“The optimum quantity of money” de Milton Friedman.

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Chapter 1

The Optimum Quantity of Money

IT IS A COMMONPLACE of monetary theory that nothing is so unimportant as the quantity of money expressed in terms of the nominal monetary unit—dollars, or pounds, or pesos. Let the unit of account be changed from dollars to cents; that will multiply the quantity of money by 100, but have no other effect. Similarly, let the number of dollars in existence be multiplied by 100; that, too, will have no other essential effect, provided that all other nominal magnitudes (prices of goods and services, and quantities of other assets and liabilities that are expressed in nominal terms) are also multiplied by 100.

The situation is very different with respect to the real quantity of money—the quantity of goods and services that the nominal quantity of money can purchase, or the number of weeks' income to which the nominal quantity of money is equal. This real quantity of money has important effects on the efficiency of operation of the economic mechanism, on how wealthy people regard themselves as being and, indeed, on how wealthy they actually are. Yet

During the roughly two decades that I have puzzled over the problems covered in this paper, I have benefited from discussions with many friends, from the reactions of students to the presentation of some of this material in class (at the University of Chicago, Columbia University, and the University of California at Los Angeles), and from the reactions of audiences at several seminars at which I have presented the central ideas (at Stanford University and Princeton University). I owe a special debt to Kenneth Arrow, who saved me from several crucial errors, and to Alvin Marty and the late D. H. Robertson, who shared my interest and helped sharpen my understanding of the problem. I am indebted for helpful comments on the first draft of this paper to Martin Bronfenbrenner, Phillip Cagan, Elaine Goldstein, Franklin D. Mills, Anna J. Schwartz, and Lester Telser.

only recently has much thought been given to what the optimum quantity of money is, and, more important, to how the community can be induced to hold that quantity of money.

When this question is examined, it turns out to be intimately related to a number of topics that have received widespread attention over a long period of time, notably (1) the optimum behavior of the price level; (2) the optimum rate of interest; (3) the optimum stock of capital; and (4) the optimum structure of capital.

The optimum behavior of the price level, in particular, has been discussed for at least a century, though no definite and demonstrable answer has been reached. Interestingly enough, it turns out that when the question is tackled indirectly, via the optimum quantity of money, a definite answer can be given. The difference is that while the conventional discussion stresses short-period adjustments, this paper stresses long-run efficiency.

In examining the optimum quantity of money, I shall start in a rather roundabout way—as befits a topic that belongs in capital theory at least as much as in monetary theory. I shall begin by examining a highly simplified hypothetical world in which the elementary but central principles of monetary theory stand out in sharp relief. Though this introduction covers familiar ground I urge the reader to be patient, since it will serve as a bridge to some unfamiliar propositions.

I. HYPOTHETICAL SIMPLE SOCIETY

Let us start with a stationary society in which there are (1) a constant population with (2) given tastes, (3) a fixed volume of physical resources, and (4) a given state of the arts. It will be simplest to regard the members of this society as being immortal and unchangeable.[1] (5) The society, though stationary, is not static. Aggregates are constant, but individuals are subject to uncertainty and change. Even the aggregates may change in a stochastic way, provided the mean values do not. (6) Competition reigns.

To this fairly common specification, let us add a number of special provisions: (7) Any capital goods which exist are infinitely durable, cannot be reproduced or used up, and require no maintenance (like Ricardo's original, indestructible powers of the soil). More important, (8) these capital goods though owned by individuals in the sense that the rents they yield go to their owners, cannot be bought and sold. (They are like human capital in our society.)

(9) Lending or borrowing is prohibited and the prohibition is effectively enforced.

(10) The only exchange is of services for money, or money for services, or

1. This is equivalent to regarding the community as having a constant distribution of persons by age, sex, etc. Each of our infinitely long-lived individuals stands, as it were, for a family line in the alternative population of changing individuals but unchanging aggregates.

services for services. Items (7) and (8) in effect rule out all exchange of commodities.

(11) Prices in terms of money are free to change, in the sense that there are no legal obstacles to buyers' and sellers' trading at any price they wish. There may be institutional frictions of various kinds that keep prices from adjusting instantaneously and fully to any change. In that sense there need not be "perfect flexibility" whatever that much overused term may be taken to mean.

(12) All money consists of strict fiat money, i.e., pieces of paper, each labelled "This is one dollar."

(13) To begin with, there are a fixed number of pieces of paper, say, 1,000.

The purpose of conditions (7), (8) and (9) is, of course, to rule out the existence of a market interest rate. We shall relax these conditions later.

II. INITIAL EQUILIBRIUM POSITION

Let us suppose that these conditions have been in existence long enough for the society to have reached a state of equilibrium. Relative prices are determined by the solution of a system of Walrasian equations. Absolute prices are determined by the level of cash balances desired relative to income. $MV = P(Y/P)$

Why, in this simple, hypothetical society, should people want to hold money? The basic reason is to serve as a medium of circulation, or temporary abode of purchasing power, in order to avoid the need for the famous "double coincidence" of barter. In the absence of money, an individual wanting to exchange A for B must find someone who wants to exchange precisely B for A. In a money economy, he can sell A for money, or generalized purchasing power, to anyone who wants A and has the purchasing power. The seller of A can then buy B for money from anyone who has B for sale, regardless of what the seller of B in turn wishes to purchase. This separation of the act of sale from the act of purchase is the fundamental productive function of money. It gives rise to the "transactions" motive stressed in the literature.

A second reason for holding money is as a reserve for future emergencies. In the actual world, money is but one of many assets that can serve this function. In our hypothetical world, it is the only such asset. This reason corresponds to the "asset" motive for holding money.

It is worth noting that both reasons depend critically on characteristic (5) of our economy, the existence of individual uncertainty. In a world that is purely static and individually repetitive, clearing arrangements could be made once and for all that would eliminate the first reason, and there would be no unforeseen emergencies to justify holding money for the second reason.

How much money would people want to hold for these reasons? Clearly, this question must be answered not in terms of nominal units but in terms of real quantities, i.e., the volume of goods and services over which people wish to have command in the form of money. I see no way to give any meaningful answer to this question on an abstract level. The amount will depend on the details of the institutional payment arrangements that characterize the equilibrium position reached, which in turn will depend on the state of the arts, on tastes and preferences, and on the attitudes of the public toward uncertainty.

It is easier to say something about the amount of money people would want to hold on the basis of empirical evidence. If we identify the money in our hypothetical society with currency in the real world, then the quantity of currency the public chooses to hold is equal in value to about one-tenth of a year's income, or about 5.2 weeks' income.[2] That is, desired velocity is about ten per year.

If we identify money in our hypothetical society with all non-human wealth in the real world, then the relevant order of magnitude is about three to five years' income.[3] That is, desired velocity is about .2 to .3 per year.

Since we are only provisionally treating our money as the equivalent of all wealth, I shall use the first comparison, and assume, therefore, that the equilibrium position is defined by an absolute level of prices which makes nominal national income equal to $10,000 per year, so that the $1,000 available to be held amounts to one-tenth of a year's income. This is an average. Particular individuals may hold cash equal to more or less than 5.2 weeks' income, depending on their individual transactions requirements and asset preferences. As always, nominal national income has several faces: the value of final services consumed, the value of productive services rendered, and the sum of the net value added by the enterprises in the community. In our hypothetical society all of the difficult problems of national income accounting are by-passed, so we need not distinguish between different concepts of national income.

III. EFFECT OF A ONCE-AND-FOR-ALL CHANGE IN THE NOMINAL QUANTITY OF MONEY

Let us suppose now that one day a helicopter flies over this community and drops an additional $1,000 in bills from the sky, which is, of course, hastily

2. For the U.S., currency was a little over four weeks' income (personal disposable income) in the 1890's and is currently slightly under four weeks' income. It has ranged in that period from 2.1 weeks in 1917 to 8.2 weeks in 1948. In Israel, it is about the same as in the U.S. In Japan, it is about five weeks' income, in Yugoslavia, about six weeks. In a study of 27 countries, Morris Perlman found the highest figure to be fourteen weeks' (Belgium) and the lowest, two weeks' (Chile).

3. In 1958, the total national wealth of the United States was roughly four times net national product, and about 5.3 times personal disposable income. Since the wealth figure includes all government wealth, the first figure seems more relevant. Currency in the preceding footnote excluded for the U.S., and I believe also for the other countries, currency held by the Treasury and Federal Reserve. See Raymond Goldsmith, *The National Wealth of the United States in the Postwar Period* (Princeton, N.J.: Princeton University Press, 1962), p. 112.

collected by members of the community. Let us suppose further that everyone is convinced that this is a unique event which will never be repeated.

To begin with, suppose further that each individual happens to pick up an amount of money equal to the amount he held before, so that each individual finds himself with twice the cash balances he had before.

If every individual simply decided to hold on to the extra cash, nothing else would happen. Prices would remain what they were before, and income would remain at $10,000 per year. The community's cash balances would simply be 10.4 weeks' income instead of 5.2.

But this is not the way people would behave. Nothing has occurred to make the holding of cash more attractive than it was before, given our assumption that everyone is convinced the helicopter miracle will not be repeated. (In the absence of that assumption, the appearance of the helicopter might increase the degree of uncertainty anticipated by members of the community, which, in turn, might change the demand for real cash balances.)

Consider the "representative" individual who formerly held 5.2 weeks' income in cash and now holds 10.4 weeks' income. He could have held 10.4 weeks' income before if he had wanted to—by spending less than he received for a sufficiently long period. When he held 5.2 weeks' income in cash, he did not regard the gain from having $1 extra in cash balances as worth the sacrifice of consuming at the rate of $1 per year less for one year, or at the rate of [illegible]nts less per year for ten years. Why should he now, when he holds 10.4 weeks' income in cash? The assumption that he was in a stable equilibrium position before means that he will now want to raise his consumption and reduce his cash balances until they are back at the former level. Only at that level is the sacrifice of consuming at a lower rate just balanced by the gain from holding correspondingly higher cash balances.

