediately below this to `600`.
- Change **Stable Manifold Divisions per Step:** to `10`.
- Change **Number of Steps:** immediately below to `100`.
- Change the text item **Initial Distance Along Eigenvector:** so that it reads `1e-07`.

Lastly, make sure that the text item **Period to Find:** is still set to the value `5`.

We are now ready to begin the computation of the stable and unstable manifolds. First select the button item **Clear** in the **Fixed Point** panel. This action resets the text item **# Found** to the value `0`. Now press **Find** in the panel. If **DsTool** does not find two period orbits of period five, then continue to select **Find**. Once the periodic points are found, choose the

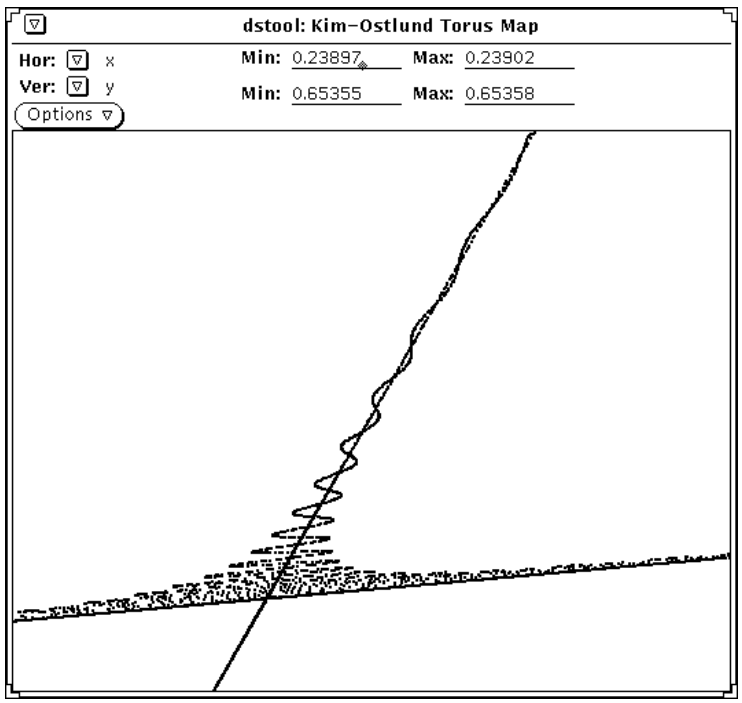

Figure 1.19: A homoclinic tangle

button item **Add 1-D Manifold**. When the manifolds have been computed, we want to blow up a region about a saddle point. Using the text items **Min:** and **Max:** located in the **2-D Image** window, change the x-range to $[0.23897, 0.23902]$ and change the y-range to $[0.65355, 0.65358]$. Now select **Refresh** and the homoclinic tangle will appear.*(Figure 1.19)*

1.3.5 Saving and Loading Data

The Kim-Ostlund torus map undergoes many bifurcations as the parameters are changed [4], [5], [6]. In investigating the behavior of a complicated dynamical system, researchers often find it convenient to save system information associated with interesting dynamics. In this section we will save information about the torus map to a file and then retrieve the information.

If necessary, open the **Selected Point** panel using MENU on the **Settings** menu button of the **DsTool command** window. Clear the **2-D Image**

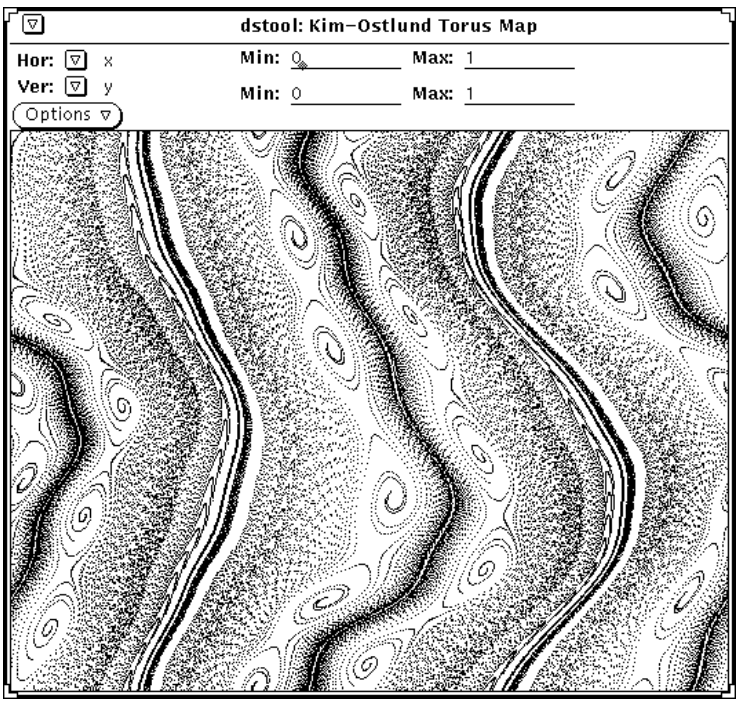

Figure 1.20: A phase portrait on the torus

window with Control-MIDDLE. Then change the value of **wx** to `0.5` and change the value of **wy** to `0.8`. Generate a phase portrait for these parameter values. *(See figure 1.20 .)*

Suppose that you wish to save this phase portrait. With **DsTool** you have several options. The first option (the default) is to save only the information *about* the phase portrait. This includes the name of the dynamical system being studied, the parameter values, values of some tolerances used in numerical computations, and a few lines worth of other information. *The default option does not save any data points.* The advantage of this option is that it does not require the storage of the large amount of data which actually comprise the phase portrait – yet the phase portrait may be fully reconstructed.

Using MENU on the **File** menu button of the **command** window, select the menu entry **Save...** to open the **Save** panel. Edit the **Directory:** and **Filename:** fields appropriately to set the name of the data file in some direc-

tory where you have write access. Options specify exactly which data is saved. Use SELECT to toggle these choices on or off. Choices are boxed within darker rectangles when they have been selected. In this case, SELECT the option **Current Settings**. Now click SELECT on the **Save** button at the bottom of the panel to actually save the data.

To reload this data at some later point, open the **Load** panel by selecting with MENU the **Load...** menu entry button of the **File** menu button on the **command** window. Edit the **Directory:** and **Filename:** fields to the same choices you made before. Maintain the option **DsTool Format** right now. (The **DsTool** Reference Manual describes how to import unformatted data into a **DsTool** memory object.) Clicking SELECT on the **Load** button will then restore the **DsTool** settings.

We remark that saving and loading trajectory data is significantly slower than merely saving the **Current Settings** values. The length of time needed to save or load data depends on the number of data points, the dimension of the phase space, and the particular machine running **DsTool**.

The pre-installed models of **DsTool** are contained in the directories $DSTOOL/src/models and $DSTOOL/src/models/contrib/def. The doc and data subdirectories of $DSTOOL/src/models/contrib have further information on these. In addition, many user panels below the directory $DSTOOL/contrib have doc subdirectories describing them. These include the **Parser** allowing one to enter a new model without recompilation, and the **Browser** for inspecting numerical values.

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