Note that there are two different questions for the individual:

(1) To what level will he want ultimately to reduce his cash balances? Since the appearance of the helicopter did not change his real income or any other basic condition, we can answer this unambiguously: to their former level.

(2) How rapidly will he want to return to the former level? To this question, we have no answer. The answer depends on characteristics of his preferences that are not reflected in the stationary equilibrium position.

We know only that each individual will seek to reduce his cash balances at some rate. He will do so by trying to spend more than he receives. But one man's expenditure is another man's receipt. The members of the community as a whole cannot spend more than the community as a whole receives—this is precisely the accounting identity underlying the multiple faces of national income. It is also a reflection of the capital identity: the sum of individual cash balances is equal to the amount of cash available to be held. Individuals as a whole cannot "spend" balances; they can only transfer them. One man can spend more than he receives only by inducing another to receive more than he spends.

It is easy to see what the final position will be. People's attempts to spend more than they receive will be frustrated, but in the process these attempts will bid up the nominal value of services. The additional pieces of paper do not alter the basic conditions of the community. They make no additional productive capacity available. They alter no tastes. They alter neither the apparent nor actual rates of substitution. Hence the final equilibrium must be a nominal income of $20,000 instead of $10,000, with precisely the same flow of real services as before.

It is much harder to say anything about the transition. To begin with, some producers may be slow to adjust their prices and may let themselves be induced to produce more for the market at the expense of non-market uses of resources. Others may try to make spending exceed receipts by taking a vacation from production for the market. Hence, measured income at initial nominal prices may either rise or fall during the transition. Similarly, some prices may adjust more rapidly than others, so relative prices and quantities may be affected. There might be overshooting and, as a result, a cyclical adjustment pattern. In short, without a much more detailed specification of reaction patterns than we have made, we can predict little about the transition. It might vary all the way from an instantaneous adjustment, with all prices doubling overnight, to a long drawn out adjustment, with many ups and downs in prices and output for the market.

We can now drop the assumption that each individual happened to pick up an amount of cash equal to the amount he had to begin with. Let the amount each individual picks up be purely a chance matter. This will introduce initial distribution effects. During the transition, some men will have net gains in consumption, others net losses in consumption. But the ultimate position will be the same, *not only for the aggregate, but for each individual separately*. After picking up the cash, each individual is in a position that he could have attained earlier, if he had wished to. But he preferred the position he had attained prior to the arrival of the helicopter. Nothing has occurred to change the ultimate alternatives open to him. Hence he will eventually return to his former position. The distributional effects vanish when equilibrium is re-attained.[4]

The existence of initial distributional effects has, however, one substantive implication: the transition can no longer, even as a conceptual possibility, be instantaneous, since it involves more than a mere bidding up of prices. Let prices

4. This conclusion depends on the assumption of infinitely lived people, but not on any assumption about the extent or quality of their foresight. The basic point, to put it in other terms, is that their permanent income or wealth is unchanged. Their having picked up more or less than their pro-rata share of cash is a transitory event that has purely transitory effects.

See G. C. Archibald and R. G. Lipsey, "Monetary and Value Theory: A Critique of Lange and Patinkin," *Review of Economic Studies*, vol. 26 (1958), pp. 1–22; R. W. Clower and M. L. Burstein, "On the Invariance of Demand for Cash and Other Assets," *ibid.*, vol. 28 (1960), pp. 32–36; Nissan Liviatan, "On the Long-Run Theory of Consumption and Real Balances," *Oxford Economic Papers* (July, 1965), pp. 205–18; Don Patinkin, *Money, Interest, and Prices*, 2nd edition, New York: Harper and Row (1965), pp. 50–59.

double overnight. The result will still be a disequilibrium position. Those individuals who have picked up more than their pro-rata share of cash will now have larger real balances than they want to maintain. They will want to "spend' the excess but over a period of time, not immediately. (Indeed, given continuous flows and only services to purchase, they can spend a finite extra amount immediately only by spending at an infinite rate for an infinitesimal time unit.)

On the other hand, those individuals who have picked up less than their pro-rata share have lower real balances than they want to maintain. But they cannot restore their cash balances instantaneously, since their stream of receipts flows at a finite time rate. They will have some desired rate at which they wish to build up their balances. Hence, even if all prices adjusted instantaneously and everyone had perfect foresight, there would still be an equilibrium path of adjustment to the initial differential disturbance of real balances. This path defines the rate at which the relative gainers transfer their excess balances to the relative losers. The relative gainers will have a higher than equilibrium level of consumption and a lower level of production during the period of adjustment. The relative losers will have a lower than equilibrium level of consumption, and a higher level of production.

This analysis carries over immediately from a change in the nominal quantity of cash to a once-and-for-all change in preferences with respect to cash. Let individuals on the average decide to hold half as much cash, and the ultimate result will be a doubling of the price level, a nominal income of $20,000 a year with the initial $1,000 of cash.

IV. BASIC PRINCIPLES ILLUSTRATED

Our simple example embodies most of the basic principles of monetary theory:

(1) The central role of the distinction between the *nominal* and the *real* quantity of money.

(2) The equally crucial role of the distinction between the alternatives open to the individual and to the community as a whole.

These two distinctions are the core of all monetary theory.

(2a) An alternative way to express (2) is the importance of accounting identities: the *flow* identity that the sum of expenditures equals the sum of receipts (or, the value of final services acquired equals the value of productive services rendered) and the *stock* identity that the sum of cash balances equals the total stock of money in existence.

(3) The importance of attempts, summarized in the famous distinction between *ex ante* and *ex post*. At the moment when the additional cash has been picked up, desired spending exceeds anticipated receipts (*ex ante*, spending exceeds recipts). *Ex post*, the two must be equal. But the *attempt* of individuals

to spend more than they receive, even though doomed to be frustrated, has the effect of raising total nominal expenditures (and receipts).

(4) The distinction between the final position and the transition to the final position: between long-run statics and short-run dynamics.

(5) The meaning of the "real balance" effect and its role in producing a transition from one stationary equilibrium position to another.

Our example also embodies two essential empirical generalizations of long-run monetary theory:

(1) The nominal amount of money is determined primarily by conditions of supply.

(2) The real amount of money is determined primarily by conditions of demand—by the functional relation between the real amount of money demanded and other variables in the system.

V. EFFECT OF A CONTINUOUS INCREASE IN QUANTITY OF MONEY

Let us now complicate our example by supposing that the dropping of money, instead of being a unique, miraculous event, becomes a continuous process, which, perhaps after a lag, becomes fully anticipated by everyone. Money rains down from heaven at a rate which produces a steady increase in the quantity of money, let us say, of 10 per cent per year. The path of the quantity of money is shown in Figure 1, M_0 being the initial quantity of money ($1,000 in our ex-

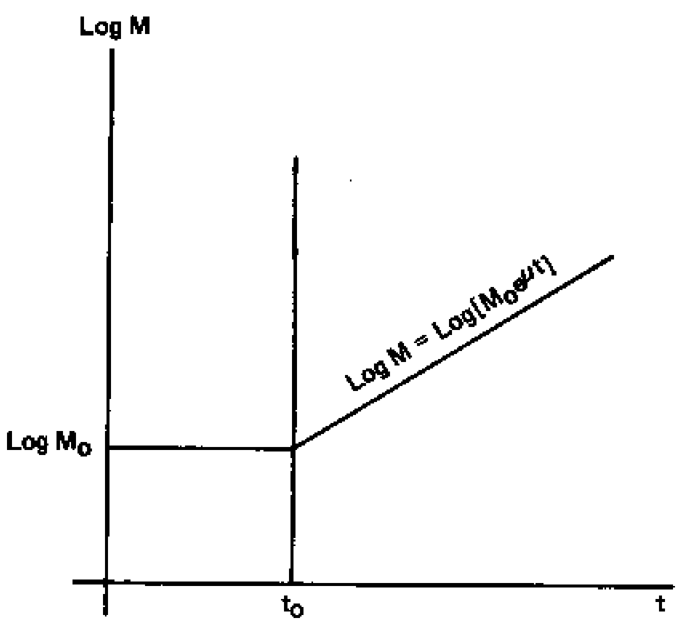

FIG. 1

ample), t_0 the date at which the money starts to rain from heaven, and μ the rate of growth of the quantity of money (10 per cent per year in our example). Mathematically,

$$M(t) = M_0 e^{\mu t} \tag{1}$$

The distribution of the additional nominal balances among individuals does not matter for our purposes, provided that an individual is not able to affect the amount of additional cash he receives by altering the amount of cash balances he holds. The simplest assumption is that each individual gets a share of the new nominal balances equal to the percentage of nominal balances he initially held, and that this share, once determined, remains constant, whatever his future behavior. The reason for this assumption will become clear. Even with this assumption, there may be distributional effects, by contrast with the once-and-for-all case, if final equilibrium cash balances are distributed differently than initial balances. For the moment, however, we shall neglect any distributional effects.

Individuals could respond to this steady monetary downpour as they did to the once-and-for-all doubling of the quantity of money, namely, by keeping real balances unchanged. If they did so, and responded instantaneously and without friction, all real magnitudes could remain unchanged. Prices would behave in precisely the same manner as the nominal money stock. They would rise from their initial level at the rate of 10 per cent per year, as shown in Figure 2. Nominal income, defined as the value of services and excluding the bonanza

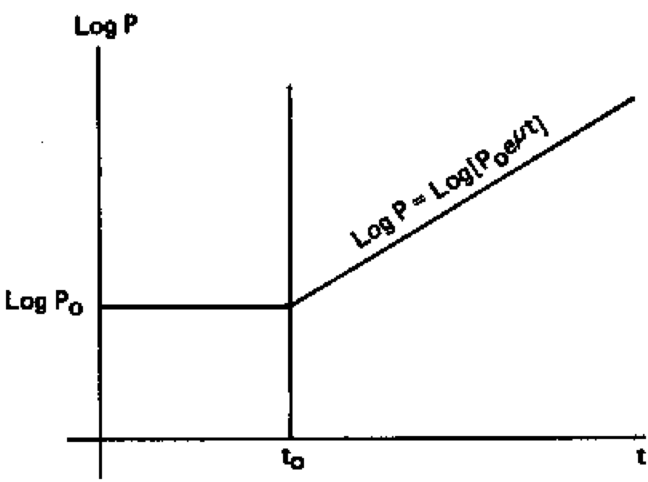

FIG. 2

from the sky, would behave in the same way; its time path could be represented by the same line. The bonanza, if included, would raise nominal income from

$$Y_1(t) = Y_0 e^{\mu t} \tag{2}$$

to

$$\begin{aligned} Y_2(t) &= Y_0 e^{\mu t} + \mu M(t) \\ &= (Y_0 + \mu M_0) e^{\mu t}, \end{aligned} \tag{3}$$

or, in terms of our example, from a value of \$10,000 to a value of \$10,100 at $t = t_0$, the additional \$100 representing the annual rate at which the quantity of money is initially being increased, i.e., at $t = t_0$.

However, given instantaneous adjustment and unchanged real balances, individuals would not regard any of this additional \$100 as available for purchasing services. All of it would have to be added to nominal cash balances in order to keep them at the initial one-tenth of a year's income. So no real magnitude would be affected.

If individuals did not respond instantaneously, or if there were frictions, the situation would be different during a transitory period. The state of affairs just described would emerge finally when individuals succeeded in restoring and maintaining initial real balances.

One natural question to ask about this final situation is, "What raises the price level, if at all points markets are cleared and real magnitudes are stable?" The answer is, "Because everyone confidently anticipates that prices will rise." There is an old saying that difference of opinion makes a horse race. And so it is in any market involving the trading of existing assets. If there are wide differences of opinion about the course of prices on the stock market, for example, there will be heavy trading, possibly with little change in prices. If there is widespread agreement, then prices can be marked up or down with little actual trading.

In our example, prices rise, though markets are continuously cleared, because everybody knows that they will. All demand and supply curves in nominal terms rise at the rate of 10 per cent per year, and so do the market-clearing prices.

A related question is, "What makes the solution stable?" The answer is the potential effect of departures. Let prices (and nominal income) for whatever reason momentarily rise less than 10 per cent per year. Cash balances will then rise relative to income. The attempts to restore them to their former level will raise prices as in the once-and-for-all example. The converse is true if prices momentarily rise more than 10 per cent per year.

While individuals could respond to the steady monetary downpour as they did to the once-and-for-all doubling of the quantity of money, by keeping all real magnitudes unchanged, they will not in fact do so. To each individual separately, it looks as if he can do better. It looks to him as if, by reducing his cash balances, he can use for consumption some of the money he gets from the helicopter instead of simply adding all of it to his nominal cash holdings. It looks to him as if, for each dollar by which he reduces his cash balances, he can get ten cents extra a year to spend on consumption.[5]

Put differently, the individual will regard as available for spending on consumption, and for adding to nominal cash balances, the nominal amount he receives for his productive services plus the amount of cash he gets from the helicopter. When he got nothing from the helicopter and cash balances amounted to 5.2 weeks' income (for the representative individual), he added nothing to his nominal cash balances, yet they remained constant in real as well as nominal terms because prices were stable. Storage costs and depreciation costs were zero, as it were. He did not try to add to his balances because he regarded the sacrifice involved in consuming at the rate of \$1 (or one cent) less for a year as just (over) balancing the satisfaction from having \$1 (or one cent) more in the form of cash balances. Had half his cash balances suddenly been destroyed (as in the opposite of the once-and-for-all increase), he would have tried to add to them because, while the sacrifice from consuming at a lower rate presumably would not be affected, the satisfaction from having an extra \$1 (or one cent) in cash balances would be higher when he had only half the real quantity of balances.[6] He would have continued trying to save at some rate until his cash balances were restored to 5.2 weeks' income, at which point he would again have been in equilibrium.

When the representative individual is getting cash from the helicopter, he can keep his *real* cash balances at 5.2 weeks' income from the sale of services only by adding all the extra cash to his *nominal* balances to offset rising prices. But now, if he is willing to lower his cash balances by \$1 initially (and by $\$1 \cdot e^{.1t}$ at each point in time), he can consume at the initial extra rate of \$1.10 per year (and at the rate of $1 + .10e^{.1t}$ per year at each point in time).[7] Since he was just on the margin when the extra consumption was at the rate of \$1 per year, he will now be over the margin and will try to raise his consumption. Storage and depreciation costs are now ten cents per dollar per year, instead of zero, so he will try to hold a smaller real quantity of money. Let us suppose, to be specific, that when prices are rising at 10 per cent a year, he desires to hold $\frac{1}{12}$ instead of $\frac{1}{10}$ of a year's proceeds from the sale of services in cash balances, ie., $4\frac{1}{3}$ instead of 5.2 weeks' income.

We are now back to our earlier problem. While to each individual separately

5. This makes clear why it is necessary to assume that the amount of extra cash the individual receives is not related to his cash balance behavior. If it were—for example, if the amount he received were not only proportional to the initial level of his balances, as assumed above, but also altered through time in such a way as to be proportional to his cash balances at each point in time—then he would get a return from his balances that would just offset the cost. The once-and-for-all solution outlined (unchanging real balances) would be the correct solution.

6. It is enough, of course, to suppose only that the satisfaction from having extra cash balances rises relative to the sacrifice from consuming at a lower rate.

7. I am indebted to Don Roper for correcting an error in this parenthesis in my initial draft.

it looks as if he can consume more by reducing cash balances, the community as a whole cannot. Once again, the helicopter has changed no real magnitude, added no real resources to the community, changed none of the physical opportunities available. The attempt of individuals to reduce cash balances will simply mean a further bidding up of prices and income, so as to make the nominal stock of money equal to $\frac{1}{12}$ instead of $\frac{1}{10}$ of a year's nominal income. The equilibrium path of prices (and of the nominal value of services rendered) will be like the dotted line in Figure 3, parallel to the solid line but higher by an amount

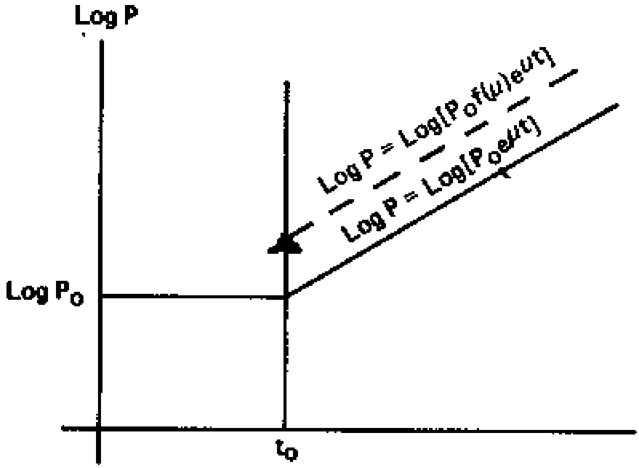

FIG. 3

depending on the size of μ. In our illustrative example, the level of prices would be 20 per cent higher than that shown by the solid line, since an increase of nominal income by 20 per cent would reduce cash balances from 5.2 to $4\frac{1}{3}$ week's income ($5.2 \div 1.2 = 4\frac{1}{3}$).

Once the community is on this path, it can stay there. Since both prices and nominal income are rising at 10 per cent a year, real income is constant. Since the nominal quantity of money is also rising at 10 per cent a year, it stays in a constant ratio to income—equal to $4\frac{1}{3}$ weeks' of income from the sale of services.

Attaining this path requires two kinds of price increase: (1) a once-and-for-all rise of 20 per cent, to reduce real balances to the level desired when it costs ten cents per dollar per year to hold cash; (2) an indefinitely continued rise in prices at the rate of 10 per cent per year to keep real balances constant at the new level.

Something definite can be said about the transition process this time. During the transition, the average rate of price rise must exceed 10 per cent. Hence, the rate of price rise must overshoot its long-term equilibrium level. It must display a cyclical reaction pattern. In Figure 4, the horizontal solid line is the ultimate equilibrium path of the rate of price change. The three broken curves illustrate

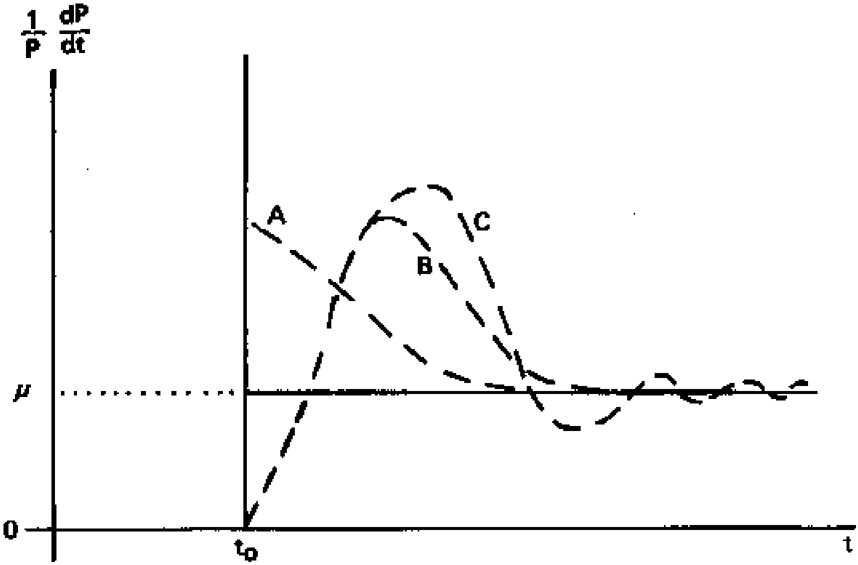

FIG. 4

alternative possible transitional paths: curve A shows a single overshooting and then gradual return to the permanent position, curves B and C show an initial undershooting, then overshooting followed by either a gradual return (curve B) or a damped cyclical adjustment (curve C).

This necessity for overshooting in the rate of price change and in the rate of income change (though not necessarily in the level of either prices or income) is in my opinion the key element in monetary theories of cyclical fluctuations. In practice, the need to overshoot is reinforced by an initial undershooting (as in curves B and C of Figure 4). When the helicopter starts dropping money in a steady stream—or, more generally, when the quantity of money starts unexpectedly to rise more rapidly—it takes time for people to catch on to what is happening. Initially, they let actual balances exceed long-run desired balances. They do so partly because they delay the adjustment of actual to desired balances; partly because they may take initial price rises as a harbinger of subsequent price declines, an anticipation which raises desired balances; and partly because the initial impact of increased money balances may be on output rather than prices, which further raises desired balances. As people catch on, prices must for a time rise even more rapidly, to undo an initial increase in real balances as well as to produce a long-run decline.

While this one feature of the transition is clear, little can be said about the details without much more precise specification of the reaction patterns of the members of the community and of the process by which they form their anticipations of price movements.

We can now refine somewhat our description of the final equilibrium path. We have implicitly been treating the real flow of services as if it were the same on the final equilibrium path as it was initially. This is wrong for two reasons.

First, and less important for our purposes, there may be permanent distributional effects. On the final path, some individuals may be receiving more cash from the helicopter than they require to keep their real cash balances constant, given their share in the downpour and their tastes. Others may be receiving less than they require. The first group is enriched relative to the second and will play a larger role in determining the structure of production. Distributional effects will be absent if, on the final path, the new money happens to be distributed among individuals in proportion to their desired holdings of cash balances.[8]

Second, and more important, real cash balances are at least in part a factor of production. To take a trivial example, a retailer can economize on his average cash balances by hiring an errand boy to go to the bank on the corner to get change for large bills tendered by customers. When it costs ten cents per dollar per year to hold an extra dollar of cash, there will be a greater incentive to hire the errand boy, that is, to substitute other productive resources for cash. This will mean both a reduction in the real flow of services from the given productive resources and a change in the structure of production, since different productive activities may differ in cash-intensity, just as they differ in labor- or land-intensity.

VI. WELFARE EFFECTS (X)

To each individual separately, the money from the sky seems like a bonanza, a true windfall gain. Yet, when the community has adjusted to it, each individual separately is worse off—if we abstract from the distributional effects noted in the second preceding paragraph. He is worse off in two respects. (1) He is poorer because the representative individual now has a reserve for emergency equal to $4\frac{1}{6}$ weeks' income (which is also his usual consumption) rather than 5.2 weeks'. (2) He has a lower real income because productive resources have been substituted for cash balances, raising the price of consumption services relative to the price of productive services.

The loss on wealth account is the counterpart to non-pecuniary consumption returns from cash balances—it reflects the role of wealth as an argument in the utility function. The loss on income account is the counterpart to the productive services rendered by cash balances—it reflects the role of cash balances as an argument in the production function.

We can get a rough measure of the magnitude of the loss along usual consumer surplus grounds. In doing so, however, we must take into account two

8. Note that desired holdings of cash balances on the final path need not be proportional to initial holdings. Hence this condition need not be the same as the condition assumed in the second paragraph of this section.

components of the loss. An individual who holds a dollar in cash balances pays two prices: (1) the annual cost imposed by the rate of price inflation; (2) the once-and-for-all cost of refraining from \$1 of consumption to accumulate the dollar of cash balances, or, equivalently, of abstaining from the dollar of consumption he could enjoy at any time by reducing his balances by a dollar.

Before the continuous downpour started, the first price was zero; but the second was still present. At his initial position, therefore, he must have valued the utility of the services he received by holding an extra dollar as much as the utility he would have gotten from raising his consumption by \$1 per year for a year. In the new equilibrium position, this second price is the same, but, in addition, he must pay ten cents per year indefinitely per dollar of real balances that he holds. Accordingly, he must regard a dollar of his now lower cash balances as worth this extra price. The *average* value he attaches to a dollar of the real cash balances that have disappeared is therefore one dollar's worth of consumption (the same before and after) plus approximately five cents a year indefinitely (the average of zero and ten cents). In our numerical example, cash balances decline from 5.2 to 4.33 weeks' consumption, or by $\frac{13}{15}$ of a week's consumption. Therefore, the continuous downpour has cost the community the equivalent of $\frac{13}{15}$ of a week's consumption plus $\frac{1}{20} \cdot \frac{13}{15} = \frac{13}{300}$ of a week's consumption per year indefinitely. (Expressed in the equivalent United States magnitudes, this is about \$10 billion plus \$500 million a year indefinitely.) Since we have not yet introduced an interest rate, we have as yet no way of combining these two components of cost.

The reason for the loss in welfare is clear: the existence of external effects, or a difference between cost to an individual and cost to all individuals affected. Consider the initial position of constant prices. For an individual to add one dollar to his cash balances he would have to consume \$1 less—at the rate, say, of \$2 a year less for six months, or \$1 a year less for a year, or fifty cents a year less for two years. But were any individual to do so, he would make the price level slightly lower than it would otherwise be. This would have the external effect of yielding capital gains to all other holders of money, trivial to each but enabling them in the aggregate to consume precisely \$1 more while keeping their real balances constant. Total consumption would not change. The individual who adds to cash balances confers a benefit on his fellows for which he cannot collect compensation. The rate at which he can substitute cash balances for consumption thus differs from the rate at which it is technically possible to do so.

The situation is the same with the other component of cost, the ten cents a year required to hold a dollar of real balances when prices are rising at the rate of 10 per cent a year. This component too is an apparent cost to the individual, but is balanced by uncompensated gains to others, so that the cost to all together is zero per year, not ten cents per year.[9]

9. In our example, this can be seen most easily by considering a representative individual who gets just enough money from the helicopter so that, when he adds it to his cash balances, he can just maintain the real balances he desires. His consumption is equal to his income from

VII. EFFECT OF A CONTINUOUS DECREASE IN THE QUANTITY OF MONEY

When prices are stable, one component of the cost is zero—namely, the annual cost—but the other component is not—namely, the cost of abstinence. This suggests that, perhaps, just as inflation produces a welfare loss, deflation may produce a welfare gain.

Suppose therefore that we substitute a furnace for the helicopter. Let us introduce a government which imposes a tax on all individuals and burns up the proceeds, engaging in no other functions. Let the tax be altered continuously to yield an amount that will produce a steady decline in the quantity of money at the rate of, say, 10 per cent a year. It does not matter for our purposes what the tax is, as long as an individual cannot affect his tax by altering his cash balances.

By precisely the same reasoning as before, the final equilibrium path will be the dotted line in Figure 5—prices decline at a rate of 10 per cent a year, but at a

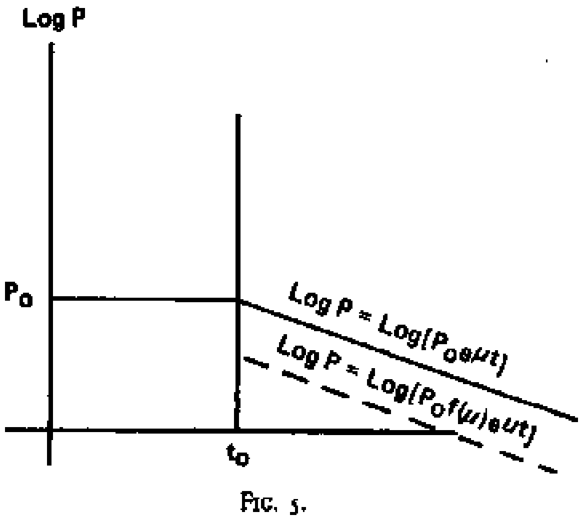

FIG. 5.

services. Suppose he now were to add an extra (real) dollar to his balances. The process of adding this (real) dollar lowers prices a trifle and enables the rest of the community to consume one (real) dollar more—this is the external effect described in the preceding paragraph of the text. But, in addition, the individual thereafter will have to consume ten (real) cents less than his income from services to maintain intact the higher level of real balances. The rest of the community will find that, at the slightly lower price level, they are receiving cash from the helicopter at a rate of ten (real) cents per year more than they need to keep their real balances intact. They therefore can, and will, spend ten (real) cents more per year on consumption than they receive from services—thereby providing the extra cash to the individual who was assumed to have added to his balances. His cost is precisely counterbalanced by their gain.

lower level than the solid line linked to the initial price level. When prices are declining, a dollar of cash balances yields a positive return. The real services that a dollar of balances will command grow at a rate of 10 per cent per year. This makes cash balances more attractive and thus raises the quantity individuals want to hold. Prices must decline not only in proportion to the quantity of money (which follows a path like the solid line in Figure 5) but by enough more to raise real balances (or the ratio of money to income) to the desired level—say, to 6.24 weeks' income. Figure 6 shows the demand curve for real balances implicit in this and the earlier examples.

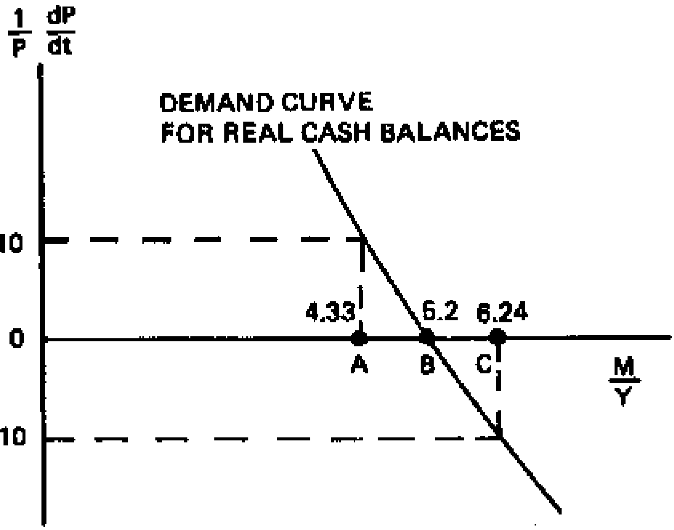

FIG. 6.

At the new equilibrium, with cash balances equal to 6.24 weeks' income, every individual is richer (if we neglect distributional effects) than he was before—he has a larger reserve for emergencies. The other real resources available to the community are the same as before. It looks, therefore, as if everyone is better off than before, and as if the higher the rate of price decline, the greater the welfare gain.

But the appearance is misleading, as we can see by considering what happens if we increase the rate of decline of prices. Beyond some point, it pays individuals to hold extra balances to benefit from their increasing purchasing power even if it costs something to do so. The retailer dispenses with an errand boy to economize on cash balances, which is a gain, but, at some point, he must hire guards to protect his cash hoard. It pays him to do so because of their rising real value. The extra real balances not only do not save productive resources, they absorb them. Similarly, on the asset side, cash will be held beyond the point at which additional cash brings non-pecuniary returns in security and satisfaction from being wealthy. The amount held will, at the margin, reduce utility—because of concern about the safety of the cash, perhaps, or because of pecuniary costs of storing and guarding the cash.

For a sufficiently small rate of decline in prices, it seems clear that there will be a net benefit; for a sufficiently large rate of decline, a net loss. What is the optimum rate of decline?

The real returns or costs to an individual from holding cash balances can be classified under four items:

(1) The rise or decline in the purchasing power of a dollar. What matters is not the actual rise or decline but the anticipated rise or decline. This item we can represent by

$$-\left(\frac{1}{P}\frac{dP}{dt}\right)^{*}, \tag{4}$$

where the asterisk indicates anticipated value. If a decline in prices is anticipated, this is positive and represents a return; if a rise in price is anticipated, this is negative and represents a cost. For any individual, the anticipated rate of price decline does not depend on his own holdings of cash balances, so average and marginal return or cost are equal.

(2) The productive services rendered per year by a dollar of cash balances as a factor of production. The value of these services does depend on the amount of cash balances the individual holds, so one must distinguish between average and marginal return. The relevant magnitude is the marginal return, which we may designate

$$MPM, \tag{5}$$

(2) cust/$ / year dimension igual a la de (i).

or marginal product of money. Since this is product per dollar per year, it, like (4), has the dimensions of the reciprocal of time, that is, of an interest rate. Like (4) also, it can be positive, and thus a return, or negative, and thus a cost. It is natural to assume diminishing marginal returns throughout.

(3) The non-pecuniary consumption services to the holder of cash balances. Let us suppose that we can express the marginal value of these services in a money equivalent, as cents per year per dollar of balances. Designate this marginal return

$$MNPS, \tag{6}$$

or marginal non-pecuniary services. Again it may be positive or negative. And, again, it is natural to assume diminishing marginal returns.

(4) The cost of abstaining from a dollar of consumption. This depends on the individual's time preference or internal rate of discount of the future. Let us suppose that, at some level of real cash balances, he values the sum of the preceding three items as ten cents per year per dollar of cash balances. By consuming one dollar more (say, by consuming at the rate of one dollar per year more for a year), he would subtract a dollar from his cash balances and thereby sacrifice a

permanent consumption stream in the form of these three items of ten cents per year indefinitely. Conversely, by lowering his consumption by a total of a dollar, he could acquire an additional permanent consumption stream of 10 cents per year indefinitely. If, under those circumstances, he chooses to add to his consumption by depleting his real cash balances, his internal rate of discount is more than 10 per cent. If he chooses to keep real cash balances constant, his internal rate of discount equals 10 per cent. If he chooses to add to his real cash balances, his internal rate of discount is less than 10 per cent. Designate this internal rate of discount

$$IRD. \tag{7}$$

It, too, is marginal and has the dimensions of a percentage.

Note that the preceding paragraph defines the internal rate of discount only at the point of a constant flow of consumption. The value of the internal rate of discount at that point does not determine at how *rapid* a pace the individual will add to or subtract from his cash balances (will save or dissave), only whether he will. How much he will save or dissave depends on what happens to the *IRD* as he alters his rate of saving or dissaving, i.e., as he alters his prospective time pattern of consumption. The more he cuts down present consumption to raise his future consumption stream (in the form of the first three items), the more reluctant he will be to cut it down further, i.e., the higher will be *IRD* (this is Böhm-Bawerk's first reason for time preference). His rate of saving or dissaving at any moment will be determined by the point at which *IRD* rises enough to equal the sum of items (4), (5), and (6), which sum itself may change with the rate of saving or dissaving.[10] Where relevant, we shall distinguish the *IRD* when saving is zero from its generalized value by designating it *IRD*(0).

For the sub-set of time patterns of consumption that consists of constant levels of consumption, it is not at all clear whether the *IRD* is best considered a constant for each individual or whether it should be regarded as a function of other variables, particularly (a) the level of consumption, and (b) the ratio of wealth to income. I can see no way to say how it depends on (a), i.e., whether it can be expected to rise or fall as the level of consumption rises. I shall therefore assume it to be unaffected by (a).

Variable (b) raises a much more difficult problem. It is not clear that *IRD* should be affected by (b) at all. Whatever the wealth-income ratio, the exchange involved is a temporary reduction in consumption in return for a permanently higher consumption stream. If *IRD* is affected by (b), stability considerations

10. Item (4) will not be affected by the particular individual's rate of saving or dissaving. Whether item (5) is affected depends on whether his saving or dissaving affects the supply of productive services competitive with or complementary to cash balances. Whether item (6) is affected depends on whether the non-pecuniary utility from cash balances is affected by the level of consumption. These inter-relations enter in because the level of real cash balances at any moment of time is not affected by the *rate* of saving or dissaving. In general, it is simplest to neglect these inter-relations and treat the second and third items, like the first, as unaffected by the rate of saving or dissaving, and determined only by the level of real balances.

call for *IRD* to be higher, the higher is the ratio of wealth to income. The rationalization is that the higher this ratio is, the more provision has already been made for the future, and the less willing the individual will be to sacrifice the present for the future. The difficulty with this rationalization is that it confounds the decline in *MNPS* as wealth rises relative to income with a rise in *IRD*. It is not clear that there is any way to distinguish the two. We shall return to this puzzling and sophisticated question later.

The *IRD* enables us to translate a stock into a flow. It is the device needed to combine the two components of cost in Section VI above.

The individual will be in a position of long-run equilibrium with respect to his cash balances when

$$-\left(\frac{1}{P}\frac{dP}{dt}\right)^{*} + \underbrace{MPM + MNPS}_{\text{gains without external effects}} = IRD(0). \qquad (8)$$

If we assume for the moment that $IRD(0)$ is a positive and constant number, we can see how this equation summarizes our earlier analysis. Let prices be rising and anticipated to rise. Then the first term is negative. Cash balances must be small enough to yield a positive marginal return in productive services and nonpecuniary services, not only to offset the first term but also to balance the right-hand side. Reduce the anticipated rate of price rise and the left side will exceed the right. An increase in cash balances will now bring down *MPM* + *MNPS*, and thereby produce a new balance. Let prices be anticipated to fall, and the first term becomes positive. If it is larger than $IRD(0)$, then cash balances will have to be sufficiently large to make *MPM* + *MNPS* negative.

Of the four terms in equation (8), *MPM* and *MNPS* are gains to the individual that involve no external effects on others. The individual gets all the benefits. As we have seen, the rate of price rise or fall is a cost or return to the individual of altering his cash balance that confers or imposes a precisely compensating return or cost on others. Similarly, the $IRD(0)$ is a cost to the individual of altering his cash balances that confers a precisely compensating benefit on others. In our simple society, if he reduces his consumption, all others will be able to consume a bit more. If all individuals simultaneously seek to reduce consumption to add to cash balances, they will lower prices, raising real cash balances without reducing total consumption.

Note that this conclusion does not hold for a commodity money (say, gold) which is produced under conditions of constant cost. The attempt by an individual to hold cash balances would initially tend to lower prices, but this would in turn divert resources to gold production and leave prices unchanged. In effect, the individual consumes a dollar less and the resources so released are diverted to producing an additional dollar's worth of gold. There are no external benefits conferred. However, if the commodity money is fixed in quantity and incapable of being produced, then the same conclusion would hold for it as for our fiat money.

It follows that cash balances of the fiat money will be at their optimum level in real terms when

$$-\left(\frac{1}{P}\frac{dP}{dt}\right)^{*} = IRD(0), \tag{9}$$

so that

$$MPM + MNPS = 0. \tag{10}$$

In words, under our assumptions, it costs nothing to provide an extra dollar of real balances. All that is required is a slightly lower price level. Hence cash balances will be at their optimum when they are held to satiety, so that the real return from an extra dollar held is zero.

This solution is for an individual. What of the community? If $IRD(0)$ were a constant for each individual separately, and also the same constant for all individuals, this solution would carry over to the community as well: the optimum quantity of money would be attained by a rate of price decline that would be equal to the common value of $IRD(0)$.

This conclusion raises three problems. First, why should different individuals have the same $IRD(0)$? Second, how could one know whether they had the same $IRD(0)$, and what its value would be, from observable market phenomena? Or, alternatively, how could one know whether equation (9) was satisfied? Third, if they do not all have the same $IRD(0)$, or if this is not a constant but a function of other variables, what then is the policy that yields the optimum quantity of money?

VIII. INTERNAL RATES OF DISCOUNT FOR DIFFERENT INDIVIDUALS

One possible theoretical justification for regarding $IRD(0)$ as both a constant and the same for all individuals is that, under the conditions assumed in our simple society, the "rational" individual will have an $IRD(0) = 0$, i.e., he will not discount the future.

The more obvious reasons for discounting the future relative to the present are absent. (1) One reason is anticipation of a higher future than present consumption. If marginal utility of consumption declines with the level of consumption at each point of time,[11] then, even if present consumption and future consumption are valued alike for a stable consumption stream, future consumption will be valued less than present consumption when the consumption stream is expected to rise. However, we have defined $IRD(0)$ for a stable consumption stream.

11. Or, more generally, the rate at which individuals are willing to substitute future for present consumption rises as the ratio of future consumption to present consumption rises.

(2) A second reason is limited life. This will produce a discount on future consumption if the individual attaches less importance to his heirs' utility from consumption than to his own. In that case, he will attach a lower value to consumption beyond his own lifetime than to consumption during his lifetime.

(3) Uncertainty of length of life will cause him to extend this discount on future consumption to periods less than his "expected" (i.e., average) length of life.

Our assumptions rule out both (2) and (3) by treating the individuals as immortal and unchangeable.

Are there any other "rational" reasons for discounting the future? As I interpret the literature, it answers in the negative—that is why the term "underestimation" of the future is so often used as a synonym for a positive internal rate of discount.

The appeal of this conclusion can be seen very clearly in our simple economy. By reducing his consumption temporarily—say, by $1 a year for a year—one of our immortal individuals can acquire an asset (a dollar of cash balances) that will yield him services that he regards as worth, say, ten cents a year indefinitely. By a temporary sacrifice, he can permanently raise his level of consumption. Suppose at time t_0 he does not do so. At some later time t will he not reproach himself for not having done so? He will say to himself: "Had I been sensible enough to make a temporary sacrifice years ago, it would be long past by now, but I would be enjoying today, and *forever after*, a higher level of consumption. I was a fool not to have made the sacrifice then." And this retrospective judgment does not involve any knowledge the individual did not have available at time t_0. His failure to make the temporary sacrifice then therefore conflicts with one characteristic it is natural to assign to "rational" behavior: behaving in a way that one does not later regret on the basis of data initially available.[12]

Even if the individual reasons in this way, it does not mean that there is no limit to the amount he will save at time t_0, only that he will save something. As he saves, he brings into play reason (1) for discounting future consumption. The "rational" man, on this logic, will regard a unit of present *utility* as equal to a unit of future *utility*. He will not necessarily regard a unit of present consumption as equal to a unit of future consumption.

This conception of rational behavior underlies conclusions such as that reached by Maurice Allais,[13] that the optimum real interest rate is zero and the optimum stock of capital in a stationary state is that at which the marginal productivity of capital is zero. It underlies in a more sophisticated way also the more recent work

12. The need to specify the same data is clear. Consider an individual offered a $2 to $1 wager that a coin he then and later regards as fair will come up heads. He takes the wager, betting that it will come up tails. Suppose that it happens to come up heads, so that he loses the $1. Ex post, he will regret having lost, but not having made the wager, because, on the basis of the data he could have had when he made the wager, it was an advantageous wager.

13. *Economie et Intérêt*, Paris: Librarie des Publications Officielle (1947).

on "golden growth paths" which regard the highest possible level of consumption per capita as an optimum.[14]

If one accepts this line of reasoning and supposes that the individuals in our hypothetical society behave rationally, the solution to our problem is immediate. The optimum situation is reached with a constant quantity of money and an ultimately stable price level. Equation (9) is then satisfied and, hence, so is equation (10). Individuals separately will try to accumulate cash balances up to the point at which the marginal yield of cash is zero. Their attempts will produce a price level that makes real cash balances sufficiently large to have a zero marginal yield.

However, I find it hard to accept this conclusion. Generalized to a world in which other forms of capital assets exist, it implies that a stationary equilibrium is possible only with capital satiety, i.e., a zero marginal yield of real capital. (The existence of such a situation would answer the second question raised above—the observable market phenomenon that would give the common value of $IRD(0)$.) A positive marginal yield of capital, however small, would be a sufficient condition for growth. This seems to me inconsistent with experience. Much, if not most, of human experience has consisted of a roughly stationary state—Europe in part of the middle ages, for example, and surely Japan for centuries prior to the nineteenth. Was the marginal yield on capital zero in those communities?

If it was positive, the present analysis would have to explain the lack of growth by either a lesser regard for one's heirs than for oneself, or by irrational behavior—by selfishness or short-sightedness. Neither appeals to me strongly as a satisfactory explanation. Yet I must confess that I have found no other.[15]

Nonetheless, it seems worth examining the effects of an $IRD(0)$ not equal to zero for every individual, but positive at least for some, leaving open whether such a situation is to be explained by selfishness, short-sightedness, or some still undiscovered reason for discounting the future.

In order to examine these effects, we must complicate our simple society. In that society, corresponding to any steady rate of growth or decline of the quantity of money, there will be an equilibrium position in which each individual adjusts his cash balances to satisfy equation (8). This is a stable position whether IRD is the same or different for different individuals, and market phenomena give no evidence of what the value of IRD is for any one individual. All we know is that, for all alike, given their levels of cash balances,

$$IRD(0) - MPM - MNPS = -\left(\frac{1}{P}\frac{dP}{dt}\right)^{*}. \tag{11}$$

14. Edmund S. Phelps, *Golden Rules of Economic Growth: Studies of Efficiency and Optimal Investment*, New York: W. W. Norton (1966).

15. For a while, I thought I had a rational explanation for an $IRD(0) > 0$ in a somewhat different model of individual behavior than the usual one. But Kenneth Arrow has persuaded me that, while this model (summarized in Appendix A) may be richer and more appealing than the usual one, it yields the same conclusion about rational behavior.

We need additional information to evaluate the individual terms on the left-hand side of the equation. We can get such additional information by relaxing some of our intital conditions.

IX. INTRODUCTION OF LENDING AND BORROWING

As a first step, let us relax condition (9), on page 2 above, to permit lending and borrowing, while retaining all other conditions. These other conditions mean that borrowing will be of only two kinds, (a) to finance extra consumption, or (b) to finance the holding of cash balances as a productive resource.

To simplify matters, let us suppose that there is only a single kind of debt instrument, namely, a promise to pay $1 a year indefinitely, a perpetuity or "consol."[16] Let us suppose also that productive enterprises are like corporations in our world—separate entities distinguishable from the individuals who are the ultimate wealth-owners, consumers, and sellers of productive resources. The only permanent asset enterprises have title to, under our assumptions, is cash, and they acquire this cash by borrowing from individuals. In this way, total cash balances can be divided into two parts:

$$M_e = \text{cash balances of enterprises}$$
$$M_w = \text{cash balances of ultimate wealth-holders.}$$

The counterparts of M_e in the portfolios of ultimate wealth-holders are then the debt instruments issued by the business enterprises.

We shall suppose also that all debt instruments are homogeneous, whether issued by enterprises or individuals, are regarded as default-free, and are traded in a free market like that in which services trade.

Let us call the individual debt instrument a "bond," and let B be the number of debt instruments, i.e., the number of perpetuities each promising to pay $1 a year. Let P_B be the price of a debt instrument, and r_B the reciprocal of P_B, or $1/P_B$, which is an interest rate.

If P_B is anticipated to remain constant on the average, though subject to variations, then r_B is the anticipated pecuniary return to a lender per dollar loaned and the anticipated pecuniary cost to a borrower per dollar borrowed.[17] However, just as the holding of money balances yields non-pecuniary returns in the form of a feeling of security and pride of possession, so also the possession of a bond may yield similar non-pecuniary returns and the issuance of a bond

16. This involves no essential loss of generality—if we consider only positive long-term interest rates and if the transactions costs of buying and selling perpetuities can be neglected since a short-term loan can always be broken into a purchase and subsequent sale of a perpetuity.

17. Treating the return to the lender and the cost to the borrower as equal assumes that both have the same anticipations and also that transactions costs of borrowing and lending can be neglected.

may involve non-pecuniary costs. The marginal non-pecuniary services yielded by a dollar's worth of bonds presumably depends on the stocks of both money and bonds held by the individual, and so does the marginal non-pecuniary services yielded by money balances. What matters is not the nominal value of the two stocks, but their real value, which we may represent by expressing the value of the stocks of both money and bonds as a ratio to income available for purchasing consumption goods (i.e., after debt service or inclusive of interest yield, but before savings or dissavings).

Let

value of stock of Mone
income.

$$m_i = \frac{M_i}{Y_i}, \qquad \nu_i = \frac{B_i P_B}{Y_i} \tag{12}$$

represent these ratios for money and bonds respectively for individual i, where M_i and B_i are the nominal amount of money and the number of bonds, respectively, held by individual i, and Y_i is his nominal income per unit time. Let

$$MNPS_M(m_i, \nu_i), \qquad MNPS_B(m_i, \nu_i) \tag{13}$$

be the marginal value of non-pecuniary services, measured in cents per unit time per dollar of capital value, yielded by money and bonds to individual i when his holdings of them are m_i and ν_i. Note that ν_i may be positive or negative and that $(m_i + \nu_i)$ may be negative.

Intuitively, money seems to be a more efficient carrier of non-pecuniary services of the kind under consideration than bonds (this is the central idea imbedded in Keynesian liquidity preference). To represent this feature, we shall assume

$$MNPS_M(m_i, \nu_i) \text{ and } MNPS_B(m_i, \nu_i) \text{ have the same sign,} \tag{14a}$$

and

$$| MNPS_M(m_i, \nu_i) | \geqslant | MNPS_B(m_i, \nu_i) | \tag{14b}$$

for all values of m_i and ν_i, the equality sign holding only when $MNPS_M$ is zero. In words, if money yields positive marginal non-pecuniary services, so do bonds; if money yields negative marginal non-pecuniary services, so do bonds. When both yield positive services, an individual who is compensated for any loss of pecuniary return will always prefer a portfolio which has \$1 more of money and \$1 less of bonds. If he is sated with one, he is sated with both, and therefore indifferent to bonds and money. When both yield negative returns, he will prefer the bonds to the money.[18] That is, money dominates bonds in the provi-

18. This seems a reasonable translation of our intuition when $MNPS_M$ is positive—the only region with which we have much experience. If money is superior because it gives greater security or more ready availability of resources for emergencies, this advantage should

sion of non-pecuniary services. This condition looks innocuous, yet it turns out to be critical.

A. Quantity of Money Constant

Let us revert to the first case considered, with neither helicopter nor furnace, in which the quantity of money is constant. Also, let us neglect M_e for a time by assuming that $MPM = 0$ for all values of M_e, so that M_e is also 0 (i.e., cash balances do not enter the production function).

Suppose lending and borrowing are introduced into our earlier society when it is in an equilibrium position with different individuals having different values of $IRD(0)$. Consider two individuals, Mr. Swinger, or S for short, who is willing to give up 20 cents a year indefinitely to raise his rate of consumption by $1 a year for one year [$IRD(0) = .20$], and Mr. Rational, or R for short, who would not be willing to reduce his permanent consumption stream at all in order to get a temporary increase in consumption of $1 a year for a year [$IRD(0) = 0$].

$s_i = \frac{S_i}{Y_i}$

At the initial position,

$$\begin{aligned} \text{For } S\text{: } & MNPS_M(m_S, 0) = IRD(0) = .20 \\ \text{For } R\text{: } & MNPS_M(m_R, 0) = IRD(0) = .00. \end{aligned} \tag{15}$$

Each can now acquire or issue bonds by saving or dissaving. Let s_i equal the amount individual i saves expressed as a fraction of income available for consumption (the base of m_i and v_i). Then both initially and at every later moment, if all individuals act as if P_B will remain constant on the average,[19] each will save up to the point at which

$$MNPS_M(m_i, v_i) - IRD(s_i) = r_B + MNPS_B(m_i, v_i), \tag{16}$$

or, subtracting $MNPS_B$ from all terms, at which

$$MNPS_M(m_i, v_i) - MNPS_B(m_i, v_i) = IRD(s_i) - MNPS_B(m_i, v_i) = r_B. \tag{17}$$

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decline as $MNPS_M$ approaches zero. When it is zero, the individual is sated with liquidity, hence would be indifferent, if compensated for any difference in pecuniary returns, between money and bonds.

The specified condition is more conjectural when $MNPS_M$ is negative. Presumably such negative non-pecuniary services reflect costs of safeguarding money, or worry over being robbed, etc. It seems plausible that bonds would be less worrisome, easier to safeguard, etc. which is the reason for the absolute value relation making bonds preferable under such circumstances.

However, if this is so, it raises a question about the positive side because then there will be some range for which this advantage of bonds will more than compensate for the higher liquidity of money, so the break-even point need not be zero but may be higher. Since only the positive side is particularly relevant for what follows, I have suppressed my misgivings about this point.

19. It is, of course, the uncertainty about P_B which makes cash more "liquid" than bonds and largely explains, under our assumptions, the inequalities (14b).

Equations (16) and (17) are simplified versions of equation (8), simplified because they assume

$$\left(\frac{1}{P}\frac{dP}{dt}\right)^* = MPM = 0,$$

amplified because they include bonds and admit the possibility of non-zero saving.

At the initial point, when $v_t = 0$, and $s_t = 0$ for both, we know from (14) and (15) that the first two expressions of (17) [$MNPS_M - MNPS_B$ and $IRD - MNPS_B$] are positive for S and zero for R. Hence there will be some range of positive rates of interest at which it will be mutually advantageous for S to borrow from R. How much S will want to borrow and R to lend will depend on the precise interest rate and on their tastes. When borrowing takes place, each shifts from a constant consumption stream to a changing one—declining for S, because current consumption is raised by his borrowing to a higher level than he can expect to maintain, rising for R, because current consumption is reduced by his loan to a lower level than he plans to maintain; which will lower the IRD for S and raise it for R. Since, on our assumptions, $MNPS_M$ and $MNPS_B$ depend only on m_t and v_t and not their rate of change, and since, for a given r_B, m_t and v_t are fixed at a given moment of time (except possibly for an initial reshuffling considered below), the change in IRD is what limits the amount that lenders are willing to lend and borrowers are willing to borrow at each interest rate.

If bonds yielded no non-pecuniary services, the size of $IRD(0)$ would determine which individuals would be savers and which dissavers, as it does in the example of S and R because the $IRD(0)$ of $R = 0$. Individuals with a high internal rate of discount would borrow from those with a low internal rate of discount, and borrowing and lending would be at a level at which all $IRD(s_t)$'s were equal. This is no longer necessarily true when bonds yield non-pecuniary returns. An individual who values such non-pecuniary returns highly relative to the non-pecuniary returns from money may save even though his $IRD(0)$ is relatively high. But, at each interest rate, some will be borrowers, some lenders. At lower and lower interest rates, more will be borrowers and each of these will be willing to borrow more, while fewer will be lenders and each will be willing to lend less. Hence there will be some interest rate, at each point in time, at which equation (17) will be satisfied for each individual, and at which the quantity of bonds demanded is equal to the quantity supplied. But at that interest rate, the $IRD(s_t)$'s need not be equal.

What initial effect, if any, will the introduction of borrowing and lending have on the demand for cash balances? For Mr. Swinger, his IRD is now lower, hence he will want to hold larger cash balances. Indeed, he may want to borrow precisely to accumulate cash balances. For Mr. Rational, his IRD is now higher, hence he will want to hold lower cash balances. Indeed, he may finance his lending by drawing down cash balances.

Insofar as people want to borrow to hold higher cash balances or lend to hold lower cash balances, this can occur at an instant of time by a reshuffling of cash and securities, a transfer of stocks of cash for stocks of securities. Insofar as they want to borrow to raise the current level of consumption of currently produced services or lend to reduce the current level of consumption and raise the future level, this is a transfer of flows for flows and must occur over time.

Let there be an instantaneous reshuffling of cash and securities. Will the real amount of cash demanded remain the same, rise, or fall? I see no way of knowing. That depends on the precise structure of tastes for cash balances on the part of those with high $IRD(0)$'s and those with low $IRD(0)$'s. If the real amount of cash demanded is higher, that will require and produce a reduction in prices; if lower, a rise in prices. For simplicity, let us assume that the real amount of cash demanded is unchanged and hence that there is no change in prices.[20]

As the borrowing and lending process proceeds, some members of the community accumulate bonds, others accumulate an obligation to pay interest on the bonds. What will be the final stationary equilibrium position?

That position is defined by the satisfaction of equation (17) for all individuals at a value of $s_i = 0$. Three sets of forces put in motion by the process of borrowing and lending may contribute to the attainment of such an equilibrium.

(1). *Changing distribution of wealth.* As the process proceeds, the lenders accumulate wealth and hence have higher and higher incomes available for consumption (if Y_i^0 is the original income of lender i from the sale of services, his income becomes $Y_i^0 + B_i$, and B_i is positive), while borrowers decumulate and hence have lower and lower incomes available for consumption. Suppose that for each individual all terms in (17) remained unchanged in the process. Then at each interest rate s_i would be unchanged. But positive s_i's would be applied to larger and larger bases, and negative s_i's to smaller and smaller bases. The absolute demand for bonds would shift to the right and the absolute supply of bonds to the left, forcing down r_B, which would increase the number of borrowers and decrease the number of lenders. The asymptotic limit would be that at which only those individuals who had the lowest common value of $IRD(0) - MNPS_B(m_i, v_i)$ would have funds left from which to save. The value of r_B would be equal to that lowest value and there would be no net saving or dissaving.[21]

20. Note that the total consolidated transferable wealth of the community remains throughout equal to M, since the positive value of bonds to their holders is precisely offset by the negative value to their issuers. The question therefore is whether there is any reason to expect the average desired wealth–income ratio to be higher or lower after the introduction of lending and borrowing than before its introduction.

21. This analysis continues to assume that each individual expects P_B to be constant on the average, which is an unsatisfactory assumption given the declining values of r_B. Similarly we continue to assume

$$\left(\frac{1}{P}\frac{dP}{dt}\right)^* = 0,$$

which may also be unsatisfactory.

(2). *Changing values of non-pecuniary services.* As the process proceeds, the ratio of wealth to income available for consumption $(m_t + v_t)$ is likely to grow for the savers and decline for the dissavers.[22] For the savers, this will tend to lower both $MNPS_B$ and $MNPS_M$, and so, for a given interest rate, require a reduction in $IRD(s_t)$ for equilibrium.[23] This will be produced by a reduction in the fraction of income saved. For borrowers, the effect will be to raise both $MNPS_B$ and $MNPS_M$ and so to require a higher $IRD(s_t)$ for equilibrium. This will be produced by a reduction in the fraction of income dissaved. Both the supply of bonds and the demand for bonds will decline on this account. There is no way of saying what, on this score alone, will happen to the interest rate; we can only

22. This ratio, call it w, is equal to

$$w = \frac{M_t + V_t}{Y_t + B_t} = \frac{M_t + V_t}{Y_t + r_B V_t}, \qquad (a)$$

where $V_t = B_t P_B$.
Differentiate with respect to V, dropping subscripts for simplicity. This gives

$$\frac{dw}{dV} = \left[Y - rM + (Y + rV) \frac{dM}{dV} \right] \Big/ (Y + rV)^2. \qquad (b)$$

so $\frac{dw}{dV} > 0$ if

$$\frac{dM}{dV} \geqslant \frac{rM - Y}{Y + rV} = \frac{r(M+V)}{Y + rV} - 1. \qquad (c)$$

In general $(dM/dV) > 0$, i.e., as wealth and income increase, so will desired money holdings but by less than the increase in wealth. Hence (c) will be satisfied if

$$\frac{r(M+V)}{Y + rV} \leqslant 1 \qquad (d)$$

or

$$\frac{Y + rV}{M + V} \geqslant r, \qquad (e)$$

i.e., the ratio of total income (including income from human services) to total wealth is greater than the rate of interest, which seems a condition very likely to be satisfied.

If income were defined inclusive of that component of non-pecuniary services of money that can be measured, namely, its excess over the non-pecuniary services of bonds, or r_B, the wealth–income ratio would necessarily move in the direction indicated. For then the wealth–income ratio (call it w') would be:

$$w' = \frac{M_t + V_t}{Y_t + r_B(M_t + V_t)} = \frac{W}{Y_t + rW}.$$

On our assumptions, for the individual, as W grows, so do M_t and V_t. But

$$\frac{dw'}{dW} = \frac{Y_t}{(Y_t + rW)^2} > 0.$$

23. As implied in the preceding footnote, after the initial reshuffling of cash and securities, borrowers may be expected to be reducing their cash balances—financing their extra consumption by both borrowing and drawing down cash balances—and lenders to be adding to their cash balances—using their savings to add to both their bond holdings and their cash. Borrowers are getting poorer in wealth and having lower incomes available for consumption. The reverse is true for lenders. The initial reshuffling simply corrects an initial stock disequilibrium produced by the prior forcible suppression of lending and borrowing.

say that the volume of lending and borrowing will be reduced. If this factor alone were at work, equilibrium would be attained by changes in the $MNPS_M$ and $MNPS_B$ that would bring (17) into equality for each individual at $s_i = 0$, with $IRD(0) - MNPS_B(m_i, v_i)$ equal for all individuals.

(3). *Changing internal rates of discount.* A third possible equilibrating factor is changes in $IRD(0)$, the internal rate of discount when saving is zero. It is sometimes argued that this rate should depend on the level of consumption. The savers, when they reach equilibrium, will have a higher level of consumption than initially, while the dissavers will have a lower level. The usual relation supposed is that the lower the level of consumption, the higher the internal rate of discount ("The poor are more short-sighted than the rich"). However, this would produce a disequilibrating movement as the process went on, because it would increase the gap between $IRD(0)$ and $MNPS_B$ for dissavers to close by dissaving. Moreover, it is hard to see on theoretical grounds any reason why the internal rate of discount should be systematically related to the level of consumption, when the level of consumption is constant over time. If current consumption is low, so is future consumption; hence, if current needs are regarded as urgent, future needs will be also. I am inclined therefore to rule out this possibility.

A more appealing possibility, though one that for reasons already suggested raises difficulties as well, is that $IRD(0)$ depends on the wealth–income ratio, rising as the wealth–income ratio rises and falling as the wealth–income ratio falls. Since the wealth–income ratio is likely to rise for savers and decline for dissavers, this will produce an equilibrating movement, tending to bring the values of $IRD(0) - MNPS_B$ together for savers and dissavers.

(4). *Final stationary equilibrium position.* Whether brought about by one or a combination of these three forces, the final stationary equilibrium will equate

$$\boxed{MNPS_M - MNPS_B = IRD(0) - MNPS_B = r_B} \tag{18}$$

for all individuals. Moreover, because of our assumption that M, the nominal stock of money, is fixed, the price level will be stationary when (18) is satisfied, so

$$\left(\frac{1}{P}\frac{dP}{dt}\right) = \left(\frac{1}{P}\frac{dP}{dt}\right)^* = 0. \tag{19}$$

The final equilibrium price level need not of course be the same as the price level immediately after the introduction of borrowing and lending. General considerations suggest that it should be higher, i.e., that real balances should be lower as a fraction of income. There is now an additional means of providing for emergencies, so the utility of cash balances for this purpose should be less. Of the three forces listed as tending toward stationary equilibrium, the first (the changing distribution of wealth) clearly works in this direction, since the individuals who initially are led to hold lower money balances come to play a more and more dominant role in the final position. Neither of the other forces has a similarly unambiguous effect on desired cash balances.

We can readily reintroduce money as a productive resource and drop the assumption that $M_e = MPM = 0$. At every moment, businesses will acquire that stock of cash balances for which $MPM = r_B$, provided that they anticipate P_B will be constant on the average. No non-pecuniary elements enter in, so this is an easy problem.

The final equilibrium will then be characterized by

$$MPM = MNPS_M - MNPS_B = IRD(0) - MNPS_B = r_B \qquad (20)$$

for every business enterprise and every individual separately. (I.e., MPM stands for a set of MPM's, one for each business enterprise. Similarly, the next two expressions each stand for a set, one for each individual, so that, if written out in full, (20) would contain $n_e + 2n_w + 1$ expressions linked by equality signs, where n_e is the number of enterprises and n_w the number of ultimate wealth-holders.) The variables that enable this solution to be attained are: the division of the fixed nominal money stock among enterprises and individuals, the price level, which permits the real money stock to be whatever is desired, the rate of interest, and the volume of bonds issued and held by different individuals.

Equation (20) takes us one step in the direction of separating out the terms on the left-hand side of (11)—when prices are constant we can evaluate MPM as equal to r_B. But we still cannot separate out $IRD(0)$ from the non-pecuniary services of bonds, and hence cannot determine separately the non-pecuniary services of money. Let us see what happens when we reintroduce changes in the quantity of money.

B. *Quantity of Money Changes at a Steady Rate*

Let us now substitute for $M = M_0$ a steady exponential rate of change: $M(t) = M_0 e^{\mu t}$, where μ can be positive or negative.

In the final position of stationary state equilibrium, by reasoning precisely the same as we used before we introduced bonds,

$$\frac{1}{P}\frac{dP}{dt} = \left(\frac{1}{P}\frac{dP}{dt}\right)^* = \mu. \qquad (21)$$

Equation (20) must now be changed to include the effect of changing prices. Consider first for both enterprises and individuals the alternatives of issuing a fraction of a bond to hold an extra dollar of cash, or acquiring a fraction of a bond with \$1 of cash. In this case, the effect of $\mu \neq 0$ cancels out. If prices are rising, the asset depreciates in value but so does the liability. Hence it must still be true that

$$MPM = MNPS_M - MNPS_B = r_B. \qquad (22)$$

This is the condition for portfolio balance, i.e., stock equilibrium.

However, for the individual, the acquisition of money or bonds by saving now involves a different set of costs or returns. Let him save an additional dollar to acquire a dollar of cash balances or a dollar's worth of the bond. The anticipated gain to him from the extra dollar of cash balance is

$$MNPS_M - \left(\frac{1}{P}\frac{dP}{dt}\right)^*,$$

and from the extra dollar of bond

$$MNPS_B + r_B - \left(\frac{1}{P}\frac{dP}{dt}\right)^*,$$

since

$$\left(\frac{1}{P}\frac{dP}{dt}\right)^*$$

is the loss he experiences in the purchasing power of his cash or bond. In either case the cost is $IRD(0)$. So we have

$$\boxed{MNPS_M - \left(\frac{1}{P}\frac{dP}{dt}\right)^* = MNPS_B + r_B - \left(\frac{1}{P}\frac{dP}{dt}\right)^* = IRD(0)} \qquad (23)$$

Subtract

$$MNPS_B - \left(\frac{1}{P}\frac{dP}{dt}\right)^*$$

from all terms and we have, with the order rearranged,

$$MNPS_M - MNPS_B = IRD(0) - MNPS_B + \left(\frac{1}{P}\frac{dP}{dt}\right)^* - r_B. \qquad (24)$$

This is the condition for zero savings, i.e., for flow equilibrium.

Combining (22) and (24), the conditions for full equilibrium are

$$\boxed{MPM = MNPS_M - MNPS_B = IRD(0) - MNPS_B + \left(\frac{1}{P}\frac{dP}{dt}\right)^* - r_B,} \qquad (25)$$

which reduces to (20) when

$$\left(\frac{1}{P}\frac{dP}{dt}\right)^* = 0.$$

Suppose now that we start with a position in which (20) is satisfied for $\mu = 0$ and introduce a positive μ. How will this affect the final equilibrium, i.e., when

$$\left(\frac{1}{P}\frac{dP}{dt}\right)^* = \mu?$$

To begin with, at the same r_B, equations (22) remain satisfied, i.e., there is no effect on portfolio balance. However, equations (24) are now out of equilibrium: the middle expression is now higher than the others: the cost of acquiring either bonds or money by saving now exceeds the gain therefrom. Hence, there will be attempted dissaving, an attempted reduction in the volume of real cash balances, an attempted reduction in the amount of bonds held, and an attempted increase in the amount of bonds issued. As before, these attempts cannot succeed, but they will produce a higher price level (over and above the rise from the increasing quantity of money), which lowers the real quantity of money to be held, and also a higher rate of interest, which lessens the desire both to reduce bond holdings and to issue more bonds. It is not clear what will happen to the aggregate value of bonds outstanding. The higher rate of interest will have lowered the value of the bonds initially outstanding, but will have offset both the initial desire to reduce the amount of bonds held and the initial desire to issue more bonds. It is clear that bonds decline less in attractiveness than cash because of the rise in interest rates. Total wealth held in the final equilibrium position must decline, since this is, after consolidating accounts, equal simply to real cash balances. However, the volume of bonds outstanding will tend to be larger relative to the amount of cash balances, and conceivably could be larger in absolute real amount.

As in our simpler example, there is clearly a welfare loss from inflation: with r_B higher, *MPM* is higher because a smaller real volume of cash is being held for productive purposes. Thus there is a lower real flow of consumer services and total wealth is lower, so the community has lost some non-pecuniary services from wealth.

Let μ be negative and the reverse effects follow: the price level will fall (beyond that required by the change in the quantity of money) and r_B will also fall. For small rates of price decline there will clearly be a welfare gain. So long as $r_B > 0$, so is *MPM*, and additional business cash balances will add to the flow of consumer services. Similarly, so long as $r_B > 0$, so is $MNPS_M$, hence the additional wealth adds to the welfare of ultimate wealth-holders. Let us now try higher and higher rates of price decline until we reach a rate at which, in equilibrium, $r_B = 0$.

At this equilibrium, we know from equation (22) that

$$MNPS_M - MNPS_B = 0. \tag{26}$$

How can that be? From equations (14), only if $MNPS_M = MNPS_B = 0$. But, this means that, from equations (25),

$$IRD(0) + \left(\frac{1}{P}\frac{dP}{dt}\right)^* = 0 \tag{27}$$

or

$$IRD(0) = -\mu. \tag{28}$$

We finally have a market measure of the internal rate of discount—that rate of steady price decline that makes the nominal interest rate equal to zero. Moreover, this situation is clearly an optimum: further increases in the rate of price